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ESTIMATION OF THE UNKNOWN PARAMETERS OF THE GENERALIZED BURR DISTRIBUTION

BY

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ABSTRACT

Bayesian and non-Bayesian methods of estimation are used to make inferences given a random sample of observations from a generalized Burr distribution (GBD). Complete and type II censored samples are considered and inferences are made on the unknown parameters. The one-parameter GBD is also studied. A numerical illustration is provided by means of a numerical example.

INTRODUCTION

Burr (1942) introduced twelve types of cdf's which might be useful for fitting distributions to the data. The twelfth type is the so-called "generalized Burr distribution" abreviated to GBD. All other types can be easily derived from the GBD by simple transformations. Further, many other distributions such as the Lomax and Logistic are special cases of the GBD. Moreover, Lewis (1981) has shown that the two most common failure time distributions, the Weibull and exponential, are special limiting cases of the GBD.

This paper is concerned with the estimation of the unknown parameters of the GBD. Its pdf and cdf are, respectively,

$$f(x) = kc\, x^{c-1} (1+x^c)^{-(k+1)}, \qquad c > o,\ k > o,\ o < x \infty, \tag{1.1}$$

and

$$F(x) = 1-(1+x^c)^{-k}, \tag{1.2}$$

where c and k are the shape parameters, Al-Marzoung and Ahmad (1985) estimated the two unknown parameters k and c. Ahmad (1985) estimated the two unknown parameters from the ratio $\ell n\, x_{(r)} / \ell n\, x_{(s)}$, where $x_{(i)}$ is the i th order statistics for a random sample of size n drawn from a GBD.

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In the case when c is known, Lewis (1981) and Radwan (1990) obtained the MLE of k for the one-parameter GBD, based on a type II censored sample. Radwan (1990) also obtained the uniformly minimum variance unbiased estimate of k.

Papadopoulos (1978) obtained the Bayes estimate of k, when c is known, by considering two prior distributions of k (uniform and gamma) using a complete sample. Evans and Ragab (1983) obtained the Bayes estimates of k and c, based on a type II censored sample. They assumed that c is restricted to be a finite discrete number and that the prior distribution of k has the gamma pdf. We believe that the discrete prior is not suitable to represent the general prior belief about c.

Based on a type II censored sample and using Lindley's approximate expansion, Radwan (1990) obtained approximate Bayes estimates of c and k. He used the gamma prior for k and the exponential prior for c. He illustrated his results numerically, by taking samples of large sizes.

The purpose of this paper is to obtain ML estimates and exact Bayes estimates for the two unknown parameters k and c. We take the gamma prior for both c and k (which is rich enough to enable a convenient presentation of any prior beliefs about k and c). Clearly, our Bayesian results, based on the gamma priors, would be more realistic than those of Evans & Ragab (1983) and we generalize the results of Radwan (1990) who employed the exponential prior. In addition, our Bayes estimats are exact, not approximate.

To illustrate the new theoretical results numerically, we use samples of different sizes (small, moderate and large) and type II censoring with different uncensored items, covering also the case of a complete sample. A numerical illustration shows that these methods are practical and they are applicable to experiments in which the GBD is the appropriate statistical model.

2. MAXIMUM LIKELIHOOD ESTIMATION

Let $X_1, X_2, \ldots, X_n$ be n i.i.d. r.v'.s with pdf given by (1.1) and suppose that $X_{(1)}, X_{(2)}, \ldots, X_{(r)}$ satistfying $0 < X_{(1)} < X_{(2)} < \ldots < X_{(r)} < \infty$, represent the observed life times of the first r items to fail in this random sample of n items put on a test (type II censoring). The likelihood function is

$$L(k,c \mid \underline{x}_{(r)}) = n!\,\{(n-r)!\}^{-1} c^r k^r x(c)\, e^{-ky(c)}, \tag{2.1}$$

where

$$x(c) = \prod_{i=1}^{r} x_{(i)}^{c-1} (1+x_{(i)}^{c})^{-1} \quad \text{and} \quad y(c) = \sum_{i=1}^{r} \ell n(1+x_{(i)}^{c}) + (n-r)\,\ell n(1+x_{(r)}^{c}). \tag{2.1a}$$

Taking the natural logarithm of (2.1), differentiating it with respect to k and c respectively and equating the results to zero, at $k = \hat{k}$ and $c = \hat{c}$, we have

$$r\,\hat{k}^{-1} - y(\hat{c}) = 0, \tag{2.2}$$

and

$$r\,\hat{c}^{-1} + \sum_{i=1}^{r} \ell n\, x_{(i)}(1+x_{(i)}^{\hat{c}})^{-1} - \hat{k}\, y'(\hat{c}) = 0, \tag{2.3}$$

where

$$y(\hat{c}) = \sum_{i=1}^{r} \ell n(1+x_{(i)}^{\hat{c}}) + (n-r)\,\ell n(1+x_{(r)}^{\hat{c}}), \tag{2.3a}$$

and

$$y'(\hat{c}) = \sum_{i=1}^{r} x_{(i)}^{\hat{c}}\, \ell n\, x_{(i)}(1+x_{(i)}^{\hat{c}})^{-1} + (n-r)\, x_{(r)}^{\hat{c}}\, \ell n\, x_{(r)}(1+x_{(r)}^{\hat{c}})^{-1}. \tag{2.3b}$$

The value $\hat{k}$ is determined by

$$\hat{k} = r\,\{y(\hat{c})\}^{-1} \tag{2.4}$$

and substituting it in (2.3), we have

$$\hat{c}^{-1} + r^{-1} \sum_{i=1}^{r} \ell n\, x_{(i)}(1+x_{(i)}^{\hat{c}})^{-1} - y'(\hat{c})\,\{y(\hat{c})\}^{-1} = 0, \tag{2.5}$$

which is solved to obtain the MLE $\hat{c}$. Using (2.2) and (2.3), the elements of the information matrix, evaluated at $k = \hat{k}$ and $c = \hat{c}$, are

$$-\,\partial^2 \ell n\, L/\partial k^2 = r\,\hat{k}^{-2} \tag{2.6}$$

$$-\,\partial^2 \ell n\, L/\partial k\,\partial c = y'(\hat{c}) \tag{2.7}$$

$$-\,\partial^2 \ell n\, L/\partial c^2 = r\,\hat{c}^{-2} + \sum_{i=1}^{r} x_{(i)}^{\hat{c}}\, (\ell n\, x_{(i)})^2 (1+x_{(i)}^{\hat{c}})^{-2} + \hat{k}\, y''(\hat{c}), \tag{2.8}$$

where

$$y''(\hat{c}) = \sum_{i=1}^{r} x_{(i)}^{\hat{c}}(\ell n\, x_{(i)})^2 (1+x_{(i)}^{\hat{c}})^{-2} + (n-r)\, x_{(r)}^{\hat{c}}(\ell n\, x_{(r)})^2(1+x_{(r)}^{\hat{c}})^{-2}$$

The inverse of the information matrix is the approximate variance-covariance matrix of $\hat{k}$ and $\hat{c}$, with

$$\text{var}\,(\hat{k}) = D^{-1}\,[\,r\,\hat{c}^{-2} + y^*\,(\hat{c})\,\{y\,(\hat{c})\}^{-1}] \tag{2.9}$$

$$\text{var}\,(\hat{c}) = \{rD\}^{-1}\;\{y\,(\hat{c})\}^2 \tag{2.10}$$

and $$\text{cov}\,(\hat{k},\hat{c}) = -D^{-1}\; y'(\hat{c}), \tag{2.11}$$

where

$$D = r^{-1}\,\{y(\hat{c})\}^{-2}\,[rc^{-2} + y^*\,(\hat{c})\,\{(\hat{c})\}^{-2}] - \{y'(c')\}^2 \tag{2.12a}$$

and

$$y^*(\hat{c}) = \{r+y(\hat{c})\}\,\sum_{i=1}^{r} x_{(i)}^{\hat{c}}\,(\ell n\, x_{(i)})^2\,(1+x_{(r)}^{\hat{c}})^{-2}$$

$$+\; r(n-r)\;x_{(r)}^{\hat{c}}\,(\ell n\, x_{(r)})^2\,(1+x_{(r)}^{\hat{c}})^{-2} \tag{2.12b}$$

Numerical methods and computer facilities are needed to solve these equations.

3. BAYESIAN ESTIMATION

Let $X_1, X_2,\ldots,X_n$ be n i.i.d. r.v's having the pdf and cdf given by (1.1) and (1.2), respectively. Let $O < X_{(1)} < X_{(2)} < \ldots < X_{(r)} < \infty$, be the observed life times of items put on a test (type I censoring). The pair (k,c) is treated as unknown. The informative conjugate prior family for k and c is

$$g(k,c) = g(k \mid c)\; g(c). \tag{3.1}$$

Assuming that both of the distributions on the right hand side of (3.1) are gamma (which is rich enough family to enable the presentation of any prior beliefs about k and c), the informative prior family for k and c is

$$g(k,c) = \delta^{\gamma+1}\,\alpha^{\beta+1}\{\Gamma(\gamma+1)\,\Gamma(\beta+1)\}^{-1}\,e^{-\delta ck}\,k^{\gamma}\,e^{-\alpha c}\,c^{\gamma+\beta+1},$$

$$\gamma > -1,\ \delta > 0,\ \beta > -1,\ \alpha > 0,\ 0 < c < \infty \text{ and } 0 < k < \infty, \tag{3.2}$$

so that

$$g(k \mid c) = (\delta c)^{\gamma+1} \{\Gamma(\gamma+1)\}^{-1} k^{\gamma} e^{-\delta ck}, \tag{3.3}$$

and

$$g(c) = \alpha^{\beta+1} \{\Gamma(\beta+1)\}^{-1} c^{\beta} e^{-\alpha c} \tag{3.4}$$

where the prior parameters γ,δ,β and α are chosen according to the prior knowledge about k and c. Taking the limiting values for γ,δ,β and α, an appropriate non-informative prior for k and c is

$$g(k,c) \propto (kc)^{-1} \tag{3.5}$$

The joint posterior pdf of k and c is

$$f(k,c|\underline{x}_{(r)}) = \{\Gamma(\gamma+r+1)\}^{-1} \exp[-k\{1+y(c)(\delta c)^{-1}\}]\, k^{\gamma+r} e^{-\alpha c} c^{\beta} x(c)\, I^{-1}, \tag{3.6}$$

where x(c) and y(c) are given in (2.1), and

$$I = \int_0^{\infty} e^{-\alpha c} c^{\beta} x(c).\{1+ y(c)(\delta c)^{-1}\}^{-(\gamma+r+1)} dc \tag{3.6a}$$

Now, differentiating (3.6) with respect to k and c respectively, and equating the results to zero, at $k = k_o$ and $c = c_o$ respectively, the joint posterior mode (k_o, c_o) is the solution of the equations

$$k_o = (\gamma+r)\, \delta c_o \{\delta c_o + y(c_o)\}^{-1}, \tag{3.7}$$

and

$$(\gamma+r)\{\delta c_o+y(c_o)\}^{-1}\{y(c_o)\, c_o^{-1} - y'(c_o)\} + \sum_{i=1}^{r} \ln x_{(i)}(1+ x_{(i)}^{c_o})^{-1} + \beta c_o^{-1} - \alpha = 0, \tag{3.8}$$

where $y(c_o)$ and $y'(c_o)$ are given by (2.1a) and (2.3b) respectively, evaluated at c_o. We need numerical methods and computer facilities to obtain c_o and k_o from equations (3.7) and (3.8).

Further, integrating (3.6) from zero to infinity for c and also for k, one can obtain the univariate marginal posterior pdf's of k and c as

$$f(k|\underline{x}_{(r)}) = \{\Gamma(\gamma+r+1)\}^{-1} k^{\gamma+r} \int_0^\infty \exp[-k\{1+y(c)(\delta c)^{-1}\}] e^{-\alpha c} c^{\beta} x(c)\, dc\, \Gamma^{1}, \tag{3.9}$$

and

$$f(c|\underline{x}_{(r)}) = e^{-\alpha c} c^{\beta} x(c) \{1+y(c)(\delta c)^{-1}\}]^{(\gamma+r+1)} . \Gamma^{1} . \tag{3.10}$$

From (3.9), the ℓ<u>th</u> posterior moment of k is

$$\mu'_{\ell}(k) = \Gamma(\gamma+r+\ell+1) \{\Gamma(\gamma+r+1)\}^{-1} \int_0^\infty e^{-\alpha c} c^{\beta} x(c) \{1+y(c)(\delta c)^{-1}\}^{-(\gamma+r+\ell+1)} dc.\ \Gamma^{1} \tag{3.11}$$

Similarly, from (3.10), the ℓ<u>th</u> posterior moment of c is

$$\mu'_{\ell}(c) = \int_0^\infty e^{-\alpha c} c^{\beta+\ell} x(c) \{1+y(c)(\delta c)^{-1}\}^{-(\gamma+r+1)} dc.\ \Gamma^{1}. \tag{3.12}$$

In particular, from (3.11) and (3.12), under a squared-error loss function the posterior means (Bayes estimates) and variances (estimated risks)of k and c respectively, are

$$\mu'_{1}(k) = (\gamma+r+1) \int_0^\infty e^{-\alpha c} c^{\beta} x(c) \{1+y(c)(\delta c)^{-1}\}^{-(\gamma+r+2)} dc.\ \Gamma^{1}$$

$$\mathrm{var}(k) = (\gamma+r+1)(\gamma+r+2) \int_0^\infty e^{-\alpha c} c^{\beta} x(c) \{1+y(c)(\delta c)^{-1}\}^{-(\gamma+r+3)} dc.\ \Gamma^{1} - \{\mu'_{1}(k)\}^{2}$$

$$\mu'_{1}(c) = \int_0^\infty e^{-\alpha c} c^{\beta+1} x(c) \{1+y(c)(\delta c)^{-1}\}^{-(\gamma+r+1)} dc.\ \Gamma^{1},$$

and

$$\mathrm{var}(c) = \int_0^\infty e^{-\alpha c} c^{\beta+2} x(c) \{1+y(c)(\delta c)^{-1}\}^{-(\gamma+r+1)} dc.\ \Gamma^{1} - \{\mu'_{1}(c)\}^{2}$$

Moreover, differentiating (3.9) and (3.10) with respect to k and c respectively, and equating the results to zero, at $k=k^*$ and $c=c^*$ respectively, we have the marginal posterior modes k^* and c^*, which are the solutions of the equations

$$\int_0^\infty [(\gamma+r)- k^*\{1+y(c)(\delta c)^{-1}\}]\, e^{-\alpha c}\, c^\beta\, \bar{e}^{k^*\{1+y(c)(\delta c)^{-1}\}} x(c)\, dc=0 \tag{3.13}$$

and

$$(\gamma+r+1)\{\delta c^*+y(c^*)\}^{-1}\, \{y(c^*)\, c^{*-1}-y\prime(c^*)\} + \sum_{i=1}^{r} \ell n\, x_{(i)}\, (1+ x_{(i)}^{c^*})^{-1}$$

$$+ \beta c^{*-1} - \alpha = 0 \tag{3.14}$$

For the non-informative prior, using (3.5), the joint posterior pdf of k and c reduces to

$$f(k,c|\underset{\sim}{x}_{(r)}) = \{\Gamma(r)\}^{-1}\, x(c)\, c^{r-1}\, k^{r-1}\, e^{-ky(c)}.\, I^{*-1}, \tag{3.15}$$

where

$$I^* = \int_0^\infty x(c)\, c^{r-1}\, \{\gamma(c)\}^r\, dc.$$

The marginal posterior pdf and ℓth posterior moments of k and c are, respectively

$$f(k|\underset{\sim}{x}_{(r)}) = \{\Gamma(r)\}^{-1}\, k^{r-1} \int_0^\infty \bar{e}^{ky(c)} x(c)\, c^{r-1}\, dc.\, I^{*-1},$$

$$f(c|\underset{\sim}{x}_{(r)}) = x(c)\, c^{r-1}\, \{y(c)\}^{-r}.\, I^{*-1},$$

$$\mu\prime_\ell(k) = \Gamma(r+\ell)\{\Gamma(r)\}^{-1} \int_0^\infty x(c)\, c^{r-1}\, \{y(c)\}^{-(r+\ell)}\, dc.\, I^{*-1}, \tag{3.16}$$

and $$\mu\prime_\ell(c) = \int_0^\infty x(c)\, c^{r+\ell-1}\, \{y(c)\}^{-r}\, dc.\, I^{*-1}. \tag{3.17}$$

Under a squared-error loss function, it is easy to deduce the posterior means (Bayes estimates) and variances of k and c. Further, from (3.15), the joint posterior mode (k,c) is obtained as the solution of the equations

$$\bar{k} = (r-1)\, \{y(\bar{c})\}^{-1}, \tag{3.18}$$

and

$$\bar{c}^{-1} +(r-1)^{-1} \sum_{i=1}^{r} \ell n\, x_{(i)}\, \{1+x_{(i)}^{\bar{c}}\}^{-1} - y\prime(\bar{c})\, \{y(\bar{c})\}^{-1} = 0 \tag{3.19}$$

The marginal posterior modes for k and c are the solutions of the equations

$$\int_0^\infty \{r-1-k^*\, y(c)\}\, \bar{e}^{k^* y(c)}\, x(c)\, c^{r-1}\, dc = 0 \tag{3.20}$$

and

$$(r-1)\, x(c^*)\, y(c^*) - rc^*\, y'(c^*)\, x(c^*) + c^*\, y(c^*)\, x'(c^*) = 0, \tag{3.21}$$

where

$$x'(c^*) = \prod_{i=1}^{r} (c^*-1-x_{(i)}^{c^*})\, x_{(i)}^{c^*-2}\, (1+x_{(i)}^{c^*})^{-2}$$

Next we consider the situation when c is assumed to be known. In this case, the posterior pdf, given by (3.6), becomes

$$f(k|\underline{x}_{(r)}) = \{\Gamma(\gamma+r+1)\}^{-1}\, k^{\gamma+r} e^{-k\{\delta c+y(c)\}} \{\delta c+y(c)\}^{\gamma+r+1}, \tag{3.22}$$

with posterior ℓth non-central moment

$$\mu'_\ell(k) = \Gamma(\gamma+r+\ell+1)\{\Gamma(\gamma+r+1)\}^{-1}(\delta c+y(c))^{-\ell}, \tag{3.23}$$

In particular, the posterior mean (Bayes estimate) and variance (estimated risk) of k are, respectively

$$\mu'_1(k) = (\gamma+r+1)\{\delta c+y(c))^{-1},$$

and

$$\mathrm{var}(k) = (\gamma+r+1)\{\delta c+y(c))^{-2}.$$

Differentiating (3.22) with respect to k and equating the result to zero, at $k=k'$, we obtain the posterior mode k', as

$$k' = (\gamma+r)\{\delta c+y(c))^{-1},$$

which is the MLE $\hat{k}$, given by (2.4), when $\gamma=0$, $\delta=0$ and c is known,

NUMERICAL ILLUSTRATION

The main object of the present section is to illustrate numerically most of the theoretical results of the two proceeding sections. This numerical illustration has been achieved using a personal computer, with the following specifications :

(i) Six sets of artificial data have been generated from the generalized Burr distribution with parameters c=4 and k=2, for samples of sizes 10,15,20,25,30 and 35. Another hypothetical example given by Papadopoulos (1978) is also used.

(ii) We have used censored type II samples with 50% and 70% uncensored items and complete samples to have an idea about the effect of changes of the sample size and the number of censored items on the properties of the new estimators.

(iii) Numerical computations are carried out to obtain the maximum likelihood estimates and their approximate variance-covariance matrix and also different Bayes estimates such as univariate and bivariate modes, Bayes estimates and risks with respect to the squared error loss. We also illustrate another set of Bayes estimates, based on the univariate posterior distribution of k, when c is known (c=5) .

(iv) Different prior parameters, including a non-informative prior, have been chosen and used.

(v) A trial and error technique has been used to obtain the maximum likelihood estimates and different posterior modes. Simpson's rule for numerical integration has been used to obtain Bayes etimates and risks with respect to squared error loss (see Mc cormik & Salvadari (1971)).

(vi) The results based on the six generated data are listed in El-Wakeel (1992), while the results based on Papadopoulos' example are displayed in tables (1), (2), (3) and (4).

Based on the analysis of the seven sets of the artificial data, we have the following conclusions :

(a) The estimated value of any unknown parameter is within two sigma of the value of the corresponding population parameter.

(b) The suggested iterative technique needs a reasonably short time and the use of a personal computer makes these methods practical and applicable to experiments in which the generalized Burr distribution is the appropriate statistical model.

(c) Most of the variances of the MLE's decrease when n or r increase. The covariance of the pair $(\hat{k}, \hat{c})$ are positive and it decreases when n or r increase.

(d) The estimated values of the joint mode of the pair (k,c) and their univariate modes depend on the choice of the prior parameters. A comparison of the estimated values of the joint and univariate modes of k and its MLE's (using non-informative priors) show that they are close to each other. This is not true for c.

(e) Bayes estimates and risks of k ad c are sensitive to the assumed values of the prior parameters. Most of the Bayes estimates are more efficient than the MLE's.

(f) If c is known, the MLE of k and the corresponding Bayes estimate of k (in the case of non-informative prior) are very close. Further, the estimated values of the mode of k and the Bayes estimate of k are close to each other in all samples. The Bayes risks of k decreases as n and r increase.

All these comments are based only on the present numerical illustration. To investigate this problem thoroughly, more Monte-carlo simulation is needed and substantial computer calculations are needed to arrive at concrete conclusions.

Table (1)

Maximum likelihood estimates of c & k and their approximate variance-covariance matrix using Papadopoulos' data (n=10)

r	MLE		Variances & covariance		
	$\hat{c}$	$\hat{k}$	var($\hat{c}$)	var($\hat{k}$)	cov($\hat{c}$,$\hat{k}$)
5	8.499	3.408	2.785	49.056	44.870
7	6.850	3.341	1.586	15.331	22.274
10	6.1999	10.656	2.013	5.948	5.903

Table (2)

Joint modes of (c,k) and their univariate modes using Papadopoulos' data (n=10)

r	Prior parameters				Joint modes		Univariate modes	
	α	β	γ	δ	c_o	k_o	c^*	k^*
5	1	-1	-1	1	1.7257	1.0830	1.2229	5.1867
	1	-1	7	5	5.1650	10.1790	3.3939	5.3029
	1	-1	9	8	4.3497	12.3504	2.6354	5.3142
	3	-1	7	1	9.0486	7.9278	5.8767	5.1763
	5	7	-1	1	5.7257	2.2078	5.2229	5.0144
	8	9	-1	1	4.7257	2.0166	4.2230	5.1146
	1	7	-1	3	8.5067	3.3837	8.1354	5.3295
	NIP				3.7286	3.2087	3.1176	4.0460
7	1	-1	-1	1	3.4435	2.5000	1.0863	2.9950
	1	-1	7	5	6.0407	12.0930	2.0669	3.0630
	1	-1	9	8	4.8478	14.5030	1.3338	3.0689
	3	-1	7	1	10 .6230	9.6660	3.8908	2.9894
	5	7	-1	1	7.4435	3.6592	4.9137	2.0289
	8	9	-1	1	6.4436	3.4502	3.9136	2.2786
	1	7	-1	3	9.6695	5.1540	7.7460	3.1978
	NIP				5.8917	3.0860	2.8282	2.3364
10	1	-1	-1	1	6.1872	5.0008	3.5599	0.7892
	1	-1	7	5	7.4878	15.0155	3.3841	0.8069
	1	-1	9	8	5.8836	17.1926	3.7472	0.8086
	3	-1	7	1	11.6750	16.8680	3.7158	0.7877
	5	7	-1	1	10.1872	6.0577	2.4401	0.6402
	8	9	-1	1	9.1872	5.8495	1.4401	0.6579
	1	7	-1	3	7.6108	8.9626	5.8600	0.8787
	NIP				9.1943	10.4190	2.0940	0.6156

Table (3)

Bayes estimates and risks of c & k

using Papadopoulos' data (n=10)

r	Prior parameters				Joint modes		Univariate modes	
	α	β	γ	δ	$\tilde{c}$	$\tilde{k}$	$v(\tilde{c})$	$v(\tilde{k})$
5	1	-1	-1	1	1.6759	1.9892	0.5972	0.7914
	1	-1	7	5	2.7676	9.9792	0.6432	7.6604
	1	-1	9	8	1.6764	9.4877	0.5377	6.0010
	3	-1	7	1	2.2351	5.1719	0.2184	2.0577
	5	7	-1	1	1.7205	0.8853	0.7229	0.1566
	8	9	-1	1	1.3992	0.8864	0.1554	0.1559
	1	7	-1	3	6.4937	1.9613	3.1990	0.7694
	NIP				7.9424	3.3013	23.5980	2.1798
7	1	-1	-1	1	3.4751	2.6929	1.6515	1.0409
	1	-1	7	5	3.2420	11.3645	2.0142	8.6102
	1	-1	9	8	1.6763	12.2325	1.5873	10.6571
	3	-1	7	1	2.2934	9.2416	0.9040	0.7781
	5	7	-1	1	3.3973	1.2148	1.3973	0.2263
	8	9	-1	1	3.4430	1.2149	1.4430	0.2109
	1	7	-1	3	6.6750	2.7056	0.7055	1.0457
	NIP				6.2272	4.2797	39.5310	2.9815
10	1	-1	-1	1	2.6219	3.6863	0.9912	3.5470
	1	-1	7	5	2.7699	13.3318	0.6137	9.7617
	1	-1	9	8	1.6761	14.2483	0.5471	8.9106
	3	-1	7	1	2.3294	6.3551	0.9420	4.6785
	5	7	-1	1	3.1205	2.4790	1.2567	0.5610
	8	9	-1	1	3.2970	1.6812	1.2784	0.4375
	1	7	-1	3	6.4069	3.7745	2.3951	1.4247
	NIP				5.1962	2.3808	2.0940	4.4393

Table (4)

MLE, modes, Bayes estimates and risks

of k when c known (c=5)

using Papadopoulos' data (n=10)

r	MLE	prior parameters		modes	Bayes estimates	Bayes risks
		γ	δ			
5	6.53029	5	2	0.92888	1.02177	0.09491
		5	3	0.63429	0.69772	0.4423
		6	2	1.02177	1.11466	0.10354
		6	3	0.69772	0.76114	0.04828
		NIP		5.22423	6.53029	8.52893
7	7.63367	5	2	1.09920	1.19080	0.10908
		5	3	0.75391	0.81674	0.05131
		6	2	1.19080	1.28241	0.11786
		6	3	0.81674	0.87956	0.05526
		NIP		6.54315	7.63367	8.32471
10	7.52211	5	2	1.32399	1.41225	0.12465
		5	3	0.91859	0.97983	0.06000
		6	2	1.41225	1.50052	0.13244
		6	3	0.97983	1.04107	0.06375
		NIP		6.76989	7.52211	5.65821

(*) C does not depend on k , see (4.10 d).

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