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ESTIMATION OF THE PARAMETERS AND RELIABILITY FUNCTION OF A GENERALIZED LIFE TESTING MODEL

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SUMMARY

This paper obtains a new estimates of the parameters of the generalized life testing model by using maximum likelihood, Bayesian and Lindley Bayes approximation procedures. The risks of these estimates were compared to the risks of moment estimates (EL-Shahat and Shalaby(1995)) and structural estimates (Hoq et al (1974)) based on a Monte Carlo simulation study . A class of quasi-prior distribution was used in the comparison for both Bayes procedures.The Bayes estimate of the reliability function of the generalized life model is also obtained by using the Bayes approximation form due to Lindley (1980) . The estimated risks of the approximate Bayes estimates of reliability function are compared with the corresponding estimated risks of the maximum likelihood estimates(MLE's) .

1.INTRODUCTION.

The exponential , Weibull ,gamma and truncated normal distributions are widely used in reliability and life testing problems . The following pdf of the generalized life testing model can be specialized to the above distributions:

$$f(x;\alpha,\gamma,\beta,\theta) = \frac{\beta(x-\gamma)^{\alpha-1}}{\theta^{\alpha}\Gamma(\alpha/\beta)} \exp\left[-\left[\frac{x-\gamma}{\theta}\right]^{\beta}\right] \quad ,x \geq \gamma \geq 0,\alpha,\beta,\theta >0 \qquad (1)$$

where;γ is the location parameter which is recgonized as the guarantee time or the minimum life of a product in reliability studies.θ is the scale parameter and α and β are the shape parameters.This model includes the exponential($\alpha=\beta=1$),Weibull ($\alpha=\beta$), gamma($\beta=1$) and truncated normal($\alpha=1,\beta=2$) distributions

The generalized life model distribution has been studied by Hoq et al(1974),Hoq(1983),Shalaby(1993),El-Shahat and Shalaby (1995) and El-Shahat (1995).

In this paper we assume that α and β are known constants ($\alpha=\alpha_0$,$\beta=\beta_0$) and we obtain the exact and approximate Bayes estimates of the parameters (γ,θ)of the generalised life testing model(1)using a Bayesian approach and the Bayes approximation procedure due to Lindley (1980).Also, we obtain the estimate of its reliability function ,R(t),using approximate Bayes and maximum likelihood procedures .

This paper will be organized as follows :
Section 2 describes the generalized life testing model together with the procedures for obtaining ML and Bayes estimates (under a squared-error loss function)of the unknown parameters $\gamma$and θ. Section 3 gives estimates of the unknown parameters γ and θ and the reliability (survivor)function using Lindley's Bayes'approximation procedure,LBA.Section 4 describes estimates of the unknown parameters γ and θ obtained by using a method of structural inference theory given in Hoq at al(1974)and the moment procedure obtained by El-Shahat and Shalaby(1995).The results and comparison between these estimates using Monte Carlo simulation study are presented in Section 5.

2.BAYES AND MAXIMUM LIKELIHOOD ESTIMATIONS.

2.1 Maximum Likelihood Estimation (MLE).

Given a random ordered sample $X=(x_{(1)},\ldots,x_{(n)})$ of size n from the model(1), the likelihood and log likelihood functions of the sample are:

$$l(x|\alpha_0,\beta_0,\gamma,\theta)=\left[\beta_0/\Gamma(\alpha_0/\beta_0)\right]^n\frac{1}{\theta^{n\alpha_0}}\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\alpha_0-1}\exp\left[-\sum_{i=1}^{n}\left[\frac{x_{(i)}-\gamma}{\theta}\right]^{\beta_0}\right],\quad(2)$$

where $\Gamma(a)=\int_0^\infty v^{a-1}e^{-v}dv$, $a>0$, is the complete gamma function,

$$L=C-n\alpha_0\log\theta+(\alpha_0-1)\sum_{i=1}^{n}\log(x_{(i)}-\gamma)-\sum_{i=1}^{n}\left[\frac{x_{(i)}-\gamma}{\theta}\right]^{\beta_0},\quad(3)$$

where C is a constant that does not depend on the unknown parameters γ and θ, $C=n\log(\beta_0/\Gamma(\alpha_0/\beta_0))$.

Equating the partial derivatives of L with respect to θ and γ to zero, the following equations are obtained:

$$-n\alpha_0+\frac{\beta_0}{\theta^{\beta_0}}\sum_{i=1}^{n}(x_{(i)}-\gamma)^{\beta_0}=0\ .\quad(4)$$

$$-(\alpha_0-1)\sum_{i=1}^{n}(x_{(i)}-\gamma)^{-1}+\frac{\beta_0}{\theta^{\beta_0}}\sum_{i=1}^{n}(x_{(i)}-\gamma)^{\beta_0-1}=0\quad(5)$$

Equations(3)&(4)can be reduced to(6)in terms of γ (the MLE of γ) only :

$$-(\alpha_o -1)\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{-1}+\frac{n\alpha_o \sum_{i=1}^{n}(x_{(i)}-\hat{\gamma})^{\beta_o -1}}{\sum_{i=1}^{n}(x_{(i)}-\hat{\gamma})^{\beta_o}}=0 \qquad (6)$$

Equation(6)can be solved iteratively for $\hat{\gamma}$,using the Newton-Raphson procedure.Then $\hat{\theta}$ (the MLE of θ)can be calculated from equation(4).

The observed Fisher information matrix at $(\hat{\gamma},\hat{\theta})$, $\hat{I}_{(\hat{\theta},\hat{\gamma})}$

$$-\begin{bmatrix} \partial^2 L/\partial\theta^2\Big|_{\theta=\hat{\theta},\gamma=\hat{\gamma}} & \partial^2 L/\partial\theta\partial\gamma\Big|_{\theta=\hat{\theta},\gamma=\hat{\gamma}} \\ \partial^2 L/\partial\theta\partial\gamma\Big|_{\theta=\hat{\theta},\gamma=\hat{\gamma}} & \partial^2 L/\partial\gamma^2\Big|_{\theta=\hat{\theta},\gamma=\hat{\gamma}} \end{bmatrix}, \text{ is}$$

$$\hat{I}_{(\hat{\theta},\hat{\gamma})} = \begin{bmatrix} \frac{\beta_o(\beta_o+1)}{\hat{\theta}^{\beta_o+2}}\sum_{i=1}^{n}(x_{(i)}-\hat{\gamma})^{\beta_o}-\frac{n\alpha_o}{\hat{\theta}^2} & \frac{\beta_o^2}{\hat{\theta}^{\beta_o+1}}\sum_{i=1}^{n}(x_{(i)}-\hat{\gamma})^{\beta_o-1} \\ \frac{\beta_o^2}{\hat{\theta}^{\beta_o+1}}\sum_{i=1}^{n}(x_{(i)}-\hat{\gamma})^{\beta_o-1} & \frac{\beta_o(\beta_o-1)}{\hat{\theta}^{\beta_o+1}}\sum_{i=1}^{n}(x_{(i)}-\hat{\gamma})^{\beta_o-2} +(\alpha_o-1)\sum_{i=1}^{n}(x_{(i)}-\hat{\gamma})^{\beta_o-1} \end{bmatrix} \qquad (7)$$

The reliability function of the generalized life testing model is:

$$R(t)=\frac{1}{\Gamma(\alpha_0/\beta_0)}\Gamma\left[\frac{\alpha_0}{\beta_0},\left(\frac{t-\gamma}{\theta}\right)^{\beta_0}\right]$$

$$=1-\gamma\left[\alpha_0/\beta_0,\left(\frac{t-\gamma}{\theta}\right)^{\beta_0}\right]/\Gamma(\alpha_0/\beta_0)\quad,t>\gamma>0\ , \qquad (8)$$

$$\text{where } \Gamma(a,b)=\int_b^{\infty} v^{a-1}e^{-v}dv\quad,a>0\ , \qquad (9)$$

and

$$\gamma(a,b)=\Gamma(a)-\Gamma(a,b) \qquad (10)$$

are the incomplete gamma functions .

The MLE of the reliability function ,$\hat{R}(t)$,is obtained by substituting (θ,γ) from (4)and (6) into (8) .

2.2 Bayes Estimates(BE's)

Jefferys(1961)suggested a class of quasi-prior distribution,$g(\theta,\gamma)$,for choosing prior distributions when one has little information relative to the likelihood function.For the generalized life testing model we assume that a quasi-joint prior distribution of (θ,γ) is :

$$g(\theta,\gamma)\ \alpha\ \theta^{-d}\quad,0<\gamma<\infty\ ,\theta>0,d\geq 0 \qquad (11)$$

Combining the prior distribution,$g(\theta,\gamma)$,with the likelihood function $l(x|\alpha_0\ ,\beta_0\ ,\gamma,\theta)$using Bayes theorem ,the joint posterior distribution of (θ,γ) is :

$$Q(\eta|x,\alpha_0,\beta_0)=\frac{K}{\theta^{n\alpha_0+d}}\exp\left[-\sum_{i=1}^{n}\left[\frac{x_{(i)}-\gamma}{\theta}\right]^{\beta_0}\right]\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\alpha_0-1},$$
$$\theta>0,\ 0<\gamma<\infty \qquad (12)$$

where $\eta=(\theta,\gamma)$ and K =normalizing constant to make $Q(\eta|x,\beta_0,\alpha_0)$ a proper pdf, i.e.,

$$K^{-1}=\frac{\beta_0^{\,n-1}}{(\Gamma(\alpha_0/\beta_0))^{n}}\Gamma((m-1)/\beta_0)\int_0^{x_{(1)}}\left[s(x,\gamma)\right]^{(1-m)/\beta_0}\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{(\alpha_0} \qquad (13)$$

where $s(x,\gamma)=\sum_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\beta_0}$ and $m=n\alpha_0+d$.

The squared-error loss function Bayes estimator of a given function is the posterior mean of that function.Thus,the squared-error loss function Bayes estimator of an arbitrary function $u(\eta)$ is:

$$\hat{u}^{*}(\eta)=E\left[u(\eta)|x\right]=\int_0^{\infty}\int_0^{x_{(1)}}u(\eta)Q(\eta|x,\alpha_0,\beta_0)\,d\gamma\, d\theta . \qquad (14)$$

If $u(\eta)=\gamma$, then the Bayes estimate $\hat{\gamma}^{*}$ is given as follows :

$$\hat{\gamma}^{*}=\frac{K}{\beta_0}\Gamma((m-1)/\beta_0)\int_0^{x_{(1)}}\gamma\left[s(x,\gamma)\right]^{((1-m)/\beta_0)}\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{(\alpha_0-1)}d\gamma .$$

If $u(\eta)=\theta$, then the Bayes estimate $\hat{\theta}^{*}$ is given as follows

$$\hat{\theta}^{*} = \frac{K}{\beta_o}\Gamma((m-2)/\beta_o)\int_0^{x_{(1)}} \left[s(x,\gamma)\right]^{((2-m)/\beta_o)} \prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{(\alpha_o-1)} d\gamma . \quad (16)$$

The integrals in(15)and(16) can not be obtainable in a closed form and a numerical integration computer routine is required for computing $\hat{\gamma}^{*}$ and $\hat{\theta}^{*}$ .Also,if $u(\eta)=R(t)$,the Bayes estimate

$$\hat{R}^{*}(t)=\int_0^{\infty}\int_0^{x_{(1)}} R(t)Q(\eta|x,\alpha_o,\beta_o)\, d\gamma d\theta,$$

is of a more complicated form and requires numerical integration computer routines for computation which might not converge for the given data.In the following section,the LBA procedure,which obviates the need to evaluate integrals,is used to obtain the Bayes estimates of γ,θ and $R(t)$.

3.The BAYES APPROXIMATION FORM OF LINDLEY (LBA).

Lindley(1980)showed that the posterior S-expectation (the squared error loss function Bayes estimator)of an arbitrary function $u(\eta)$,$E[u(\eta)|x]$,can be asymptotically approximated by:

$$E[u(\eta)|x]\sim\left[U+\frac{1}{2}\sum_i\sum_j\left[u_{ij}+2u_i\rho_j\right]\sigma_{ij}+\frac{1}{2}\sum_i\sum_j\sum_k\sum_h L_{ijk}\sigma_{ij}\sigma_{kh}u_h\right]_{\hat{\eta}}+O\left[\frac{1}{n^2}\right] \quad (17)$$

or smaller

$i,j,k,h=1,2,\ldots,M;\eta=(\eta_1,\eta_2,\ldots,\eta_M);U=u(\eta);L=L(\eta)=\log$ likelihood;

$u_i=\partial U/\partial\eta_i;u_{ij}=\partial^2 U/\partial\eta_i\partial\eta_j;L_{ijk}=\partial^3 L/\partial\eta_i\partial\eta_j\partial\eta_k;\rho=\rho(\eta)=\log(g(\eta));\rho_j=\partial\rho/\partial\eta_j;$

σ_{ij} is the (i,j)th element of the matrix $\hat{I}^{-1}_{ij}(\eta)$. Expression(17) is to be evaluated at the MLE's $(\hat{\eta})$.

For the 2-parameter case, $\eta=(\theta,\gamma)$; (17) reduces to :

$$E[u(\eta)|x]=U+\frac{1}{2}\Big[\sum_{i=1}^{2}\sum_{j=1}^{2}u_{ij}\sigma_{ij}+L_{30}B_{12}+L_{21}C_{12}+L_{12}C_{21}+L_{03}B_{21}+2(\rho_1 A_{12}+\rho_2 A_{21})\Big] \quad (18)$$

Where $B_{ij}=(u_i\sigma_{ii}+u_j\sigma_{ij})\sigma_{ii}, i\neq j$, (19)

$C_{ij}=3u_i\sigma_{ii}\sigma_{ij}+u_j(\sigma_{ii}\sigma_{jj}+2\sigma^2_{ij})$, $i\neq j$, (20)

and $A_{ij}=u_i\sigma_{ii}+u_j\sigma_{ji}$, $i\neq j$.

Now ,to apply Lindley's form(18),we first obtain the σ_{ij} elements of the inverse of the matrix $\hat{I}_{ij(\hat{\theta},\hat{\gamma})}$, i,j=1,2,which can be shown to be

$$\sigma_{11}=\left[\frac{\beta_0(\beta_0-1)}{\hat{\theta}^{\beta_0}}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_0-2}+(\alpha_0-1)\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{-2}\right]/D , \quad (21)$$

$$\sigma_{12}=\sigma_{21}=-\frac{\beta_0^2}{\hat{\theta}^{\beta_0+1}}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_0-1}/D , \quad (22)$$

$$\sigma_{22}=\left[\frac{\beta_0(\beta_0+1)}{\hat{\theta}^{\beta_0+2}}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_0}-\frac{n\alpha_0}{\hat{\theta}^2}\right]/D \quad (23)$$

$$\frac{\beta_o^2}{\hat{\theta}^{2\beta_o+2}}\left[(\beta_o^2-1)\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o-2}\right.$$

$$\left.\left[\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o-1}\right]^2\right]+\frac{\beta_o}{\hat{\theta}^{\beta_o+2}}\left[(\alpha_o-1)(\beta_o+1)\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{-2}\right.$$

$$\left.(\beta_o-1)\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o-2}\right]-\frac{n\alpha_o(\alpha_o-1)}{\hat{\theta}^2}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{-2}, \qquad (24)$$

hermore:

$$\log g(\theta,\gamma)/\partial\theta=\frac{-d}{\theta}, \quad \rho_2=\partial\log g(\theta,\gamma)/\partial\gamma=0,$$

$$\frac{2n\alpha_o}{\hat{\theta}^3}+\frac{\beta_o(\beta_o+1)(\beta_o+2)}{\hat{\theta}^{\beta_o+3}}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o}, \qquad (25)$$

$$\frac{\beta_o^2}{\hat{\theta}^{\beta_o+2}}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o-1}, \quad L_{12}=\frac{\beta_o(\beta_o-1)}{\hat{\theta}^{\beta_o+1}}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o-2}, \qquad (26)$$

$$\frac{\ldots(\beta_o-1)(\beta_o-2)}{\hat{\theta}^{\beta_o}}\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{\beta_o-3}-2(\alpha_o-1)\sum_{i=1}^{n}\left[x_{(i)}-\hat{\gamma}\right]^{-3}. \qquad (27)$$

itution of the above values in(18)yields the Bayes estimate

Lindley'method,denoted by U of a function $U=U(\theta,\gamma)$.All functions of equation(18)are to be evaluated at the MLE's$(\hat{\theta},\hat{\gamma})$.

If in (18) $U(\theta,\gamma)=\theta$,then

$$\tilde{\theta}=\theta+\frac{1}{2}\left[\sigma_{11}(L_{30}\sigma_{11}+3L_{21}\sigma_{12}+L_{21}\sigma_{22})+\sigma_{21}(2L_{12}\sigma_{21}+L_{03}\sigma_{22})-\frac{2d\sigma_{11}}{\theta}\right], \tag{28}$$

evaluated at $(\hat{\theta},\hat{\gamma})$.

If in (18) $U(\theta,\gamma)=\gamma$, then

$$\tilde{\gamma}=\gamma+\frac{1}{2}\left[L_{30}\sigma_{12}\sigma_{11}+L_{21}(\sigma_{11}\sigma_{22}+2\sigma_{12}^{2})+3L_{12}\sigma_{22}\sigma_{21}+L_{03}\sigma_{22}^{2}\right], \tag{29}$$

evaluated at $(\hat{\theta},\hat{\gamma})$.

If in (18) $U(\theta,\gamma) = R(t)$, then

$$\tilde{R}(t)=R(t)+\frac{1}{2}\left[\sum_{i=1}^{2}\sum_{j=1}^{2}u_{ij}\sigma_{ij}+L_{30}B_{12}+L_{21}C_{12}+L_{12}C_{21}+L_{03}B_{21}\right.$$

$$\left.-\frac{2d}{\theta}(u_{1}\sigma_{11}+u_{2}\sigma_{21})\right], \tag{30}$$

evaluated at $(\hat{\theta},\hat{\gamma})$,

where $u_{12}=u_{21}=\dfrac{\beta_o^{2}}{\hat{\theta}^{2}\,\Gamma(\alpha_o/\beta_o)}\left[\dfrac{t-\hat{\gamma}}{\hat{\theta}}\right]^{\frac{\alpha_o-1}{\beta_o}}\left[\left[\dfrac{t-\hat{\gamma}}{\hat{\theta}}\right]^{\beta_o}-\dfrac{\alpha_o}{\beta_o}\right]\exp\left[-\left[\dfrac{t-\hat{\gamma}}{\hat{\theta}}\right]^{\beta_o}\right]$,

$u_{11}=\dfrac{\beta_o}{\hat{\theta}^{2}\,\Gamma(\alpha_o/\beta_o)}\left[\dfrac{t-\hat{\gamma}}{\hat{\theta}}\right]^{\beta_o}\left[\beta_o\left[\dfrac{t-\hat{\gamma}}{\hat{\theta}}\right]^{\beta_o}-\alpha_o-1\right]\exp\left[-\left[\dfrac{t-\hat{\gamma}}{\hat{\theta}}\right]^{\beta_o}\right]$,

ISSR, CAIRO UNIV., VOL. 40, NO.2, 1996 PP.151-171

$$u_{22} = \frac{\beta_0^2}{\hat\theta^2\,\Gamma(\alpha_0/\beta_0)}\left[\frac{t-\hat\gamma}{\hat\theta}\right]^{\frac{\alpha_0-2}{\beta_0}}\left[\left[\frac{t-\hat\gamma}{\hat\theta}\right]^{\beta_0} + \frac{1-\alpha_0}{\beta_0}\right]\exp\left[-\left[\frac{t-\hat\gamma}{\hat\theta}\right]^{\beta_0}\right],$$

$$u_1 = \frac{\beta_0}{\hat\theta\,\Gamma(\alpha_0/\beta_0)}\left[\frac{t-\hat\gamma}{\hat\theta}\right]^{\alpha_0}\exp\left[-\left[\frac{t-\hat\gamma}{\hat\theta}\right]^{\beta_0}\right], \text{ and}$$

$$u_2 = \frac{\beta_0}{\hat\theta\,\Gamma(\alpha_0/\beta_0)}\left[\frac{t-\hat\gamma}{\hat\theta}\right]^{\frac{\alpha_0-1}{\beta_0}}\exp\left[-\left[\frac{t-\hat\gamma}{\hat\theta}\right]^{\beta_0}\right]$$

4. MOMENT AND STRUCTURAL ESTIMATORS OF θ AND γ .

4.1 Moment Estimators (ME's)

El-Shahat and Shalaby(1995)obtained the ME's $\tilde\theta$ and $\tilde\gamma$ from computing the following equations:

$$\tilde\theta = S_x\Gamma_0\left[\Gamma_0\Gamma_2-\Gamma_1^2\right]^{-1/2}, \tag{31}$$

$$\tilde\gamma = \bar{x} - \tilde\theta\,\Gamma_1\Gamma_0^{-1}, \tag{32}$$

Where $\Gamma_i = \Gamma(\frac{\alpha_0+i}{\beta_0})$,i=0,1,2 ,$\bar{x}$ be the sample mean $\sum_{i=1}^{n} x_{(i)}/n$ and S_x^2 be the sample variance $\sum_{i=1}^{n}(x_{(i)}-\bar{x})^2/n$.

4.2 Structural Estimators (STE's).

By the use of a method of structural inference theory, the marginal pdf's of θ and γ have been obtained by Hoq et al(1974), they are given as :

$$q_1(\theta)=\frac{K_1}{\theta^{n\alpha_0+1}}\int_0^{x_{(1)}}\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\alpha_0-1}\exp\left[-\sum_{i=1}^{n}\left[\frac{x_{(i)}-\gamma}{\theta}\right]^{\beta_0}\right]d\gamma,\ \theta>0 \quad (33)$$

$$q_2(\gamma)=\frac{K_1}{\beta_0}\Gamma(n\alpha_0/\beta_0)\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\alpha_0-1}\Big/\left[\sum_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\beta_0}\right]^{(n\alpha_0/\beta_0)}$$

$$0<\gamma<x_{(1)},\quad (34)$$

where

$$K_1=\int_0^{x_{(1)}}\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\alpha_0-1}\Big/\left[\sum_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\beta_0}\right]^{(n\alpha_0/\beta_0)}d\gamma \quad (35)$$

Applying the previous results, the structural estimates $\hat{\hat{\theta}}$ and $\hat{\hat{\gamma}}$ can be obtained as follows :

$$\hat{\hat{\theta}}=\int_0^{\infty}\theta q_1\,d\theta$$

$$=\frac{K_1}{\beta_0^{\ n}}\Gamma((n\alpha_0-2)/\beta_0)\int_0^{x_{(1)}}\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\alpha_0-1}\exp\left[-\sum_{i=1}^{n}\left[\frac{x_{(i)}-\gamma}{\theta}\right]^{\beta_0}\right]d\gamma,\quad (36)$$

and

$$\hat{\hat{\gamma}}=\frac{K_1}{\beta_0}\Gamma(n\alpha_0/\beta_0)\int_0^{x_{(1)}}\gamma\prod_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\alpha_0-1}\Big/\left[\sum_{i=1}^{n}\left[x_{(i)}-\gamma\right]^{\beta_0}\right]^{n\alpha_0/\beta_0}d\gamma \quad (37)$$

he solution of (36)and(37)requires numerical integration .

5.NUMERICAL RESULTS AND COMPARISONS.

In order to compare the different estimators of the arameters and reliability function of the generalized life esting model(1)obtained in the above sections,a large simulation tudy was performed. 2000 random samples of sizes n=15,30,50,70, 00,150 are generated as follows:

i) Using $\alpha=3.99$,$\beta=3.52$,$\gamma=6.12$ and $\theta=6.412$ (the mle's of α,β,γ nd θ applied to an example of El-Shahat and Shalaby(1995))as he true values .We generated random samples of different sizes rom a generalized life model(1) by observing that if Z is niform(0,1) then X which is obtained by solving the equation,

$=\gamma(\frac{\alpha}{\beta},(\frac{X-\gamma}{\theta})^{\beta})/\Gamma(\alpha/\beta)$ follows a generalized life model(1).The NAG ORKSTATION LIBRARY(1986)is used in the generation of the uniform ariaties and solving the above equation.

ii) The maximum likelihood,Bayes,approximate Bayes,moment and tructural estimates of θ and γ can be computed from the solution f(4)and(6)and by using(15),(16),(28),(29),(31),(32),(36)and(37).

ii) The ML and LBA estimates of $R(t_o)$ are computed at some alues of t_o(chosen to be 7, 11,13,15.5)by replacing γ and θ by $\hat{\gamma}$ nd $\hat{\theta}$ in (8) and (30),respectively.

The well known shape and prior parameters used throughout he computations are chosen to be; $\alpha_o=3.99$,$\beta_o=3.52$ and d=2.3 .

v) The squared deviations $(EST(\gamma)-\gamma)^2$,$(EST(\theta)-\theta)^2$ and $(EST(R(t_o))-R(t_o))^2$ are computed,for different sample sizes n, where each of EST(γ)and EST(θ) stands for an estimate (ML,Bayes, Bayes approximation of Lindley,moment,structural) and EST($R(t_o)$) stands for (MI ,LBA).

(v) The above steps are repeated 2000 times and the estimated risks computed by averaging the square deviations over the 2000 repetitions . The computational results are displayed in Tables (I),(II) .

The simulations were performed on an IBM compatible computer with NAG FORTARAN WORKSTATION LIBRARY(1986).Double precision accuracy was used for all computation.

Based on samples of sizes n=15,30,50,70,100,150,the estimated reliabilities $\hat{R}(t_o)$:($\hat{\gamma}$=6.9658,6.2347,6.0710,6.2031, 6.1584,6.2326,$\hat{\theta}$=6.1241,6.5686,6.6867,6.6263,6.6781,6.5604) and $\bar{R}(t_o)$:$\bar{\gamma}$=5.5892,5.6413,5.7083,5.9463,5.9783,6.1146,$\bar{\theta}$=7.2655, 7.1841,7.0638,6.8933,6.8657,6.6834) together with the true reliability $R(t_o)$:(γ=6.12,θ=6.412) are drawn in Figures (1)-(6)

CONCLUDING REMARKS

1- In this article,Lindley's form is based on the expansion about the MLE's .

2- Tierney and Kadane(1986) have pointed out that Lindley's approximation form might not work very well for small samples . Indeed,Lindley's form is an approximation of $O(n^{-2})$.It is, therefore,for small n(n<30) .The Bayes estimates obtained by Lindley's approximation form had larger estimated risks,but it decreases with increasing sample size(see Tables I,II).

3- Table(I)shows that the MLE's have the smallest estimated risks as compared with Bayes,Bayes approximation due to Lindley, moment and structural estimates.

4- Both Bayes and Bayes approximation of Lindley estimators of γ and θ have smaller risks than the moment estimates. Also,the Bayes estimators perform better than the structural estimates.The Bayes estimates perform better than approximate Bayes estimate.

5-Table(II) gives the estimated risks of $\hat{R}(t)$ and $\breve{R}(t)$,we draw the following conclusions from the results .

* The Lindley's approximate Bayes estimator of R(t) have smaller risk than the ML estimator at t=13 and are very inclose to each other at larger t values,especially at larger sample sizes .

* For the smaller t values considered,MLE performs much better than Lindley's approximate Bayes estimators,especially at smaller values of sample sizes .

* The performance of each estimator depends on the value of t . Bayes estimator had smaller estimated risks for t values far from γ,and ML procedures had smaller estimated risks at t values near γ,for all n considered .

In all cases the estimated reliabilities curves of $\hat{R}(t)$ are closer to the true reliability curve of R(t),especially at smaller values of both sample size n($n \leq 30$) and t values than the estimated curves of $\breve{R}(t)$.

The estimated reliability curves of $\breve{R}(t)$ get closer to both the true reliability curve of R(t) and estimated reliability curves of $\hat{R}(t)$ as either t value or sample size n increases .

TABLE (I): Estimated Risks Of The Estimates Of γ And θ For Different Sample Sizes n.

n	para	MLE	ME	STE	BE	LBAE
15	γ	0.5319	1.3916	1.3998	1.3210	2.2258
	θ	0.6069	1.2887	1.3344	1.2363	2.3141
30	γ	0.4038	0.6515	0.6635	0.6343	0.6276
	θ	0.4227	0.6103	0.6239	0.6004	0.6817
50	γ	0.2947	0.3893	0.3825	0.3743	0.3545
	θ	0.2938	0.3655	0.3648	0.3558	0.3633
70	γ	0.2517	0.2709	0.2647	0.2609	0.2598
	θ	0.2302	0.2543	0.2517	0.2475	0.2580
100	γ	0.1742	0.1923	0.1850	0.1835	0.1872
	θ	0.1615	0.1752	0.1703	0.1686	0.1765
150	γ	0.1224	0.1307	0.1236	0.1228	0.1287
	θ	0.1117	0.1183	0.1126	0.1117	0.1187

TABLE (II) : Estimated Risks Of The Estimates Of R(t) for Selected Values Of t and n.

n	t	MLE	LBAE	n	t	MLE	LBAE
15	7.0	3.7606E-6	1.9256E-2	70	7.0	9.2527E-7	9.3568E-4
	11.0	5.7303E-3	8.5413E-2		11.0	1.5435E-3	3.5549E-3
	13.0	9.2821E-3	9.6390E-3		13.0	2.0523E-3	1.8418E-3
	15.5	7.4992E-4	8.6640E-3		15.5	1.7438E-4	2.5170E-4
30	7.0	2.4466E-6	5.1024E-3	100	7.0	6.1377E-7	4.5828E-4
	11.0	3.1236E-3	1.9791E-2		11.0	1.1460E-3	2.0409E-3
	13.0	4.7304E-3	3.6847E-3		13.0	1.3939E-3	1.2975E-3
	15.5	3.9100E-4	8.5824E-4		15.5	1.1765E-4	1.5357E-4
50	7.0	1.3617E-6	1.8187E-3	150	7.0	3.9101E-7	2.0646E-4
	11.0	2.0128E-3	6.5173E-3		11.0	7.8807E-4	1.1763E-3
	13.0	2.8096E-3	2.4107E-3		13.0	9.2374E-4	8.7979E-4
	15.5	2.3838E-4	3.9103E-4		15.5	7.8694E-5	9.4572E-5

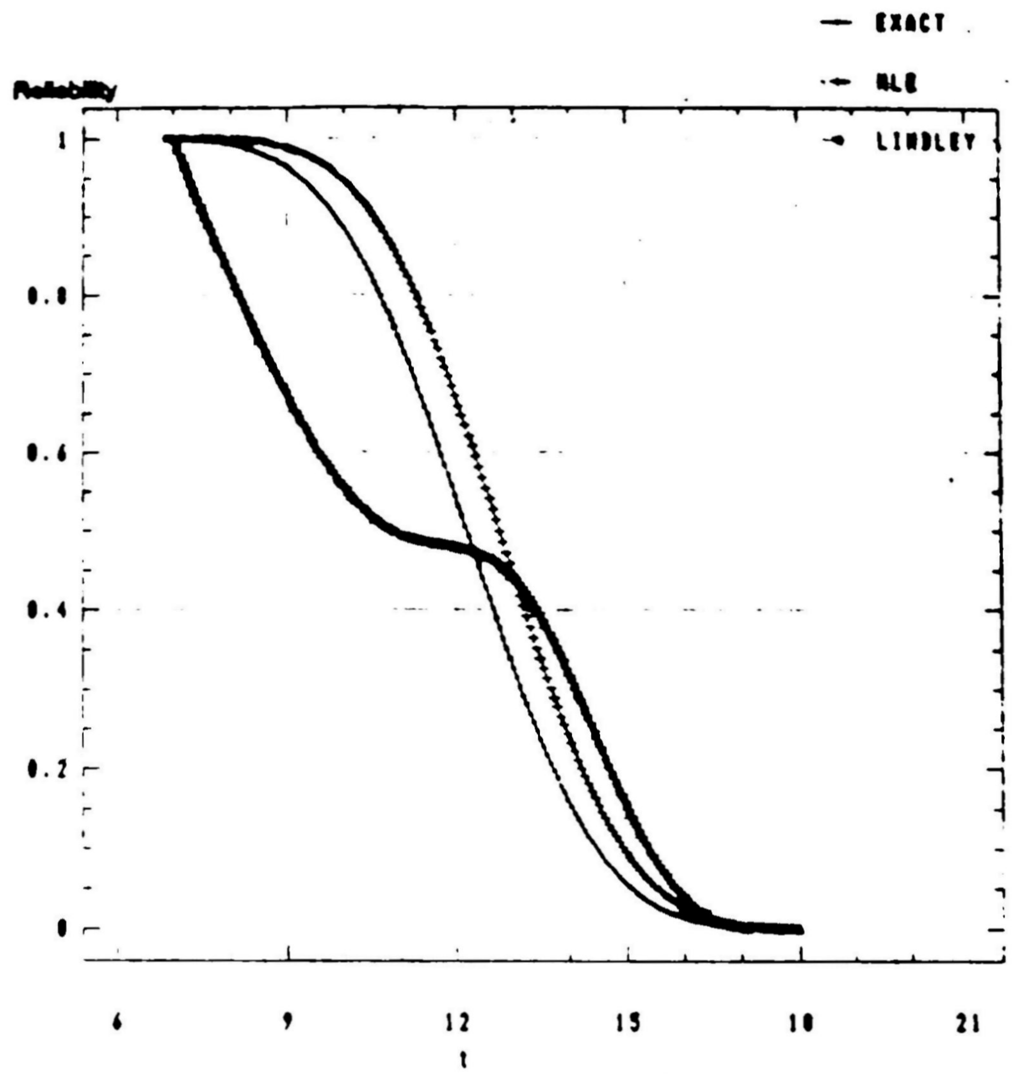

Figure (1) : Estimated reliability as compared with the actual R(t) based on sample of size15

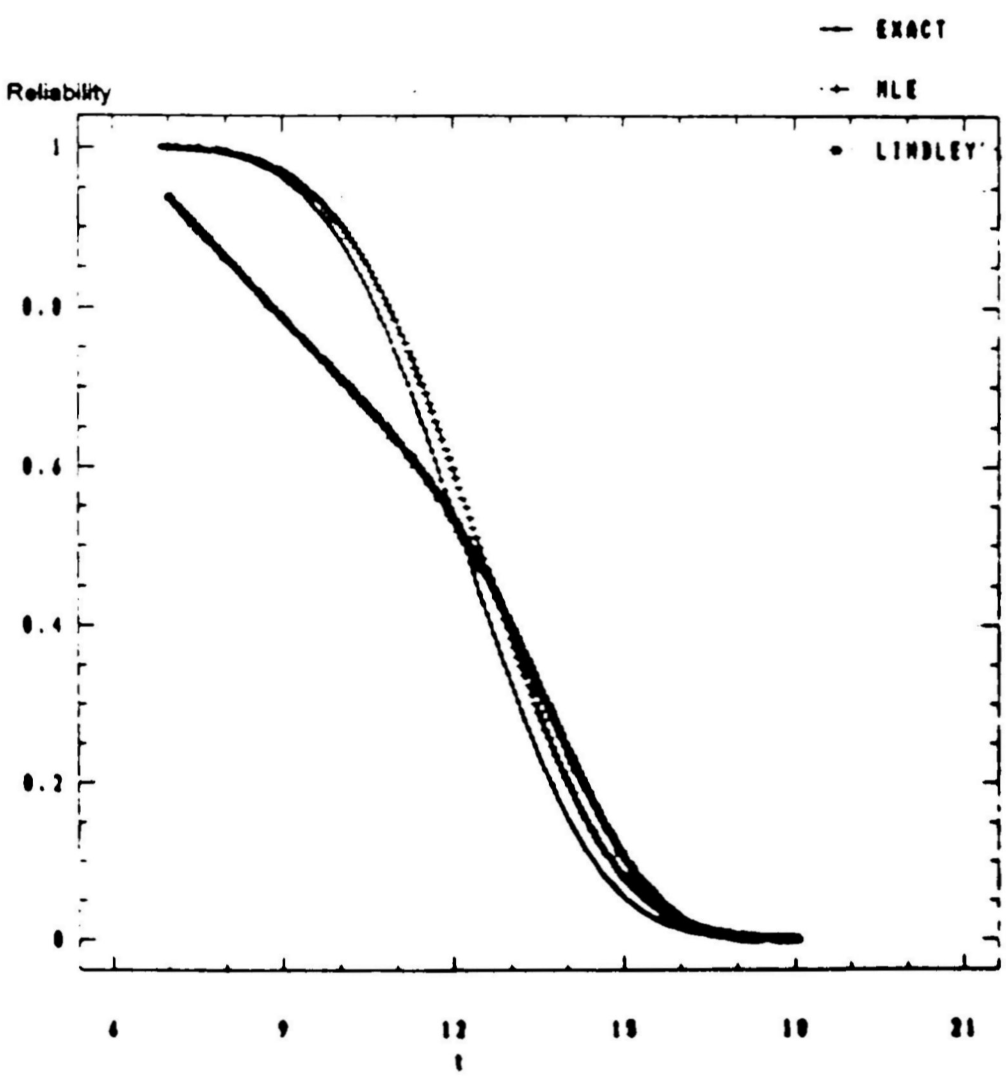

Figure (2) Estimated reliability as compared with the actual R(t) based on sample of size 30

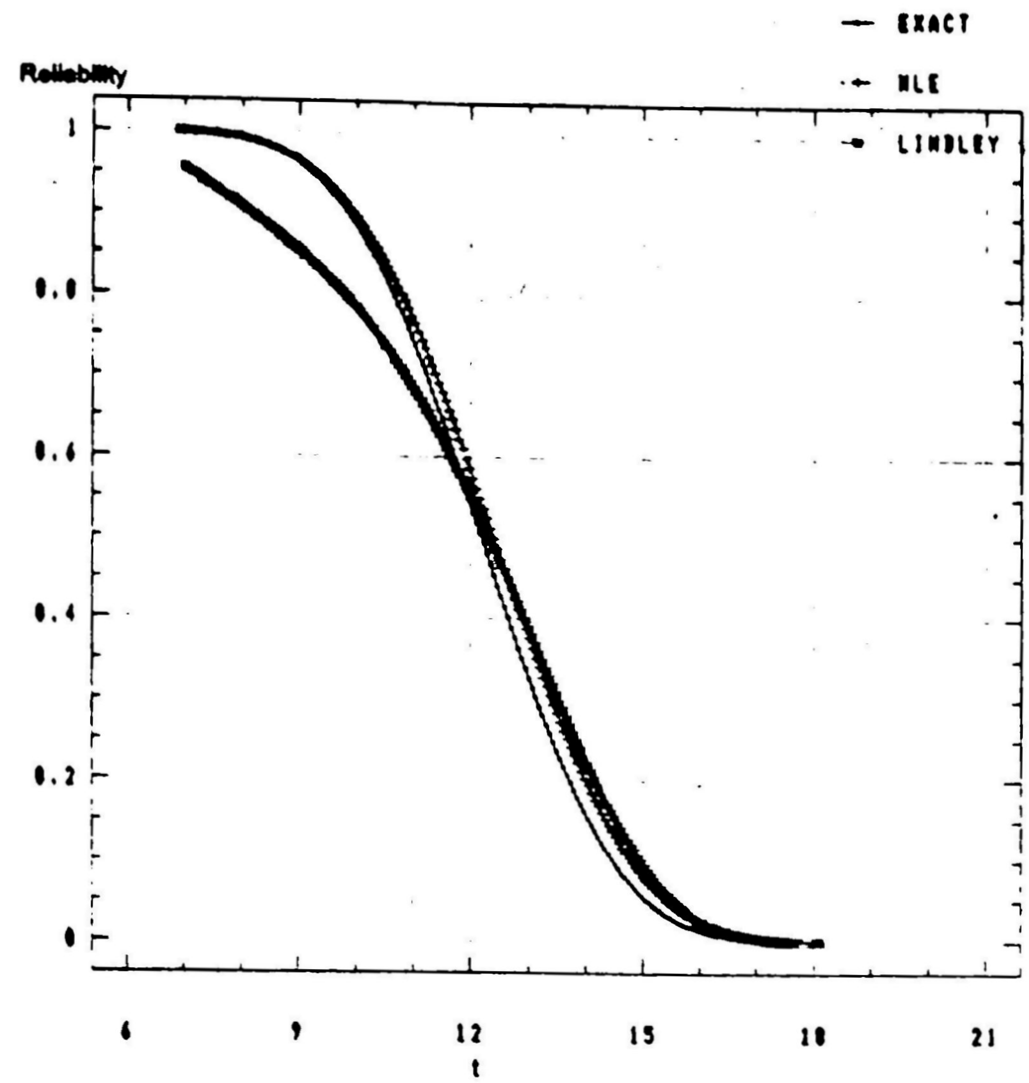

Figure (3) : Estimated reliability as compared with the actual R(t) based on sample of size 50

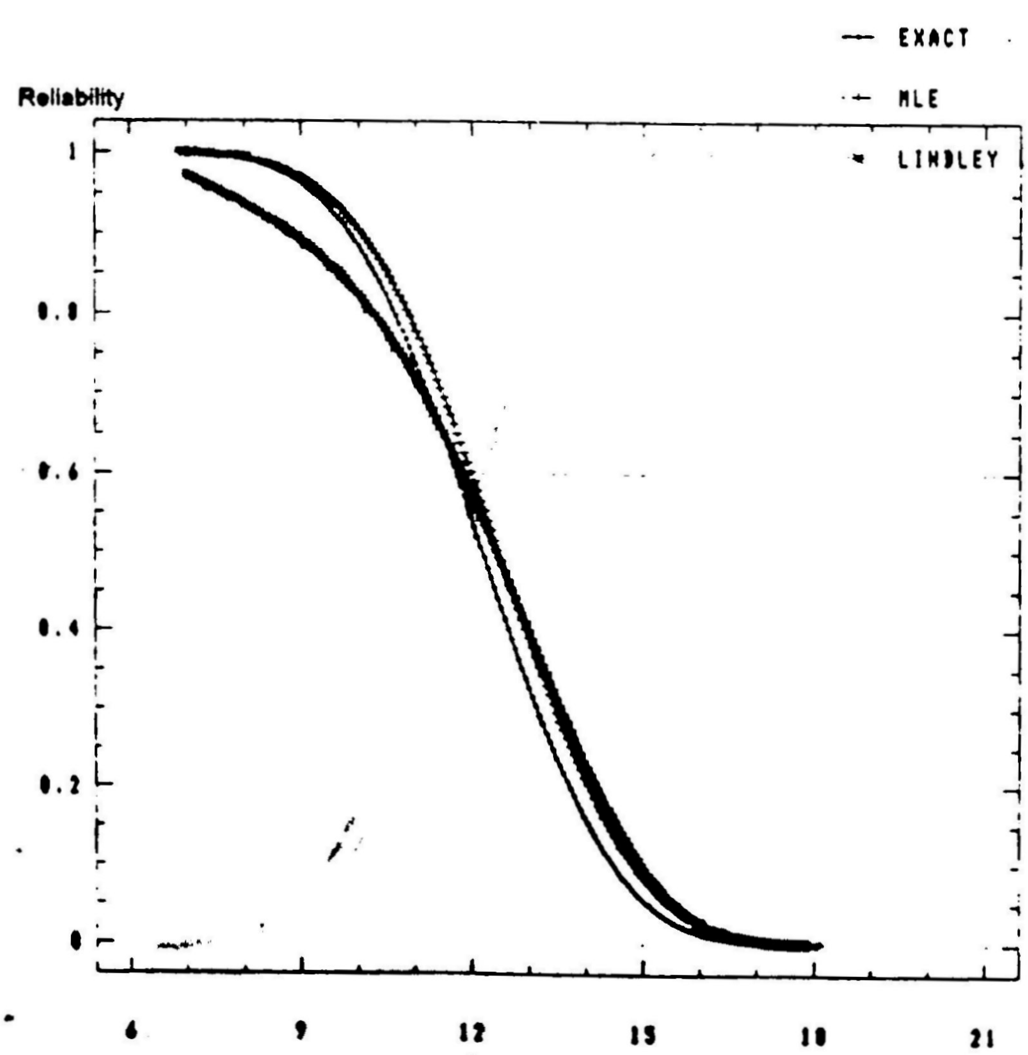

Figure (4) : Estimated reliability as compared with the actual R(t) based

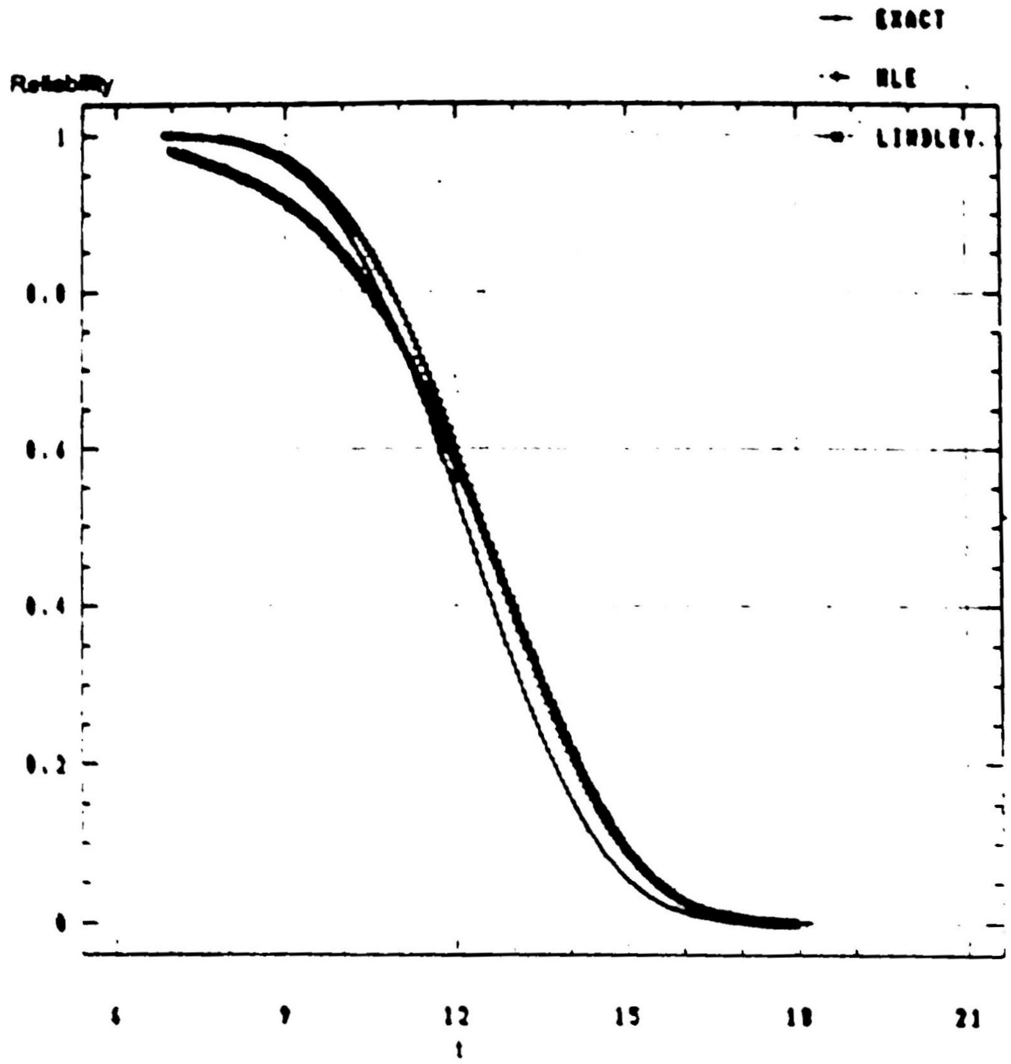

Figure (5) : Estimated reliability as compared with the actual R(t) based on sample of size100

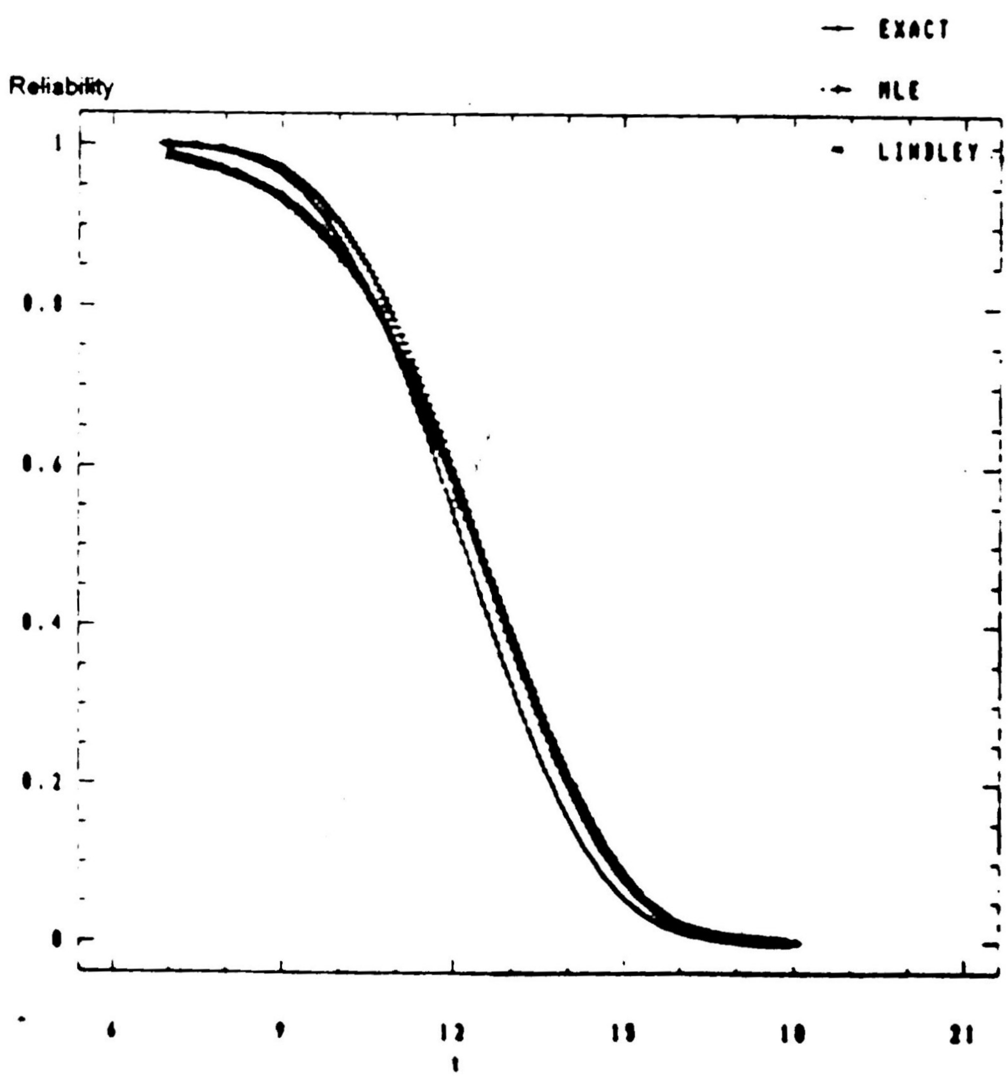

Figure (6) . Estimated reliability as compared with the actual R(t) based on sample of size150

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