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ON THE GOODNESS OF FIT TESTS FOR THE BURR FAILURE MODEL

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ABSTRACT

The Burr type XII distribution yields a wide range of values of skewness and kortosis and it can be fitted to almost any given set of data arising from a unimodel distribution, so special attention has been focused on it. It has become an important family of distributions for Reliability studies.

It is often of interest to determine whether a set of data can be considered to have come from a population belonging to this family. Goodness of fit tests are designed for checking the validity of a null hypothesis, which is a statment about the form of the distribution function of the parent population from which the sample is drawn. Ideally, the hypothesized distribution is completely specified, if the hypothesis states that the distribution belong to some family of distributions. The unknown parameters must be estimated from the sample data in order to perform a test.

The complete sample procedures of goodness of fit tests are inappropriate for use with censored samples and critical values obtained from published tables of the distributions of the test statistics based on complete samples. Also, the goodness of fit tests for censored data are inappropriate when parameters of the hypothesized distribution are estimated from the data used for the test.

Three of the best known tests for goodness of fit are the Kolmogorov-Smirnov test (KS), the Cramer-Von Mises test (CVM) and the Anderson-Darling test (AD). These tests become extremely conservative when they are used in cases when the hypothesized distribution contains unknown parameters which must be estimated from the sample data.

In this paper, we provide the tables of critical values of modified Kolmogorov-Smirnov (KS), Cramer-Von Mises (CVM) and Anderson-Darling (AD) tests for the Burr distribution with unknown parameters in the case of type II censored samples. The powers of these tests are given for a number of alternative distributions. Also, we obtain, numerically, the sampling distributions for the three test statistics, (KS, CVM and AD) in the case of censored samples, if the underlying distribution is Burr type XII with two unknown parameters. We have used the Pearson system and the Least Squares to derive their distributions. These distributions may be utilized to obtain critical values for each test statistics, to be used with small, medium and large sample sizes.

1. INTRODUCTION

The Burr family of distributions is widely used in life testing studies. The two common survival or failure-time distributions, the Weibull and the Exponential, are both special cases or limiting cases of the Burr type XII (see Lewis (1981)), denoted by Burr(c,k), where c and k are the shape parameters. The distribution function of Burr (c,k) is

$$F(x) = 1 - (1 + x^{c})^{-k}, \quad x > 0, c, k > 0 \tag{1}$$

Goodness of fit tests are employed to determine how well the observed sample data "fits" some proposed model. The main problem is that of testing the hypothesis about the distribution function, F(x), of the form

$$H_0 : F(x) = F_0(x), \tag{2}$$

where $F_0(x) = P(X < x)$ is a specified family of cumulative distribution functions. When $F_0(x)$ is completely specified (i.e. contains no unknown parameters) and the data are uncensored, the tests are all distributionfree and percentage points for the various test statistics are generally known. This is no longer the case when data are censored or when $F_o(x)$ involves unknown parameters. Three standard goodness of fit tests, called Kolmogorov-Smirnov, Cramer-Von Mises and Anderson-Darling tests, can be used if the hypothesized population distribution contains no unknown parameters.

The test statistic for the Kolmogorov-Smirnov test is

$$D = \sup_x | S(x) - F_o(x) | \tag{3}$$

The test statistic for the Cramer-Von Mises test is

$$W_n^2 = \sum_{i=1}^{n} [F_o(x_{(i)}) - \frac{i-0.5}{n}]^2 + \frac{1}{12n} \tag{4}$$

and the test statistic for the Anderson-Darling test is

$$A_n^2 = - n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) [\ln F_o(x_i) + \ln (1-F_o(x_{n-i+1}))], \tag{5}$$

where x_i is the ith order statistic. In the sequal, we refer to them as the KS statistic, CVM statistic and AD statistic, respectively.

Most of the literature on goodness of fit tests for censored data deals with a special aspect of goodness of fit to a completely specified distribution. Barr and Davidson (1973), Koziol and Byar (1975) and Dufour and Maag (1978) discussed the Kolmogorov-Smirnov statistic so modified, while Pettitt and Stephens (1976) and Smith and Bain (1976) introduced censored versions of the Cramer-Von Mises statistic.

Lilliefors (1967) constructed tables of critical values for the KS statistic to test for normality when the mean and the variance are estimated. The construction of tables for these goodness of fit tests usually requires Monte Carlo techniques. Subsequent papers have extended their work to include tests for the Exponential, Weibull, Extreme Value, Laplace, Gamma, Logistic, Rayleigh, Lognormal, two parameter Exponential and Inverse Gaussian distributions. For an extensive survey of goodness of fit tests, one may refer to Abd-Elfattah (1994).

In this paper, we obtain the tables of critical values for modified (KS), (CVM) and (AD) tests for the Burr distribution with unknown parameters in the case of complete and type II censored samples. Tables of critical values are obtained for sample sizes 10 (5) 25, 50 and with type II censoring at 80%, 90% and 100%.

Using Burr type XII distribution with two unknown parameters, we also aim to obtain the sampling distributions of the three test statistics (KS), (CVM) and (AD) see Abd-Elfattah (1994). These distributions will be used to study the properties of these three test statistics and to obtain the critical values. We shall use two methods to obtain, numerically, the sampling distributions of these test statistics. The first is the Pearson system technique, the second technique is to find a best fit to the cumulative distribution function using the Least Squares approach.

2. GOODNESS OF FIT TESTS

For a fixed sample size, we generate the random deviates x_1, x_2, ..., x_n from the Burr distribution with parameters c and k. David and Johnson (1948) demonstrated that the distributions of KS, CVM and AD statistics do not depend on the values of the parameters of the hypothesized distribution. i.e., without any loss of generality, a study of the distributions of KS, CVM and AD statistics can be conducted by fixing c=k=1.

The generated random samples were then used to estimate the two shape parameters of the Burr (c,k) distribution by the method of maximum likelihood. The maximum likelihood estimators of c and k are obtained by solving the equations:

$$\frac{r}{k} - (n-r)\log(1+x_r^c) - \sum_{i=1}^{r} \log(1+x_i^c) = 0 \qquad (6)$$

$$\frac{r}{k} + \sum_{i=1}^{r} \log x_i - (n-r)\, k\, x_r^c \frac{\log x_r}{1+x_r^c} - (k+1) - \sum_{i=1}^{r} \frac{x_i^c \log x_i}{1+x_i^c} = 0 \qquad (7)$$

For a solution of equations (6) and (7), the use of a suitable iterative method such as the Newton-Raphson method and computer facilities are required. The resulting maximum likelihood estimators of the parameters were then used to determine the hypothesized cumulative distribution function, $F_0(x)$, of the Burr (c,k) distribution.

The appropriate test statistic was calculated for some given values of n. Thus KS, CVM and AD statistics were calculated, for each of the samples of sizes n = 10, 15, 20, 25 and 50 and with type II censoring at 80%, 90% and 100%.

The procedure for calculating the appropriate test statistics was repeated 5000 times, thus generating 5000 independent values of the appropriate test statistics. These 5000 values for KS, CVM, and AD statistics were then ranked and the 5%, 10%, 15%, (5%) 95%, 96%, 97%, 98% and 99% quantiles were obtained. These provided the critical values for the particular test and the sample size used, for both complete and type II consored samples. The critical values for the modified KS, CVM And AD test statistics are listed in tables (1), (2) and (3) respectively.

3. Power study

The power function plays an important role in hypothesis testing. It will be our standard in assessing the goodness of a test or in comparing two competing tests.

In this section, we compare the power of the three test statistics (KS, CVM and AD statistics), using some alternative distributions. These power comparisons were made using Monte Carlo simulation. We have generated 5000 pseudo-random samples of size n from each of the selected alternative distributions. We then calculated each of the three test statistics and compared them with their respective critical values and counted the number of rejections of the null hypothesis. The above procedure was repeated for all cases of complete and censored sample sizes considered.

The alternative distributions considered were the Logistic, Double-Exponential, Standard Normal, the Weibull with shape parameter 1.0, 2.0, Gamma with shape parameter 2.0, Uniform, Beta (2,2), Beta (2,3), and Chi-square distribution with 1 degree of freedom. The results of the power study are presented in tables (4), (5) and (6). It should be noted that all power comparisons were made at the 0.05 and the 0.01 levels of significance.

Tables (4), (5) and (6) show that The power of the tests is quite good against Exponential, Gamma, Beta (2,2), Beta (2,3) and Chi-square alternatives, and it gets better as the sample size increases. The power of the tests in decreasing order are those of Cramer-Von Mises, Kolmogorov-Smirnov and Anderson-Darling tests. With no prior knowledge of the alternative distribution, it may be advisable to use the Cramer-Von Mises or Kolmogorov-Smirnov statistics since their power were reasonably good for all alternatives. The power against alternatives that are more nearly like the Burr in shape is not very good, for small sample sizes.

4. The curve fitting using Pearson's method

The Pearson system of distributions entails a wide range of frequency curves that fit data encountered in most practical situations. The criterion for fixing the distribution family in a particular case is

$$K_p = \frac{\beta_1 (\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \qquad (8)$$

where β_1 and β_2 are the measures of skewness and kortosis respectively. This value of K_p differs for various types of Pearson curves.

Now, to apply Pearson's technique we generate the random deviates x_1, x_2, ..., x_n from the Burr (c,k) distribution with parameters c and k with c = k = 1. The generated random samples were then used to estimate the two shape parameters of the Burr distribution using Maximum Likelihood methods. The three test statistics, KS, CVM and AD were calculated for the given value of the samples of sizes n = 10, 15, 20, 25 and 50 and with type II censoring at 80%, 90% and 100%.

The procedure for calculating the appropriate test statistics was repeated 5000 times, thus generating 5000 independent values for each of KS, CVM and AD statitics. From these values, the mean, standard deviation, coefficient of variation, variance, skewness, kortosis and Pearson's coefficient were calculated for each statistic and sample size. The Pearson's types of distributions have been listed in tables (7), (8) and (9). From these tables, we have:

(i) Table (7) reveals that the sampling distributions of KS are of types I, II and IV in Pearson's system of curves for the cases of complete and censored semple sizes.

(ii) Table (8) of CVM shows that the sampling distributions of CVM test statistic are of types I, III and IV in Pearson's system for the cases of complete and censored sample size. (iii) Table (9) of AD test statistic shows that, the sampling distributions of AD test statistic are of types I and III in Pearson's system.

5. The curve fitting using the method of least squares

The method of Least Squares can be employed to fit a straight line or a curve to a set of data points. This method of adjusting observations makes the sum of the squares of the assumed figures a minimum. The theory underlying the method is that errors are distributed in accordance with the normal curve of error.

Assume that the regression line of variable Y on variable X has the form $\beta_0 + \beta_1 X$. Then we can write the linear first order model as

$$Y = \beta_0 + \beta_1 X + \epsilon, \tag{9}$$

where ϵ is the error term and β_0 and β_1 are the unknown parameters of the model. We use the information provided by the observations on (Y,X) to give us estimates b_0 and b_1 of β_0 and β_1; thus we can write

$$\hat{Y} = b_0 + b_1 X \tag{10}$$

We recall our problem and suppose that Y = F(x), where F(x) is the empirical distribution of a test statistic from KS, CVM and AD, and suppose that x represents the corresponding critical value

$$F(x) = b_o + b_1 x \tag{11}$$

This method of fitting is applied to fit the cumulative distribution function of the three test statistics as a linear model. We used SPSS/PC to estimate the unknown parameters of the model.

Tables (7), (8) and (9) represent values of b_o, b_1, R^2, multiple R, adjusted R^2, standard error, calculated F, standard errors of b_o and b_1 for the KS, CVM and AD tests respectively. These tables show that the values of R^2 are close to unity except for a complete sample of size n = 50 of the CVM and AD test statistics.

To obtain the set of critical values for the three test statistics, we use the equation

$$x_p = \frac{1}{b_1} (F(x_p) - b_o), \tag{12}$$

which represents the relationship between the critical values of the test statistic and its corresponding percentiles. Equation (12) is used to calculate the critical values for the three test statistics at the same specifie quantiles.

TABLE (1)

Critical Values of KS (D)

N : Complete Sample Size, R : Censored Sample Size and α : Level of Signif

N	N=10			N=15			N=20			N=25			N=50		
α R	R=8	9	10	12	14	15	16	18	20	20	23	25	40	45	50
.95	.04	.04	.02	.03	.02	.01	.03	.02	.01	.02	.01	.00	.02	.01	.00
.90	.09	.07	.05	.06	.04	.02	.05	.03	.01	.04	.02	.01	.03	.02	.00
.85	.12	.10	.07	.10	.06	.04	.08	.05	.02	.06	.03	.01	.04	.03	.01
.80	.14	.12	.09	.12	.08	.05	.10	.07	.03	.08	.04	.02	.05	.04	.01
.75	.15	.14	.10	.13	.10	.06	.12	.09	.04	.10	.06	.03	.06	.04	.01
.70	.16	.15	.11	.15	.11	.08	.14	.11	.04	.12	.07	.03	.07	.05	.01
.65	.18	.16	.12	.16	.12	.08	.15	.12	.06	.14	.09	.04	.08	.06	.02
.60	.18	.17	.12	.17	.13	.09	.17	.14	.07	.16	.10	.04	.09	.07	.02
.55	.19	.17	.13	.18	.14	.10	.18	.15	.08	.18	.11	.05	.10	.08	.02
.50	.20	.18	.14	.19	.15	.10	.19	.16	.08	.19	.13	.06	.12	.09	.02
.45	.21	.19	.14	.20	.16	.11	.21	.18	.09	.21	.14	.07	.13	.11	.03
.40	.22	.20	.15	.21	.17	.12	.22	.19	.09	.23	.15	.07	.15	.12	.03
.35	.23	.21	.16	.23	.18	.12	.23	.20	.10	.24	.16	.08	.17	.13	.03
.30	.24	.22	.16	.24	.19	.13	.25	.21	.11	.26	.17	.09	.18	.14	.04
.25	.26	.23	.17	.25	.21	.14	.26	.23	.12	.27	.18	.10	.20	.15	.04
.20	.27	.25	.18	.26	.22	.15	.28	.24	.12	.29	.20	.10	.22	.17	.05
.15	.29	.26	.19	.28	.24	.16	.29	.26	.13	.31	.22	.11	.24	.18	.06
.10	.31	.29	.21	.30	.26	.17	.32	.29	.15	.33	.24	.13	.26	.20	.06
.05	.34	.32	.24	.34	.30	.20	.35	.32	.17	.37	.27	.14	.30	.23	.08
.04	.35	.33	.25	.35	.32	.20	.36	.33	.17	.38	.28	.15	.31	.24	.08
.03	.36	.35	.25	.36	.34	.21	.37	.35	.18	.38	.29	.16	.32	.25	.08
.02	.38	.37	.27	.38	.36	.22	.39	.37	.19	.40	.31	.17	.33	.26	.09
.01	.41	.41	.30	.40	.41	.24	.41	.39	.21	.41	.34	.18	.35	.28	.11

TABLE (2)

Critical Values of CVM (W)

N : Complete Sample Size, R : Censored Sample Size and α : Level of Signif.

N	N=10			N=15			N=20			N=25			N=50		
α R	R=8	9	10	12	14	15	16	18	20	20	23	25	40	45	50
.95	.03	.03	.03	.04	.03	.02	.06	.04	.02	.08	.04	.02	.25	.12	.02
.90	.04	.04	.03	.05	.04	.03	.08	.06	.03	.12	.06	.03	.33	.16	.03
.85	.04	.04	.03	.06	.05	.03	.10	.07	.03	.15	.07	.03	.40	.19	.03
.80	.05	.05	.04	.07	.05	.03	.11	.08	.03	.18	.08	.03	.47	.23	.03
.75	.06	.05	.04	.08	.06	.04	.13	.10	.04	.21	.09	.04	.53	.26	.03
.70	.06	.06	.04	.09	.06	.04	.15	.11	.04	.24	.10	.04	.58	.29	.04
.65	.07	.06	.04	.10	.07	.04	.17	.12	.04	.27	.11	.04	.64	.32	.04
.60	.07	.07	.05	.11	.08	.05	.19	.13	.04	.30	.13	.05	.69	.36	.04
.55	.08	.07	.05	.12	.09	.05	.21	.15	.05	.33	.14	.05	.75	.39	.05
.50	.09	.08	.05	.13	.09	.05	.23	.17	.05	.37	.15	.05	.81	.42	.05
.45	.09	.08	.06	.14	.10	.06	.25	.19	.06	.40	.17	.06	.87	.46	.05
.40	.10	.09	.06	.16	.11	.06	.28	.20	.06	.44	.19	.06	.94	.50	.06
.35	.11	.10	.07	.17	.13	.06	.31	.23	.06	.49	.21	.07	1.0	.54	.06
.30	.12	.11	.07	.20	.14	.07	.34	.26	.07	.54	.24	.07	1.1	.59	.07
.25	.14	.12	.08	.22	.16	.08	.38	.30	.08	.59	.27	.08	1.2	.65	.07
.20	.15	.13	.08	.25	.18	.08	.42	.34	.09	.66	.31	.09	1.3	.73	.08
.15	.18	.15	.10	.29	.22	.09	.48	.39	.10	.74	.36	.10	1.4	.18	.09
.10	.21	.19	.11	.34	.27	.11	.65	.47	.11	.84	.44	.11	1.6	.92	.11
.05	.27	.26	.13	.44	.40	.13	.68	.63	.15	.98	.58	.14	1.8	1.1	.13
.04	.29	.29	.14	.47	.44	.14	.71	.67	.16	1.0	.63	.15	1.9	1.2	.14
.03	.32	.34	.15	.51	.50	.16	.76	.73	.17	1.1	.69	.16	1.9	1.2	.14
.02	.35	.41	.16	.56	.58	.18	.83	.80	.20	1.1	.76	.17	2.0	1.3	.16
.01	.41	.52	.19	.63	.75	.22	.94	.93	.23	1.3	.97	.19	2.3	1.5	.17

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TABLE (3)

Critical Values of AD (A)

N : Complete Sample Size, R : Censored Sample Size and α : Level of Signi

N	N=10			N=15			N=20			N=25			N=50		
α R	8	9	10	12	14	15	16	18	20	20	23	25	40	45	50
.95	-.47	.03	.19	-.56	.11	.17	-.43	.07	.17	.24	.09	.16	.87	.52	.16
.90	-.32	.09	.21	-.36	.19	.19	-.06	.23	.19	.31	.24	.19	2.2	.94	.18
.85	-.22	.14	.23	-.19	.25	.21	.24	.35	.21	.76	.36	.21	3.2	1.3	.20
.80	-.14	.19	.24	-.03	.32	.23	.53	.51	.22	1.2	.48	.23	4.1	1.7	.22
.75	-.05	.24	.26	.11	.38	.25	.82	.64	.24	1.7	.58	.25	5.0	2.0	.23
.70	.04	.30	.27	.27	.43	.26	1.1	.78	.26	2.2	.70	.26	5.8	2.3	.25
.65	.14	.34	.29	.42	.49	.28	1.4	.94	.28	2.7	.81	.28	6.7	2.7	.27
.60	.24	.39	.30	.56	.56	.30	1.7	1.1	.29	3.3	.95	.30	7.6	3.0	.29
.55	.33	.44	.32	.74	.63	.32	2.0	1.3	.31	3.8	1.1	.32	8.5	3.3	.30
.50	.45	.49	.34	.92	.70	.33	2.3	1.5	.33	4.4	1.2	.34	9.5	3.7	.32
.45	.57	.54	.36	1.1	.79	.35	2.6	1.7	.36	5.0	1.4	.36	10.5	4.2	.34
.40	.69	.61	.38	1.3	.90	.37	3.0	2.0	.37	5.6	1.6	.39	11.5	4.6	.36
.35	.83	.68	.40	1.6	1.0	.40	3.4	2.2	.40	6.3	1.8	.41	12.6	5.1	.39
.30	1.0	.76	.43	1.8	1.2	.42	3.9	2.6	.43	7.1	2.1	.44	13.8	5.7	.41
.25	1.2	.86	.47	2.1	1.4	.46	4.4	2.9	.47	7.9	2.4	.48	15.3	6.4	.45
.20	1.4	.96	.51	2.5	1.6	.50	5.0	3.4	.52	8.8	2.8	.52	17.0	7.1	.49
.15	1.7	1.1	.56	3.1	1.9	.55	5.8	4.0	.57	10.0	3.2	.57	18.9	8.0	.54
.10	2.1	1.3	.64	3.8	2.3	.63	6.8	4.9	.67	11.7	3.8	.65	21.4	9.2	.62
.05	2.9	1.7	.75	4.9	3.1	.77	8.6	6.3	.85	13.8	4.9	.78	25.5	11.1	.73
.04	3.1	1.9	.79	5.4	3.3	.82	9.1	6.6	.91	14.5	5.3	.82	26.7	11.5	.76
.03	3.4	2.0	.84	6.1	3.6	.89	9.8	7.2	.98	15.4	5.7	.88	28.1	12.2	.81
.02	3.8	2.4	.93	7.1	4.0	.99	10.5	8.1	1.1	16.5	6.2	.96	29.8	13.1	.88
.01	4.5	2.8	1.0	8.9	5.1	1.2	12.3	9.3	1.3	19.2	7.2	1.1	34.1	14.2	1.0

Table (4)

Power of modified KS test for Burr distribution

H_0 : Burr Distribution

H_1 : Another Distribution

Level of significance $\alpha = .05, .01$

Alternative	10			15			20			25			50		
	8	9	10	12	14	15	16	18	20	20	23	25	40	45	50
Logistic	.057	.095	.095	.015	.041	.232	.004	.004	.266	.001	.008	.266	.001	.001	.252
	.011	.018	.142	.004	.006	.116	.001	.002	.129	.001	.001	.125	.001	.001	.120
Expone-	.864	.849	.854	.807	.782	.828	.736	.712	.797	.648	.685	.799	.515	.676	.988
ntial	.707	.688	.830	.764	.709	.802	.683	.644	.762	.599	.606	.750	.397	.559	.951
Normal	.102	.129	.362	.033	.060	.303	.015	.015	.333	.001	.031	.303	.001	.002	.313
	.065	.028	.188	.011	.009	.169	.002	.002	.182	.001	.002	.170	.001	.001	.165
Uniform	.063	.095	.288	.024	.038	.237	.004	.008	.241	.001	.173	.461	.001	.001	.251
	.012	.020	.140	.008	.003	.123	.001	.001	.110	.001	.105	.340	.001	.001	.116
Weibull	.062	.088	.278	.020	.038	.256	.003	.004	.398	.001	.171	.274	.001	.001	.013
with shape 1	.015	.020	.129	.006	.003	.127	.001	.001	.309	.001	.106	.133	.001	.001	.115
Weibull	.061	.100	.290	.019	.043	.236	.003	.012	.245	.001	.167	.265	.001	.001	.189
with shape 2	.013	.020	.123	.006	.006	.122	.001	.001	.108	.001	.100	.130	.001	.001	.081
Gamma	.761	.793	.889	.698	.782	.883	.634	.656	:870	.507	.739	.879	.451	.636	.900
	.599	.603	.809	.548	.510	.846	.456	.503	.825	.387	.576	.831	.312	.488	.868
Beta (2,2)	.914	.918	.945	.866	.884	.925	.807	.826	.920	.741	.819	.921	.579	.689	.900
	.892	.890	.923	.837	.831	.915	.770	.780	.893	.710	.768	.893	.499	.611	.870
Beta (2,3)	.925	.925	.950	.875	.899	.925	.825	.848	.925	.773	.843	.925	.625	.724	.900
	.900	.900	.925	.850	.850	.925	.798	.809	.900	.747	.795	.900	.560	.650	.875
Chi-square	.724	.839	.860	.694	.755	.778	.629	.662	.736	.545	.594	.673	.171	.205	.607
with 1 d.f	.593	.826	.848	.606	.736	.768	.552	.651	.704	.505	.573	.636	.146	.172	.452

Entries are probability of rejecting H_0 when the random sample in actually from the stated alternative distribution. For each alternative distribution, the upper entry is for $\alpha = 0.05$ and the lower entry is for $\alpha = .01$.

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Table (5)

Power of modified CVM test for Burr distribution

H_0 : Burr Distribution

H_1 : Another Distribution

Level of significance α = .05, .01

Alternative	10			15			20			25			50		
	8	9	10	12	14	15	16	18	20	20	23	25	40	45	50
Logistic	.137	.176	.494	.054	.071	.459	.012	.007	.443	.001	.018	.477	.001	.001	.497
	.054	.035	.314	.013	.012	.229	.002	.001	.270	.001	.001	.336	.001	.001	.373
Expone-	.971	.950	.990	.980	.979	.963	.925	.991	.995	.953	.936	.947	.955	.998	.993
ntial	.989	.987	.991	.993	.988	.996	.997	.997	.995	.996	.995	.998	.998	.991	.992
Normal	.194	.227	.554	.079	.107	.565	.033	.034	.569	.001	.049	.557	.001	.001	.675
	.087	.052	.384	.031	.023	.320	.008	.004	.350	.001	.004	.428	.001	.001	.546
Uniform	.143	.166	.489	.046	.069	.458	.014	.015	.385	.001	.356	.999	.001	.001	.483
	.051	.027	.307	.018	.008	.239	.001	.001	.221	.001	.208	.993	.001	.001	.359
Weibull	.148	.161	.489	.040	.070	.479	.009	.011	.100	.001	.362	.455	.001	.001	.488
with shape 1	.057	.030	.305	.016	.009	.240	.001	.001	.984	.001	.204	.315	.001	.001	.367
Weibull	.138	.170	.496	.040	.075	.477	.011	.028	.367	.001	.352	.455	.001	.001	.511
with shape 2	.048	.026	.313	.014	.015	.240	.001	.004	.196	.001	.203	.313	.001	.001	.383
Gamma	.879	.902	.974	.888	.942	.998	.891	.917	.999	.848	.984	.999	.948	.999	.999
	.755	.697	.947	.762	.762	.987	.748	.785	.995	.713	.913	.999	.849	.989	.999
Beta (2,2)	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
Beta (2,3)	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
Chi-square	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
with 1 d.f	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.998	.999

Entries are probability of rejecting H_0 when the random sample in actually from the stated alternative distribution. For each alternative distribution, the upper entry is for α = 0.05 and the lower entry is for α = .01.

Table (6)

Power of modified AD test for Burr distribution

H_0 : Burr Distribution

H_1 : Another Distribution

Level of significance $\alpha = .05, .01$

Alternative	10			15			20			25			50		
	8	9	10	12	14	15	16	18	20	20	23	25	40	45	50
Logistic	.004	.088	.546	.001	.028	.491	.001	.001	.489	.001	.001	.534	.001	.001	.550
	.001	.020	.367	.001	.004	.264	.001	.001	.280	.001	.001	.325	.001	.001	.372
Expone-	.105	.789	.999	.035	.893	.999	.001	.254	.999	.001	.895	.999	.001	.532	.999
ntial	.024	.428	.999	.001	.380	.999	.001	.027	.999	.001	.460	.999	.001	.269	.999
Normal	.033	.192	.682	.007	.061	.655	.001	.002	.650	.001	.010	.682	.001	.001	.809
	.006	.065	.514	.001	.017	.450	.001	.001	.456	.001	.001	.504	.001	.001	.667
Uniform	.004	.079	.538	.001	.016	.496	.001	.001	.430	.001	.001	.521	.001	.001	.547
	.001	.018	.363	.001	.002	.277	.001	.001	.232	.001	.001	.312	.001	.001	.365
Weibull	.004	.084	.540	.001	.018	.509	.001	.001	.494	.001	.001	.519	.001	.001	.546
with shape 1	.001	.019	.365	.001	.003	.276	.001	.001	.282	.001	.001	.318	.001	.001	.366
Weibull	.006	.086	.547	.001	.025	.502	.001	.001	.417	.001	.001	.512	.001	.001	.567
with shape 2	.001	.017	.375	.001	.004	.272	.001	.001	.206	.001	.001	.313	.001	.001	.392
Gamma	.683	.999	.415	.004	.092	.433	.001	.018	.456	.001	.028	.469	.001	.001	.517
	.641	.988	.395	.001	.053	.411	.001	.007	.438	.001	.014	.456	.001	.001	.508
Beta (2,2)	.999	.999	.999	.999	.999	.999	.998	.999	.999	.907	.999	.999	.997	.999	.999
	.951	.999	.999	.776	.999	.999	.748	.999	.999	.375	.999	.999	.623	.999	.999
Beta (2,3)	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
Chi-square	.506	.662	.858	.494	.762	.921	.483	.701	.963	.464	.811	.981	.479	.850	.998
with 1 d.f	.488	.648	.855	.448	.742	.920	.448	.677	.962	.421	.795	.980	.430	.837	.998

Entries are probability of rejecting H_0 when the random sample in actually from the stated alternative distribution. For each alternative distribution, the upper entry is for $\alpha = 0.05$ and the lower entry is for $\alpha = .01$.

Table (7)

The fitted linear regression model

for the c.d.f. of KS statistic $Y = F(x) = A + Bx$

n	r	Multiple R	R^2	Adjusted R^2	Standard error	calcul. F	Estimate of $\hat{B}$	Estimate of $\hat{A}$	Standard error $\hat{B}$	Standard error $\hat{A}$	Type
	8	.96779	.93663	.93399	.09924	354.70	-2.69947	1.06438	.14333	.03797	IV
10	9	.95752	.91684	.91338	.11262	264.62	-2.63236	1.01006	.16182	.04305	IV
	10	.93179	.86823	.86274	.13458	158.14	-3.28471	.96541	.26120	.05164	IV
	12	.96719	.93545	.93276	.09502	347.79	-2.54854	1.00863	.13666	.03626	II
15	14	.94398	.89109	.88655	.11972	196.37	-2.69437	.96083	.19227	.03399	IV
	15	.94983	.90218	.89811	.11113	221.35	-3.9837	.93719	.26776	.03961	II
	16	.97322	.94716	.94496	.08055	430.22	-2.52874	.99591	.12192	.03141	II
20	18	.96556	.93230	.92948	.09119	330.49	-2.44117	.91458	.13428	.03487	II
	20	.94187	.88712	.88242	.11925	188.62	-2.08007	.85373	.29708	.04146	II
	20	.97054	.94195	.93953	.08071	389.40	-2.24090	.94450	.11356	.03029	II
25	23	.91288	.83336	.82641	.13961	120.02	-2.36322	.81879	.21571	.04655	II
	25	.92232	.85068	.84446	.13526	136.73	-4.37219	.80336	.37391	.04345	II
	40	.96733	.93572	.93305	.08546	349.39	-2.68548	.87055	.14367	.03062	II
50	45	.95983	.92127	.91799	.09674	280.84	-3.32356	.85994	.19832	.03453	II
	50	.87197	.76033	.75035	.14563	76.14	-8.76597	.75398	1.0046	.04593	I

Table (8)

The fitted linear regression model

for the c.d.f. of CVM statistic $Y = F(x) = A+Bx$

n	r	Multiple R	R^2	Adjusted R^2	Standard error	calcul. F	Estimate of $\hat{B}$	Estimate of $\hat{A}$	Standard error $\hat{B}$	Standard error $\hat{A}$	Type
	8	.84290	.71048	.69843	.19230	58.90	-1.81600	.70901	.23663	.06591	III
10	9	.75662	.57247	.55466	.21596	32.14	-1.18356	.01006	.20878	.06935	III
	10	.81639	.66650	.65260	.20510	47.96	-3.44845	.71874	.49793	.06909	III
	12	.80583	.64937	.63476	.20786	44.45	-1.15379	.70601	.17306	.06072	III
15	14	.74423	.55389	.53530	.22404	29.80	-.82494	.55991	.15112	.06856	III
	15	.79948	.63918	.62414	.20004	42.51	-3.03484	1.65947	.46544	.06683	III
	16	.88047	.77522	.76585	.16149	82.77	-.79278	.72561	.08714	.05695	I
20	18	.82475	.68021	.66669	.20583	51.05	-.74146	.68941	.10377	.06041	I
	20	.68092	.46366	.44131	.26080	20.75	-1.89927	.61384	.41697	.06944	III
	20	.88272	.77920	.77000	.16059	84.69	-.60898	.77170	.06617	.05277	I
25	23	.75109	.56413	.54597	.23789	31.06	-.60430	.63595	.10843	.06396	III
	25	.78231	.61201	.95584	.22753	37.86	-3.03037	.70534	.49252	.06832	III
	40	.93100	.86677	.86122	.13536	156.14	-.40743	.29454	.03261	.04919	I
50	45	.70072	.49100	.46980	.24416	23.15	-.29965	.65116	.06226	.06531	IV
	50	.33411	.11163	.07461	.32007	3.02	-.22248	.46493	.12811	.06625	III

Table (9)

The fitted linear regression model

for the c.d.f. of AD statistic $Y = F(x) = A+Bx$

n	r	Multiple R	R^2	Adjusted R^2	Standard error	calcul F	Estimate B	Estimate $\hat{A}$	Standard $\hat{B}$	Standard $\hat{A}$	Type
	8	.81419	.66291	.64886	.21836	47.20	-.12964	.59630	.01887	.05250	III
10	9	.71123	.61032	.59409	.21851	37.59	-.19788	.62918	.03228	.05826	III
	10	.79533	.63254	.61723	.20915	41.31	-.72315	.77922	.11251	.07294	III
	12	.76125	.57950	.56198	.23879	33.07	-.05667	.58450	.00985	.05839	III
15	14	.72189	.52113	.50118	.23252	26.12	-.09620	.56216	.01882	.06191	III
	15	.21570	.66537	.65143	.19419	47.72	-.59416	.70449	.08601	.06997	III
	16	.98628	.80331	.79512	.16768	98.02	-.05671	.69860	.00573	.04858	III
20	18	.23319	.69420	.68146	.19952	54.48	-.06624	.65424	.00897	.05711	III
	20	.71449	.51050	.49010	.24499	25.03	-.32649	.64469	.07725	.06996	III
	20	.88586	.78474	.77577	.16408	87.49	-.03717	.71045	.00397	.04754	III
25	23	.79650	.63441	.61918	.21756	41.65	-.07609	.63721	.01179	.05997	III
	25	.80678	.65089	.63634	.20669	44.75	-.60765	.74077	.09084	.06842	III
	40	.91816	.84302	.83648	.14629	128.89	-.02441	.79674	.00215	.04658	I
50	45	.88141	.77689	.76759	.16718	83.57	-.05145	.74664	.00563	.05276	I
	50	.35884	.12876	.09246	.32775	3.55	-.05262	.51092	.02794	.06846	III

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