THE EGYPTIAN STATISTICAL JOURNAL
ISSR, CAIRO UNIV., VOL. 40, NO.1, 1996

SHOCK MODELS WITH NEXBRC PROPERTIES

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Abstract

The new exponential better than renewal in convex (NEXBRC) class of life distributions, which is an intermediate class between HNBUE and GHNBUE, and its dual class NEXWRC are studied. Relationships with other classes of life distributions are presented. A shock model in which shocks are arriving according to a nonhomogeneous Poisson process, is also studied. A shock model for NEXBRC (NEXWRC) based on probability generating function is also established. The Laplace transform characterization for this class is given.

1 INTODUCTION

In many reliability applications, various classes of life distributions and their duals have been introduced to describe several types of deterioration or improvement that accompany ageing. It has been found very useful to classify life distribution using the concept of stochastic ordering. For definitions of several calsses of life distributions e.g. IFR, IFRA, NBU, DCCS, NBUC, NBUAS, NBUFR, NBUE, HNBUE, GHNBUE, DMRL and their duals, see [1-7].

Let X be a non-negative random variable representing a device life with distribution $F(t)$ and survival function $\bar{F}(t) = 1 - F(t)$ and let $X_{w(t)}$ be the stationary renewal of the equipment with distribution $W(t)$ and survival function

$$\bar{W}(t) = \mu^{-1} \int_0^\infty \bar{F}(u)du \qquad t \geq 0, \mu > 0 \tag{1.1}$$

where $\mu = \int_0^\infty \bar{F}(u)du$

Let X_{EX} be exponantial life of the equipment with distribution function $F_{EX}(X)$ and survival function

$$\bar{F}_{EX}(X) = e^{-X/\mu} \qquad \mu > 0, X \geq 0 \tag{1.2}$$

We define F_{EX} a new exponential better than renewal $\bar{W}(t)$ in convex ordering (NEXBRC) ifand only if(iff) $X_{W(t)}$ is smaller than X_{EX} in convex ordring.

Deshpande, et al. [5] introduced another set of classes in terms of stochastic dominance. Their motivation was to relate the use of stochastic dominance in applied economics to notions of ageing in reliability theory. One of their interesting class is harmonic new better than used of thrid order HNBUE(3). However, they did not consider closure of these classes under reliability operations, or shock models. Abouammoh and Ahmed [1] studied HNBUET (HNBUE(2)) closure properties under some reliability operations such as convolutions, mixtures, coherent systems and homogeneous Poisson shock models.

The main theme of this paper is to investigate further this class NEXBRC (HNBUE(3)). In section 2 definitions and relationships are considered. Shock models for nonhomogeneous Poisson process are studied in section 3. In section 4 the Laplace transforms characterization for this class is established.

2 DEFINITION and RELATIONSHIPS

Definition 2.1 (Stoyan [8]): Let X and Y be two random variables with distributions F and G, respectively. If

$$\int_x^\infty \bar{F}(u)du \leq \int_x^\infty \bar{G}(u)du \qquad for all x \geq 0$$

then X is smaller than Y in convex ordering ($X \leq_c Y$).

Definition 2.2 A non-negative random variable X with distribution F is said to be new exponential better than renewal in convex ordering (NEXBRC) if $X_{W(t)} \leq_c X_{EX}$ denoted by $X \in$ NEXBRC (or F$\in$ NEXWRC). Its dual class is new exponential worse than renewal in convex ordering (NEXWRC) which is defined by $X_{W(t)} \geq_c X_{EX}$

$$\int_x^\infty \frac{1}{\mu} \int_t^\infty \bar{F}(v)dvdt \leq (\geq) \int_x^\infty e^{-y/\mu}dy, \qquad x,t \geq 0. \qquad (2.1)$$

Theorem 2.1 $X \in$ NEXBRC(NEXWRC) iff

$$\int_u^\infty \int_t^\infty \bar{F}(v)dvdt \leq (\geq)\mu^2 e^{-u/\mu}, \qquad u,t \geq 0. \qquad (2.2)$$

Proof. It is obvious from (1.1), (1.2), and (2.1) □.

Theorem 2.2 If $X \in$ HNBUE then $X \in$ NEXBRC.

Proof. Since $X \in$ HNBUE,

$$\int_t^\infty \bar{F}(v)dv \leq (\geq)\mu e^{-t/\mu}, \qquad t \geq 0. \qquad (2.3)$$

then integrating (2.3) with respect to t over $(u,\infty), u > 0$ gives (2.2) which is NEXBRC condition. □

Theorem 2.3 If $X \in$ NEXBRC then $X \in$ GHNBUE.

Proof. Since $X \in$ NEXBRC,

$$\int_u^\infty \int_t^\infty \bar{F}(v)dvdt \leq \mu^2 e^{-u/\mu}, \qquad u,t \geq 0. \qquad (2.4)$$

by letting u tend to zero in (2,4), we have

$$\int_0^\infty \int_t^\infty \bar{F}(v)dvdt \leq \mu^2, \qquad t \geq 0. \qquad (2.5)$$

which (2.5) is the GHNBUE conditions □.

Then we have the following implications.

HNBUE $\Longrightarrow$ NEXBRC $\Longrightarrow$ GHNBUE

A similar chain of implications hold for the corresponding dual classes.

3 NONHOMOGENEOUS POISSON SHOCK MODELS

Let $\bar{H}(t)$ be the survival function of a device which is subject to a sequence of independent shocks occuring randomly in time. Let $N = \{N(t), t \geq 0\}$ be a general counting process during [0, t] and $\{\bar{P}_k\}_{k=0}^{\infty}$ the probability that the device survives the first k shocks. $\bar{P}_k$ is assumed to be decreasing in k and $\bar{P}_0 = 1$. Then the survival probability that the device survives beyond time t can be expressed in the form

$$\bar{H}(t) = \sum_{k=0}^{\infty} P\{N(t) = k\}\bar{P}_k \tag{3.1}$$

For $P\{N(t) = k\} = \frac{(\lambda t)^k}{k!}e^{-\lambda t}$, this shock model was considered by Essary et al.[10]. A-Hameed and Proschan [11] and [12] have studied this model for cases when N is a nonhomogeneous Poisson process and N is a birth process respectively. These authors have considered the cases where $\bar{P}_k$, $k = 0, 1, \ldots$ has the following discrete properties IFR, IFRA, NBU and DMRL.

In this section we consider the shock model given by (3.1) such that shocks occur according to a non-homogeneous Poisson process with mean value function $\Lambda(t)$ and event rate $\lambda(t) = \Lambda'(t)$ both defined on $[0, \infty)$, $\Lambda'(0)$ is taken as right derivative of $\Lambda(t)$ at t=0. Thus the shock model (3.1) is reduced to the form

$$\bar{H}(t) = \sum_{k=0}^{\infty} e^{-\Lambda(t)}\frac{\Lambda^k(t)}{k!}\bar{P}_k \tag{3.2}$$

Now we prove that the discrete NEXBRC property of $\bar{P}_k$, $k = 0, 1, 2, \ldots$ is preserved for $\bar{H}(t)$ under the model (3.2).

Definition 3.1 A discrete distribution or its survival $\bar{P}_k = 1 - P_k$, $k = 0, 1, 2, \ldots$, is called discrete NEXBRC (NEXWRC) if

$$\sum_{j=k}^{\infty}\sum_{i=j}^{\infty} \bar{P}_i \leq (\geq) \mu^2(1 - 1/\mu)^k; \qquad k = 0, 1, 2, \ldots \tag{3.3}$$

Theorem 3.1 The survival function $\bar{H}(u)$ in (3.2) is NEXBRC if $\{\bar{P}_k\}_{k=0}^{\infty}$ has the discrete NEXBRC property (3.3), $\Lambda'(u) > 0$ for $u \geq 0$ and

$$e^{-\Lambda(u)/\mu} \leq \Lambda'^2(u)e^{-u/\mu} \qquad u > 0 \tag{3.4}$$

Proof. Using definition (3.1) $\{\bar{P}_k\}_{k=0}^{\infty}$ is NEXBRC, then (3.3) can be written

$$\bar{P}_k + 2\bar{P}_{k+1} + 3\bar{P}_{k+2} + \cdots \leq \mu^2(1-1/\mu)^k; \qquad k = 0,1,2,\ldots \tag{3.5}$$

multiplying both sides of (3.5) by kernel $e^{-\Lambda(u)}\frac{(\Lambda(u))^k}{k!}$ and summing over $k = 0,1,2,\ldots$ gives

$$\bar{H}(u) \leq \mu^2 e^{-\Lambda(u)/\mu} - \sum_{j=1}^{\infty}(j+1)\bar{H}_j(u) \tag{3.6}$$

where

$$\bar{H}_j(u) = \sum_{k=0}^{\infty} e^{-\Lambda(u)}\frac{\Lambda^k(u)}{k!}\bar{P}_{k+j}, \qquad j = 1,2,\ldots \tag{3.7}$$

In fact $\bar{H}(u)$ is NEXBRC if

$$\int_u^{\infty}\int_t^{\infty}\bar{H}(v)dvdt \leq \mu^2 e^{-u/\mu}, \qquad u,t \geq 0. \tag{3.8}$$

therefore

$$\int_u^{\infty}\int_t^{\infty}\sum_{k=0}^{\infty} e^{-\Lambda(v)}\frac{\Lambda^k(v)}{k!}\bar{P}_k dvdt \leq \mu^2 e^{-u/\mu}$$

i.e

$$\int_u^{\infty}\sum_{k=0}^{\infty}\bar{P}_k\left(\int_t^{\infty} e^{-\Lambda(v)}\frac{(\Lambda(v))^k}{k!}\frac{d\Lambda(v)}{\Lambda'(v)}\right)dt \leq \mu^2 e^{-u/\mu}, \tag{3.9}$$

By using the second mean value theorem (see Gradshtein, and Ryzhik [17]) and noting that the funciton $\Lambda'(u)$ is bounded and monotonic increasing function for $u \in [t,\infty)$, we get

$$\int_u^{\infty}\sum_{k=0}^{\infty}\bar{P}_k\left(\frac{1}{\Lambda'(t)}\int_t^{\infty} e^{-\Lambda(v)}\frac{\Lambda^k(v)}{k!}d\Lambda(v)\right)dt \leq \mu^2 e^{-u/\mu},$$

$$\int_u^{\infty}\sum_{k=0}^{\infty}\bar{P}_k\left(\frac{1}{\Lambda'(t)}\sum_{j=0}^{k} e^{-\Lambda(v)}\frac{\Lambda^j(t)}{j!}\right)dt \leq \mu^2 e^{-u/\mu},$$

$$\sum_{k=0}^{\infty}\bar{P}_k\sum_{j=0}^{k}\left(\int_u^{\infty} e^{-\Lambda(t)}\frac{\Lambda^j(t)}{j!}\frac{d\Lambda(t)}{\Lambda'^2(t)}\right) \leq \mu^2 e^{-u/\mu},$$

Using the second mean value theorem once again we get

$$\sum_{k=0}^{\infty}\frac{\bar{P}_k}{\Lambda'^2(u)}\sum_{j=0}^{k}\left(\int_u^{\infty}e^{-\Lambda(t)}\frac{\Lambda^j(t)}{j!}d\Lambda(t)\right)\leq \mu^2 e^{-u/\mu},$$

$$\sum_{j=0}^{\infty}\frac{1}{\Lambda'^2(u)}\left(\sum_{i=0}^{j}e^{-\Lambda(u)}\frac{\Lambda^j(u)}{i!}\right)\sum_{k=j}^{\infty}\bar{P}_k\leq \mu^2 e^{-u/\mu},$$

$$\frac{1}{\Lambda'^2(u)}\sum_{i=0}^{\infty}e^{-\Lambda(u)}\frac{\Lambda^i(u)}{i!}\sum_{j=i}^{\infty}\sum_{k=j}^{\infty}\bar{P}_k\leq \mu^2 e^{-u/\mu},$$

Therefore

$$\frac{1}{\Lambda'(u)^2}\sum_{i=0}^{\infty}e^{-\Lambda(u)}\frac{\Lambda^i(u)}{i!}[\bar{P}_i+2\bar{P}_{i+1}+\cdots\leq \mu^2 e^{-u/\mu},$$

$$\bar{H}(u)+2\bar{H}_1(u)+3\bar{H}_2(u)+\cdots\leq \Lambda'(u)^2\mu^2 e^{-u/\mu}$$

i.e.

$$\bar{H}(u)\leq \Lambda'^2(u)\mu^2 e^{-u/\mu}-\sum_{j=1}^{\infty}\bar{H}_j(j+1)(u). \tag{3.10}$$

where $\bar{H}_j(u)$ is defined above by (3.7). From (3.6) and (3.10) we conclude that $\bar{H}(u)$ satisfies the assumption of NEXBRC property if

$$\mu^2 e^{-u/\mu}-\sum_{j=1}^{\infty}\bar{H}_j(j+1)(u)\leq \Lambda'^2(u)\mu^2 e^{-u/\mu}-\sum_{j=1}^{\infty}\bar{H}_j(j+1)(u).$$

Therefore (3.4) is satisfied and the proof is completed.

For the dual property we present the following theorem whose proof is omitted.

Theorem 3.2 The survival function $\bar{H}(u)$ in model (3.2) is (NEXWRC) if $\{\bar{P}_k\}_{k=0}^{\infty}$ is NEXWRC, $\Lambda'(0)>0$, $u\geq 0$ and condition (3.4) is satisfied with the inequality sign reversed.

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4 GENERATING FUNCTIONS SHOCK MODELS

Let $p_0 = 1 - \bar{P}_0$ and $p_i = \bar{P}_{i-1} - \bar{P}_i, i = 1, 2, \ldots$ be the probability mass function (pmf) of a nonnegative random variable X. The probability generating function of X is

$$\psi(\theta) = E[\theta^X] = \sum_{j=0}^{\infty} \theta^j P_j.$$

Observe that

$$\psi(\theta) = 1 - \sum_{j=0}^{\infty} (1-\theta)\theta^j \bar{P}_j. \quad (4.1)$$

For a geometric random variable X with parameter θ, i.e., with pmf as follows:

$$f_j = P(X = j) = (1-\theta)\theta^j \quad (4.2)$$

Relation (4.1) may be written in the form

$$\psi(\theta) = 1 - \sum_{j=0}^{\infty} P(X = j)\bar{P}_j, \quad k = 0, 1, 2, \ldots \quad (4.3)$$

Let $X_j, j = 1, 2, \ldots, n$ be iid random variables with common pmf given by (4.2). The variable $V = \sum_{j=1}^{n} X_j$ has the negative binomial distribution,

$$P(V = n + j) = \binom{n+j-1}{j} \theta^j (1-\theta)^n. \quad j = 0, 1, \ldots \quad (4.4)$$

Next define

$$B_n(\theta) = \begin{cases} \sum_{j=0}^{\infty} \binom{n+j-1}{j} \theta^j (1-\theta)^n \bar{P}_j, & \text{for } n = 1, 2, \ldots \\ 1 & \text{for } n = 0. \end{cases} \quad (4.5)$$

The form (4.5) has the following interesting physical meaning. Suppose that a device is subjected to two different types of shocks I and II say. At every time unit a shock of type I occurs with probability $1-\theta$. If X_j denote the number of type I shocks between the $(j-1)^{th}$ and $\underline{jth}$, $j \in N$ of type II shocks, then X_j has geometric distribution with pmf given by (4.2) and V

has a negative binomial distribution given by (4.4). Hence $B_n(\theta), n \in N$, represents the probabilities that the device survives n shocks of type I, where $\bar{P}_j$ represents the probability that the device survives the first j shocks of type II.

Using the form (4.5) Abouammoh and Hendi [16] found conditions for discrete life distributions, namely, NBURFR and NBARFR in terms of $B_n(\theta)$. Next, we translate the NEXBRC (NEXWRC) properties in terms of $B_n(\theta)$.

Theorem 4.1: Let $B_n(\theta)$ be given by (4.5), $\{\bar{P}_j\}_{j=0}^{\infty}$ is NEXBRC iff

$$B_n(\theta) \leq \mu^2 \sum_{k=0}^{\infty}(1-1/\mu)^k \binom{n+k-1}{k} \theta^k(1-\theta)^n - \sum_{j=0}^{\infty}(j+1)B_{n,j}(\theta),$$

where

$$B_{n,j}(\theta) = \sum_{k=0}^{\infty} \binom{n+k-1}{k} \theta^k(1-\theta)^n \bar{P}_{k+j}, \qquad n,j = 1,2,\ldots,.$$

Proof: Since $\bar{P}_K$ is NEXBRC,

$$\sum_{j=k}^{\infty}\sum_{i=j}^{\infty} \bar{P}_i \leq \mu^2(1-1/\mu)^k \tag{4.6}$$

,i.e,

$$\bar{P}_k + 2\bar{P}_{k+1} + 3\bar{P}_{k+2} + \cdots \leq \mu^2(1-1/\mu)^k; \quad k = 0,1,2,\ldots \tag{4.7}$$

Multiplying both sides of(4.7) by the kernel $\binom{n+k-1}{k} \theta^k(1-\theta)^n$ and taking summation over $k = 0,1,\ldots$ we get

$$B_n(\theta) + 2B_{n,1}(\theta) + \cdots \leq \mu^2 \sum_{k=0}^{\infty}(1-1/\mu)^k \binom{n+k-1}{k} \theta^k(1-\theta)^n$$

$$B_n(\theta) \leq \mu^2 \sum_{k=0}^{\infty}(1-1/\mu)^k \binom{n+k-1}{k} \theta^k(1-\theta)^n - \sum_{j=0}^{\infty}(j+1)B_{n,j}(\theta) \quad n,j = 1,2,\ldots \tag{4.8}$$

For the dual property we present the following theorem whose proof is omitted.

Theorem 4.2: Let $B_n(\theta)$ be given by (4.5), $\{\bar{P}_j\}_{j=0}^{\infty}$ is NEXBRC is satisfied with inequality sign of (4.8) reversed.

5 LAPLACE TRANSFORMS FOR NEXBRC

Here we establish necessary and sufficient conditions for a life distribution to have the NEXBRC property by using the Laplace transform. These conditions may be used to investigate the closure under some reliability operations.

Now let F be a distribution function such that $F(0-)=0$ and

$$\phi(s)=\int_0^\infty e^{-su}dF(u),\quad s>0$$

be the Laplace transform of $F(x)$. Define

$$a_n(s)=\frac{(-1)^n}{n!}\frac{d^n}{ds^n}\left(\frac{1-\phi(s)}{s}\right),\quad n\geq 0, s\geq 0 \tag{5.1}$$

let $\alpha_{n+1}(s)=s^{n+1}a_n(s)$ for $n\geq 0$ and $s\geq 0$ and $\alpha_0(s)=1$. The transforms $\alpha_n(s)$ can be written in the forms

$$a_n(s)=\frac{1}{n!}\int_0^\infty u^n e^{-su}\bar{F}(u)du \tag{5.2}$$

and

$$\alpha_{n+1}(s)=\frac{1}{n!}\int_0^\infty s(su)^n e^{-su}\bar{F}(u)du,\quad n\geq 0, s\geq 0 \tag{5.3}$$

In fact Vinogradov [13] has characterized the IFR property in terms of $\alpha_n(s)$ while Block and Savits [14] have obtained similar characterizations for IFRA, DMRL, NBU and NBUE properties. Abouammoh et al. [15] have characterized the NBUFR and NBAFR properties similarly.

Theorem 5.1: Let F be a life distribution with $F(0-)=0$, then F has the NEXBRC property if and only if

$$\sum_{j=n+1}^{\infty}\sum_{i=j+1}^{\infty}\alpha_{i+1}(s)\leq\frac{\mu^2 s^2}{(1+\frac{1}{s\mu})^{n+1}},\qquad n\geq 0, s>0 \tag{5.4}$$

Proof. First, we prove the necessary condition. Let $F\in$ NEXBRC then (5.3) is satisfied, form which we deduce that

$$\sum_{j=n+1}^{\infty}\sum_{i=j+1}^{\infty}\alpha_{i+1}(s)=\sum_{j=n+1}^{\infty}\sum_{i=j+1}^{\infty}s\int_0^\infty e^{-su}\frac{(su)^i}{i!}\bar{F}(u)du,\quad n\geq 0, s>0$$

$$= s^2 \sum_{j=n+1}^{\infty} \int_0^{\infty} \left(\frac{1}{s} \sum_{i=j+1}^{\infty} e^{-su} \frac{(su)^i}{i!} \right) \bar{F}(u)du$$

$$= s^2 \sum_{j=n+1}^{\infty} \int_0^{\infty} \int_0^{u} e^{-sv} \frac{(sv)^j}{j!} \bar{F}(u)dvdu$$

$$= s^2 \sum_{j=n+1}^{\infty} \int_0^{\infty} e^{-sv} \frac{(sv)^j}{j!} \int_v^{\infty} \bar{F}(u)dudv$$

$$= s^3 \int_0^{\infty} (\frac{1}{s} \sum_{j=n+1}^{\infty} e^{-sv} \frac{(sv)^j}{j!}) \int_v^{\infty} \bar{F}(u)dudv$$

$$= s^3 \int_0^{\infty} (\int_0^{v} e^{-st} \frac{(st)^n}{n!} dt) \int_v^{\infty} \bar{F}(u)dudv$$

$$= s^3 \int_0^{\infty} e^{-st} \frac{(st)^n}{n!} \int_t^{\infty} \int_v^{\infty} \bar{F}(u)dudvdt$$

since $F \in$ NEXBRC, i.e. (2.2) is satisfied, then

$$\sum_{j=n+1}^{\infty} \sum_{i=j+1}^{\infty} \alpha_{i+1}(s) \leq s^3 \int_0^{\infty} e^{-st} \frac{(st)^n}{n!} \mu^2 e^{-t/\mu} dt$$

$$\leq s^2 \mu^2 \int_0^{\infty} \frac{(st)^n}{n!} e^{-st(1+\frac{1}{s\mu})} dt$$

which gives the condition in (5.4). This completes the necessary condition part.

To prove that the condition (5.4) is sufficient, we may rewrite (5.4) in the form:

$$\sum_{j=n+1}^{\infty} \sum_{i=j+1}^{\infty} s \int_0^{\infty} e^{-su} \frac{(su)^i}{i!} \bar{F}(u)du \leq \frac{\mu^2 s^2}{(1+\frac{1}{s\mu})^{n+1}}$$

$$\leftrightarrow \sum_{j=n+1}^{\infty} s \int_0^{\infty} \left(\sum_{i=j+1}^{\infty} e^{-su} \frac{(su)^i}{i!} \right) \bar{F}(v)dv \leq \frac{\mu^2 s^2}{(1+\frac{1}{s\mu})^{n+1}}$$

$$\leftrightarrow s^2 \int_0^\infty \left(\sum_{j=n+1}^{\infty} e^{-su} \frac{(su)^i}{i!} \right) du \int_u^\infty \bar{F}(v) dv \leq \frac{\mu^2 s^2}{(1+\frac{1}{s\mu})^{n+1}}$$

which gives

$$\int_0^\infty G_n(u) \int_u^\infty F(v) dv \leq \frac{\mu^2}{(1+\frac{1}{s\mu})^{n+1}}, \qquad (5.5)$$

where

$$G_n(u) = \sum_{j=n+1}^{\infty} \frac{(su)^j}{j!} e^{-su}.$$

It is obvious that

$$G_n(u) = s \int_0^u \frac{(sv)^n}{n!} e^{-sv} dv = P(\sum_{i=1}^{n+1} Y_i \leq u)$$

where $Y_1, Y_2, \ldots, Y_{n+1}$ are mutually independent random variables and exponentiall distributed with rate s. Hence $G_n(u)$ represents a gamma distribution function with parameters $(n+1, s)$ and its characteristic function is given by $\phi_{n+1}(w) = (1 - \frac{iw}{s})^{-(n+1)}$

Letting $\frac{(n+1)}{s} \longrightarrow t$, it can be shown that

$$\lim_{n\to\infty} \phi_{n+1}(w) = \exp(iwt).$$

that is

$$G_n(w) \longrightarrow G(u) = \begin{cases} 0 & \text{for } u < t \\ 1 & \text{for } u \geq t \end{cases} \qquad (5.6)$$

Taking the limit for both sides in (5.6) as $n \to \infty$.

$$\lim_{n\to\infty} \int_0^\infty G_n(u) \int_u^\infty F(v) du dv \leq \lim_{n\to\infty} \frac{\mu^2}{(1+\frac{t}{\mu(n+1)})^{n+1}},$$

which gives

$$\int_t^\infty \int_u^\infty \bar{F}(v) dv du \leq \mu^2 e^{-t/\mu}, \qquad u, t \geq 0,$$

which completes the proof of the sufficient condition part.

The corresponding Laplace transforms of the dual properties HNRWUE is given in the following theorem whose proof is omitted.

Theorem 5.2: Let F be a life distribution with $F(0-) = 0$. Then F is NEXWRC iff (5.4) is satisfied with inequality sign reversed.

Acknowledgement:

The authors are grateful to Professor A. A. Alzaid, King Saud University, Riyadh, Saudi Arabia for valuable comments and stimulating discussion when preparing this paper.

ISSR, Cairo Univ., Vol.,40, No. 1, 1996, PP.1-14

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