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THE ESTIMATION OF MULTIPLE - OUTPUT GENERALIZED BOX-COX COST FUNCTION AND CHOICE AMONG SOME FLEXIBLE FUNCTIONAL FORMS

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ABSTRACT

In this paper we estimate a multiple – output generalization of Khaled's BOX-COX model to study the cost structure and growth in productivity in Egyptian Textile industry. The estimated model includes several commonly used some flexible functional forms as special or limiting cases. Total factor productivity is estimated parametrically rather than being computed as the residual of growth in outputs minus growth in inputs. These findings were robust across cost function specifications, casting some doubt on the importance of choosing among flexible functional forms.

1. Introduction

In recent years a great deal of researcher has been directed to the modeling and measurement of parametric productivity. The literature has devoted considerable attention to the specification of increasingly general cost model. For example, Khaled (1978); Pollak et al. (1984) and Diewert et al (1987). The above studies deal with single. output technologies. However, one expects it to be a simple exercise to generalize the suggested specifications to a multiple-output environment.

Despite this observation, almost all empirical multiple-output models have been based on the translog approximation [see, e.g., Caves et al. (1981), Ferrier and Lovell (1988)]. The purpose of the present paper is

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to estimate a multiple-output generalization of Khaled's Box-Cox model. The proposed specification is applicable to total as well as variable cost functions, and in the case of a variable cost function, it allows the straightforward introduction of multiple fixed factors. It yields the translog a generalized leontief and generalized square-root quadratic as special or limiting cases so that the relative performance of these models can be evaluated in a multiple-output environment.

The proposed model is applied to analyze the cost structure and productivity growth of Textile industry in Egypt over period (1987-1997).

Textile & Weaving industry arises in Egypt since long time . This industry has all basic structures for support it. These structures include: material such as cotton, active labor, and local & foreign markets. This industry concentrate on cotton, wool, and other materials. It is appear from the analysis of the materiality of this sector that it contributes about 28.2 % from the total industrial production, and about 22.8 % from the total of industrial sails. In addition , it contributes about 49 .2 % from total labor. On the anothег side, this sector provides about 38.8% from the added value of industry sector in Egypt.

The study is based on the data of El-Sharkia for Textile and Weaving company over the period (1987 -1997) as a sample for this sector.

The paper is organized as follows: In section 2 we present the multiple-output Box-Cox cost function, and review its economic properties. In section 3 we implement the model to estimate the structure of Textile costs and productivity growth in Egypt, also empirical results are discussed in this section. Finally, section 4 concludes the summary and conclusions.

2- A generalized Box–Cox (GBC) cost function for multiple–output technologies :

In Textile industry there exits production function relating the flow of gross output (**Y**) to the services of four inputs : capital (**K**), labor (**L**), energy (**E**) and all other intermediate materials (**M**). In this section we present a straightforward multiple-output generalization of the specification suggested by (Khaled (1978)). It encompasses the generalized leontief (**GL**) as a special case and the multiple-output translog (**TL**) as a limiting case, the adequacy of which can be empirically evaluated. The proposed specification directly applies to both unrestricted and restricted cost functions due to the symmetric treatment of outputs and fixed factors.

To be precise, we consider the following cost function specification, allowing for technical change:

$$TC \equiv \left[1+\lambda G(P)\right]^{1/\lambda}\left[\prod_{h=1}^{H} Y_h^{\beta_h(Y,P)}\right]\left[e^{T(t,p)}\right] \qquad (1)$$

where

$$G(P) \equiv \alpha_0 + \sum_{i=1}^{N}\alpha_i P_i(\lambda) + \sum_{i=1}^{N}\sum_{j=1}^{N}\delta_{ij}\, P_i(\lambda)P_j(\lambda), \qquad (2)$$

$$\beta_h(Y,P) \equiv \beta_h + \sum_{t=1}^{H}\frac{\theta_{th}}{2}\ln Y_t + \sum_{i=1}^{N}\phi_i \ln P_i, \qquad (3)$$

$$P_i(\lambda) \equiv \left(P_i^{\lambda/2}-1\right)/\lambda/2, \qquad (4)$$

$$T(t,p) \equiv t\left(\tau + \sum_{i=1}^{N}\tau_i \ln P_i\right) \qquad (5)$$

The vector P consists of the prices of N variable inputs, i.e., $P=(P_1,P_2,\ldots\ldots\ldots,P_N)$. The vector Y is assumed to consist of R outputs and

(H-R) fixed factors, where $H \geqq R$. For H=R, eqs. (1)-(5) describe on unrestricted total cost model. If H >R, the model should be interpreted as a short-run restricted cost function[1]

Technical change is captured via equ. (5). It is said to be input i saving, i neutral or i using depending on whether τ_i is less than, equal to, or greater than zero. Note that, this is not the most general specification possible. Indeeded, it implies a rate of cost diminution given by

$$\frac{\partial \ln TC}{\partial t} = \tau + \sum_{i=1}^{N} \tau_i \ln P_i \qquad (6)$$

Technical change is Hicks neutral[2] at a constant exponential rate of τ if $\tau = 0$ for all i.

By assumption, $\delta_{ij} = \delta_{ji}$ and $\theta_{ih} = \theta_{hi}$ Moreover, linear homogeneity in input prices can be shown to require the following restrictions on the parameters.

$$\sum_{i=1}^{N} \alpha_i = 1 + \lambda \alpha_0, \qquad (7)$$

$$\sum_{j=1}^{N} \delta_{ij} = \frac{\lambda}{2} \alpha_i \forall i, \qquad (8)$$

$$\sum_{i=1}^{N} \phi_{hi} = 0 \quad \forall h, \qquad (9)$$

$$\sum_{i=1}^{N} \tau_i = 0. \qquad (10)$$

Imposing these restrictions on (1) yields, after some algebra,

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(1) Note that, in total there are (N+H-R) factors of production. To keep notation as simple as possible we do not use different symbols for outputs and fixed factors.

(2) This definition assumes movement along an expansion path and corresponds with the notion of "extended Hicks neutral technical change" discussed by Blackorby, Lovell, and Thursby (1976).

$$TC = \left[\frac{2}{\lambda}\sum_{i=1}^{N}\sum_{j=1}^{N}\delta_{ij}P_i^{\lambda/2}P_j^{\lambda/2}\right]^{1/\lambda}\left[\prod_{h=1}^{H} Y_h^{\beta_h(Y,P)}\right]\left[e^{T(t,p)}\right]. \qquad (11)$$

The GBC cost model (11) includes several special cases. If we set $\lambda=1$ we obtain a generalized leontief, (GL) as

$$TC = 2\sum_{i=1}^{N}\sum_{J=1}^{N}\delta_{ij}P_i^{1/2}P_j^{1/2}\left[\prod_{h=1}^{H} Y_h^{\beta_h(y,p)}\right]\left[e^{T(t,p)}\right]. \qquad (12)$$

Also, when $\lambda=2$, we obtain the (GSR) as

$$TC = \left[\sum_{i=1}^{N}\sum_{J=1}^{N}\delta_{ij}P_i\,P_j\right]^{1/2}\left[\prod_{h=1}^{H} Y_h^{\beta_h(y,p)}\right]\left[e^{T(t,p)}\right]. \qquad (13)$$

Moreover, it is easily shown that when λ approaches zero, relation (11) converges to the following translog specification[*]

$$\ln TC = \alpha_0 + \sum_{i=1}^{N}\alpha_i \ln P_i + 1/2\sum_{i=1}^{N}\sum_{j=1}^{N}\delta_{ij}\ln P_i \ln P_j + \sum_{h=1}^{H}\beta_h \ln Y_h$$
$$+ \sum_{h=1}^{H}\sum_{\iota=1}^{H}\frac{\theta_{\iota h}}{2}\ln Y_\iota \ln Y_h + \sum_{h=1}^{H}\sum_{i=1}^{N}\phi_{hi}\ln P_i \ln Y_h$$
$$+ \tau t + \sum_{i=1}^{N}\tau_i t \ln P_i . \qquad (14)$$

The properties of (11) are easily derived. First, to obtain derived demand system corresponding to the proposed cost function, we differentiate (11) with respect to the exogenous input prices and then employ shephard's Lemma [*], which yields the input - output equations.

$$Xi = \left[\frac{2}{\lambda}\sum_{j=1}^{N}\delta_{ij}\left(P_i/P_j\right)^{\lambda/2}\right]\left[\prod_{h=1}^{H} Y_h^{\lambda\beta(Y,P)}\right]$$

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[*] To see this, solve (1) for G(P) and note that the result is the Box-Cox transformation of the expression $\left[TC / \sum_{h=1}^{H} Y_h^{\beta(Y,P)}\right]$ Therefore, for λ approaching zero G(P) converges to $\left[\ln TC = \sum_{h=1}^{H}\beta_h(Y,P)\ln Y_h\right]$. Using this result together with (2) and (3), and noting that $P_i(\lambda)$ Converges to $\ln P_i$, we find that (11) converges to (14) when λ approaches to zero.

[*] For further discussion, see Diewert (1971)

$$\cdot\left[e^{\lambda T(t,p)}\right](C/P_i)^{1-\lambda} + \sum_{h=1}^{H}(\phi_{hi}\ln Y_h + \tau_i t)\frac{C}{Pi} \quad (15)$$

where unit cost $C \equiv C/Y$.

Second, the Allen partial elasticities of substitution σ_{ij} can be calculated to be.

$$\sigma_{ij} = 1-\lambda+\delta_{ij}\frac{(P_iP_j)^{\lambda/2}}{S_iS_j}Z+\lambda\frac{F_j(Y,t)}{S_j}$$
$$+\lambda\left[1-\frac{F_j(Y,t)}{S_j}\right]\frac{F_i(Y,t)}{S_i}, \quad i \neq j, \quad (16)$$

and

$$\sigma_{ii} = 1-\lambda+\delta_{ii}\frac{P_i^{\lambda}}{S_i^2}Z+\lambda\frac{F_i(Y,t)}{S_i}$$
$$+\lambda\left[1-\frac{F_i(Y,t)}{S_i}\right]\frac{F_i(Y,t)}{S_i}+\frac{\lambda}{2}\left[1-\frac{F_i(Y,t)}{S_i}\right]\frac{1}{S_i}-\frac{1}{S_i}, \quad (17)$$

where

$$Z=\left[\frac{2}{\lambda}\sum_{i=1}^{N}\sum_{j=1}^{N}\delta_{ij}P_i^{\lambda/2}P_j^{\lambda/2}\right]^{-1}$$

$$F_i(Y,t)=\sum_{h=1}^{H}\phi_{hi}\ln Y_h+\tau_i t, \quad i=1,....,N$$

$$S_i = P_i X_i/C, \quad i=1,....,N$$

The associated input price elasticities η_{ij} are computed as

$$\eta_{ij} = S_j\sigma_{ij} \quad , i, J = 1,........., N \quad (18)$$

Third, the cost elasticity with respect to the elements of the vector Y are found to be given by

$$\epsilon_h = \frac{\partial \ln C}{\partial \ln Y_h} = \beta_h + \sum_{t=1}^{H}\phi_{th}\ln Y_t + \sum_{i=1}^{N}\phi_{hi}\ln P_i. \quad (19)$$

These elasticities can be used to drive expression describing the degree of returns to scale. If the cost model describes a restricted cost function (i.e H>R), Caves and et al. (1981) have shown that the degree of returns to scale is

$$RTS = \left(1 - \sum_{h=R+1}^{N} \epsilon_h\right) \Big/ \sum_{k=1}^{R} \epsilon_h . \qquad (20)$$

We have increasing returns to scale if RTS is greater than, equal to, or smaller than one. If (11) and (15) refer to are unrestricted total cost function, i.e h = R, (20) reduces to the inverse of the sum of output cost elasticities, Viz.

$$RTS = 1 \Big/ \sum_{h=1}^{H} \epsilon_h \qquad (21)$$

Finally, we again follow Caves et al. (1981) and defined two productivity measures. The first one, W_1, is defined as the common rate at which outputs can grow over time with all inputs held constant. One easily shows that.

$$W_1 = - \epsilon_t \Big/ \sum_{h=1}^{R} \epsilon_h , \qquad (22)$$

where ϵ_t is the rate of cost diminution, given by (6). A second index, W_2, is the common rate at which inputs can be reduced over time with all outputs held at a fixed level. One finds.

$$W_2 = - \epsilon_t \Big/ \left(1 - \sum_{h=R+1}^{H} \epsilon_h\right) . \qquad (23)$$

Note that these two productivity indices will differ, unless there are constant returns to scale. Indeed, RTS =1 implies $W_1=W_2$ [see equation (20)].

3- Costs and productivity in Egyptian Textile operations: Estimation and empirical results.

Earlier we noted that the GBC form takes on the GSR, GL, and TL cost functions as special or limiting cases according as $\lambda = 2$, $\lambda = 1$ and $\lambda = 0$, respectively. We now examine whether the most general GBC model permits us to discriminate among the various flexible functional forms.

As a case study, gross output Quantity and input, output coefficients in Textile manufacturing in El-Sharkia for Textile and Weaving company , 1987 - 1997, are shown in Table 1.

TABLE (1)

GROSS OUTPUT QUANTITY AND INPUT/OUTPUT COEFFICIENTS IN TEXTILE ANUFACTURING IN EL-SHARKIA FOR TEXTILE AND WEAVING COMPANY, 1987 - 1997*

Year	Y_1	Y_2	K_1/Y_1	K_2/Y_2	L_1/Y_1	L_2/Y_2	E_1/Y_1	E_2/Y_2	M_1/Y_1	M_2/Y_2
1986/1987	22150408	3242097	0.709237	0.1429969	0.0290365	0.00497873	0.0141599	0.0024279	0.6009166	0.1027328
1987/1988	38389827	2752706	0.4317887	0.033352	0.0195638	0.00151116	0.0195638	0.00151116	0.4786025	0.0369685
1988/1989	46426578	5801963	0.2491776	0.0355872	0.0140116	0.00200113	0.0299501	0.0042818	0.5640613	0.0805586
1989/1990	64809422	2996711	0.192043	0.0093103	0.0123696	0.00059968	0.013645	0.0117376	0.3426239	0.294759
1990/1991	71799203	3372981	0.188383	0.0092861	0.0107633	0.00053056	0.203036	0.00100084	0.5927568	0.0292191
1991/1992	85567000	2221000	0.3428875	0.0091372	0.0103679	0.00027628	0.0101764	0.00046778	0.6329813	0.016567
1992/1993	88746000	2558000	0.6249047	0.0185467	0.0066825	0.00270939	0.0292495	0.00091100	0.593955	0.0176281

1993/1994	8256000	1739000	0.7244164	0.1933036	0.00768802	0.00205147	0.0151225	0.0040352	0.5147041	0.1373438
1994/1995	118104000	3385000	0.5909694	0.0174376	0.0062699	0.00018500	0.0147958	0.00043657	0.6245914	0.0184296
1995/1996	131227000	3329000	0.7831631	0.0203846	0.00381626	0.00099331	0.0179766	0.0004679	0.6171306	0.0160629
1996/1997	95121000	5124000	0. 816644	0.0464959	0.00731454	0.00041645	0.0170179	0.00096892	0.489040	0.0278435

* **Source:**

The management of El-Sharkia company for Textile and Weaving in Zagazig.

Remark:

Y_1 and Y_2 represent the gross output of cotton Textile and wool Textile respectively .
K_1 and K_2 represent the capital which are used in cotton Textile and wool Textile respectively .
L_1 and L_2 represent the labor which are used in cotton Textile and wool Textile respectively .
E_1 and E_2 represent the energy which are used in cotton Textile and wool Textile respectively .
M_1 and M_2 represent all other intermediate materials which are used in cotton Textile and wool Textile respectively .

Estimation results(*) for the generalized Box-Cox (GBC) and generalized leontief (GL) models are presented in Table (2). The estimates are satisfactory. In both specifications, the majority of coefficients is significantly different from zero.

Moreover, both models fitted the data very well, with $R^{2(*)}$ of 0.934 and 0.917 for the GBC and GL respectively.

Not that the parameter vectors β, θ, ϕ and τ are quite similar in the two specifications. On the other hand, the large differences in the estimated δ_{ij} are not suprising. since imposing $\lambda=1$ in the GL case, directly affects the order of magnitude of the vector δ via the restrictions (7) and (8).

(*) Not that, the sample Log-likelihood function is:

$$\ln L = -\frac{(N-1)T(\ln 2\Pi + 1)}{2} - \frac{T}{2}\ln|\hat{\Omega}| + \sum_{t=1}^{T} \ln\|J_t\|,$$

$\|J_t\| = 1 - (1-\lambda)(C_t/C_t^*)^{\lambda}$. where C_t and C_t^* are the minimum cost and unit cost respectively and T is the total number of observations in each equation.

- For further discussion, see Berendt (1977).

(*) Not that,

Likelihood- ratio test statistic = $-T \ln(1-\tilde{R}^2)$,

generalized $R^2 = \tilde{R}^2 = \{1 - \exp[2(L_0 - L_{max})/T\}$,

Where L_0 is the sample maximum of the lagarthim of the above likelihood function when all δ_{ij}, ϕ_i and T_i are constrained to zero; L_{max} is the maximum when all these coefficients are included in the model.

TABLE (2)

ESTIMATED GENERALIZED BOX-COX (GBC) AND GENERALIZED LEONTIEF (GL) SHORT- RUN VARIABLE COST FUNCTIONS

(Asymptotic stand and errors in parentheses)

Model / Parameter	GBC		GL	
	Estimate	Standard Error	Estimate	Standard Error
δ_{LL}	0.3141	(0.0112)[a*]	0.6213	(0.0112)[a]
δ_{LE}	0.0218	(0.0109)[a]	0.0931	(0.129)[a]
δ_{EE}	0.0032	(0.0230)	-0.0152	(0.0129)
β_R	0.5123	(0.1276)[a]	0.4989	(0.2101)[a]
β_F	0.4121	(0.0426)[a]	0.3712	(0.0812)[a]
β_K	-0.1329	(0.0723)	-0.0968	(0.0521)
θ_{RR}	0.3216	(0.0871)[a]	0.2241	(0.0898)[a]
θ_{RF}	0.3712	(0.1113)[a]	0.3123	(0.2139)[a]
θ_{RK}	-0.0976	(0.0576)	-0.1168	(0.0689)
θ_{FF}	-1.6176	(0.4783)[a]	-1.2987	(0.4987)[a]
θ_{FK}	0.0235	(0.0326)	0.0784	(0.0527)
θ_{KK}	0.2187	(0.0611)[a]	0.4112	(0.0498)[a]
ϕ_{RL}	-0.2019	(0.0098)[a]	-0.2102	(0.0098)[a]
ϕ_{FL}	-0.0044	(0.0108)	-0.0071	(0.0131)
ϕ_{KL}	0.0442	(0.0107)[a]	0.0412	(0.0129)[a]
τ_T	0.0210	(0.0006)[a]	-0.0311	(0.0006)[a]
τ_L	0.0051	(0.0005)[a]	0.0071	(0.0007)[a]
λ	0.5988	(0.2101)[a]	1	

* [a] Indicates significant at the 5 percent level.

A Wald-test was used to test the appropriateness of the GL model. The test statistic, estimated to be 5.74 has a χ^2 distribution with 1 degree of freedom. This implies that the GL can be rejected at the 5% significance level (critical value 3.84) but not at the 1% level (critical value 6.63).

Several other restricted versions of the GBC model were estimated. homotheticity, implying all elements. If the vector ϕ is equal to zero, was decisively rejected. This automatically implies rejection of homogeneity in inputs and constant returns to scale since these are nested within the homothetic version. We also tested for neutrality of technical change, which would imply $\tau_t = 0$. It was rejected in favor of energy-saving technical progress, as suggested by the positive sign of τ_t. Apparently, technical progress has been driven by the massive electrification program rather than by improvements in labor productivity.

Strictly speaking the translog model (TL) cannot be obtained by imposing the restriction $\lambda \rightarrow 0$ directly on the estimation procedure. For purposes of comparison we did estimate a (TL) model, however. Estimation results are in Table 3. To ease the comparison with the (GBC) and (GL) models not that, given our two-variable factor model, the restrictions (7) and (8) reduce in the TL case to $\alpha_L + \alpha_E = 1$ and $\delta_{LL} = \delta_{LE} = -\delta_{EL}$, respectively, Therefore, the (GBC) parameters λ, δ_{LL}, δ_{LE}, δ_{EE}, are in the TL model replaced by the free parameters α_L, δ_{LL} and α_0

TABLE (3)

ESTIMATION RESULT TRANSLOG SHORT-RUN VARIABLE COST FUNCTION

(Asymptotic standard errors in parentheses)

Parameter	Estimate	Standard error
α_0	0.2131	(0.0112)[a]
α_L	0.9131	(0.0052)[a]
δ_{LL}	0.0121	(0.0087)
β_R	0.5002	(0.0361)[a]
β_F	0.3123	(0.1092)[a]
β_K	-0.0839	(0.0421)
θ_{RR}	0.1982	(0.1729)
θ_{RF}	0.5231	(0.1725)[a]
θ_{RK}	-0.2138	(0.2102)
θ_{FF}	-2.3618	(1.1262)[a]
θ_{FK}	0.1021	(0.3123)
θ_{KK}	0.5823	(0.1812)[a]
ϕ_{RL}	-0.1095	(0.0101)[a]
ϕ_{FL}	-0.0203	(0.0211)
ϕ_{KL}	0.0621	(0,0121)[a]
τ_T	-0.0191	(0.0022)[a]
τ_{Tt}	0.0062	(0.0008)[a]

a Indicates significant at the 5 percent level.

The estimates in Tables 2 and 3 capture information about the production process and the evaluation of technical change over time.

In Table 4 we present estimates of the elasticity of substitution between labor and energy and the associated input price elasticities for each of the three specifications (GBC, GL, TL) and for each subperiods several observations are in order. First, consider the differences between specifications. The GBC and GL yield very similar estimates except for σ_{LE} in the final subperiod. The TL specification shows less varibiltiy in σ_{LE} over time and consistently higher price elasticities. Second, it is clear that despite these minor differences the economic implications of the three models are very similar. Labor demand is very inelastic through out the sample period.

Estimated elasticities range between - 0.04 and - 0.229. Energy price elasticities also point at inelastic demand; they vary from -0.5 to - 0.76 depending on the subperiod and the specification.

TABLE (4)

ESTIMATED PRICE AND SUBSTITUTION ELASTICITIES

Elasticity of substitution between labor and energy σ_{LE}				Price elasticity of demand for labor η_{LL}			Price elasticity of demand for energy η_{EE}		
Period	GBC	GL	TL	GBC	GL	TL	GBC	GL	TL
1987-1989	1.1019	1.0213	0.8738	-0.2311	-0.213	-0.229	-0.7210	-0.611	-0.7213
1990-1994	0.7981	0.8421	0.8014	-0.0721	-0.0291	-0.0521	-0.6221	-0.5191	-0.07829
1995-1997	0.06239	0.6219	0.9213	-0.0472	-0.0731	-0.0932	-0.6821	-0.6112	-0.7989

Estimates of the cost elasticity with respect to capital stock are reported in Table 5. Again, note that all these cost function specifications yield qualitatively similar results.

TABLE (5)
ESTIMATED COST ELASTICITY WITH RESPECT TO CAPITAL STOCK ($\in_K$)

(Asymptotic standard errors in parentheses)

model / Period	GBC	GL	TL
1987-1989	-0.2131	-0.2431	-0.1410
	(0.0628)	(0.0612)	(0.0721)
1990-1994	-0.2319	-0.2117	-0.2239
	(0.0892)	(0.0603)	(0.0791)
1995-1997	-0.0985	-0.0631	-0.0369
	(0.0319)	(0.0573)	(0.0339)

Despite not being Legally allowed to do so, the desire to prevent a further decline in output and the existence of a soft budget constraint may have induced the firm to price under marginal cost. However, it is even not clear that this was an entirely deliberate policy.

Next, consider Table (6). There we present estimates of returns to scale indicator RTS as well as the two productivity indices W_1 and W_2 with respect to scale economies, the differences between specifications are again reasonably small. The reported point estimates suggest economies of scale in the first and second subperiods. The final subperiod indicates slight diseconomies of scale.

It should be noted, however, that the estimated scale economies early in the sample period and the diseconomies in the final decade are mild and that, more importantly, they were not significantly different from one.

TABLE (6)

ESTIMATED SCALE ECONOMIES AND PRODUCTIVITY

Economics of scale indicator. RTS				Productivity growth index. $W_1\%$			Productivity growth index $W_2\%$		
Period	GBC	GL	TL	GBC	GL	TL	GBC	GL	TL
1987-1989	1.2176	1.1553	1.1607	2.0161	1.9625	1.8771	1.6558	1.6987	1.6172
	(0.3814)	(0.3714)	(0.4011)	(0.2623)	(0.2518)	(0.3621)	(0.4941)	(0.5316)	(0.6423)
1990-1994	1.3378	1.2338	1.4439	2.3720	1.9415	1.8142	1.7731	1.5736	1.2565
	(0.4279)	(0.4053)	(0.4326)	(0.3217)	(0.3081)	(0.3982)	(0.5489)	(0.5551)	(0.6017)
1995-1997	0.9181	0.8805	0.8475	1.7263	1.5553	1.6123	1.8803	1.7664	1.9137
	(0.3589)	(0.3472)	(0.3319)	(0.4196)	(0.4076)	(0.5117)	(0.5821)	(0.5977)	(0.5741)

Finally consider the estimated indices of technical change. They are presented as average annual percentage increases in productivity over the different subperiods. First, note that the indices W_1 and W_2 lead to some what different results. This is not surprising since they would have yielded the some result if and only if RTS=1 would have prevailed throughout the sample period. Depending on the specification and the subperiod, average annual increases inproductivity range between 1.3% and 2.4% Furthermore, note that there is no conclusive evidence that technical progress has slowdown over time. Although W_1 suggests a slowdown over the last subperiod, this conclusion is not supported by the evaluation in W_2. It should be noted that the absence of more variation in the productivity changes over time may be due to the particular method used to incorporate technical progress into the empirical cost models. As indicated in section 2 we imposed a lot of structure on the evolution of technical change by making it independent of outputs and capital stock. The advantage of this procedure is that it reduces the technical problems (e.g., multicollinearity associated with more general specifications of productivity growth. The cost of the procedure is that by imposing (too) much structure on the specification, some interesting aspects related to differences in technical progress over time may not be identifiable.

To conclude this empirical section we may note that, contrary to the findings of Diwert and Wales (1987), we did not find important quantitative deferences in estimated economic characteristics of the firm between different specifications of the cost function.

4- Conclusion

In this paper we estimated a multiple-output generalization of khaled's **Box-Cox** cost function specification. It generates the generalized **GL** leontief as a special case and the **TL** as a limiting case. The proposed specification captures both total and restricted cost models. If it used to describe a restricted cost function the introduction of multiple fixed factors is straightforward.

We applied the model to study the cost structure and the evolution of productivity growth in Textile operations in Egypt over period (1987-1997). The proposed cost model was estimated using iterative three stage least squares. We found input price and substitution elasticities well with in the range of estimates reported in the literature. The results further suggested economies of scale early in the sample period. No economies of scale were found for the final years of the sample period, however. Average annual rates of productivity growth, obtained on the basis of the generalized **Box-Cox** model, ranged between 1.3% and 2.4% depending on the subperiod and the productivity index used.

To compare the performance of the proposed specification with the transloge and leontief models we also estimated the parameters of these other cost functions. Contrary to the findings of Diewert and wales (1987) we did not find important quantitative differences in the estimates of the technological characteristics of the firm between the three specifications considered in this paper. As there is obviously no guarantee that this will be the case in other applications we view this paper as either providing a general alternative cost model in a multi-output environment, or as providing a tool for evaluating the relative performance of different empirical models

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ملخص البحث

تقدير دالة التكاليف فى ضوء نموذج بوكس-كوكس المعمم للناتج المتعدد مع المقارنة ببعض الأشكال الدالية الأخرى.

يقدم هذا البحث نموذج مقترح لتقدير داله التكاليف فى ضوء نموذج بوكس-كوكس المعمم للناتج المتعدد وذلك بهدف دراسة كل من مكونات التكاليف ومعدلات النمو فى انتاجية الوحدات الصناعية.

حيث يعتبر النموذج المقترح حاله عامة للعديد من الأشكال الدالية المعروفة فى هذا المجال، مثل نموذجى ليونيتيف المعمم (GL) الجذور التربيعية المعممة (GSR) كحالات خاصة ونموذج اللوغاريتمات المحوله (TL) كحاله محدده.

وقد تم تطبيق نتائج الدراسة عن بيانات تكاليف الأنتاج لشركة الشرقية للغزل والنسيج كإحدى الشركات التابعة لقطاع الغزل والنسيج فى جمهورية مصر العربية والبيانات عبارة عن سلسلة زمنية للفترة من ١٩٨٧ حتى ١٩٩٧. وقد أكدت نتائج الدراسة مدى التطابق بين نتائج مقدارات النموذج المقترح ومقدرات الاشكال الدالية الأخرى المقارنة، مما يؤكد أمكانية الاعتماد على النموذج المقترح فى دراسة تكاليف الإنتاج وكذلك دراسة معدلات النمو السنوية للمنشأة الصناعية.