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A COMPARISON OF APPROXIMATIONS TO PERCENTILES OF THE NONCENTRAL F-DISTRIBUTION

ABSTRACT

Various approximations to percentiles of the noncentral F-distribution are examined for their accuracy over a wide range of values of the parameters of the distribution.

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1. INTRODUCTION AND SUMMARY

It is widely recognized that the noncentral F-distribution is of considerable theoretical and practical importance in many mathematical and statistical applications. For instance, there are many applications of the noncentral F-distribution including evaluation of power function or sensitivity function of the analysis of variance F-test (Graybill, 1976, pp. 128-130; Itô, 1980), determination of sample size in an experimental design (Odeh and Fox, 1991), derivation of the nonnull distribution and power of the Hotelling's T^2 (Anderson, 1984, pp. 161-163; Tiku, 1985), in discriminant analysis (Rao, 1970; Lachenbruch, 1975) and certain engineering problems in communication theory (Price, 1964). Genizi and Soller (1979) give certain interesting applications of a mixture of two noncentral F-distributions.

In many applications involving the noncentral F-distribution one has to compute its percentiles involving the evaluation of the inverse probability functions (see, *e.g.*, Bagui, 1996). However, the evaluation of such inverse functions is extremely tedious involving slow and expensive techniques of numerical iteration such as the Newton-Raphson procedure (see, *e.g.*, Ralston and Wilf, 1967; Carnahan *et al.*, 1969). There are a number of approximations for computing the percentage points of these distributions, at arbitrary probability levels, available in the literature. The applicability of several of these approximations is further enhanced by ease of their computational simplicity. The purpose of this paper is to compare these approximations to determine their accuracy. Some of these approximations were previously investigated by Tiku (1966), Mudholkar *et al.* (1976), Tiku and Yip (1978) and Cox and Reid (1987). A brief description of each procedure is given and appropriate tables comparing their accuracy, calculated for each procedure, are presented. A more comprehensive set of tables is given in Sahai and Ojeda (1998).

2. APPROXIMATIONS

The noncentral F-distribution was derived by Tang (1938) though Patnaik (1948) seems to have been the first to call it by this name. There are various approximations of the noncentral F-distribution discussed in the literature which can be used to compute the percentiles of the distribution. Some of the important ones are considered here.

In this paper, $F'_{\nu_1, \nu_2}(\lambda)$ will be used to denote a noncentral F-variate with ν_1 and ν_2 degrees of freedom and the noncentrality parameter λ. In addition, $F'_{\nu_1, \nu_2; \alpha}(\lambda)$ will denote its 100 α-th percentile defined by

$$\Pr\left[F'_{\nu_1, \nu_2}(\lambda) \leq F'_{\nu_1, \nu_2; \alpha}(\lambda)\right] = \alpha. \qquad (2.1)$$

Patnaik (1949) suggested an approximation of

$$F'_{\nu_1,\nu_2}(\lambda) = \left[\chi'^2_{\nu_1}(\lambda) / \nu_1\right] / \left[\chi^2_{\nu_2} / \nu_2\right] \qquad (2.2)$$

by first approximating the distribution of $\chi'^2_{\nu_1}(\lambda)$ by that of $c\chi^2_f$ where c and f, obtained by equating the first two moments of the two variables, are

$$c = \left(\nu_1 + 2\lambda\right) / \left(\nu_1 + \lambda\right) \quad \text{and} \quad f = \left(\nu_1 + \lambda\right)^2 / \left(\nu_1 + 2\lambda\right).$$

Then the distribution of $F'_{\nu_1,\nu_2}(\lambda)$ is approximated by that of $\left(cf/\nu_1\right)F_{f,\nu_2} = \left(1+\lambda/\nu_1\right)F_{f,\nu_2}$.

Laubscher (1960) proposed a square root transformation by expressing

$$\left[\left(\nu_1/\nu_2\right)F'_{\nu_1,\nu_2}(\lambda)\right]^{1/2} = \left[2\chi'^2_{\nu_1}(\lambda)\right]^{1/2} / \left[2\chi^2_{\nu_2}\right]^{1/2}, \qquad (2.3)$$

and then approximating the distributions of $\left[2\chi'^2_{\nu_1}(\lambda)\right]^{1/2}$ and $\left(2\chi^2_{\nu_2}\right)^{1/2}$ by appropriate normal random variables. A theorem due to Fieller (1932) <<which states that if X and Y are independent and normally distributed with means μ_x and μ_y and standard deviations σ_x and σ_y respectively, then the ratio $R = \left(\mu_x V - \mu_y\right) / \left(\sigma_x^2 V^2 + \sigma_y^2\right)^{1/2}$ where $V = Y/X$ is nearly normally distributed with zero mean and unit variance, provided the probability of X being negative is very small, <<can then be applied to the variable $\left[\left(\nu_1/\nu_2\right)F'_{\nu_1,\nu_2}(\lambda)\right]^{1/2}$ to show that the transformed random variable

$$\frac{\left(2\nu_2 - 1\right)^{1/2}\left[\left(\nu_1/\nu_2\right)F'_{\nu_1,\nu_2}(\lambda)\right]^{1/2} - \left[2\left(\nu_1+\lambda\right) - \left(\nu_1+2\lambda\right)/\left(\nu_1+\lambda\right)\right]^{1/2}}{\left[\left(\nu_1/\nu_2\right)F'_{\nu_1,\nu_2}(\lambda) + \left(\nu_1+2\lambda\right)/\left(\nu_1+\lambda\right)\right]^{1/2}} \qquad (2.4)$$

has approximately a standard normal distribution.

Laubscher (1960) further considered as a possible normalizing transformation

$$\left(\nu_2/2-2\right)^{1/2}\text{Cosh}^{-1}\left\{\frac{\nu_1\left(\nu_2-2\right)^{1/2}\left[F'_{\nu_1,\nu_2}(\lambda)+\left(\nu_2/\nu_1\right)\right]}{\nu_2\left(\nu_1+\nu_2-2\right)^{1/2}}\right\}, \tag{2.5}$$

which is approximately normal with mean

$$\left(\nu_2/2-2\right)^{1/2}\text{Cosh}^{-1}\left\{\frac{\nu_1+\nu_2+\lambda-2}{\left(\nu_2-2\right)^{1/2}\left(\nu_1+\nu_2-2\right)^{1/2}}\right\}$$

and variance one. This approximation, however, is not very satisfactory and is included here for the sake of completeness.

In addition, Laubcher (1960) proposed an improved normal approximation of $F'_{\nu_1,\nu_2}(\lambda)$ by using the second derivative term in the Taylor series for expectations. Thus, the transformed random variable

$$\left(\nu_2/2-2\right)^{1/2}\left[\text{Cosh}^{-1}\left\{\frac{\left(\nu_2-2\right)^{1/2}\left(1+\frac{\nu_1}{\nu_2}F'_{\nu_1,\nu_2}(\lambda)\right)}{\left(\nu_1+\nu_2-2\right)^{1/2}}\right\}\right.$$
$$\left.-\text{Cosh}^{-1}\left\{\frac{\left(\nu_2-2\right)^{1/2}\left(1+\frac{\nu_1}{\nu_2}\mu'_1\right)}{\left(\nu_1+\nu_2-2\right)^{1/2}}\right\}\right]+\left(\frac{\mu_1'+\nu_2/\nu_1}{\nu_2-4}\right)\mu_2^{-1/2} \tag{2.6}$$

may be better approximated by a standard normal distribution, where μ'_1 and μ_2 are the first two moments of $F'_{\nu_1,\nu_2}(\lambda)$.

We will call the approximations (2.4), (2.5) and (2.6) as Laubscher's 1st, 2nd and 3rd approximations respectively.

Laubscher (1960) and Severo and Zelen (1960) independently proposed that $\left\{\chi^{2}_{\nu_1}(\lambda)\right\}^{1/3}$ and $\left\{\chi^{2}_{\nu}\right\}^{1/3}$ be each approximated by a normal distribution as in Abdel-Aty (54) and Wilson-Hilferty (1931) approximations of noncentral and central χ^2 distributions respectively. By applying the Fieller theorem as stated earlier to the transformed random variable $\left[\left(\nu_1/\nu_2\right)F'_{\nu_1,\nu_2}(\lambda)\right]^{1/3}$, it follows that

$$\frac{\left(1-2/9\nu_2\right)\left[\nu_1 F'_{\nu_1,\nu_2}(\lambda)/\left(\nu_1+\lambda\right)\right]^{1/3}-\left[1-2\left(\nu_1+2\lambda\right)/9\left(\nu_1+\lambda\right)^2\right]}{\left\{\left[2\left(\nu_1+2\lambda\right)/9\left(\nu_1+\lambda\right)^2\right]+\left(2/9\nu_2\right)\left[\nu_1 F'_{\nu_1,\nu_2}(\lambda)/\left(\nu_1+\lambda\right)\right]^{2/3}\right\}^{1/2}} \qquad (2.7)$$

is approximately a standard normal distribution.

An approximation similar in spirit to that of (2.7) can be obtained by first approximating $F'_{\nu_1,\nu_2}(\lambda)$ by a central F-distribution as in Patnaik's approximation (2.2) considered above and then approximating the central F-distribution by a unit normal distribution via Paulson's (1942) approximation (see, *e.g.*; Sahai and Thompson, 1974). The resulting approximation, however, is slightly more complicated and does not result in any appreciable increase in accuracy.

Tiku (1965) proposed approximating the distribution of $F'_{\nu_1,\nu_2}(\lambda)$ by that of cF_{f,ν_2} where b, c and f, obtained by equating the first three moments of the two variables, are

$$b = -\left[\nu_2/\left(\nu_2-2\right)\right]\left(c-1-\lambda/\nu_1\right),$$

$$c = \left(f/\nu_1\right)(H/K)\left[1/\left(2f+\nu_2-2\right)\right],$$

$$f = (1/2)\left(\nu_2-2\right)\left\{\left[H^2/\left(H^2-4K^3\right)\right]^{1/2}-1\right\}$$

$$H = 2\left(\nu_1+\lambda\right)^3+3\left(\nu_1+\lambda\right)\left(\nu_1+2\lambda\right)\left(\nu_2-2\right)+\left(\nu_1+3\lambda\right)\left(\nu_2-2\right)^2$$

and

$$K = \left(\nu_1 + \lambda\right)^2 + \left(\nu_2 - 2\right)\left(\nu_1 + 2\lambda\right).$$

Tiku and Yip (1978) considered a four-moment approximation similar to above. The procedure consists of approximating the distribution of $F'_{\nu_1,\,\nu_2}(\lambda)$ by that of $b + c\,F_{f_1,f_2}$, where b, c, f and f_2 are obtained by equating the first four moments of the two variables. The expressions for μ'_1 and the central moments μ_2, μ_3 and μ_4 of $F'_{\nu_1,\,\nu_2}(\lambda)$ are given in Pearson and Tiku (1970). The values of b, c, f and f_2, obtained by equating the first four moments of $F'_{\nu_1,\,\nu_2}(\lambda)$ and $b + c\,F_{f_1,f_2}$, are:

$$f_2 = 2\left[3 + \frac{\beta_2 + 3}{\beta_2 - \left(\frac{3+1}{1.5\,\beta_1}\right)}\right], \quad f_1 = \frac{1}{2}\left(f_2 - 1\right)\left[\sqrt{\left\{1 + \frac{32\left(f_2 - 4\right)/\left(f_2 - 6\right)2}{\beta_1 - 32\left(f_2 - 4\right)/\left(f_2 - 6\right)^2}\right\}} - 1\right],$$

$$c = \sqrt{\left\{\frac{f_1\left(f_2 - 2\right)2\left(f_2 - 4\right)}{2f_2^2\left(f_1 + f_2 - 2\right)}\mu_2\right\}}, \quad \text{and} \quad b = \mu_1' - \frac{f_2}{f_2 - 2}c,$$

with

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \quad \text{and} \quad \beta_2 = \mu\frac{\mu_4}{\mu_2^2}.$$

Mudholkar *et al.* (1976) proposed an approximation of

$$F'_{\nu_1,\,\nu_2}(\lambda) = \left[\chi'^2_{\nu_1}(\lambda)/\nu_1\right] / \left[\chi^2_{\nu_2}/\nu_2\right]$$

by first approximating the distribution of $\chi'^2_{\nu_1}(\lambda)$ by that of $b + c\chi^2_f$ where b, c and f are obtained by equating the first three moments of the two variables. Then the distribution of $F'_{\nu_1,\nu_2}(\lambda)$ is approximated by that of $b' + c'F_{f,\nu_2}$ where b' and c' are obtained by equating the first two moments of the two variables. The resulting values of b', c' and f are

$$b' = -\left[\nu_2/(\nu_2-2)\right](c-1-\lambda/\nu_1),$$

$$c' = \left\{(f/\nu_1)/\left[f^2+(\nu_2-2)f\right]^{1/2}\right\}\left[(\nu_2-2)(\nu_1+2\lambda)+(\nu_1+\lambda)^2\right]^{1/2}$$

and

$$f = (\nu_1+2\lambda)^3/(\nu_1+3\lambda)^2.$$

Using an approximation for the noncentral χ^2, Cox and Reid (1987) proposed an approximation

$$\Pr\left[F'_{\nu_1,\nu_2}(\lambda)\le t\right] \approx Pr\left[F_{\nu_1,\nu_2} \le t/(1+\lambda/\nu_1)\right].$$

The above approximation is, of course, valid to $O\left(\nu_1^{-1}\right)$ for $\lambda = O(1)$ as $\nu_1 \Rightarrow \infty$.

3. RESULTS

The percentiles of $F'_{\nu_1,\nu_2}(\lambda)$ calculated for the various approximations as well as their exact values, for selected values of α, ν_1, ν_2, and λ, are shown in Table 1. The results show that the four moment central F-approximation of Tiku and Yip is the most accurate. It is considerably more accurate than the Tiku's three-moment approximation, although much more difficult to compute. Severo-Zelen's normal approximation seems to be the easiest to compute although slightly less accurate than Patnaik's approximation for large values of ν_2. Patnaik's approximation is relatively simple to compute and gives a rather satisfactory results for both lower and upper percentiles. The two-stage approximation of Mudholkar *et* compares favorably in accuracy with that of Tiku's three-moment approximation and is much simpler to compute. The accuracy of both approximations increases with ν_1 and increases with ν_2, and both approximations are less accurate in the lower tail of the distribution. The 1st Laubscher and Severo-Zelen have similar performance. Although they give rather satisfactory results for lower percentiles, especially when both ν_1 and ν_2 are

large, their performance is extremely poor for upper percentiles. Both 2nd and 3rd Laubscher are undefined for $\nu_2 \leq 4$. Although they perform rather poorly for small values of ν_2, their performance improves steadily as ν_2 increases. For both lower and upper percentiles, Cox-Reid approximation performs very poorly for small values of ν_1, but its performance improves considerably as ν_1 increases.

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TABLE 1
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUT
$\alpha = 0.05$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar-*et al.*	Cox
						$\nu_1 = 3$				
5	1	0.1622	0.0953	0.7477	12.7511	0.1540	0.1401	0.1387	1.1408	0.1
	4	0.4162	0.3310	1.5369	6.6105	0.4087	0.3363	0.3341	0.3459	0.2
	10	1.1222	1.0455	0.8149	3.2310	1.1185	1.0275	1.0264	1.0458	0.8
	16	1.9288	1.8731	0.6904	2.0144	1.9275	1.8430	1.8425	1.8624	0.7
	25	3.2047	3.1856	0.8371	1.2200	3.2075	3.1346	3.1344	3.1524	1.0
20	1	0.1702	0.0958	0.5539	0.6899	0.1609	0.1460	0.1403	0.1462	0.1
	4	0.4547	0.3493	0.6238	0.5464	0.4454	0.3582	0.3420	0.3623	0.2
	10	1.3011	1.1804	1.3129	1.0624	1.2948	1.1641	1.1482	1.1765	0.9
	16	2.3193	2.2016	2.2512	1.8885	2.3150	2.1789	2.1669	2.1962	0.7
	25	3.9898	3.8656	3.8393	3.3363	3.9872	3.8579	3.8501	3.8786	1.0
50	1	0.1721	0.0959	0.5183	0.5916	0.1626	0.1474	0.1371	0.1475	0.1
	4	0.4645	0.3538	0.6478	0.5619	0.4549	0.3639	0.3296	0.3659	0.2
	10	1.3528	1.2181	1.4391	1.2391	1.3460	1.2023	1.1592	1.2088	0.8
	16	2.4426	2.3028	2.4927	2.2292	2.4377	2.2806	2.2406	2.2910	0.7
	25	2.2614	4.1239	4.2840	3.9491	4.2582	4.0992	4.0664	4.1138	1.0
100	1	0.1727	0.0959	0.5093	0.5682	0.1632	0.1480	0.1357	0.1480	0.1
	4	0.4680	0.3553	0.6557	0.5700	0.4582	0.3660	0.3230	0.3670	0.2
	10	1.3719	1.2319	1.4784	1.2961	1.3649	1.2163	1.1569	1.2199	0.8
	16	2.4898	2.3411	2.5705	2.3400	2.4847	2.3191	2.2588	2.3251	0.7
	25	4.3698	4.2183	4.4349	4.1556	4.3665	4.1940	4.1386	4.2032	1.0

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTI
$\alpha = 0.05$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox-H
						$\nu_1 = 10$				
5	1	0.3319	0.3217	0.9702	3.4045	0.3317	0.3314	0.3313	0.3314	0.330
	4	0.4353	0.4248	0.7812	2.5631	0.4352	0.4313	0.4312	0.4315	0.420
	10	0.6663	0.6589	0.6046	1.7137	0.6667	0.6572	0.6572	0.6584	0.60
	16	0.9129	0.9107	0.5471	1.2840	0.9138	0.9020	0.9019	0.9039	0.78
	25	1.2965	1.3036	0.5552	0.9379	1.2983	1.2852	1.2851	1.2875	1.05
20	1	0.3982	0.3772	0.5284	0.4999	0.3974	0.3974	0.39720	0.3974	0.39
	4	0.5265	0.5022	0.6160	0.5568	0.5256	0.5196	0.51830	0.5198	0.50
	10	0.8217	0.7956	0.8619	0.7617	0.8210	0.8053	0.80320	0.8065	0.72
	16	1.1449	1.1195	1.1538	1.0218	1.1443	1.1237	1.12171	1.1259	0.93
	25	1.6571	1.6349	1.6330	1.4592	1.6568	1.6333	1.63180	1.6367	1.26
50	1	0.4190	0.3942	0.5490	0.5185	0.4181	0.4181	0.4175	0.4181	0.41
	4	0.5556	0.5265	0.6558	0.6054	0.5547	0.5476	0.5439	0.5478	0.53
	10	0.8741	0.8409	0.9413	0.8671	0.8732	0.8542	0.8474	0.8550	0.75
	16	1.2268	1.1922	1.2760	1.1850	1.2261	1.2003	1.1929	1.2019	0.98
	25	1.7922	1.7577	1.8252	1.7139	1.7916	1.7609	1.7510	1.7635	1.32
100	1	0.4269	0.4006	0.5567	0.5268	0.4261	0.4259	0.4251	0.4259	0.42
	4	0.5668	0.5357	0.6695	0.6230	0.5658	0.5581	0.5532	0.5584	0.54
	10	0.8948	0.8586	0.9685	0.9034	0.8939	0.8734	0.8631	0.8738	0.77
	16	1.2599	1.2214	1.3185	1.2415	1.2592	1.2309	1.2189	1.2319	1.00
	25	1.8484	1.8085	1.8944	1.8041	1.8478	1.8133	1.8011	1.8151	1.35

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.05$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 20$					
0.3875	0.3895	0.5721	1.6225	0.3880	0.3875	0.3875	0.3875	0.3873	0.3875
0.4452	0.4477	0.5231	1.4230	0.4458	0.4447	0.4447	0.4447	0.4427	0.4448
0.5659	0.5705	0.4676	1.1179	0.5668	0.5645	0.5645	0.5646	0.5533	0.5646
0.6912	0.6983	0.4455	0.9252	0.6923	0.6890	0.6890	0.6892	0.6640	0.6891
0.8836	0.8951	0.4467	0.7433	0.8852	0.8809	0.8809	0.8813	0.8300	0.8810
0.4946	0.4877	0.5552	0.5139	0.4941	0.4945	0.4945	0.4945	0.4943	0.4945
0.5693	0.5618	0.6165	0.5650	0.5692	0.5684	0.5682	0.5684	0.5649	0.5682
0.7285	0.7208	0.7555	0.6869	0.7285	0.7254	0.7250	0.7256	0.7016	0.7256
0.8960	0.8887	0.9079	0.8243	0.8960	0.8911	0.8907	0.8915	0.8474	0.8914
1.1565	1.1506	1.1508	1.0469	1.1565	1.1500	1.1495	1.1507	1.0592	1.1502
0.5346	0.5240	0.6024	0.5707	0.5344	0.5344	0.5344	0.5344	0.5342	0.5345
0.6159	0.6042	0.6746	0.6371	0.6157	0.6147	0.6140	0.6147	0.6105	0.6148
0.7908	0.7779	0.8365	0.7899	0.7906	0.7866	0.7850	0.7867	0.7631	0.7869
0.9763	0.9628	1.0131	0.9593	0.9762	0.9696	0.9676	0.9699	0.9157	0.9699
1.2670	1.2535	1.2947	1.2313	1.2668	1.2577	1.2556	1.2585	1.1447	1.2581
0.5511	0.5389	0.6202	0.5923	0.5510	0.5510	0.5509	0.5510	0.5507	0.5510
0.6353	0.6218	0.6962	0.6638	0.6351	0.6339	0.6329	0.6339	0.6294	0.6341
0.8171	0.8018	0.8666	0.8278	0.8169	0.8123	0.8097	0.8124	0.7868	0.8126
1.0107	0.9944	1.0528	1.0091	1.0105	1.0030	0.9994	1.0032	0.9441	1.0033
1.3153	1.2983	1.3499	1.3002	1.3151	1.3045	1.3004	1.3050	1.1801	1.3048

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.05$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 30$					
0.4079	0.4135	0.4371	1.1014	0.4086	0.4079	0.4079	0.4079	0.4079	0.4079
0.4482	0.4544	0.4180	0.9979	0.4490	0.4481	0.4481	0.4481	0.4473	0.4481
0.5307	0.5385	0.3954	0.8434	0.5317	0.5303	0.5303	0.5303	0.5263	0.5303
0.6152	0.6247	0.3868	0.7343	0.6163	0.6145	0.6145	0.6145	0.6052	0.6145
0.7441	0.7565	0.3902	0.6213	0.7455	0.7431	0.7431	0.7432	0.7236	0.7431
0.5351	0.5327	0.5709	0.5287	0.5351	0.5350	0.5350	0.5350	0.5349	0.5350
0.5883	0.5858	0.6173	0.5694	0.5883	0.5880	0.5880	0.5880	0.5867	0.5881
0.6987	0.6964	0.7164	0.6581	0.6987	0.6977	0.6976	0.6977	0.6902	0.6978
0.8128	0.8108	0.8213	0.7535	0.8129	0.8114	0.8109	0.8111	0.7938	0.8111
0.9884	0.9874	0.9855	0.9044	0.9885	0.9858	0.9856	0.9861	0.9491	0.9859
0.5869	0.5811	0.6325	0.6036	0.5869	0.5869	0.5869	0.5869	0.5868	0.5869
0.6457	0.6394	0.6867	0.6549	0.6456	0.6453	0.6451	0.6453	0.6436	0.6453
0.7682	0.7614	0.8023	0.7654	0.7682	0.7667	0.7662	0.7668	0.7572	0.7668
0.8956	0.8886	0.9245	0.8831	0.8956	0.8930	0.8923	0.8931	0.8708	0.8931
1.0929	1.0859	1.1160	1.0684	1.0929	1.0890	1.0880	1.0892	1.0411	1.0891
0.6098	0.6023	0.6569	0.6329	0.6097	0.6097	0.6097	0.6097	0.6096	0.6097
0.6709	0.6629	0.7142	0.6880	0.6708	0.6705	0.6702	0.6705	0.6686	0.6705
0.7990	0.7902	0.8363	0.8067	0.7990	0.7973	0.7963	0.7973	0.7866	0.7974
0.9327	0.9233	0.9655	0.9331	0.9326	0.9296	0.9281	0.9297	0.9046	0.9297
1.1404	1.1306	1.1684	1.1323	1.1403	1.1356	1.1337	1.1358	1.08159	1.1357

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.05$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Muddholkar *et al.*	Cox -Reid
						$\nu_1 = 50$				
	1	0.4249	0.4332	0.3281	0.6744	0.4258	0.4249	0.4249	0.4249	0.4249
	4	0.4501	0.4589	0.3243	0.6377	0.4510	0.4501	0.4501	0.4501	0.4499
5	10	0.5010	0.5109	0.3206	0.5768	0.5021	0.5009	0.5010	0.5010	0.4999
	16	0.5525	0.5635	0.3210	0.5288	0.5537	0.5524	0.5524	0.5524	0.5499
	25	0.6305	0.6433	0.3273	0.4739	0.6318	0.6303	0.6303	0.6303	0.6249
	1	0.5717	0.5728	0.5874	0.5458	0.5718	0.5717	0.5717	0.5717	0.5717
	4	0.6057	0.6069	0.6184	0.5739	0.6058	0.6057	0.6057	0.6057	0.6053
20	10	0.6749	0.6763	0.6822	0.6322	0.6750	0.6747	0.6746	0.6747	0.6726
	16	0.7452	0.7470	0.7478	0.6924	0.7453	0.7448	0.7448	0.7448	0.7399
	25	0.8523	0.8547	0.8487	0.7856	0.8524	0.8516	0.8515	0.8516	0.8407
	1	0.6377	0.6356	0.6648	0.6398	0.6377	0.6377	0.6377	0.6377	0.6377
	4	0.6758	0.6736	0.7011	0.6747	0.6758	0.6757	0.6757	0.6757	0.6752
50	10	0.7534	0.7511	0.7758	0.7468	0.7534	0.7531	0.7530	0.7531	0.7502
	16	0.8327	0.8304	0.8526	0.8212	0.8327	0.8320	0.8318	0.8320	0.8253
	25	0.9539	0.9517	0.9707	0.9359	0.9539	0.9527	0.9525	0.9528	0.9378
	1	0.6689	0.6653	0.6978	0.6786	0.6689	0.6689	0.6689	0.6689	0.6689
	4	0.7089	0.7051	0.7363	0.7162	0.7089	0.7088	0.7087	0.7088	0.7082
100	10	0.7907	0.7866	0.8156	0.7938	0.7906	0.7902	0.7900	0.7902	0.7869
	16	0.8744	0.8701	0.8973	0.8741	0.8744	0.8735	0.8731	0.8735	0.8656
	25	1.0027	0.9983	1.0231	0.9979	1.0027	1.0012	1.0006	1.0013	0.9837

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.10$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severop Zelen	Tiku	Tiku - Yip	Muddholkar *et al.*	Cox - Reid
						$\nu_1 = 3$				
	1	0.2689	0.2205	1.3401	7.0625	0.2652	0.2530	0.2517	0.2535	0.2511
	4	0.6251	0.5690	0.8047	3.4688	0.6226	0.5698	0.5677	0.5763	0.4395
5	10	1.5351	1.4854	0.7331	1.5876	1.5351	1.4710	1.4701	1.4834	0.8162
	16	2.5356	2.4958	1.0127	0.9936	2.5379	2.4783	2.4778	2.4913	1.1929
	25	4.0942	4.0693	1.6282	0.7151	4.1003	4.0473	4.0471	4.0592	1.7580
	1	0.2771	0.2245	0.5448	0.5567	0.2740	0.2611	0.2564	0.2612	0.2572
	4	0.6656	0.6000	0.8308	0.6599	0.6639	0.6056	0.5931	0.6082	0.4501
20	10	1.7133	1.6457	1.8217	1.4880	1.7135	1.6326	1.6210	1.6398	0.8358
	16	2.9126	2.8490	3.0025	2.5541	2.9134	2.8317	2.8231	2.8415	1.2216
	25	4.8315	4.7757	4.9114	4.3132	4.8328	4.7566	4.7511	4.7683	1.8002
	1	0.2791	0.2256	0.5351	0.5163	0.2762	0.2631	0.2547	0.2632	0.2587
	4	0.6761	0.6080	0.8611	0.7097	0.6748	0.6155	0.5901	0.6166	0.4527
50	10	1.7647	1.6914	1.9246	1.6732	1.7656	1.6802	1.6510	1.6839	0.8407
	16	3.0305	2.9582	3.1909	2.8801	3.0322	2.9426	2.9164	2.9481	1.2287
	25	5.0828	5.0144	5.2534	4.8724	5.0848	4.9970	4.9762	5.0046	1.8107
	1	0.2798	0.2259	0.5324	0.5074	0.2770	0.2639	0.2540	0.2639	0.2592
	4	0.6798	0.6108	0.8695	0.7256	0.6787	0.6191	0.5878	0.6197	0.4536
100	10	1.7837	1.7081	1.9548	1.7295	1.7850	1.6981	1.6589	1.7000	0.8424
	16	3.0755	2.9997	3.2494	2.9814	3.0776	2.9851	2.9473	2.9882	1.2311
	25	5.1826	5.1087	5.3667	5.0535	5.1851	5.0924	5.0590	5.0970	1.8143

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.10$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 10$					
0.4373	0.4301	0.6113	1.9116	0.4377	0.4370	0.4370	0.4370	0.4362	0.4371
0.5695	0.5616	0.5531	1.4302	0.5701	0.5668	0.5667	0.5669	0.5552	0.5671
0.8574	0.8503	0.5551	0.9638	0.8585	0.8513	0.8512	0.8521	0.7931	0.8517
1.1601	1.1548	0.6271	0.7471	1.1620	1.1528	1.1527	1.1541	1.0311	1.1531
1.6268	1.6249	0.7907	0.5994	1.6299	1.6193	1.6193	1.6209	1.3880	1.6195
0.5015	0.4901	0.6025	0.5418	0.5016	0.5010	0.5008	0.5010	0.4998	0.5010
0.6570	0.6440	0.7391	0.6503	0.6572	0.6530	0.6521	0.6532	0.6361	0.6531
1.0043	0.9904	1.0662	0.9371	1.0046	0.9949	0.9934	0.9956	0.9088	0.9947
1.3769	1.3632	1.4248	1.2664	1.3772	1.3648	1.3634	1.3661	1.1814	1.3645
1.9597	1.9474	2.0032	1.7965	1.9601	1.9463	1.9452	1.9481	1.5904	1.9460
0.5214	0.5085	0.6229	0.5719	0.5217	0.5209	0.5204	0.5209	0.5196	0.5209
0.6847	0.6698	0.7732	0.7050	0.6851	0.6804	0.6778	0.6804	0.6613	0.6802
1.0532	1.0365	1.1295	1.0396	1.0537	1.0426	1.0381	1.0430	0.9447	1.0418
1.4522	1.4351	1.5237	1.4179	1.4527	1.4382	1.4334	1.4390	1.2281	1.4372
2.0819	2.0650	2.1516	2.0256	2.0824	2.0655	2.0612	2.0669	1.6532	2.0646
0.5290	0.5155	0.6296	0.5823	0.5294	0.5284	0.5279	0.5284	0.5271	0.5284
0.6954	0.6796	0.7842	0.7229	0.6958	0.6908	0.6874	0.6909	0.6708	0.6905
1.0724	1.0546	1.1503	1.0729	1.0730	1.0612	1.0548	1.0615	0.9583	1.0602
1.4824	1.4638	1.5561	1.4679	1.4831	1.4675	1.4602	1.4680	1.2459	1.4662
2.1323	2.1134	2.2042	2.1035	2.1330	2.1147	2.1074	2.1155	1.6771	2.1133

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.10$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 20$					
0.4867	0.4860	0.4450	0.9581	0.4876	0.4866	0.4866	0.4866	0.4865	0.4867
0.5584	0.5578	0.4426	0.8313	0.5595	0.5581	0.5581	0.5581	0.5560	0.5581
0.7071	0.7070	0.4629	0.6693	0.7085	0.7061	0.7061	0.7062	0.6950	0.7062
0.8600	0.8607	0.5029	0.5740	0.8618	0.8586	0.8585	0.8587	0.8340	0.8586
1.0938	1.0959	0.5830	0.4950	1.0961	1.0919	1.0919	1.0922	1.0425	1.0919
0.5856	0.5818	0.6395	0.5812	0.5858	0.5856	0.5856	0.5856	0.5853	0.5856
0.6729	0.6688	0.7211	0.6529	0.6731	0.6724	0.6723	0.6724	0.6689	0.6724
0.8565	0.8521	0.8969	0.8112	0.8567	0.8547	0.8544	0.8548	0.8362	0.8547
1.0474	1.0432	1.0829	0.9815	1.0477	1.0447	1.0444	1.0449	1.0034	1.0446
1.3420	1.3383	1.3728	1.2495	1.3423	1.3384	1.3381	1.3388	1.2543	1.3383
0.6218	0.6166	0.6779	0.6370	0.6219	0.6217	0.6217	0.6217	0.6214	0.6217
0.7151	0.7094	0.7676	0.7214	0.7152	0.7144	0.7140	0.7144	0.7102	0.7144
0.9125	0.9062	0.9605	0.9058	0.9127	0.9103	0.9093	0.9104	0.8878	0.9102
1.1193	1.1127	1.1648	1.1030	1.1195	1.1158	1.1145	1.1160	1.0653	1.1156
1.4401	1.4336	1.4842	1.4127	1.4403	1.4354	1.4340	1.4357	1.3317	1.4351
0.6367	0.6309	0.6919	0.6567	0.6389	0.6366	0.6365	0.6366	0.6363	0.6366
0.7325	0.7261	0.7844	0.7454	0.7327	0.7318	0.7312	0.7318	0.7272	0.7317
0.9359	0.9288	0.9837	0.9389	0.9362	0.9335	0.9319	0.9336	0.9090	0.9333
1.1497	1.1421	1.1952	1.1459	1.1500	1.1459	1.1438	1.1460	1.0908	1.1455
1.4826	1.4747	1.5264	1.4714	1.4828	1.4772	1.4748	1.4775	1.3635	1.4768

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL F-DISTRIBUTIO
$\alpha = 0.10$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar et al.	Cox - Re
						$\nu_1 = 30$				
	1	0.5043	0.5054	0.3882	0.6600	0.5054	0.5043	0.5043	0.5043	0.5043
	4	0.5538	0.5551	0.3946	0.6049	0.5551	0.5538	0.5538	0.5538	0.5538
5	10	0.6549	0.6566	0.4166	0.5260	0.6563	0.6546	0.6546	0.6546	0.6546
	16	0.7577	0.7600	0.4468	0.4740	0.7594	0.7572	0.7572	0.7572	0.7572
	25	0.9140	0.9173	0.5018	0.4262	0.9162	0.9133	0.9133	0.9134	0.9134
	32	1.0367	1.0408	0.5498	0.4049	1.0392	1.0359	1.0359	1.0361	1.0361
	1	0.6199	0.6183	0.6570	0.6022	0.6200	0.6198	0.6198	0.6198	0.6198
	4	0.6811	0.6795	0.7156	0.6550	0.6813	0.6810	0.6810	0.6810	0.6810
20	10	0.8072	0.8056	0.8377	0.7663	0.8074	0.8067	0.8066	0.8067	0.8067
	16	0.9366	0.9350	0.9642	0.8827	0.9368	0.9356	0.9355	0.9357	0.9356
	25	1.1346	1.1333	1.1592	1.0633	1.1348	1.1331	1.1330	1.1332	1.1331
	32	1.2908	1.2898	1.3139	1.2071	1.2911	1.2891	1.2890	1.2893	1.2892
	1	0.6659	0.6630	0.7057	0.6708	0.6660	0.6658	0.6658	0.6658	0.6658
	4	0.7320	0.7289	0.7703	0.7326	0.7321	0.7318	0.7317	0.7318	0.7318
50	10	0.8687	0.8653	0.9048	0.8621	0.8688	0.8679	0.8676	0.8679	0.8679
	16	1.0096	1.0062	1.0444	0.9973	1.0097	1.0083	1.0078	1.0083	1.0082
	25	1.2263	1.2228	1.2602	1.2069	1.2264	1.2243	1.2237	1.2244	1.2243
	32	1.3979	1.3946	1.4317	1.3737	1.3980	1.3955	1.3949	1.3957	1.3956
	1	0.6859	0.6824	0.7243	0.6962	0.6860	0.6858	0.6858	0.6858	0.6858
	4	0.7541	0.7504	0.7915	0.7611	0.7542	0.7539	0.7537	0.7539	0.7539
100	10	0.8956	0.8915	0.9309	0.8974	0.8957	0.8947	0.8941	0.8947	0.8948
	16	1.0419	1.0376	1.0759	1.0398	1.0420	1.0404	1.0395	1.0404	1.0403
	25	1.2674	1.2630	1.3005	1.2607	1.2676	1.2651	1.2640	1.2652	1.2651
	32	1.4465	1.4420	1.4792	1.4368	1.4466	1.4437	1.4425	1.4438	1.4437

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTIO
$\alpha = 0.10$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Re
						$\nu_1 = 50$				
	1	0.5188	0.5213	0.3422	0.4298	0.5201	0.5188	0.5188	0.5188	0.5188
	4	0.5495	0.5521	0.3505	0.4118	0.5509	0.5495	0.5495	0.5495	0.5493
5	10	0.6114	0.6144	0.3695	0.3834	0.6130	0.6114	0.6114	0.6114	0.6104
	16	0.6739	0.6772	0.3909	0.3627	0.6756	0.6738	0.6738	0.6738	0.6714
	25	0.7683	0.7722	0.4264	0.3416	0.7702	0.7681	0.7681	0.7681	0.7630
	1	0.6505	0.6506	0.6739	0.6230	0.6507	0.6505	0.6505	0.6505	0.6505
	4	0.6891	0.6892	0.7115	0.6575	0.6893	0.6891	0.6891	0.6891	0.6887
20	10	0.7673	0.7674	0.7881	0.7281	0.7675	0.7671	0.7671	0.7671	0.7652
	16	0.8465	0.8467	0.8661	0.8003	0.8467	0.8462	0.8462	0.8462	0.8418
	25	0.9667	0.9672	0.9848	0.9106	0.9670	0.9663	0.9663	0.9663	0.9566
	1	0.7079	0.7068	0.7344	0.7055	0.7080	0.7079	0.7079	0.7079	0.7079
	4	0.7499	0.7488	0.7760	0.7458	0.7500	0.7500	0.7499	0.7500	0.7495
50	10	0.8355	0.8343	0.8608	0.8280	0.8356	0.8353	0.8353	0.8353	0.8328
	16	0.9224	0.9212	0.9473	0.9120	0.9225	0.9221	0.9220	0.9221	0.9161
	25	1.0548	1.0536	1.0793	1.0405	1.0549	1.0542	1.0540	1.0542	1.0440
	1	0.7347	0.7330	0.7599	0.7381	0.7347	0.7347	0.7347	0.7347	0.7346
	4	0.7784	0.7766	0.8032	0.7806	0.7785	0.7783	0.7783	0.7783	0.7778
100	10	0.8674	0.8656	0.8915	0.8674	0.8675	0.8672	0.8671	0.8672	0.8643
	16	0.9581	0.9562	0.9817	0.9562	0.9582	0.9577	0.9575	0.9577	0.9507
	25	1.0965	1.0945	1.1197	1.0921	1.0966	1.0958	1.0954	1.0958	1.0803

TABLE 1 (CONTINUED)
PROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.90$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 3$					
4.7947	0.2205	19.7810	4.6438	0.2652	4.8062	4.8082	4.8059	4.8260	4.8052
8.1356	0.5690	30.7970	9.2081	0.6226	8.1752	8.1781	8.1705	8.4454	8.1745
14.5478	1.4854	51.8300	17.3604	1.5351	14.5934	14.5947	14.5846	15.6844	14.5932
20.8503	2.4958	72.5110	25.1499	2.5379	20.8909	20.8916	20.8817	22.9234	20.8909
30.2375	4.0693	103.337	36.6284	4.1003	30.2708	30.2711	30.2624	33.7818	30.2708
3.1324	0.2245	4.1200	3.1404	0.2740	3.1381	3.1433	3.1381	3.1734	3.1432
5.1426	0.6000	6.2880	5.2796	0.6639	5.1601	5.1714	5.1595	5.5535	5.1728
8.7723	1.6457	10.2060	9.0343	1.7135	8.7911	8.7998	8.7896	10.3137	8.8008
12.2159	2.8490	13.9330	12.5699	2.9134	12.2327	12.2387	12.2308	15.0739	12.2393
17.2451	4.7757	19.3870	17.7211	4.8328	17.2592	17.2628	17.2571	22.2141	17.2632
2.8866	0.2256	3.5400	2.8379	0.2762	2.8906	2.8983	2.8906	2.9290	2.8972
4.6991	0.6080	5.3720	4.7118	0.6748	4.7090	4.7258	4.7089	5.1257	4.7254
7.9044	1.6914	8.6250	7.9521	1.7656	7.9097	7.9232	7.9096	9.5192	7.9240
10.8963	2.9582	11.6750	10.9625	3.0322	10.8972	10.9070	10.8974	13.9126	10.9084
15.2135	5.0144	16.0870	15.3013	5.0848	15.2106	15.2170	15.2110	20.5028	15.2186
2.8097	0.2259	3.3790	2.7474	0.2770	2.8131	2.8215	2.8131	2.8525	2.8203
4.5603	0.6108	5.1180	4.5434	0.6787	4.5673	4.5851	4.5672	4.9919	4.5851
7.6308	0.7081	8.1840	7.6306	1.7850	7.6304	7.6439	7.6305	9.2706	7.6467
10.4769	2.9997	11.0410	10.4818	3.0776	10.4703	10.4793	10.4706	13.5494	10.4837
14.5592	5.1087	15.1510	14.5690	5.1851	14.5470	14.5521	14.5478	19.9675	14.5574

TABLE 1 (CONTINUED)
PROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.90$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 10$					
3.6256	0.4301	13.0615	3.9913	0.4377	3.6258	3.6258	3.6258	3.6271	3.6258
4.5965	0.5616	16.2638	5.2623	0.5701	4.5985	4.5986	4.5984	4.6164	4.5984
6.5065	0.8503	22.5501	7.6841	0.8585	6.5108	6.5110	6.5103	6.5948	6.5108
8.3969	1.1548	28.7664	10.0334	1.1620	8.4021	8.4022	8.4012	8.5732	8.4021
11.2164	1.6249	38.0353	13.4991	1.6299	11.2218	11.2218	11.2207	11.5409	11.2218
2.1281	0.4901	2.5148	2.1647	0.5016	2.1282	2.1283	2.1282	2.1304	1.1283
2.6815	0.6440	3.1145	2.7421	0.6572	2.6823	2.6831	2.6823	2.7114	2.6831
3.7381	0.9904	4.2600	3.8335	1.0046	3.7400	3.7411	3.7399	3.8735	3.7412
4.7617	1.3632	5.3703	4.8854	1.3772	4.7640	4.7649	4.7637	5.0355	4.7650
6.2670	1.9474	7.0044	6.4284	1.9601	6.2694	6.2702	6.2691	6.7786	6.2702
1.8995	0.5085	2.1102	1.8994	0.5217	1.8995	1.8997	1.8995	1.9021	1.8997
2.3879	0.6698	2.6074	2.3961	0.6851	2.3880	2.3892	2.3880	2.4208	2.3893
3.3083	1.0365	3.5460	3.3255	1.0537	3.3082	3.3099	3.3082	3.4583	3.3101
4.1899	1.4351	4.4467	4.2130	1.4527	4.1894	4.1910	4.1895	4.4958	4.1914
5.4750	2.0650	5.7611	5.5048	2.0824	5.4741	5.4753	5.4742	6.0520	5.4757
1.8269	0.5155	1.9969	1.8198	0.5294	1.8269	1.8271	1.8269	1.8295	1.8271
2.2944	0.6796	2.4651	2.2922	0.6958	2.2942	2.2953	2.2942	2.3285	2.2956
3.1704	1.0546	3.3445	3.1723	1.0730	3.1693	3.1709	3.1693	3.3265	3.1717
4.0051	1.4638	4.1842	4.0087	1.4831	4.0031	4.0044	4.0031	4.3244	4.0055
5.2159	2.1134	5.4044	5.2211	2.1330	5.2129	5.2138	5.2131	5.8213	5.2151

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL F-DISTRIBUTION
$\alpha = 0.90$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar et al.	Cox - Reid
						$\nu_1 = 20$				
	1	3.3668	0.4860	11.6316	3.8682	0.4876	3.3668	3.3668	3.3668	3.3670
	4	3.8448	0.5578	13.2061	4.4766	0.5595	3.8450	3.8450	3.8450	3.8480
5	10	4.7939	0.7070	16.3300	5.6676	0.7085	4.7946	4.7946	4.7945	4.8100
	16	5.7375	0.8607	19.4341	6.8381	0.8618	5.7385	5.7386	5.7384	5.7720
	25	7.1472	1.0959	24.0703	8.5727	1.0961	7.1485	7.1485	7.1483	7.2150
	32	8.2408	1.2812	27.6664	9.9116	1.2806	8.2422	8.2423	8.2420	8.3373
	1	1.8832	0.5818	2.1463	1.9221	0.5858	1.8832	1.8832	1.8832	1.8835
	4	2.1473	0.6688	2.4328	2.1951	0.6731	2.1474	2.1475	2.1474	2.1526
20	10	2.6638	0.8521	2.9933	2.7269	0.8567	2.6641	2.6643	2.6641	2.6908
	16	3.1707	1.0432	3.5436	3.2474	1.0477	3.1712	3.1714	3.1711	3.2289
	25	3.9207	1.3383	4.3581	4.0161	1.3423	3.9213	3.9215	3.9212	4.0361
	32	4.4986	1.5722	4.9859	4.6079	1.5758	4.4993	4.4995	4.4992	4.6640
	1	1.6461	0.6166	1.7678	1.6523	0.6219	1.6461	1.6461	1.6461	1.6465
	4	1.8755	0.7094	2.0018	1.8840	0.7152	1.8755	1.8756	1.8755	1.8817
50	10	2.3205	0.9062	2.4563	2.3323	0.9127	2.3203	2.3207	2.3203	2.3522
	16	2.7538	1.1127	2.8994	2.7682	1.1195	2.7536	2.7540	2.7536	2.8226
	25	3.3908	1.4336	3.5512	3.4084	1.4403	3.3904	3.3908	3.3905	3.5282
	32	3.8793	1.6892	4.0513	3.8990	1.6958	3.8788	3.8791	3.8788	4.0771
	1	1.5686	0.6309	1.6593	1.5694	0.6369	1.5686	1.5686	1.5686	1.5691
	4	1.7865	0.7261	1.8782	1.7883	0.7327	1.7864	1.7866	1.7864	1.7932
100	10	2.2076	0.9288	2.3016	2.2104	0.9362	2.2071	2.2074	2.2071	2.2415
	16	2.6159	1.1421	2.7129	2.6194	1.1500	2.6152	2.6155	2.6152	2.6898
	25	3.2139	1.4747	3.3158	3.2180	1.4828	3.2128	3.2131	3.2129	3.3623
	32	3.6711	1.7404	3.7769	3.6755	1.7486	3.6697	3.6699	3.6698	3.8853

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.90$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid
						$\nu_1 = 30$				
	1	3.2798	0.5054	11.1564	3.8293	0.5054	3.2798	3.2798	3.2798	3.2799
	4	3.5963	0.5551	12.1980	4.2266	0.5551	3.5963	3.5963	3.5963	3.5973
5	10	4.2266	0.6566	14.2721	5.0118	0.6563	4.2268	4.2268	4.2268	4.2321
	16	4.8546	0.7600	16.3380	5.7884	0.7594	4.8549	4.8549	4.8549	4.8669
	25	5.7938	0.9173	19.4272	6.9432	0.9162	5.7943	5.7943	5.7942	5.8191
	1	1.7960	0.6183	2.0183	1.8346	0.6200	1.7960	1.7960	1.7960	1.7962
	4	1.9682	0.6795	2.2051	2.0118	0.6813	1.9682	1.9682	1.9682	1.9700
20	10	2.3079	0.8056	2.5739	2.3609	0.8074	2.3080	2.3080	2.3080	2.3176
	16	2.6434	0.9350	2.9382	2.7051	0.9368	2.6436	2.6436	2.6436	2.6653
	25	3.1417	1.1333	3.4795	3.2157	1.1348	3.1419	3.1420	3.1419	3.1867
	1	1.5517	0.6630	1.6441	1.5588	0.6659	1.5517	1.5517	1.5517	1.5519
	4	1.6998	0.7289	1.7954	1.7079	0.7321	1.6998	1.6998	1.6998	1.7020
50	10	1.9904	0.8653	2.0925	2.0004	0.8688	1.9904	1.9905	1.9904	2.0024
	16	2.2759	1.0062	2.3845	2.2873	1.0097	2.2758	2.2759	2.2758	2.3028
	25	2.6976	1.2228	2.8161	2.7111	1.2264	2.6974	2.6976	2.6974	2.7533
	1	1.4699	0.6824	1.5347	1.4722	0.6860	1.4699	1.4699	1.4699	1.4701
	4	1.6099	0.7504	1.6755	1.6125	0.7542	1.6099	1.6099	1.6099	1.6124
100	10	1.8838	0.8915	1.9512	1.8868	0.8957	1.8836	1.8837	1.8836	1.8969
	16	2.1519	1.0376	2.2214	2.1551	1.0420	2.1515	2.1516	2.1515	2.1815
	25	2.5468	1.2630	2.6196	2.5503	1.2676	2.5462	2.5463	2.5462	2.6083

TABLE 1 (CONTINUED)
PROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.90$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 50$					
3.2100	0.5213	10.7768	3.7991	0.5201	3.2100	3.2100	3.2100	3.2100	3.2100
3.3986	0.5521	11.3973	4.0328	0.5509	3.3986	3.3986	3.3986	3.3989	3.3986
3.7752	0.6144	12.6359	4.4979	0.6130	3.7752	3.7752	3.7752	3.7765	3.7752
4.1510	0.6772	13.8722	4.9604	0.6756	4.1511	4.1511	4.1511	4.1542	4.1511
4.7138	0.7722	15.7232	5.6506	0.7702	4.7139	4.7139	4.7139	4.7207	4.7139
1.7234	0.6506	1.9130	1.7609	0.6507	1.7234	1.7234	1.7234	1.7234	1.7234
1.8243	0.6892	2.0226	1.8646	0.6893	1.8243	1.8243	1.8243	1.8248	1.8243
2.0250	0.7674	2.2405	2.0704	0.7675	2.0250	2.0250	2.0250	2.0275	2.0250
2.2244	0.8467	2.4571	2.2747	0.8467	2.2244	2.2244	2.2244	2.2303	2.2244
2.5216	0.9672	2.7801	2.5791	0.9669	2.5217	2.5217	2.5217	2.5344	2.5217
1.4697	0.7068	1.5388	1.4767	0.7080	1.4697	1.4697	1.4697	1.4698	1.4697
1.5556	0.7488	1.6267	1.5630	0.7500	1.5556	1.5556	1.5556	1.5562	1.5556
1.7258	0.8343	1.8008	1.7341	0.8356	1.7258	1.7258	1.7258	1.7291	1.7258
1.8944	0.9212	1.9733	1.9034	0.9225	1.8943	1.8944	1.8943	1.9020	1.8944
2.1448	1.0536	2.2297	2.1550	1.0549	2.1448	2.1448	2.1448	2.1614	2.1448
1.3819	0.7330	1.4261	1.3844	0.7347	1.3819	1.3819	1.3819	1.3819	1.3819
1.4625	0.7766	1.5073	1.4651	0.7785	1.4625	1.4625	1.4625	1.4632	1.4625
1.6220	0.8656	1.6681	1.6248	0.8675	1.6220	1.6220	1.6220	1.6258	1.6220
1.7796	0.9562	1.8270	1.7824	0.9582	1.7795	1.7795	1.7795	1.7883	1.7795
2.0132	1.0945	2.0627	2.0162	1.0966	2.0130	2.0131	2.0130	2.0322	2.0131

TABLE 1 (CONTINUED)
PROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.95$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox- Reid	Exact
				$\nu_1 = 30$					
7.143	0.0953	34.116	8.6369	0.1540	7.1534	7.1566	7.1531	7.2126	7.1540
11.949	0.3310	52.554	16.3730	0.4087	11.9845	11.9892	11.9803	12.6221	11.9863
21.035	1.0455	87.731	30.0605	1.1185	21.0757	21.0778	21.0679	23.4410	21.0766
29.923	1.8731	122.310	42.1045	1.9275	29.9592	29.9602	29.9510	34.2599	29.9596
43.140	3.1856	173.845	62.3088	3.2075	43.1692	43.1696	43.1618	50.4882	43.1694
4.057	0.0958	5.243	4.0789	0.1609	4.0546	4.0579	4.0546	4.1312	4.0619
6.507	0.3493	7.782	6.6061	0.4454	6.4899	6.4949	6.4907	7.2296	6.5042
10.786	1.1804	12.310	10.9611	1.2948	10.7535	10.7557	10.7566	13.4264	10.7636
14.790	2.2016	16.589	15.0263	2.3150	14.7536	14.7547	14.7584	19.6231	14.7604
20.604	3.8856	22.827	20.9236	3.9872	20.5672	20.5676	20.5732	28.9183	20.5712
3.646	0.0959	4.404	3.5942	0.1626	3.6406	3.6432	3.6406	3.7200	3.6489
5.785	0.3538	6.482	5.7380	0.4549	5.7537	5.7527	5.7543	6.5100	5.7697
9.418	1.2481	10.105	9.3599	1.3460	9.3589	9.3495	9.3617	12.0900	9.3712
12.748	2.3028	13.465	12.6826	2.4377	12.6776	12.6660	12.6824	17.6701	12.6867
17.510	4.1239	18.295	17.4373	4.2582	17.4336	17.4224	17.4407	26.0401	17.4399
3.520	0.0960	4.177	3.4537	0.1632	3.5137	3.5155	3.5137	3.5940	3.5223
5.563	0.3553	6.129	5.4881	0.4582	5.5276	5.5216	5.5280	6.2896	5.5440
8.998	1.2319	9.503	8.8981	1.3649	8.9279	8.9078	8.9296	11.6806	8.9407
12.116	2.3411	12.608	12.0022	2.4847	12.0309	12.0059	12.0342	17.0717	12.0409
16.542	4.2183	17.040	16.4136	4.3665	16.4462	16.4198	16.4514	25.1583	16.4534

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.95$

ν_2	λ	Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid
						$\nu_1 = 10$				
	1	5.2055	0.3217	22.1480	6.9421	0.3317	5.2057	5.2057	5.2057	5.2086
	4	6.5900	0.4247	27.5052	9.0815	0.4352	6.5917	6.5919	6.5916	6.6291
5	10	9.2969	0.6589	38.0181	13.1450	0.6667	9.3007	9.3009	9.3002	9.4701
	16	11.9663	0.9107	48.4120	17.0800	0.9138	11.9709	11.9711	11.9701	12.3112
	25	15.9403	1.3036	63.9084	22.8802	1.2983	15.9450	15.9451	15.9440	16.5727
	1	2.5789	0.3772	3.0368	2.6286	0.3974	2.5787	2.5788	2.5787	2.5827
	4	3.2395	0.5022	3.7319	3.3007	0.5256	3.2377	3.2379	3.2378	3.2870
20	10	4.4819	0.7956	5.0512	4.5610	0.8210	4.4775	4.4777	4.4779	4.6958
	16	5.6739	1.1195	6.3245	5.7688	1.1443	5.6680	5.6681	5.6686	6.1045
	25	7.4170	1.6349	8.1931	7.5350	1.6568	7.4101	7.4102	7.4111	8.2176
	1	2.2248	0.3942	2.4727	2.2354	0.4181	2.2244	2.2243	2.2244	2.2288
	4	2.7863	0.5265	3.0280	2.7926	0.5547	2.7829	2.7821	2.7830	2.8366
50	10	3.8245	0.8409	4.0668	3.8236	0.8732	3.8155	3.8136	3.8159	4.0523
	16	4.8059	1.1922	5.0564	4.8001	1.2261	4.7934	4.7911	4.7942	5.2680
	25	6.2248	1.7577	6.4935	6.2138	1.7916	6.2095	6.2070	6.2108	7.0915
	1	2.1153	0.4006	2.3181	2.1210	0.4261	2.1149	2.1146	2.1149	2.1194
	4	2.6459	0.5357	2.8348	2.6446	0.5658	2.6418	2.6401	2.6419	2.6974
100	10	3.6196	1.2214	3.7950	3.6078	0.8939	3.6085	3.6043	3.6087	3.8534
	16	4.5336	0.8586	4.7042	4.5146	1.2592	4.5177	4.5123	4.5183	5.0094
	25	5.8468	1.8085	6.0170	5.8203	1.8478	5.8267	5.8207	5.8278	6.7434

TABLE 1 (CONTINUED)
APPROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL F-DISTRIBUTION
$\alpha = 0.95$

ν_2	λ	patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar et al.	Cox - Reid
						$\nu_1 = 20$				
	1	4.7856	0.3895	19.6006	6.6025	0.3880	4.7856	4.7856	4.7856	4.7860
	4	5.4636	0.4477	22.2337	7.6236	0.4458	5.4638	5.4638	5.4638	5.4698
5	10	6.8063	0.5705	27.4574	9.6201	0.5668	6.8069	6.8069	6.8069	6.8372
	16	8.1385	0.6983	32.6476	11.5803	0.6923	8.1394	8.1394	8.1393	8.2046
	25	10.1259	0.8951	40.3988	14.4836	0.8852	10.1271	10.1271	10.1269	10.2558
	32	11.6664	1.0509	46.4108	16.7236	1.0376	11.6676	11.6677	11.6674	11.8511
	1	2.2298	0.4877	2.5318	2.2728	0.4945	2.2298	2.2298	2.2298	2.2304
	4	2.5408	0.5618	2.8618	2.5880	0.5692	2.5405	2.5405	2.5405	2.5490
	10	3.1448	0.7208	3.5054	3.1997	0.7285	3.1439	3.1439	3.1439	3.1862
20	16	3.7342	0.8887	4.1356	3.7965	0.8960	3.7328	3.7328	3.7329	3.8235
	25	4.6027	1.1506	5.0666	4.6760	1.1565	4.6008	4.6008	4.6010	4.7793
	32	5.2701	1.3595	5.7833	5.3519	1.3640	5.2680	5.2680	5.2683	5.5228
	1	1.8727	0.5240	2.0120	1.8838	0.5344	1.8726	1.8726	1.8726	1.8733
	4	2.1318	0.6042	2.2709	2.1407	0.6157	2.1312	2.1310	2.1312	2.1410
50	10	2.6296	0.7779	2.7711	2.6348	0.7906	2.6275	2.6270	2.6276	2.6762
	16	3.1105	0.9628	3.2566	3.1129	0.9762	3.1072	3.1065	3.1074	3.2114
	25	3.8132	1.2535	3.9682	3.8124	1.2668	3.8086	3.8078	3.8089	4.0143
	32	4.3498	1.4868	4.5127	4.3469	1.4998	4.3445	4.3437	4.3449	4.6387
	1	1.7596	0.5389	1.8664	1.7680	0.5510	1.7595	1.7594	1.7595	1.7603
	4	2.0020	0.6218	2.1053	2.0074	0.6351	2.0013	2.0008	2.0013	2.0117
100	10	2.4655	0.8018	2.5644	2.4658	0.8169	2.4627	2.4615	2.4628	2.5147
	16	2.9108	0.9944	3.0079	2.9073	1.0105	2.9063	2.9046	2.9065	3.0176
	25	3.5583	1.2983	3.6550	3.5503	1.3152	3.5519	3.5498	3.5522	3.7720
	32	4.0508	1.5432	4.1484	4.0400	1.5602	4.0433	4.0411	4.0438	4.3587

TABLE 1 (CONTINUED)
ROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.95$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 30$					
4.6454	0.4135	18.7538	6.4924	0.4086	4.6454	4.6454	4.6454	4.6456	4.6454
5.0932	0.4544	20.4956	7.1585	0.4490	5.0932	5.0932	5.0932	5.0951	5.0932
5.9837	0.5385	23.9637	8.4739	0.5317	5.9839	5.9839	5.9839	5.9943	5.9839
6.8698	0.6247	27.4178	9.7741	0.6163	6.8701	6.8701	6.8701	6.8934	6.8701
8.1938	0.7565	32.5825	11.7067	0.7455	8.1943	8.1943	8.1942	8.2421	8.1943
2.1069	0.5327	2.3566	2.1453	0.5351	2.1069	2.1069	2.1069	2.1071	2.1069
2.3082	0.5858	2.5711	2.3491	0.5883	2.3081	2.3081	2.3081	2.3110	2.3081
2.7039	0.6964	2.9937	2.7498	0.6987	2.7036	2.7036	2.7037	2.7188	2.7037
3.0934	0.8108	3.4106	3.1440	0.8129	3.0929	3.0929	3.0929	3.1266	3.0930
3.6700	0.9874	4.0290	3.7278	0.9885	3.6692	3.6692	3.6693	3.7383	3.6693
1.7432	0.5811	1.8461	1.7520	0.5869	1.7432	1.7432	1.7432	1.7434	1.7432
1.9089	0.6394	2.0126	1.9164	0.6456	1.9087	1.9086	1.9087	1.9121	1.9087
2.2322	0.7614	2.3384	2.2375	0.7682	2.2315	2.2313	2.2315	2.2495	2.2316
2.5481	0.8886	2.6576	2.5515	0.8956	2.5467	2.5464	2.5468	2.5870	2.5469
3.0126	1.0859	3.1283	3.0139	1.0929	3.0106	3.0103	3.0108	3.0931	3.0108
1.6255	0.6023	1.7004	1.6324	0.6097	1.6255	1.6255	1.6255	1.6257	1.6255
1.7796	0.6629	1.8530	1.7848	0.6708	1.7793	1.7792	1.7793	1.7831	1.7794
2.0790	0.7902	2.1506	2.0812	0.7990	2.0780	2.0775	2.0780	2.0977	2.0781
2.3703	0.9233	2.4411	2.3700	0.9326	2.3684	2.3676	2.3685	2.4124	2.3686
2.7970	1.1306	2.8678	2.7937	1.1403	2.7941	2.7931	2.7942	2.8844	2.7943

TABLE 1 (CONTINUED)
ROXIMATE AND EXACT PERCENTILES OF THE NONCENTRAL *F*-DISTRIBUTION
$\alpha = 0.95$

Patnaik	Laubscher 1st	Laubscher 2nd	Laubscher 3rd	Severo-Zelen	Tiku	Tiku - Yip	Mudholkar *et al.*	Cox - Reid	Exact
				$\nu_1 = 50$					
4.5333	0.4332	18.0773	6.4055	0.4258	4.5333	4.5333	4.5333	4.5333	4.5333
4.7995	0.4589	19.1148	6.7970	0.4510	4.7995	4.7995	4.7995	4.8000	4.7995
5.3307	0.5109	21.1858	7.5756	0.5021	5.3307	5.3307	5.3307	5.3333	5.3307
5.8606	0.5635	23.2526	8.3496	0.5537	5.8606	5.8606	5.8606	5.8666	5.8606
6.6536	0.6433	26.3472	9.5046	0.6318	6.6537	6.6537	6.6537	6.6666	6.6537
2.0049	0.5728	2.2126	2.0385	0.5718	2.0049	2.0049	2.0049	2.0049	2.0049
2.1222	0.6069	2.3381	2.1572	0.6058	2.1222	2.1222	2.1222	2.1229	2.1222
2.3549	0.6763	2.5874	2.3928	0.6750	2.3549	2.3549	2.3549	2.3588	2.3549
2.5857	0.7470	2.8348	2.6264	0.7453	2.5856	2.5856	2.5856	2.5946	2.5856
2.9292	0.8547	3.2035	2.9742	0.8524	2.9290	2.9290	2.9290	2.9484	2.9291
1.6314	0.6356	1.7052	1.6372	0.6377	1.6314	1.6314	1.6314	1.6315	1.6314
1.7266	0.6736	1.8013	1.7317	0.6758	1.7266	1.7266	1.7266	1.7275	1.7266
1.9147	0.7711	1.9915	1.9187	0.7534	1.9145	1.9145	1.9145	1.9194	1.9145
2.1004	0.8304	2.1796	2.1034	0.8327	2.1000	2.1000	2.1000	2.1113	2.1001
2.3756	0.9517	2.4586	2.3773	0.9539	2.3750	2.3749	2.3750	2.3992	2.3750
1.5067	0.6653	1.5560	1.51111	0.6689	1.5067	1.5067	1.5067	1.5086	1.5067
1.5945	0.7051	1.6434	1.5980	0.7089	1.5944	1.5944	1.5944	1.5954	1.5944
1.7674	0.7866	1.8160	1.7695	0.7906	1.7672	1.7670	1.7672	1.7727	1.7672
1.9377	0.8701	1.9862	1.9385	0.8744	1.9371	1.9369	1.9371	1.9499	1.9372
2.1892	0.9983	2.2380	2.1884	1.0027	2.1883	2.1880	2.1883	2.2158	2.1884