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Nonresponse Effect in Double Sampling : A Simulation Study

by

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Abstract

We consider estimation of the population total in double sampling for regression in the presence of nonresponse.Two estimators were examined, one allowing for nonresponse and the other based only on respondents'set. A simulation study was performed to investigate the performance of the two estimators under various response situations. The simulation results showed that, under the True Response Model (TRM), the performance of the estimator allowing for nonresponse is superior when bias, mean squares and confidence level are used as performance measures. When the model is false (FRM), this estimator is inferior though slightly.

1. Introduction

A problem frequently encountered by sampling practitioners is that of unit nonresponse. Failure of a unit to respond may be due to its refusal or inability to respond. It may be, also due to failure to locate the unit on the part of the sampler. Analytical investigations reveal that presence of nonresponse may lead to bias in the estimators, increase in variance and, sometimes, serious distortions in the confidence statements (Cochran [1]). In a simulation study, Little [3] examined the performance of several estimators of the mean under various nonresponse situations. Samdal and Swensson [4] used Monte Carlo simulation to compare three estimators of the total in the presence of nonresponse.These studies and others are, however, confined to single-phase sampling. In this paper, we investigate the effect of nonresponse in double sampling for regression by comparing two estimators for population total.

We assume that we have a population U of N units. A large frist-phase sample S_1 of size n_1 is taken from U by simple random sampling (without replacement), and characteristic x (an auxillary variable) measured for all unit in S_1. A second- phase sample S_2 of size n_2 ,is selected from S_1 , also by simple random sampling. Due to nonresponse in the second-phase sample, measurements are obtained on characteristics x and y, the variable of the study, for only some of the units in S_2.It is required to examine the effect of nonresponse on the estimate of the population total $t = \sum_{k \in U} y_k$ where y_k is the value of y for unit k in the population.

2. Estimation of the population total

2.1 Estimators of the total:

We examine the effect of nonresponse through the comparison of two estimators. The first estimator, which we shall call the respondents regression estimator is

$$\hat{t}_1 = N(\bar{y}_R + b_R(\bar{x}_1 - \bar{x}_R)) \qquad (1)$$

where $\bar{y}_R$ and $\bar{x}_R$ are,respectively, the mean of y and x for the respondents, b_R is the estimate of the regression coefficient also calculated from the respondents' set (in S_2), and $\bar{x}_1 = \sum_{S_1} x_k / n_1$ the mean of x in the first-phase sample S_1. When response is complete, $\hat{t}_1$ is the ordinary regression estimator in double sampling (see for example Cochran [1]). Estimator $\hat{t}_1$ is biased, and will remain biased even if response is complete unless some assumption is made about the presence of linear regression in the population.

Now, suppose the second-phase sample S_2 is divided into L strata, the h^{th}, S_{2h}, is of size n_{2h} and such that units within S_{2h} respond with the same probability θ_h. The S_{2h}'s are also called response homogeneity groups. Letting r_h be the set of respondents in S_{2h} and m_h its size, we have the weighted regression estimator $\hat{t}_2$ given by:

$$\hat{t}_2 = \frac{N}{n_1}\sum_{S_1}\hat{y}_k + \frac{N}{n_2}\sum_h f_n^{-1}\sum_{r_h}(y_k - \hat{y}_k)$$

$$= N\left(\sum_h W_h \bar{y}_h + b_R\left(\bar{x}_1 - \sum_h W_h \bar{x}_h\right)\right) \qquad (2)$$

where

$$\hat{y}_k = b_R x_k,\; f_h = \frac{m_h}{n_{2h}},\; W_h = \frac{n_{2h}}{n_2} \text{ and } \bar{x}_h = \sum_{r_h} x_k/m_h,\; \bar{y}_h = \sum_{r_h} y_k/m_h$$

The estimator $\hat{t}_2$ is a special case of the regression estimator given by El-Beshir [2]. It is approximately unbiased. The second form in (2) reflects the stratified nature of $\hat{t}_2$. The basic feature of $\hat{t}_2$ is that estimates of $\bar{y}_h$ and $\bar{x}_h$ from the strata (response homogeneity groups) are weighted by the relative sizes of the strata.

If we set $b_R=0$ in (2) we get the estimator

$$\hat{t}_3 = N \sum_h W_h \bar{y}_h \tag{3}$$

The estimator $\hat{t}_3$ may be looked at as a weighted estimator that does not utilize the correlation between x and y. In the context of double sampling for regression, it is meaningless to consider an estimator like $\hat{t}_3$. However, $\hat{t}_3$ is used here merely as a "control estimator" that helps in throwing more light on the effect of nonresponse on the regression estimators $\hat{t}_1$ and $\hat{t}_2$.

When response is complete, $m_h = n_{2h}$ so that

$$\sum_h W_h \bar{y}_h = \bar{y}_2 \text{ and } \sum_h W_h \bar{x}_h = \bar{x}_2$$

with $\bar{y}_2$ and $\bar{x}_2$, respectively, the mean of y and x in S_2. Also $\bar{y}_R = \bar{y}_2$ and $\bar{x}_R = \bar{x}_2$ with the result that both $\hat{t}_1$ and $\hat{t}_2$ taking the familiar form of the estimator of the total in double sampling for regression (with complete response):

$$\hat{t}^* = N(\bar{y}_2 + b^*(\bar{x}_1 - \bar{x}_2)) \tag{4}$$

which is approximately unbiased, where b^* is the regression coefficient calculated from the complete second-phase sample.

On the other hand, if the response rates f_h's are the same for all h, $\hat{t}_1$ and $\hat{t}_2$ will also be identical, since the sample will then be self weighting. Hence, as the response rates increase and differences between strata in response rates decrease $\hat{t}_1$ approaches $\hat{t}_2$. On the other hand comparison of (1) and (2) shows that the difference between $\hat{t}_1$ and $\hat{t}_2$ depends on the difference between the regression coefficients b^* and b_R as well as differences between the means of x and the means of y in the respondent set and the second-phase sample.

2.2 Variance Estimates:

It is well known [1] that if terms in $\frac{1}{n_2}$ are negligible, the variance of $\hat{t}^*$ is $V(\hat{t}^*) \simeq S_y^2 \frac{(1-\rho^2)}{n_2} + \frac{\rho^2 S_y^2}{n_1} - \frac{S_y^2}{N}$

where S_y^2 and ρ^2 are the variance of y, and the population correlation between x and y respectively. An approximately unbiased estimator of $V(\hat{t}^*)$ is :

$$\hat{V}(\hat{t}^*) = N^2 \left[\frac{\hat{S}_y^2}{n_1} - \frac{\hat{S}_y^2}{N} + \frac{\hat{S}_e^2}{n_2} - \frac{\hat{S}_e^2}{n_1} \right] \tag{5}$$

where

$$\hat{S}_y^2 = \frac{\left(\sum_{k \in S_2} y_k^2 - \left(\sum_{k \in S_2} y_k \right)^2 / n_2 \right)}{(n_2 - 1)}$$

and $\hat{S}_e^2$ is of a form identical to that of $\hat{S}_y^2$ with y replaced by the residual e where $e_k = y_k - \hat{y}_k$. On the other hand, utilizing a result given by EL-Beshir [2],it can be shown that an approximately unbiased estimator of variance of $\hat{t}_2$ is given by

$$\hat{V}(\hat{t}_2) = \frac{N}{n_1 n_2 (n_2 - 1)} \left[(N - n_1)\, Q(y) + \frac{N(n_1 - n_2)}{n_2} Q(e) \right] + \frac{N^2}{n_2^2} \sum_h w_h (n_{2h} - m_h) \hat{S}^2_{e r_h} \tag{6}$$

where:

$$Q(z) = n_2 \left\{ \sum_h w_h \sum_{r_h} z_k^2 - \frac{\left(\sum_h w_h \sum_{r_h} z_k \right)^2}{n_2} \right\} + \sum_h w_h \left[(\dot{w}_h - 1) \sum_{r_h} z_k^2 - (\dot{w}_h - w_h) \left(\sum_{r_h} z_k \right)^2 \right]$$

and where $\hat{S}e^2_{r_h}$ is the estimate of the residual variance calculated from r_h

and :

$$w_h = \frac{n_{2h}}{m_h}, \quad w^*_h = \frac{(n_{2h}-1)}{(m_h-1)}$$

The estimator $\hat{V}(\hat{t}_2)$ can be written in the more informative form:

$$\hat{V}(\hat{t}_2) = \frac{N^2}{n_2(n_2-1)} \Big[(1-f_1)\, Q(y)/n_1 + (1-f_2)\, Q(e)/n_2 + n_2(n_2-1) \sum_h W^2_h (1-f_h)\, \hat{S}e^2_{r_h}/m_h \Big] \qquad (7)$$

with $f_1 = \frac{n_1}{N}$, $f_2 = \frac{n_2}{n_1}$ the sampling fractions in phase one and phase two respectively, and f_h is the response rate in stratum h. This form of variance estimator shows that the first and second terms inside the square brackets correspond,respectively, to the first and second phase of sampling. As the sampling fraction in each phase approaches unity, the corresponding variance component approaches zero. The last term in (7) represents nonresponse. It vanishes when response rate is unity.

On the other hand if $b_R=0$, $\hat{V}(\hat{t}_2)$ reduces to $\hat{V}(\hat{t}_3)$ where $\hat{V}(\hat{t}_3)$ takes the form (6) with e replace by y. Furthermore, if response is complete then

$n_{2h} = m_h$ and $w_h = w^*_h = 1$. In this case the last term in (7) vanishes and

$Q(z) = n_2\, Q_2(z)$ where $Q_2(z)$ is the sum of squares of z in the second sample given by

$$Q_2(z) = \sum_h \sum_{r_h} z^2_k - \frac{(\sum_h \sum_{r_h} z_k)^2}{n_2}$$

with (7) reducing to (5). As a result, if (5) is used to estimate variance when there is nonresponse, the variance will be underestimated.

On theoretical grounds therefore we expect that if the model on which $\hat{t}_2$ is based holds, $\hat{t}_2$ will provide an approximately unbiased estimator with confidence levels close to nominal level. On the other hand

$\hat{t}_1$ is expected to be more biased than $\hat{t}_2$ with possibly serious distortion in the confidence intervals. To enable a deeper look at the relative merits of $\hat{t}_1$ and $\hat{t}_2$ we performed a simulation study. The objectives are:

a. To investigate the relative performance of $\hat{t}_1$ and $\hat{t}_2$ with respect to bias, precision and confidence intervals under various response situations.
b. To determine the consequence of using $\hat{t}^*$ when there is nonresponse,i.e., using $\hat{t}_1$.

3. The simulation Study

Six factors suspected to affect estimators were chosen as parameters in the study. These factors were:

(i) The magnitude of response probabilities
(ii) The between groups (strata) differences in response probabilities
(iii) The within groups differences in response probabilities (homogeneity)
(iv) Sample size
(v) Differences in groups means
(vi) Linear correlation between x and y

3.1 Populations:

Three populations POP 1, POP 2, and POP 3 were formed. Each population was of size 2000 and consisted of four strata ST 1, ST 2, ST 3, and ST 4 with sizes 700, 500, 450, and 350 respectively.

POP 1 was constructed in such a way that y, the variable of the study, was related to the explanatory variable x according to the linear model:

$$y_i = 2 x_i + e_i \qquad i=1,2,...,2000 \qquad (8)$$

where the e_i's are normally distributed with zero mean and unit variance. The value actually used for the e_i's were drawn from the standardize normal distribution using MISL software package on IBM 3083 mainframe at King Saud University. On the other hand, the x values were generated independently according to the rule :

$$x_i = \frac{i}{100} \qquad i=1,2,...,2000 \qquad (9)$$

POP 1 was also formed so that differences among strata means were large. To achieve this, values of y in ST 2 were increased by 5, those in ST 3 by 10 and those in ST 4 by 15. This resulted in the strata means: 0.71, 6.96, 12.82, and 18.6 for ST 1, ST2, ST3, and ST 4 respectively. The correlation between x and y, ρ_{xy} in POP 1 was 0.96 indicating an almost perfect linear relation.

POP 2 was formed in exactly the same way as POP 1 the only difference being that the strata were constructed such that their means differ less. To achieve this without distorting the population total we added the quantities $\bar{y}_w$, $\bar{y}_w$ - 5, $\bar{y}_w$ -10, and $\bar{y}_w$ -15 to y values in ST 1, ST 2, ST 3, and ST 4 of POP 1 respectively, where

$$\bar{y}_w = (N_1 + 5\ N_2 + 10\ N_3 + 15\ N_4)/2000$$

and N_1, N_2, N_3, and N_4 are the sizes of ST 1, ST 2, ST 3, and ST 4 respectively. The means of ST 1, ST 2, ST 3, and ST 4 in POP 2, were respectively 6.83, 8.09, 8.94, and 9.8. For POP 2, ρ_{xy} = 0.75.

On the other hand, POP 3 was constructed in such a way that the correlation ρ_{xy} was small. This was obtained by adding 10 to y_1 and -0.01 i to y_i (i=2,...,2000) for values of y in POP 1. This keeps the population total unaffected but affects the strata means which became 19.30, -3.06, 2.82, and 8.67 for ST 1, ST 2, ST 3, and ST 4, respectively. Also ρ_{xy} = 0.33 in this case.

3.2 Sampling Scheme:

From each population, a simple random sample S_1 (without replacement) of size n_1 was selected. From S_1 a second-phase sample S_2 of size n_2 was also drawn by simple random sampling. The pairs of values (n_1 ,n_2) used were (1200,800), (800,400), and (400,200) representing respectively large, moderate, and small samples.

3.3 Response Probabilities:

Once selected, S_2 was exposed to simulated unit nonresponse. To achieve response probabilities of varying magnitudes, varying differences among strata, and varying differences within strata, the following was adopted. Units in S_2 were divided into four strata S_{2h} ,h=1,2,3,4 with units in stratum S_{21} , S_{22} , S_{23}, and S_{24} belonging, respectively, to ST 1, ST 2, ST 3, and ST 4. Let θ_{ih} be the response probability for unit i in stratum h. A starting value θ_{1h} was chosen for h=1,2,3,4 such that

$$\theta_{1h} = p_{11} + (h-1)\, p_{12}, \qquad h=1,2,3,4$$

where p_{11} and p_{12} are parameters controlling the magnitude and between strata deferences in response probabilities, respectively. The values 0.20 and 0.50 were used for p_{11} to obtain small and large response probabilities respectively, while the values 0.02 and 0.10 were used for p_{12} to achieved small and large between strata differences in response probabilities respectively. Perfect homogeneity of the θ_{ih} within strata were obtained by letting $\theta_{ih} = \theta_h$ for all i (h=1,2,3,4). This led to homogeneity (measured by intra-strata correlation) in the range 0.95-0.99 on the average. On the other hand, low homogeneity was achieved by using $\theta_{ih} \neq \theta_{jh}$ for all $i \neq j$ leading to average homogeneity in the range 0.002-0.37.

3.4 Performance Measures

Given a particular population, a particular pair of samples (S_1, S_2) and a given set of response probabilities θ_{ih} , the simulation proceeded as follows. For each unit $k \in S_{2h}$ a Bernoulli trial is carried out with probability of success (response) θ_{kh} and probability of failure (nonresponse) $1 - \theta_{kh}$. Following the notation of section 2 we denote by r_h the response set generated from S_{2h} and by m_h its size. The overall response set or the third-phase sample, r say, is of size m where m is the union of the m_h's . We may also look at r as an incomplete second-phase sample.

The stratum to which each unit k ($k \in r$) belonged, was determined and values of x and y recorded. The estimates $\hat{t}_1$, $\hat{t}_2$, and $\hat{t}_3$, their respective estimates of variances, and their confidence intervals, were then computed. This process was performed for each population and for every level of homogeneity. In each case, the procedure was repeated 500 times (the number of iterations) providing 500 generated response sets r's. No significant change in the result was observed when the number of iterations was raised from 500 to 1000.

Given that $\hat{t}^{(i)}$ is the value of one of the three estimators in the i^{th} iterations (response set), the following performance measures were computed:

$$BIAS = \sum_{i=1}^{k} (\hat{t}^{(i)} - t)/k \; ; \quad MSE = \sum_{i=1}^{k} (\hat{t}^{(i)} - t)^2/k \; ; \quad VAR = MSE - (BIAS)^2$$

where k (i.e., 500) is the number of iterations. Also calculated were the means $\bar{v}_1$,$\bar{v}_2$,and $\bar{v}_3$, over the 500 iterations of $\hat{V}(\hat{t}_1)$,$\hat{V}(\hat{t}_2)$, and $\hat{V}(\hat{t}_3)$ respectively. Furthermore the proportion of 500 confidence intervals, with $Z_{0.975}$ =1.96, that contained the total t was also obtained and provided in percentage form. This was denoted by CV95.

A computer program code in FORTRAN, written by the authors and run on IBM 3083 at King Suad University, was used in this simulation study.

4. Analysis of The Results

The estimator $\hat{t}_2$ is based on the extended response homogeneity groups ERHG (El-Beshir [2]) which postulates that units within each response group (stratum) h respond independently with the same response probability θ_h. The adjustment to nonresponse takes the form of weighting strata estimates in a way similar to that in stratified sampling. We therefore hypothesize that $\hat{t}_2$ is superior to $\hat{t}_1$ when the strata are internally homogeneous with respect to response probabilities and when strata means of y are different.

4.1 Bias and Mean Square :

We first consider the effect on bias and precision of the various factors included in the study (see section 3). Table (1) gives bias in the estimators, bias in the estimators of their variances as well as the mean square errors (MSE), when the sampled population is POP 1. These are provided for various values of p_{11} (the magnitude parameter), p_{12}(the strata differences in response probabilities parameter), and for the two levels of homogeneity. We recall that POP 1 is characterized by large between strata differences in mean of y and by a linear correlation between x and y amounting to 0.96. The sample sizes for phase one and phase two were 800 and 400 respectively which may be considered "moderated". With strong homogeneity, the range 0.95-0.99, we have the case of the true response model (TRM). The results for this case are given in the upper half of Table (1). In the lower half of the table, we have the case of the false response model (FRM).

Table (1) shows that under TRM, the bias in $\hat{t}_2$ is much less than the bias in $\hat{t}_1$. This is true irrespective of the magnitude (p_{11}) of, or the between strata difference (p_{12}) in, response probabilities. However, $\hat{t}_2$ is notably superior when between strata differences (p_{12}) is high. These results conform to the theoretical results. The effect of increasing the magnitude of response probabilities (p_{11}) is such that it reduces the bias in both $\hat{t}_1$ and $\hat{t}_2$. However, differences in bias between the weighted nonregression estimators $\hat{t}_3$ and $\hat{t}_2$ are,in general, less than differences in bias between the unweighted regression estimators $\hat{t}_1$ and $\hat{t}_2$. This implies that controlling the between strata differences in mean of y through weighting may be more effective, as far as bias reduction is concerned, than utilizing the correlation between x and y in an unweighted regression estimator. We recall that $\hat{t}_2$ involves both weighting and utilization of the regression of y on x. This makes it superior to both $\hat{t}_1$ and $\hat{t}_3$.

Similar results hold for estimators of variance, the performance of $\hat{t}_2$ is better than that of $\hat{t}_1$ under TRM. However, in general, bias in estimator of variance is less when between strata differences (p_{12}) in response probabilities are less. We also note that, with respect to variance estimation,$\hat{t}_1$ performs better than $\hat{t}_3$.

On the other hand, $\hat{t}_2$ is obviously more precise than $\hat{t}_1$ as reflected in a smaller MSE in all cases. Furthermore, the unweighted regression estimator $\hat{t}_1$ is more precise than $\hat{t}_3$. This may be attributed to the fact that $\hat{t}_1$ eliminates the variation explained by x.

A completely different picture emerges when the model is false (FRM) i.e., when the within strata homogeneity with respect to response probabilities is small. Here $\hat{t}_1$ is superior, though slightly, to $\hat{t}_2$ with respect to the three performance measures.

When sample size varies, we see from Table (2) that under TRM, bias in $\hat{t}_2$, its variance estimator, as well as the MSE, generally, decrease with increase in sample size. This confirms the fact that $\hat{t}_2$ is essentially a large sample estimator. On the other hand $\hat{t}_2$ seems to be worse, though slightly, than $\hat{t}_1$ under the FRM with respect to bias of estimator and MSE, but better with respect to estimator of variance.

4.2 Confidence Intervals :

It is well known that nonresponse in single-phase sampling can distort the confidence interval with the result that the interval falls off its center and actual confidence level deviates from the nominal level. In two-phase sampling, we see from Table (3) that,under the TRM and with POP 1 being the sampled population, the actual and nominal confidence levels are closer for $\hat{t}_2$ than for $\hat{t}_1$ when response probabilities are low (p_{11}=0.20). On the other hand, $\hat{t}_1$ and $\hat{t}_3$ lead to the same confidence intervals under TRM. In case of high (p_{11}=0.50), in general, the performances of $\hat{t}_1$ and $\hat{t}_2$ are almost the same.

Under FRM both $\hat{t}_1$ and $\hat{t}_2$ lead to confidence levels differing greatly from nominal levels specially when response probabilities are low. The situation with respect to length of interval under the FRM is similar to that under the TRM.

ISSR, Cairo Univ. Vol.39 No.2, 1995 PP.171-189

As sample size increases, under TRM, the confidence level for $\hat{t}_2$ approaches the nominal level when response probabilities are small as shown in Table (4). On the other hand,for all estimator, the length of the confidence interval decreases with increase in sample size. For $\hat{t}_1$ the confidence level seems to worsen with increase in the sample size.

4.3 Other Factors :

Since $\hat{t}_2$ is essentially a stratified sample estimator with strata given by the response groups, we expect its performance to be better than that of $\hat{t}_1$ with respect to bias and precision the more apart are the strata means of y. Table (5) provides the performance measures for the three estimators using POP 1 and POP 2 which represent large and small differences in the strata means,respectively. We observe that $\hat{t}_2$ has less bias in $\hat{t}$ and less MSE for POP 1 than For POP 2.

When the correlation between x and y is weak (see Table (6)), $\hat{t}_2$ and $\hat{t}_3$ seem to provide almost the same results for most performance measures but with both superior to $\hat{t}_1$.

It is interesting to consider the situation of complete response i.e., the case with $p_{11}= 1$, $p_{12}= 0$, and within group homogeneity equal to one. As expected, Table (7) shows that the performances of $\hat{t}_1$ and $\hat{t}_2$ are identical for all perfomance measures. This is expected since both $\hat{t}_1$ and $\hat{t}_2$ reduce to $\hat{t}^*$ when response is complete.

5. Conclusions

Analytical comparison of $\hat{t}_1$ and $\hat{t}_2$ reveals that $\hat{t}_2$ approaches $\hat{t}_1$ the higher and more homogeneous are the response rates.

Comparison of $\hat{t}_1$ and $\hat{t}_2$ under various response situations, various population structures and different sample sizes led to the following conclusions. The performance of $\hat{t}_2$ under TRM is superior to that of $\hat{t}_1$ with respect to bias (in t and V), MSE and confidence level irrespective of the magnitude of, and the differences among, response rates. Although this holds for all sample sizes used in the simulation study yet the performance of $\hat{t}_2$ seems, in general, to improve with increase in sample size meaning that it is essentially a large sample estimator.

On the other hand, under FRM, $\hat{t}_2$ is slightly inferior to $\hat{t}_1$. This result suggests that in practice, it may be safer to use $\hat{t}_2$ even when we are not sure that the ERHG model holds. This is so since at warst we expect only a silght loss. Furthermore, our simulation suggests, as expected, that the performance $\hat{t}_2$ is better the larger are the differences in strata means.

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ISSR, Cairo Univ. Vol.39 No.2, 1995 PP.171-189

Table (.)

Bias in Estimators, Variance Estimators, and MSE at Each Level of the Parameters p_{11}, p_{12}, and AH*
Using POP 1 with n_1= 800 and n_2 = 400

		Bias in $\hat{t}$				Bias in $\hat{v}$				MSE			
	p_{11}	0.20		0.50		0.20		0.50		0.20		0.50	
	p_{12}	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10
TRM	$\hat{t}_1$	0.12	0.19	0.06	0.13	0.00	-0.04	0.00	-0.01	0.30	0.30	0.19	0.20
AH :	$\hat{t}_2$	0.05	0.00	0.02	0.00	0.00	-0.02	0.00	0.00	0.27	0.27	0.18	0.18
(0.95-0.99)	$\hat{t}_3$	0.06	0.09	0.10	-0.02	-0.08	-0.17	-0.16	-0.12	0.45	0.57	0.54	0.50
	Ass**	90	129	208	249	90	129	208	249	90	129	208	249
FRM	$\hat{t}_1$	-1.22	-0.89	-0.45	-0.31	-0.02	-0.01	-0.01	0.00	1.71	0.97	0.36	0.27
AH :	$\hat{t}_2$	-1.25	-0.86	-0.48	-0.40	-0.03	-0.15	0.00	0.00	1.78	1.09	0.40	0.33
(.002-.37)	$\hat{t}_3$	0.27	0.14	0.06	0.03	-0.09	-0.14	-0.06	-0.06	0.53	0.52	0.43	0.42
	Ass	168	267	258	288	168	263	258	288	168	263	258	288

* Average Homogeneity
** Average sample size

Table (2)

Bias and MSE at Selected Level of AH, P_{11} and P_{12} for Various Sample Sizes Using POP 1

			TRM (AH: 0.95-0.99)						FRM (AH:0.002-0.37)					
			Bias in $\hat{t}$		Bias in $\hat{V}$		MSE		Bias in $\hat{t}$		Bias in $\hat{V}$		MSE	
		P_{11}	0.20	0.50	0.20	0.50	0.20	0.50	0.20	0.50	0.20	0.50	0.20	0.50
		P_{12}	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.02
$\hat{t}$	n_1	n_2												
$\hat{t}_1$	400	100	0.39	0.06	0.06	- 0.02	0.86	0.63	- 0.60	- 0.40	- 0.25	0.05	1.22	0.66
	800	400	0.19	0.06	- 0.04	0.00	0.30	0.19	- 0.89	- 0.45	- 0.01	- 0.01	0.97	0.36
	1200	800	0.20	0.03	- 0.02	0.00	0.16	0.09	- 0.92	- 0.54	- 0.01	0.01	0.94	0.37
$\hat{t}_2$	400	100	0.22	- 0.02	0.02	- 0.02	0.84	0.62	- 0.78	- 0.40	- 0.16	0.03	1.38	0.67
	800	400	0.00	0.02	- 0.02	0.00	0.27	0.18	- 0.86	- 0.48	- 0.15	0.00	1.09	0.40
	1200	800	0.00	0.01	0.00	0.00	0.11	0.09	- 1.03	- 0.56	- 0.01	0.00	1.15	0.38
$\hat{t}_3$	400	100	0.02	- 0.26	0.09	- 0.35	1.97	2.20	0.40	0.02	- 0.16	- 0.42	2.07	2.18
	800	400	0.09	0.10	- 0.17	- 0.16	0.57	0.54	0.14	0.06	- 0.14	- 0.06	0.52	0.43
	1200	800	0.04	0.02	- 0.01	0.00	0.16	0.14	0.19	0.07	- 0.01	- 0.01	0.19	0.15

Table (3)

Actual Confidence Levels (Nominal level 95%) and Average Length of Confidence Interval for Various Levels of AH, p_{11}, and p_{12} Using POP 1

		CV95 (%) (Actual Confidence Level)				Average Length of Confidence Interval			
	p_{11}	0.20		0.50		0.20		0.50	
	p_{12}	0.02	0.10	0.02	0.10	0.02	0.10	0.02	0.10
TRM	$\hat{t}_1$	93.8	88.8	94.6	95.33	2.07	1.84	1.70	1.65
AH :	$\hat{t}_2$	95.2	93.0	94.4	95.33	2.04	1.95	1.69	1.65
(0.95-0.99)	$\hat{t}_3$	93.8	90.0	85.2	90.33	2.48	2.45	2.39	2.38
	Ass	90	129	420	249	90	129	420	249
FRM	$\hat{t}_1$	24.4	41.0	79.8	87.2	1.75	1.62	1.63	1.61
AH :	$\hat{t}_2$	20.6	35.4	77.0	83.6	1.73	1.61	1.62	1.61
(.002-.37)	$\hat{t}_3$	89.6	89.6	93.0	93.4	2.39	2.37	2.38	2.49
	Ass	167.8	263	258	288.2	167.8	263	258	288.2

Table (4)

Actual Confidence Levels (Nominal level 95%) and Average Length of Confidence Interval for Selected Parameter Levels and Various Sample Sizes Using POP 1

			TRM (AH: 0.95-0.99)				FRM (AH:0.002-0.37)			
			CV95 (%)		Average Length of Confidence Interval		CV95 (%)		Average Length of Confidence Interval	
		P_{11}	0.20	0.50	0.20	0.50	0.20	0.50	0.20	0.50
		P_{12}	0.10	0.02	0.10	0.02	0.10	0.02	0.10	0.02
$\hat{t}$	n_1	n_2								
$\hat{t}_1$	400	100	91.8	94.8	3.43	3.05	82.6	91.0	3.05	0.55
	800	400	88.8	94.6	1.84	1.70	41.0	79.8	1.62	1.63
	1200	800	86.4	93.0	1.24	1.13	9.80	49.2	1.13	1.05
$\hat{t}_2$	400	100	92.0	94.4	3.68	3.04	82.6	90.6	3.06	0.54
	800	400	93.0	94.4	1.95	1.69	35.4	77.0	1.61	1.62
	1200	800	94.6	94.4	1.31	1.14	6.2	48.4	1.22	1.05
$\hat{t}_3$	400	100	94.0	89.2	5.38	5.22	93.0	92.0	5.18	1.76
	800	400	90.0	85.2	2.45	2.39	89.6	93.0	2.37	2.38
	1200	800	93.2	94.0	1.51	1.47	92.8	94.0	1.47	1.46

Table (5)
Performance Measures Using POP 1 and POP 2
(p_{11}= 0.5, p_{12}= 0.1, AH=0.96)

Population Type	$\hat{t}$	CV95	MSE	Bias in $\hat{t}$	Bias in $\hat{v}$	Average Length of Interval
POP 1	$\hat{t}_1$	95.33	0.19	0.13	0.00	1.65
	$\hat{t}_2$	95.33	0.17	0.00	0.00	1.65
	$\hat{t}_3$	95.33	0.51	0.02	0.14	2.39
POP 2	$\hat{t}_1$	94.4	0.18	0.00	0.00	0.52
	$\hat{t}_2$	95.8	0.18	0.00	0.01	0.53
	$\hat{t}_3$	95.2	0.28	-0.01	-0.01	0.64

Table (6)
Performance Measures Using POP 3
(ρ_{xy} = 0.33, p_{11}= 1, p_{12}= 0)

$\hat{t}$	CV95	MSE	Bias in $\hat{t}$	Bias in $\hat{v}$	Average Length of Interval
$\hat{t}_1$	86.0	0.84	0.15	0.31	0.88
$\hat{t}_2$	93.0	0.61	0.02	-0.06	0.92
$\hat{t}_3$	94.6	0.59	0.02	-0.06	1.90

Table (7)
Performance Measures Using POP 1
(p_{11}= 1, p_{12}= 0, AH=1)

$\hat{t}$	CV95	MSE	Bias in $\hat{t}$	Bias in $\hat{v}$	Average Length of Interval
$\hat{t}_1$	93.2	0.16	0.03	0.01	1.53
$\hat{t}_2$	93.2	0.16	0.03	0.01	1.53
$\hat{t}_3$	91.0	0.47	0.03	0.11	2.37