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Bitopological approach for generalized rough sets via minimal structures

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ABSTRACT*: *The concept of bitopological spaces was introduced by J.C.Kelly [11]. He studied some of separation axioms properties in bitopological spaces. Many authors studied the relation between rough set and topology[1, 3, 6, 10, 13, 15]. The relation between rough set and minimal structure was studied in[20,9,23]. In this paper we used the right and the left neighborhood of any relation to introduce a biminimal approximation space of uncertain sets as a mathematical tool to modify the approximations. we discussed some definitions and properties of rough sets and applied them in two minimal structures generated by using the right and left neighborhoods. Moreover, several important measures such as accuracy measure and quality of approximation would be studied. We proved that biminimal structure is more efficient and accurate in obtaining results than bitopology. Finally, we show the importance of our new approximations with medical science by applying these approximations in corona virus problem.

KEYWORDS: minimal space- biminimal structure-left and right neighborhood- M_{xR} (M_{Rx})open (closed) sets

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I. INTRODUCTION

V. Popa and T. Noiri [25] represented the concept of minimal structure (briefly M structure). They introduced the notion of M-closure and M-interior. The concept of M-open set and M-closed set was characterized . The notion of M-continuous functions on functions between minimal structures have been introduced. In [5,9, 28,40] the properties and applications of minimal structure was studied.

The concept of bitopological spaces was introduced by J.C.Kelly [11]. He studied some of separation axioms properties in bitopological spaces. In [10,12, 19, 17,22, 33, 24, 36, 29] many concepts and characterizations of topological space have been studied in bitopological spaces. The notion of biminimal structure spaces was introduced by C.Boonpok [3] He studied $m^1{}_X m_X{}^2$- closed sets and $m^1{}_X m_X{}^2$-open sets in biminimal structure spaces. C.Boonpok et.al [32] introduced gm $^{(i,j)}$ - closed sets, $m^{(i,j)}$ - $T_{(1/2)}$-spaces and $gm^{(i,j)}$ -continuity for biminimal structure spaces and investigated some of their properties . The concept of smg-closed sets and pair wise smg-closed set in biminimal structure space were introduced by E. Subaha and N. Nagaveni [34] and some of its properties were studied .

Rough set theory introduced by Pawalak [27, 28,29] has been considered as an extension of set theory. Rough set theory has been applied in many fields such as machine learning and knowledge discovery [.7, 14, 39] data mining [6, 18] decision-making support and analysis [8, 16, 17, 20, 34, 25] process control [26]expert system [38] and pattern recognition [21].The concept of topological rough sets was introduced by Wi-weger[35]. Al-Shami studied the relation between rough set and topology[1, 2, 3, 4]. The concept of Cj-neighborhoods were used to improve rough set's accuracy measure[7,8]. Also Al-Shami and others used the concepts of j-adhesion neighborhoods and ideals to generate topologies and defined a new rough set model derived from these topologies. These models have been proved to be finer than other topologies [3].

El-Sharkasy [9] represented the concept of minimal structure approximation spaces and near open sets and studied some of its applications.

In this paper, we represented the concept of biminimal structure space and studied M_{xR} M_{Rx}- open sets and $M_{xR}M_{Rx}$- closed sets in biminimal structure approximation spaces. We applied the new concepts in a very critical problem (covid-19). The rest of this article is organized as follows: Section 2 is devoted to recalling some basics and properties of minimal structure, some concepts in rough set theory and minimal structure approximation space. In section 3 we introduced the concept of biminimal structure approximation space and studied some of it's properties. Furthermore, we defined the minimal general lower(upper) approximation and investigated some of it's properties. Section 4 we discussed the definability of any set in biminimal structure approximation space and gave some examples. Section 5 applying the biminimal structure approximation space in covid-19 problem.

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II. PRELIMINARIES

In this section we reviewed some basic notions that helpful for our next sections.

Definition 2.1: [29] A family $M\subseteq P(X)$ is said to be a minimal structure on X if φ , $X \in M$, then the pair (X,M) is said to be minimal space. A set $A\in P(X)$ is called M-open set if $A\in M$, and is called M-closed set if $A^c \in M$.

Definition 2.2: [29] Let (X,M) be a minimal space and $A \subseteq X$. Then the M-interior is defined as M-int(A)=$\cup\{G:G\in M,G\subseteq A\}$, and the M-closure is defined as M-cl(A) = $\cap\{F: F^c \in M, A\subseteq F\}$.

Definition 2.3: [29] Let R be an equivalence relation on U, and $X\subseteq U$. The equivalence class of the element x is defined as $[x] = \{y : yRx \}$. The family of all equivalence classes with respect to equivalence relation R is defined as $U/R=\{[x] : x\in U\}$.

Definition2.4: [29] Let $X \subseteq U$ and R be an equivalent relation on U. Then the lower and (upper) approximation resp. is defined as $\underline{RX} =\cup\{Y\in U/R: Y\subseteq X\}$ and $\overline{RX}=\cup\{Y\in U/R: Y\cap X \neq\varphi\}$.

Proposition 2.1: [28] Let $X \subseteq U$ and R be an equivalence relation on U. Then

(1) X is R- definable (R-exact) if and only if $\overline{RX} = \underline{RX}$.

(2) X is rough (R-inexact) if and only if $\overline{RX} \neq \underline{RX}$.

Proposition 2.2: [28] Let $X,Y \subseteq U$ and R be an equivalence relation on U. Then

(1) $\underline{RX} \subseteq X \subseteq \overline{RX}$.

(2) $\underline{R\varphi} = \overline{R\varphi} = \varphi$, $\underline{RU} = \overline{RU}=U$.

(3) $\overline{R(X\cap Y)} = \underline{RX} \cap .\underline{RY}$.

(4) $\overline{R(X\cup Y)}=\overline{RX} \cup \overline{RX}$.

(5) $X \subseteq Y$ implies $\underline{RX} \subseteq \underline{RY}$.

(6) $X \subseteq Y$ implies $\overline{RX} \subseteq \overline{RY}$.

Definition 2.5: [29] Let (X,M) be a minimal space, and A⊆U. Then A has the following types of definability:

(1) A is totally definable (exact) if M-int(A)=A=M-cl(A).

(2) A is internally definable if M-int(A)=A and M-cl(A)≠A.

(3) A is externally definable if M-int(A)≠A and M-cl(A)=A.

(4) A is undefinable (rough) If M-int(A)≠A and M-cl(A)≠A.

Definition 2.6: [9] Let (U, R) be a generalized approximation space, where U be a finite nonempty universe set and R be an arbitrary binary relation on U. Let N(X)={y ∈U: xRy} is the right neighborhood of x for all x ∈U. Then the class MS(U)={φ, U, N(X)} is called a minimal structure on (U, R). The members of the minimal structure MS(U) are called MS-open sets and (U,R,MS) is called minimal structure approximation space (MSAS for short). The complement of an MS-open set is called MS-closed set. The class of all MS-closed sets is denoted by $MS^c(U)$.

Definition 2.7: [9] Let (U,R,MS) be MSAS, and A⊆U. The lower and upper approximation is defined resp. as MS(A) = ∪{G:G∈MS(U),G⊆A}, and MS(A) = ∩{F : F∈MS^c(U),A⊆F}.

Definition 2.8: A subset A of a topological space (X,τ) is called:

(1) Pre-open [13] if A⊆ int(cl(A)) and pre-closed if cl(int(A))⊆A.

(2) Semi-open [16] if A ⊆cl(int(A)) and semi-closed if int(cl(A))⊆A.

(3) γ-open [13] if A ⊆cl(int(A))∪int(cl(A)).

(4) Semi-preopen [1] (β-open [9]) if A ⊆cl(int(cl(A))), and semi-pre-closed (β-closed) if int(cl(int(A)))⊆A.

(5) α-open [23] if A⊆int(cl(int(A))) and α-closed if cl(int(cl(A)))⊆A.

Definition 2.9: let M_1 and M_2 are two minimal structure spaces, if $M_1 \subseteq M_2$.Then $M_1 \prec M_2$.

III. Biminimal approximations

In this section, we defined two minimal structure generated by any binary relation R. The first minimal structure denoted by (M_{xR}) is generated by the right neighborhood xR={y∈ X: xRy}. The second minimal structure denoted by(M_{Rx}) is generated by the left neighborhood Rx={y ∈X :yRx}. By using these minimal structures we introduced the the minimal lower and upper approximation of a subset X⊆U as M_{xR} (X)=∪{xR: xR ⊆X} , M_{xR}(X) = ∪{xR : xR ∩X ≠φ}, and M_{Rx}(X) = {Rx :Rx ⊆X}, M_{Rx}(X) =∪{Rx: Rx ∩X ≠φ}.

Definition 3.1: Let (U,R) be a generalized approximation space where U be a finite non empty universe set and R an arbitrary relation on U. The classes M_{xR}(U)={φ, U, xR} and M_{Rx}(U)={φ, U, Rx} are called the first and the second minimal on (U,R) respectively. The triple (U ,M_{xR} ,M_{Rx}) is called biminimal structure approximation space.

Definition 3.2: Let (U,M_{xR}, M_{Rx}) be a biminimal structure approximation space, and X⊆U. Then X is said to be

(1) M-Semi-rough (M-S_{12}-Rough) if $X \subseteq \overline{M_{Rx}}(\underline{M_{xR}}(X))$.

(2) M-Pre-rough(M-P_{12}-Rough) If $X \subseteq \underline{M_{xR}}(\overline{M_{Rx}}(X))$.

(3) M-Semi-Pre rough(M-β_{12}-rough) if $X \subseteq \overline{M_{Rx}}((\underline{M_{xR}}(\overline{M_{Rx}}(X)))$.

(4) M-α-rough (M-α_{12}-rough) if $X \subseteq \underline{M_{xR}}(\overline{M_{Rx}}(\underline{M_{xR}}(X)))$.

(5) M-γ-rough(M-γ_{12}-rough) if $X \subseteq (\overline{M_{Rx}}(\underline{M_{xR}}(X))) \cup (\underline{M_{xR}}(\overline{M_{Rx}}(X)))$.

The family of all M-S_{12}-rough(Resp. M-P_{12}-rough, M-β_{12}-rough, M-α_{12}-rough and M-γ_{12}-rough) set in (U,R) is denoted by M-S_{12}(U), M-P_{12}(U), M-β_{12}(U), M-α_{12}(U) and M-γ_{12}(U), and the complements of them is called(M-$S_{12}{}^{c}$-Rough, M-$P_{12}{}^{c}$-Rough, M-$\beta_{12}{}^{c}$-rough, M-$\alpha_{12}{}^{c}$-rough and M-$\gamma_{12}{}^{c}$-rough), and the family of complements is M-$S_{12}{}^{c}$-rough, M-$P_{12}{}^{c}$-rough, M-$\beta_{12}{}^{c}$-rough, M-$\alpha_{12}{}^{c}$-rough and M-$\gamma_{12}{}^{c}$-rough.

Proposition 3.1: Let (U, M_{xR}, M_{Rx}) be a biminimal structures approximation spaces. Then

(1)M-α_{12}(U)≺ M-S_{12}(U) ≺M-γ_{12}(U)≺M-β_{12}(U).

(2)M-α_{12}(U)≺M-P_{12}(U) ≺M-γ_{12}(U)≺M-β_{12}(U).

Proof :(1)Since $\underline{M_{xR}}(\overline{M_{Rx}}(\underline{M_{Rx}}(X))) \subseteq \overline{M_{Rx}}(\underline{M_{xR}}(X))$, then M-$\alpha_{12}$(U) ≺ M-$S_{12}$(U) and $\overline{M_{Rx}}(\underline{M_{xR}}(X)) \subseteq (\overline{M_{Rx}}(\underline{M_{xR}}(X))) \cup (\underline{M_{xR}}(\overline{M_{Rx}}(X)))$. So M-$S_{12}$(U) ≺M-$\gamma_{12}$(U). Also, $(\overline{M_{Rx}}(\underline{M_{xR}}(X))) \cup (\underline{M_{xR}}(\overline{M_{Rx}}(X))) \subseteq \overline{M_{Rx}}((\underline{M_{xR}}(\overline{M_{Rx}}(X)))$. Then M-$\gamma_{12}$(U)≺M-$\beta_{12}$(U). Therefore, M-$\alpha_{12}$(U)≺ M-$S_{12}$(U) ≺M-$\gamma_{12}$(U)≺M-$\beta_{12}$(U).

(2) Since $\underline{M_{Rx}}(X) \subseteq X$, then $\underline{M_{xR}}(\overline{M_{Rx}}(\underline{M_{xR}}(X))) \subseteq \underline{M_{xR}}(\overline{M_{Rx}}(X))$. Thus M-$\alpha_{12}$(U)≺M-$P_{12}$(U), and $\underline{M_{xR}}(\overline{M_{Rx}}(X)) \subseteq (\overline{M_{Rx}}(\underline{M_{xR}}(X))) \cup (\underline{M_{xR}}(\overline{M_{Rx}}(X)))$. So M-$P_{12}$(U) ≺M-$\gamma_{12}$(U). And $(\overline{M_{Rx}}(\underline{M_{xR}}(X))) \cup (\underline{M_{xR}}(\overline{M_{Rx}}(X))) \subseteq \overline{M_{Rx}}((\underline{M_{xR}}(\overline{M_{Rx}}(X)))$. Thus M-$\gamma_{12}$(U)≺M-$\beta_{12}$(U). So M-$\alpha_{12}$(U)≺M-$P_{12}$(U) ≺M-$\gamma_{12}$(U)≺M-$\beta_{12}$(U).

The following example illustrated that the reverse inclusion in Proposition 3.1 was not satisfied.

Example3.1: Let R be a binary relation defined on a non empty set U={a,b,c,d} defined by R={(a,a), (a,b), (a,c), (d,d), (d,c)}. Then

M_{xR} ={{a,b,c},{c,d}, U, φ}.

$M_{xR}{}^{c}$ ={{d}, {a,b}, U, φ}.

M_{Rx}={{a}, {d}, {a,d}, U, φ}.

$M_{Rx}{}^{c}$={{b,c,d}, {a,b,c}, {b,c}, U, φ}.

M-S_{12}-rough ={{b,c,d}, {a,b,c}, {c,d}, U, φ}.

M-α_{12}-rough={U, φ, {c,d}, {a,b,c}}.

M-β_{12}-rough={φ, U, {a}, {d}, {a,b}, {a,c}, {a,d}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {b,c,d}, {a,c,d}}.

M-P_{12}-Rough={φ, U, {a}, {d}, {a,b}, {a,c}, {a,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}}.

M-γ_{12}-rough={φ, U, {a}, {d}, {a,b}, {a,c}, {a,d}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {b,c,d}, {a,c,d}}.

So M-S_{12}-rough ⊀ M-α_{12}-rough, M-γ_{12}-rough ⊀ M-S_{12}-rough, and M-β_{12}-rough ⊀ M-γ_{12}-rough.

Definition3.3: Let (U,M_{xR}, M_{Rx}) be a biminimal structure approximation space, and X⊆U. Then the general lower(briefly $\underline{M\text{-}\lambda_{12}}(X)$) of X where M-$\lambda_{12}$(U)∈{M-$\alpha_{12}$(U), M-$S_{12}$(U), M-$\gamma_{12}$(U), M-$P_{12}$(U), M-$\beta_{12}$(U)} is defined by $\underline{M\text{-}\lambda_{12}}(X)=\cup\{G:G\in M\text{-}\lambda_{12}(U), G\subseteq X\}$, then the minimal general lower approximation is denoted by $\underline{M_{\lambda_{12}}}(X)$ for any subset X⊆U is defined as $\underline{M_{\lambda_{12}}}(X)=\underline{M\text{-}\lambda_{12}}(X)$, and the general upper(briefly $\overline{M\text{-}\lambda_{12}}(X)$) of X is defined by $\overline{M\text{-}\lambda_{12}}(X)=\cap\{H:H\in M\text{-}\lambda_{12}{}^{c}(U), H\supseteq X\}$, then the minimal general upper approximation is denoted by $\overline{M_{\lambda_{12}}}(X)$ for any subset X⊆U is defined as $\overline{M_{\lambda_{12}}}(X)=\overline{M\text{-}\lambda_{12}}(X)$.

Proposition3.2: Let (U, M_{xR}, M_{Rx}) be a biminimal structure approximation space, and $X \subseteq U$. Then

(1) $\underline{M}_{xR}(X) \subseteq \underline{M}_{\alpha_{12}}(X) \subseteq \underline{M}_{S_{12}}(X) \subseteq \underline{M}_{\gamma_{12}}(X) \subseteq \underline{M}_{\beta_{12}}(X) \subseteq X \subseteq \overline{M}_{\beta_{12}}(X) \subseteq \overline{M}_{\gamma_{12}}(X) \subseteq \overline{M}_{S_{12}}(X) \subseteq \overline{M}_{\alpha_{12}}(X) \subseteq \overline{M}_{xR}(X)$.

(2) $\underline{M}_{Rx}(X) \subseteq \underline{M}_{\alpha_{12}}(X) \subseteq \underline{M}_{P_{12}}(X) \subseteq \underline{M}_{\gamma_{12}}(X) \subseteq \underline{M}_{\beta_{12}}(X) \subseteq X \subseteq \overline{M}_{\beta_{12}}(X) \subseteq \overline{M}_{\gamma_{12}}(X) \subseteq \overline{M}_{P_{12}}(X) \subseteq \overline{M}_{\alpha_{12}}(X) \subseteq \overline{M}_{Rx}(X)$.

Proof: (1) $\underline{M}_{xR}(X)=\cup\{G: G \in M_{xR}(U), G \subseteq X\} \subseteq \cup\{G: G \in M\text{-}\alpha_{12}(U), G \subseteq X\} \subseteq \cup\{G: G \in M\text{-}S_{12}(U), G \subseteq X\} \subseteq \cup\{G: G \in M\text{-}\gamma_{12}(U), G \subseteq X\} \subseteq \cup\{G: G \in M\text{-}\beta_{12}(U), G \subseteq X\} \subseteq X \subseteq \cap\{H: H \in M\text{-}\beta_{12}{}^{c}(U), H \supseteq X\} \subseteq \cap\{H: H \in M\text{-}\gamma_{12}{}^{c}(U), H \supseteq X\} \subseteq \cap\{H: H \in M\text{-}S_{12}{}^{c}(U), H \supseteq X\} \subseteq \cap\{H: H \in M\text{-}\alpha_{12}{}^{c}(U), H \supseteq X\} \subseteq \cap\{H: H \in M_{XR}{}^{c}(U), H \supseteq X\}$. So $\underline{M}_{xR}(X) \subseteq \underline{M}_{\alpha_{12}}(X) \subseteq \underline{M}_{S_{12}}(X) \subseteq \underline{M}_{\gamma_{12}}(X) \subseteq \underline{M}_{\beta_{12}}(X) \subseteq X \subseteq \overline{M}_{\beta_{12}}(X) \subseteq \overline{M}_{\gamma_{12}}(X) \subseteq \overline{M}_{S_{12}}(X) \subseteq \overline{M}_{\alpha_{12}}(X) \subseteq \overline{M}_{xR}(X)$.

(2) $\underline{M}_{Rx}(X)=\cup\{G:G \in M_{Rx}(U),G \subseteq X\} \subseteq \cup\{G:G \in M\text{-}\alpha_{12}(U),G \subseteq X\} \subseteq \cup\{G:G \in M\text{-}P_{12}(U),G \subseteq X\} \subseteq \cup\{G:G \in M\text{-}\gamma_{12}(U),G \subseteq X\} \subseteq \cup\{G:G \in M\text{-}\beta_{12}(U),G \subseteq X\} \subseteq X \subseteq \cap\{H:H \in M\text{-}\beta_{12}{}^{c}(U),H \supseteq X\} \subseteq \cap\{H:H \in M\text{-}\gamma_{12}{}^{c}(U),H \supseteq X\} \subseteq \cap\{H:H \in M\text{-}P_{12}{}^{c}(U),H \supseteq X\} \subseteq \cap\{H:H \in M\text{-}\alpha_{12}{}^{c}(U),H \supseteq X\} \subseteq \cap\{H:H \in M_{Rx}{}^{c}(U),H \supseteq X\}$. Thus $\underline{M}_{Rx}(X) \subseteq \underline{M}_{\alpha_{12}}(X) \subseteq \underline{M}_{P_{12}}(X) \subseteq \underline{M}_{\gamma_{12}}(X) \subseteq \underline{M}_{\beta_{12}}(X) \subseteq X \subseteq \overline{M}_{\beta_{12}}(X) \subseteq \overline{M}_{\gamma_{12}}(X) \subseteq \overline{M}_{P_{12}}(X) \subseteq \overline{M}_{\alpha_{12}}(X) \subseteq \overline{M}_{Rx}(X)$.

The following example illustrated the previous proposition.

Example3.2: From Exmaple 3.1 if $X=\{a,b,d\}$, then $\underline{M}_{\alpha_{12}}(X) = \varphi$, $\underline{M}_{\beta_{12}}(X) =\{a,b,d\}$. So, $\underline{M}_{\alpha_{12}}(X) \subseteq \underline{M}_{\beta_{12}}(X)$. If $Y=\{a,b,c\}$, then $\overline{M}_{S_{12}}(Y)=U$, $\overline{M}_{\beta_{12}}(Y) =\{a,b,c\}$. So, $\overline{M}_{\beta_{12}}(Y) \subseteq \overline{M}_{S_{12}}(Y)$.

Proposition 3.3: Let (U,M_{xR}, M_{Rx}) be a biminimal structure approximation space, and $X , Y \subseteq U$. Then for all $M\text{-}\lambda_{12} \in \{M\text{-}\alpha_{12}, M\text{-}S_{12}, M\text{-}\gamma_{12}, M\text{-}P_{12}, M\text{-}\beta_{12}\}$

(1) $\underline{M}_{\lambda_{12}}(\varphi)= \varphi =\overline{M}_{\lambda_{12}}(\varphi)$, $\underline{M}_{\lambda_{12}}(U)= U =\overline{M}_{\lambda_{12}}(U)$.

(2) If $X \subseteq Y$ then $\underline{M}_{\lambda_{12}}(X) \subseteq \underline{M}_{\lambda_{12}}(Y)$.

(3) If $X \subseteq Y$ then $\overline{M}_{\lambda_{12}}(X) \subseteq \overline{M}_{\lambda_{12}}(Y)$.

(4) $\underline{M}_{\lambda_{12}}(X \cup Y) \supseteq \underline{M}_{\lambda_{12}}(X) \cup \underline{M}_{\lambda_{12}}(Y)$.

(5) $\overline{M}_{\lambda_{12}}(X \cup Y) \supseteq \overline{M}_{\lambda_{12}}(X) \cup \overline{M}_{\lambda_{12}}(Y)$.

(6) $\underline{M}_{\lambda_{12}}(X \cap Y) \subseteq \underline{M}_{\lambda_{12}}(X) \cap \underline{M}_{\lambda_{12}}(Y)$.

(7) $\overline{M}_{\lambda_{12}}(X \cap Y) \subseteq \overline{M}_{\lambda_{12}}(X) \cap \overline{M}_{\lambda_{12}}(Y)$.

(8) $\underline{M}_{\lambda_{12}}(X^{c}) = (\overline{M}_{\lambda_{12}}(X))^{c}$.

(9) $\overline{M}_{\lambda_{12}}(X^{c}) =(\underline{M}_{\lambda_{12}}(X))^{c}$.

Proof: (1) $\underline{M}_{\lambda_{12}}(\varphi)=\cup\{xR:xR \subseteq \varphi\}=\varphi=\cup\{xR:xR \cap \varphi \neq \varphi\}=\overline{M}_{\lambda_{12}}(\varphi)$. $\underline{M}_{\lambda_{12}}(U) = \cup\{xR:xR \subseteq U\}=U=\overline{M}_{\lambda_{12}}(U) =\cup\{xR:xR \cap U \neq \varphi\}$.

(2)Since $X \subseteq Y$, then $\underline{M}_{\lambda_{12}}(X)=\cup\{xR:xR \subseteq X\} \subseteq \cup\{xR:xR \subseteq X \subseteq Y\} \subseteq \cup\{xR:xR \subseteq Y\} \subseteq \underline{M}_{\lambda_{12}}(Y)$.

(3)Since $X \subseteq Y$, then $\overline{M}_{\lambda_{12}}(X)=\cup\{xR:xR \cap X \neq \varphi\} \subseteq \cup\{xR:xR \cap Y \neq \varphi\} \subseteq \overline{M}_{\lambda_{12}}(Y)$.

(4) Since $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$, then $\underline{M}_{\lambda_{12}}(X) = \cup\{xR:xR \subseteq X\} \subseteq \cup\{xR:xR \subseteq X \cup Y\} \subseteq \underline{M}_{\lambda_{12}}(X \cup Y)$. Also, $\underline{M}_{\lambda_{12}}(Y) = \cup\{xR:xR \subseteq Y\} \subseteq \cup\{xR:xR \subseteq X \cup Y\} \subseteq \underline{M}_{\lambda_{12}}(X \cup Y)$. So, $\underline{M}_{\lambda_{12}}(X \cup Y) \supseteq \underline{M}_{\lambda_{12}}(X) \cup \underline{M}_{\lambda_{12}}(Y)$.

(5) Since $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$, then $\overline{M}_{\lambda_{12}}(X) = \cup\{xR:xR \cap X \neq \varphi\} \subseteq \cup\{xR:xR \cap (X \cup Y) \neq \varphi\} \subseteq \overline{M}_{\lambda_{12}}(X \cup Y)$. Also, $\overline{M}_{\lambda_{12}}(Y) = \cup\{xR:xR \cap Y \neq \varphi\} \subseteq \cup\{xR:xR \cap (X \cup Y) \neq \varphi\} \subseteq \overline{M}_{\lambda_{12}}(X \cup Y)$. So, $\overline{M}_{\lambda_{12}}(X \cup Y) \supseteq \overline{M}_{\lambda_{12}}(X) \cup \overline{M}_{\lambda_{12}}(Y)$.

(6) Since $X\cap Y\subseteq X$ and $X\cap Y\subseteq Y$, then $\underline{M}_{\lambda_{12}}(X\cap Y) \subseteq \underline{M}_{\lambda_{12}}(X)$, and $\underline{M}_{\lambda_{12}}(X\cap Y) \subseteq \underline{M}_{\lambda_{12}}(Y)$. So, $M_{\lambda_{12}}(X\cap Y) \subseteq \underline{M}_{\lambda_{12}}(X)\cap \underline{M}_{\lambda_{12}}(Y)$.

(7) Similar to the proof of (6).

(8) $\underline{M}_{\lambda_{12}}(X^c)=\cup[\{H: H\in M\text{-}\lambda_{12}^c(U), H\subseteq X^c\}]=\cup\{(H^c): (H^c)^c \in M\text{-}\lambda_{12}^c(U), H^c \supseteq X\}]= [\cap\{H^c : H^c \in M\text{-}\lambda_{12}^c(U), H^c\}\supseteq X\}]^c=(\overline{M}_{\lambda_{12}}(X))^c$.

(9) $\overline{M}_{\lambda_{12}}(X^c)=(\cap\{H^c: H^c\in M\text{-}\lambda_{12}^c(U), H^c\subseteq X\})^c = [\cup\{F : F\in M\text{-}\lambda_{12}(U), F\subseteq X\}]^C = (\underline{M}_{\lambda_{12}}(X))^c$.

The inverse of Property (4) is not generally true, as shown in the following example.

Example 3.4: From Example 3.2 : if $M\text{-}\lambda_{12}=M\text{-}\alpha_{12}$ and $X=\{c\}$, $Y=\{d\}$. Then $M_{\alpha_{12}}(X\cup Y)=\{c,d\}$. And $\underline{M}_{\alpha_{12}}(X)=\varphi$, $\underline{M}_{\alpha_{12}}(Y)=\varphi$.Then $\underline{M}_{\alpha_{12}}(X\cup Y)\nsubseteq \underline{M}_{\alpha_{12}}(X)\cup\underline{M}_{\alpha_{12}}(Y)$.

The inverse of Property (5) is not generally true, as shown in the following example.

Example 3.5: From Example 3.2 : If $M\text{-}\lambda_{12} = M\text{-}\beta_{12}$ and $X=\{a\}$, $Y=\{d\}$. Then $M_{\alpha_{12}}(X\cup Y)=U$, and $\overline{M}_{\beta_{12}}(X)=\{a\}$, $\overline{M}_{\beta_{12}}(Y)=\{d\}$.Then $\overline{M}_{\alpha_{12}}(X\cup Y) \nsubseteq \overline{M}_{\beta_{12}}(X)\cup \overline{M}_{\beta_{12}}(Y)$.

The inverse of Property (6) is not generally true, as shown in the following example.

Example 3.6: From Example 3.2 : If $M\text{-}\lambda_{12}=M\text{-}S_{12}$ and $X=\{a,b,c\}$, $Y=\{b,c,d\}$, then $M_{S_{12}}(X\cap Y)=\varphi$, $\underline{M}_{S_{12}}(X)=\{a,b,c\}$, $\underline{M}_{S_{12}}(Y)=\{b,c,d\}$. So, $\underline{M}_{S_{12}}(X\cap Y) \nsupseteq \underline{M}_{S_{12}}(X) \cap \underline{M}_{S_{12}}(Y)$.

Proposition 3.4: Let (U,M_{xR}, M_{Rx}) be a biminimal structure approximation space, and $X\subseteq U$. Then for all $M\text{-}\lambda_{12}\in\{M\text{-}\alpha_{12}, M\text{-}S_{12}, M\text{-}\gamma_{12}, M\text{-}P_{12}, M\text{-}\beta_{12}\}$.

(1) $\underline{M}_{\lambda_{12}}(X)\neq\underline{M}_{\lambda_{12}}(\overline{M}_{\lambda_{12}}(X)) \neq \overline{M}_{\lambda_{12}}(\underline{M\lambda}_{12}(X))$.

(2) $\overline{M}_{\lambda_{12}}(X) \neq \underline{M}_{\lambda_{12}}(\overline{M}_{\lambda_{12}}(X))\neq\overline{M}_{\lambda_{12}}(\overline{M}_{\lambda_{12}}(X))$.

The following example illustrated this idea.

Example 3.7: From Example 3.2.

1) If $M\text{-}\lambda_{12}=M\text{-}\alpha_{12}$ and $X=\{c,d\}$, then $\underline{M}_{\alpha_{12}}(X)=\{c,d\}$, $\underline{M}_{\alpha_{12}}(\overline{M}_{\alpha_{12}}(X)) = \{c,d\}$.But, $\overline{M}_{\alpha_{12}}(\underline{M}_{\alpha_{12}}(X)) = U$. So, $\underline{M}_{\lambda_{12}}(\underline{M}_{\lambda_{12}}(X)) \neq \overline{M}_{\lambda_{12}}(\underline{M}_{\lambda_{12}}(X))$.

2) If $M\text{-}\lambda_{12}=M\text{-}S_{12}$ and $X=\{b\}$, then $\overline{M}_{S_{12}}(X)=\{a,b\}$, $\overline{M}_{S_{12}}(\overline{M}_{S_{12}}(X))=\{a ,b\}$. But $\overline{M}_{S_{12}}(M_{S_{12}}(X)) = \varphi$. So, $\overline{M}_{S_{12}}(\overline{M}_{S_{12}}(X)) \neq \underline{M}_{S_{12}}(\overline{M}_{S_{12}}(X))$.

Proposition 3.5: Let (U,R) be a generalized approximation space generated by any binary relation R then for any two subsets X and $Y\subseteq U$ we have $\underline{M}_{\lambda_{12}}(X-Y)\subseteq \underline{M}_{\lambda_{12}}(X)- \underline{M}_{\lambda_{12}}(Y)$ for all $M\text{-}\lambda_{12}\in\{M\text{-}\alpha_{12}, M\text{-}S_{12}, M\text{-}\gamma_{12}, M\text{-}P_{12}, M\text{-}\beta_{12}\}$.

Proof: As $X-Y=X\cap Y^c$, then $M_{\lambda_{12}}(X-Y)=M_{\lambda_{12}}(X\cap Y^c)$. From the Proposition (3.3) $M_{\lambda_{12}}(X\cap Y^c)\subseteq \underline{M}_{\lambda_{12}}(X)\cap\underline{M}_{\lambda_{12}}(Y^c)\subseteq \underline{M}_{\lambda_{12}}(X)\cap(\underline{M}_{\lambda_{12}}(Y))^c\}\subseteq\underline{M}_{\lambda_{12}}(X)- \underline{M}_{\lambda_{12}}(Y)$. So, $\underline{M}_{\lambda_{12}}(X-Y)\subseteq \underline{M\lambda}_{12}(X)- \underline{M}_{\lambda_{12}}(Y)$.

The following example showed that the inverse of Proposition 3.5 is not generally true.

Example3.8: From Example (3.1) If $M\text{-}\lambda_{12}=M\text{-}S_{12}$ and $X=\{b,c,d\}$, $Y=\{c,d\}$, then $M_{S_{12}}(X)=\{b,c,d\}$, $\underline{M}_{S_{12}}(Y)=\{c,d\}$, and $\underline{M}_{S_{12}}(X-Y)=\underline{M}_{S_{12}}(\{b\}) = \varphi$. But $\underline{M}_{S_{12}}(X)-\underline{M}_{S_{12}}(Y) =\{b\}$, then $\underline{M}_{\lambda_{12}}(X) - \underline{M}_{\lambda_{12}}(Y) \nsubseteq \underline{M}_{\lambda_{12}}(X-Y)$.

Lemma3.1: Let (U, M_{xR}, M_{Rx}) be a biminimal structure approximation space, and X⊆U. Then $(cl_{\lambda_{12}}(X))^c = Int_{\lambda_{12}}(X^c)$ for all $\lambda_{12} \in \{\alpha_{12}, S_{12}, \gamma_{12}, P_{12}, \beta_{12}\}$.

Proof: For all M-λ_{12}∈{M-α_{12}, M-S_{12}, M-γ_{12}, M-P_{12}, M-β_{12}}, let X ⊆ U. Then $(cl(X))^c = [\cap\{H : H \in M\text{-}\lambda_{12}{}^c(U), H \supseteq X\}]^c = [\cup\{H^c : H^c \in M\text{-}\lambda_{12}(U), H^c \subseteq X^c\}] = Int_{\lambda_{12}}(H^c) = Int_{\lambda_{12}}(X^c)$.

IV –Minimal generalization of rough concepts

In this section we introduced the definition of definability of approximation space in biminimal structure approximation space, and proved some of its properties. Also, we defined the accuracy measure of biminimal structure approximation space.

Definition 4.1: Let (U, M_{xR}, M_{Rx}) be a biminimal structure approximation space, and X⊆U .Then for all M-λ_{12}∈{M-α_{12}, M-S_{12}, M-γ_{12}, M-P_{12}, M-β_{12}}.

(1) X is totally minimal λ_{12}-definable (M-λ_{12}-exact) if $\underline{M}_{\lambda_{12}}(X)=X=\overline{M}_{\lambda_{12}}(X)$.

(2) X is internally minimal λ_{12}-definable if $\underline{M}_{\lambda_{12}}(X)=X$ and $\overline{M}_{\lambda_{12}}(X)\neq X$.

(3) X is externally minimal λ_{12}-definable if $\underline{M}_{\lambda_{12}}(X) \neq X$ and $\overline{M}_{\lambda_{12}}(X)=X$.

(4) X is minimally λ_{12}-undefinable (M-λ_{12}-rough) If $\underline{M}_{\lambda_{12}}(X) \neq X$ and $\overline{M}_{\lambda_{12}}(X) \neq X$.

Example 4.1: From Example 3.1, then

(1) If M-λ_{12}=M-γ_{12} and X={a}, $\underline{M}_{\gamma_{12}}(X)=\{a\}$, and $\overline{M}_{\gamma_{12}}(X)=\{a\}$. So, $\underline{M}_{\gamma_{12}}(X) = X = \overline{M}_{\gamma_{12}}(X)$. Thus, X is M-$\gamma_{12}$-exact set.

(2) If M-λ_{12}=M-S_{12} and X={c,d}, then $\underline{M}_{S_{12}}(X)=\{c,d\}$ and $\overline{M}_{\gamma_{12}}(X) = U$. So, $\underline{M}_{\gamma_{12}}(X)=X$. But $\overline{M}_{\gamma_{12}}(X)\neq X$. Therefore, X is internally minimal γ_{12}-definable.

(3) If M-λ_{12}=M-α_{12} and X={a,b}, then $\underline{M}_{\alpha_{12}}(X)=\varphi \neq X$, $\overline{M}_{\alpha_{12}}(X)=\{a, b\}=X$. So, X is externally minimalα_{12}-definable.

(4) If M-λ_{12}=M-S_{12} and X={b}, then $\underline{M}_{S_{12}}(X) = \varphi$, $\overline{M}_{S_{12}}(X) = \{a,b\}$, $\underline{M}_{S_{12}}(X) \neq X$ and $\overline{M}_{S_{12}}(X) \neq X$. So, X is M-$S_{12}$-rough set.

Definition 4.2: Let (U, M_{xR}, M_{Rx}) be a biminimal structure approximation space, then we can introduce the generalized accuracy measure for any set X⊆U (denoted by $ACC_{\lambda_{12}}$) as the following

$ACC_{\lambda_{12}} =((|\underline{M}_{\lambda_{12}}(X)|)/(|\overline{M}_{\lambda_{12}}(X)|)), \overline{M}_{\lambda_{12}}(X)\neq\varphi$, where M-$\lambda_{12}$∈{M-$\alpha_{12}$, M-$S_{12}$, M-$\gamma_{12}$, M-$P_{12}$, M-$\beta_{12}$} and |X| denoted the cardinality of the set X. The number of generalized accuracy measure for any set X⊆U is the measure of the degree of exactness. So, by this action we will figure out which is the best of our definition for M-λ_{12}- lower and M-λ_{12}-upper approximations. So,

1) $0 \leq ACC_{M_{12}} \leq ACC_{\alpha_{12}} \leq ACC_{S_{12}} \leq ACC_{\gamma_{12}} \leq ACC_{\beta_{12}} \leq 1$

2) $0 \leq ACC_{M_{12}} \leq ACC_{\alpha_{12}} \leq ACC_{P_{12}} \leq ACC_{\gamma_{12}} \leq ACC_{\beta_{12}} \leq 1$

The following example illustrated the comparison between $ACC_{S_{12}}$ and $ACC_{\beta_{12}}$ in Table(1)

Example 4.2: From Example 3.

X	$ACC_{S_{12}}$	$ACC_{\beta_{12}}$
{c,d}	1/2	1
{a,c,d}	1/2	¾
{a,b,c}	3/4	1
{b,c,d}	3/4	1

Table(1:The comparison between $ACC_{S_{12}}$ and $ACC_{\beta_{12}}$.

By using the definition of rough concepts at we can tends to exactness of many sets ,this will lead to accurate results.

Definition4.3: Let (U, M_{xR}, M_{Rx}) be a biminimal structure approximation space, and X,Y⊆U. Then for all M-λ_{12}∈{M-α_{12}, M-S_{12}, M-γ_{12}, M-P_{12}, M-β_{12}}.

(1) X $\subsetneq_{\lambda_{12}}$Y if $\underline{M_{\lambda_{12}}}$(X) ⊂ $\underline{M_{\lambda_{12}}}$(Y).
(2) X$\tilde{\subset}_{\lambda_{12}}$Y if $\overline{M_{\lambda_{12}}}$(X) ⊂$\overline{M_{\lambda_{12}}}$(Y).

Example 4.3: From Example 3.1:

(1) If M-λ_{12}=M-S_{12} and X={d}, Y={a,b}, then $\underline{M_{S_{12}}}$ (X) = φ, $M_{S_{12}}$(Y) = φ, and $M_{\gamma_{12}}$(X)⊆ $M_{S_{12}}$(Y). Thus X$\subsetneq_{S_{12}}$Y.

(2) If M-λ_{12}=M-β_{12} and X={a,b}, Y={a,d}, then $\overline{M_{\beta_{12}}}$ (X) ={a,b}, $\overline{M_{\beta_{12}}}$ (Y)=U, and $\overline{M_{\beta_{12}}}$ (X) ⊆ $\overline{M_{\beta_{12}}}$(Y). So X$\tilde{\subset}_{\beta_{12}}$Y.

Definition4.4: Let (U, M_{xR}, M_{Rx}) be a biminimal structure approximation space, and X⊆U and any element x∈U. Then for all M-λ_{12}∈{M-α_{12}, M-S_{12}, M-γ_{12}, M-P_{12}, M-β_{12}}.

(1) If x$\underset{\sim}{\in}_{\lambda_{12}}$X if and only if x ∈ $\underline{M_{\lambda_{12}}}$ (X).

(2) If x$\tilde{\in}_{\lambda_{12}}$ X if and only if x ∈ $\overline{M_{\lambda_{12}}}$(X).

Proposition4.1: Let (U,M_{xR}, M_{Rx}) be a biminimal structure approximation space, and X⊆U. Then for any x∈U, and M-λ_{12}∈{M-α_{12}, M-S_{12}, M-γ_{12}, M-P_{12}, M-β_{12}}, the following properties hold:

(1) If x$\underset{\sim}{\in}_{\lambda_{12}}$X, then x∈ X.

(2) If x $\tilde{\notin}_{\lambda_{12}}$X, then x ∉X.

Proof: (1) Let x $\in_{\lambda_{12}}$X then x∈ $\underline{M_{\lambda_{12}}}$(X) and $\underline{M_{\lambda_{12}}}$(X)⊆X. So, x∈X.

(2) Let x ∉X then x ∉$\overline{M_{\beta_{12}}}$(X) and X ⊆ $\overline{M_{\beta_{12}}}$(X). Therefore, x ∉X

The inverse of Definition 4.4 is not generally true.

Example 4.4: From Example 3.1 if M-λ_{12}=M-S_{12} and X={a,c,d},x=a, then $\underline{M_{S_{12}}}$(X)={c,d}. But x $\notin_{S_{12}}$X.

Also, if M-λ_{12}=M-S_{12} and X={b},x=a, $\overline{M_{S_{12}}}$(X)={a,b}, then x$\notin_{S_{12}}$X. But x ∉X.

V-Application of the biminimal structure approximation space on Covid-19.

In this section we applied the concept of biminimal approximation space in a very critical problem and we tried to get more accurate results by reducing the condition.

The sort of pneumonia brought about by the 2019 novel Covid disease(covid-19) is profoundly irresistible infection and the continuous flare-up has been proclaimed by WHO as a worldwide general wellbeing crisis. Coronavirus pneumonia was first announced in Wuhan then it Spread universally. This is clinical highlights of four pregnant ladies with affirmed COVID-19 pneumonia and inspect the upward transmission capability of COVID-19. Every one of the four patients had a cesarean segment in their third trimester. Let $U=\{P_1,P_2,P_3,P_4\}$ be a set of patients and $H = \{H_1,H_2,H_3,H_4,H_5,H_6\}$ be a set of symptoms represented as (H_1 is fever in admission ,H_2 is cough ,H_3 is dyspnea, H_4 is sore throat ,H_5 is diarrhea , H_6 is chest pain) , D represented the decision of the covid-19 in which y means the patient has the virus and n means the patient has no virus as shown in Table (2).

	P_1	P_2	P_3	P_4
H_1	y	y	y	y
H_2	y	y	n	y
H_3	y	N	n	y
H_4	n	N	n	y
H_5	n	N	n	n
H_6	y	N	n	n
D	y	Y	n	y

Table (2: Represented the relation between the symptoms of the patients and the decision of the covid-19).

Table (3) represented similarities between patient's symptoms where the degree of similarity μ(x,y) is defined as $\mu(x,y)=((\sum_{i=1}^{n}(a_{i}(x)=a_{i}(y)))/n)$ where n is the number of symptoms. We defined the relationship in each issue according to the expert's requirement. The first minimal strucutre M_{xR} was defined by the relation $a\Re b$ if $\mu(a,b)>0.8$ and the second minimal structure M_{Rx} was defined by the relation $a\Re b$ if $\mu(a,b)>0.6$. The patients had covid-19 were denoted by B_1, while the patients had not covid-19 were denoted by B_2.

	P_1	P_2	P_3	P_4
P_1	1	4/6	3/6	4/6
P_2	4/6	1	5/6	4/6
P_3	3/6	5/6	1	1/6
P_4	4/6	4/6	1/6	1

Table (3: Represented the degree of similarities between patient's symptoms

$M_{xR}=\{\varphi, P, \{P_1\}, \{P_2,P_3\}, \{P_4\}\}$ and $M_{Rx}=\{\varphi, P, \{P_{1,}P_2,P_4\}, \{P_3\}\}$, then $M_{xR}^{C}\}=\{\varphi, P, \{P_2,P_3,P_4\}, \{P_1,P_4\}, \{P_1,P_2,P_3\}\}$. $M_{Rx}^{C}=\{\varphi, P, \{P_3\}, \{P_1,P_2,P_3\}\}$. Now we calculate

$M\text{-}\alpha_{12}(P)=\{\varphi, p, \{p_1\}, \{p_4\} , \{p_1,p_4\}, \{p_2,p_3\}, \{p_1,p_2,p_3\}, \{p_2,p_3,p_4\}\}$.

$M\text{-}S_{12}(P)=\{\varphi, p, \{p_1\}, \{p_4\}, \{p_1,p_2\}, \{p_1,p_4\}, \{p_2,p_3\}, \{p_2,p_4\}, \{p_1,p_2,p_3\}, \{p_1,p_2,p_4\}, \{p_2,p_3,p_4\}\}$.

$M\text{-}P_{12}(P)=\{\varphi, p, \{p_1\}, \{p_4\}, \{p_1,p_3\}, \{p_1,p_4\} , \{p_2,p_3\}, \{p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_1,p_3,p_4\}, \{p_2,p_3,p_4\}\}$.

$M\text{-}\gamma_{12}(P) = \{\varphi, p, \{p_1\}, \{p_4\}, \{p_1,p_2\}, \{p_1,p_3\}, \{p_1,p_4\}, \{p_2,p_3\}, \{p_2,p_4\}, \{p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_1,p_2,p_4\}, \{p_1,p_3,p_4\}, \{p_2,p_3,p_4\}\}$,

$M\text{-}\beta_{12}(P)=\{\varphi,p,\{p_1\},\{p_2\},\{p_4\}, \{p_1,p_2\},\{p_1,p_3\}, \{p_1,p_4\}, \{p_2,p_3\}, \{p_2,p_4\}, \{p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_1,p_2,p_4\}, \{p_1,p_3,p_4\}, \{p_2,p_3,p_4\}\}$.

$M\text{-}\alpha_{12}^{c}(P) = \{\varphi, P, \{p_2,p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_2,p_3\}, \{p_1,p_4\}, \{p_4\}, \{p_4\}\}$.

$M\text{-}S_{12}^{c}(P) = \{\varphi, P, \{p_2,p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_1,p_4\}, \{p_3,p_4\}, \{p_1,p_3\}, \{p_2,p_3\}, \{p_4\}, \{p_3\}, \{p_1\}\}$.

$M\text{-}P_{12}^{c}(P) = \{\varphi, p, \{p_2,p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_2,p_3\}, \{p_2,p_4\}, \{p_1,p_4\}, \{p_1,p_3\}, \{p_4\}, \{p_1\}, \{p_3\}\}$.

$M\text{-}\gamma_{12}^{c}(P) = \{\varphi, p, \{p_2,p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_3,p_4\}, \{p_1,p_2\}, \{p_2,p_3\}, \{p_2,p_4\}, \{p_1,p_4\}, \{p_1,p_3\}, \{p_4\}, \{p_2\}, \{p_1\}, \{p_3\}\}$.

$M\text{-}\beta_{12}^{c}(P) = \{\varphi, p, \{p_2,p_3,p_4\}, \{p_1,p_3,p_4\}, \{p_1,p_2,p_3\}, \{p_3,p_4\}, \{p_1,p_2\}, \{p_2,p_3\}, \{p_2,p_4\}, \{p_1,p_4\}, \{p_1,p_3\}, \{p_4\}, \{p_2\}, \{p_1\}, \{p_3\}\}$.

$B_1=\{p_1,p_2,p_4\}$, $M_{xR}(B_1)=\{p_1,p_4\}$, $M_{xR}(B_1)= P$, $ACC_{MxR}(B_1)=(1/2)$, $M_{S_{12}}(B_1) =\{p_1,p_2,p_4\}$, $M_{S_{12}}(B_1)=P$, $ACC_{S_{12}}(B_1) = (3/4)$.

We show that $M\text{-}\alpha_{12}(U) \prec M\text{-}S_{12}(U) \prec M\text{-}\gamma_{12}(U) \prec M\text{-}\beta_{12}(U)$ is satisfied.

The accuracy of B_1 through the lower and upper approximation of M_{xR} ($ACC_{MxR}(B_1)=(1/2)$) and the accuracy of B_1 through the lower and upper of $M\text{-}S_{12}$($ACC_{S_{12}}(B_1)=(3/4)$). So we found that the accuracy of $M\text{-}S_{12}$ is greater than the accuracy of M_{xR}.

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