Bayesian and Non-Bayesian Estimation About Weibull Distribution Based on Upper Record Values

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Abstract

This paper is concerned with some Bayesian and non-Bayesian estimation problem of the unknown parameters of the Weibull distribution based on upper record values. Maximum likelihood estimators and Bayesian point and interval estimators for the parameters from Weibull distribution based on upper record values are obtained. Numerical illustration and illustrative example are presented.

Keywords and phrase: Upper record values; Maximum likelihood estimation; Bayesian interval estimation;

1-Introduction

A random variable is said to have the Weibull(θ,β) distribution if its probability density function (pdf) is given by

$$f(x;\theta,\sigma,\gamma)=\theta\beta\, x^{\beta-1}e^{-\theta x^{\beta}} \qquad x>0,\ \theta>0,\ \beta>0 \tag{1.1}$$

and its cumulative distribution function will be

$$F(x;\theta,\beta)=1-e^{-\theta x^{\beta}} \qquad x>0,\ \theta>0,\ \beta>0. \tag{1.2}$$

Since the hazard function of this distribution is a decreasing function when the shape parameter β is less than 1, a constant when β equal 1, and an increasing function when β greater than 1, the distribution becomes suitable for the area of reliability, life testing and quality control (see Johnson, et al (1995)).

Let $\{X_n, n\geq 1\}$ be a sequence of independent and identically distributed (i.i.d) random variables with cumulative function $F(x)$ and corresponding pdf $f(x)$. Set $Y_n=\max\{X_1,X_2,\cdots,X_n\}$ for $n\geq 1$, we say X_j is an upper record value of $\{X_n\}$, if $Y_{j+1}>Y_j$. By definition, X_1 is an upper record values. The indices at which the upper record values occur are called upper record times $\{U(m), m\geq 0\}$, where $U(0)=1$ and $U(m)=\min\{j: j>U(m-1), X_j>X_{U(m-1)}\}$. Then $R_m=X_{U(m)}$, $m\geq 0$ are called the upper record values.

Record values and associated statistics are of great important in several real-live problems involving weather, economic, and sport data. The statistical study of record values started with Chandler (1952) and has now spread in different directions. Interested readers may refer to Foster and Stuart (1954), Galambos (1978), Dunsmore

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(1983), Resnick (1973), Nagaraja (1988), Ahsanullah (1994) and Arnold et al. (1992, 1998) for a review of developments in this area of research. While a lot of work has been done on characterizations, asymptotic theory and generalizations, not much has been done on statistical inference based on record values. Ahsanullah (1994) studied the record values based on the Weibull distribution and obtained the best linear unbiased estimators for its parameters. El-Qasem (1996) used the upper record values to obtain the maximum likelihood estimator for the uniform, the exponential and the Pareto distribution with one parameter.

Section (2) is devoted to obtain the maximum likelihood estimators for the unknown parameters of Weibull distribution (1.1) based on the upper record values. Section (3) discussed Bayesian estimation (point and interval) of these parameters based on upper record values. Numerical Illustration for the theoretical results will be presented in sections (4).

2- Maximum Likelihood Estimation for the Unknown Parameters of the Weibull Distribution based on Upper Record Values

Let $X_1, X_2, \ldots$ be an infinite sequence of independent and identically distributed random variables having the Weibull(θ,β) distribution (1.1). Consider $R_0, R_1, R_2, \ldots, R_m$ represent the first $(m+1)$ upper records from Weibull(θ,β) distribution, the likelihood function based on $(m+1)$ upper record values $R_0, R_1, \ldots, R_m$ is given by

$$L(\theta,\beta) = \theta^{m+1}\,\beta^{m+1}\left(\prod_{i=0}^{m} r_i\right)^{\beta-1} e^{-\theta r_m^{\beta}} \qquad \theta,\beta > 0. \tag{2.1}$$

Taking the logarithm of the likelihood function (2.1) then we have

$$\ln L \propto (m+1)\ln\theta + (m+1)\ln\beta + (\beta-1)\sum_{i=0}^{m}\ln r_i - \theta r_m^{\beta}. \tag{2.2}$$

Differentiate (2.2) with respect to θ and β respectively we get

$$\frac{\partial \ln L}{\partial\theta} = \frac{(m+1)}{\theta} - r_m^{\beta} \tag{2.3}$$

and

$$\frac{\partial \ln L}{\partial\beta} = \frac{(m+1)}{\beta} + \sum_{i=0}^{m}\ln r_i - \theta\, r_m^{\beta}\ln r_m \tag{2.4}$$

Equating (2.3) and (2.4) by zero and solving with respect to $\hat{\theta}$ and $\hat{\beta}$ then the maximum likelihood estimators $\hat{\theta}$ and $\hat{\beta}$ for θ and β will be

$$\hat{\beta} = \frac{(m+1)}{m\ln r_m - \sum_{i=0}^{m-1}\ln r_i} \tag{2.5}$$

and

$$\hat{\theta} = \frac{(m+1)}{r_m^{\hat{\beta}}} \tag{2.6}$$

obtain $\hat{\beta}$ and substituting $\hat{\beta}$ in (2.6) then $\hat{\theta}$ may be obtained.

The second derivatives of the log-likelihood function (2.1) are given by

$$\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{(m+1)}{\theta^2}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{(m+1)}{\beta^2} - \theta\, r_m^{\beta} (\ln r_m)^2$$

and

$$\frac{\partial^2 \ln L}{\partial \theta\, \partial \beta} = -\, r_m^{\beta} \ln r_m .$$

Hence, the observed information defined by Fisher (1922) as the second derivatives of the log-likelihood function evaluated at the MLE of the parameters, therefore,

$$\hat{I}(\hat{\theta},\hat{\beta}) = -\begin{bmatrix} \dfrac{\partial^2 \ln L}{\partial \theta^2} & \dfrac{\partial^2 \ln L}{\partial \theta\, \partial \beta} \\ \dfrac{\partial^2 \ln L}{\partial \theta\, \partial \beta} & \dfrac{\partial^2 \ln L}{\partial \beta^2} \end{bmatrix}\Bigg|_{\theta=\hat{\theta},\, \beta=\hat{\beta}}$$

Consequently, the inverse of the observed Fisher information matrix will be

$$\hat{I}^{-1}(\hat{\theta},\hat{\beta}) = \begin{bmatrix} Va\tilde{r}(\hat{\theta}) & Co\tilde{v}(\hat{\theta},\hat{\beta}) \\ Co\tilde{v}(\hat{\theta},\hat{\beta}) & Va\tilde{r}(\hat{\beta}) \end{bmatrix}$$

For large sample the matrix $I^{-1}(\hat{\theta},\hat{\beta})$ is an approximated variance-covariance matrix of the maximum likelihood estimators $\hat{\theta}$ and $\hat{\beta}$ (see lawless 1982).

When β is known, then the maximum likelihood estimator of the parameter θ based on the first $(m+1)$ upper record values is given by

$$\hat{\theta} = \frac{(m+1)}{r_m^{\beta}} \tag{2.7}$$

The mean and the variance of the MLE $\hat{\theta}$ will be

$$E(\hat{\theta}) = \frac{m+1}{m}\theta$$

and

$$Var(\hat{\theta}) = \frac{(m+1)^2}{m^2(m-1)}\theta^2 \tag{2.8}$$

For exponential and Rayleigh distributions, the maximum likelihood estimator for the parameter θ based on the upper record values will be obtained from (2.7) by putting $\beta = 1$ and $\beta = 2$, respectively, with the variance (2.8). Note that the result obtained for exponential distribution coincide with the result obtained by El-Qasem (1996).

3- Bayesian Estimation for the Unknown Parameters of the Weibull Distribution based on Upper Record Values

Consider a bivariate prior density function for the parameters θ and β is $\pi(\theta,\beta)$, and assume that θ and β are independent. Consider now the parameter θ has a gamma prior distribution with the following pdf

$$\pi(\theta)\propto \theta^{a-1}e^{-b\theta} \qquad \theta>0,\ a,b>0 \tag{3.1}$$

while the prior density of the shape parameter β is the uniform distribution with density function given by (see Canavos and Tsokos (1973))

$$\pi(\beta)\propto \frac{1}{d-c} \qquad c<\beta<d \tag{3.2}$$

Then the joint pdf of θ and β will be obtained from (3.1) and (3.2) as follows

$$\pi(\theta,\beta)\propto \frac{\theta^{a-1}e^{-b\theta}}{d-c} \qquad \theta>0,\ c<\beta<d\ \ a,b>0 \tag{3.3}$$

Non Informative prior distribution of θ and β is given by

$$\pi(\theta,\beta)\propto \frac{\theta^{a-1}}{(d-c)} \qquad \theta>0,\ c<\beta<d,\ a>0 \tag{3.4}$$

3.1 Bivariate Posterior Density Estimation

Combining the prior pdf (3.3) with the likelihood function (2.1) via Bayesian theorem then the bivariate posterior distribution of θ and β is given by

$$\pi(\theta,\beta|\underline{r})\propto \theta^{m+a}\,\beta^{m+1}\prod_{i=0}^{m} r_i^{\beta-1}\,e^{-\theta(b+r_m^{\beta})} \quad \theta>0, c<\beta<d \tag{3.5}$$

From (3.5) the normalizing constant is given by

$$K=1/\int_c^d\int_0^\infty \theta^{m+a}\,\beta^{m+1}\prod_{i=0}^{m} r_i^{\beta-1}\,e^{-\theta(b+r_m^{\beta})}\,d\theta d\beta \tag{3.6}$$

There may be situations where it is convenient to choose a single value as an estimate of the unknown parameters. It seems sensible to choose that value with highest posterior probability. Consequently we define as a Bayesian point estimate the quantity which maximizes the joint posterior distribution; this quantity is called the joint posterior mode. Now to find the joint posterior mode for the parameters θ and β taking the logarithm for (3.5) and differentiate with respect to θ and β respectively we get

$$\begin{aligned}\frac{\partial \ln \pi(\theta,\beta|\underline{r})}{\partial\theta}&=\frac{(m+a)}{\theta}-(b+r_m^{\beta})\\ \frac{\partial \ln \pi(\theta,\beta|\underline{r})}{\partial\beta}&=\frac{(m+1)}{\beta}-\sum_{i=0}^{m}\ln r_i-\theta r_m^{\beta}\ln r_m\end{aligned} \tag{3.7}$$

Equating equation (3.7) by zero and solving for $\ddot{\theta}_1$ and $\ddot{\beta}_1$ yields the following equations

$$\ddot{\theta}_1=\frac{(m+a)}{(b+r_m^{\ddot{\beta}_1})} \tag{3.8}$$

and

$$(m+1)b+\left[(m+1)-(m+a)\hat{\hat{\beta}}_1 \ln r_m\right] r_m^{\hat{\hat{\beta}}_1} - \hat{\hat{\beta}}_1 (b + r_m^{\hat{\hat{\beta}}_1}) \sum_{i=0}^{m} \ln r_i = 0 \qquad (3.9)$$

From equation (3.9) and by using numerical technique one can get the value of the Bayesian point estimator $\hat{\hat{\beta}}_1$. Substituting by the value $\hat{\hat{\beta}}_1$ in equation (3.8) then the value of $\hat{\hat{\theta}}_1$ may be obtained.

Non informative prior (3.4) is used to obtain joint posterior mode for the parameters θ and β as follows

$$\hat{\hat{\theta}}_1 = \frac{m+a}{r_m^{\hat{\hat{\beta}}_1}} \quad \text{and} \quad \hat{\hat{\beta}}_1 = \frac{(m+1)}{(m+a)\ln r_m - \sum_{i=0}^{m} \ln r_i}$$

which are approximately the same as the maximum likelihood estimators. When β is known, then the Bayesian point estimator for the parameter θ based on the Gamma prior distribution (3.1) will be

$$\hat{\hat{\theta}}_1 = \frac{(m+a)}{(b + r_m^{\beta})} \qquad (3.10)$$

For the non informative prior (3.4), Bayesian point estimator for θ is given by $\hat{\hat{\theta}}_1 = \frac{(m+a)}{r_m^{\beta}}$ which is the same as the MLE of θ when $a = 1$. For exponential distribution, the posterior mode will be obtained from (3.10) when $\beta = 1$. Put $\beta = 2$ in (3.10) one may obtain the Bayesian point estimator for the parameter θ for Rayleigh distribution.

3.2 Univariate Posterior density Estimation

Since the bivariate posterior distribution $\pi(\theta, \beta | \underline{r})$ is available, one may obtained the marginal posterior distribution for both parameters θ and β. Using these marginal distributions one can obtain some measures as the posterior mean and the posterior variance. For instance, when squared error loss function is used, the posterior mean used as the Bayes estimator for the parameter of interest and posterior variance used as Bayes risk. In this section we will derive another Bayesian estimator for unknown parameters from Weibull distribution.

3.2.1Univariate Posterior Estimation for the Parameter θ

From the bivariate posterior distribution of θ and β (3.5), integrating out β yields the marginal posterior distribution of θ as follows

$$\pi_1(\theta | \underline{r}) = K_1 \theta^{m+a} e^{-\theta b} \int_c^d \beta^{m+1} \left[\prod_{i=0}^{m} r_i\right]^{\beta-1} e^{-\theta r_m^{\beta}} d\beta \quad \theta > 0 \quad (3.11)$$

where

$$K_1 = 1/\Gamma(m+a+1)\int_c^d \frac{\beta^{m+1}\left[\prod_{i=0}^{m} r_i\right]^{\beta-1}}{\left(b+r_m^{\beta}\right)^{m+a+1}} d\beta$$

Bayesian Posterior mean for the parameter θ will be

$$\hat{\hat{\theta}}_2 = E(\theta|\underline{r}) = \int_0^\infty \theta\,\pi_1(\theta|\underline{r})d\theta$$

$$= K_1\Gamma(m+a+2)\int_c^d \frac{\beta^{m+1}}{c(b+r_m^{\beta})^{m+a+2}}\left[\prod_{i=0}^{m} r_i\right]^{\beta-1} d\beta \tag{3.12}$$

The second moment of the parameter θ is given by

$$E(\theta^2|\underline{r}) = \int_0^\infty \theta^2\,\pi_1(\theta|\underline{r})d\theta$$

$$= K_1\Gamma(m+a+3)\int_c^d \frac{\beta^{m+1}}{c(b+r_m^{\beta})^{m+a+3}}\left[\prod_{i=0}^{m} r_i\right]^{\beta-1} d\beta \tag{3.13}$$

Numerical technique is needed to solve (3.12) and (3.13). With respect to the quadratic loss function, the Bayesian point estimation is the posterior mean (3.12) and the posterior risk is the variance of the parameter θ.

A HPD interval $(q1, q2)$ for the parameter θ with probability cover τ will be obtained by solving the following equations

$$P\left[q1 < \theta < q2|\underline{r}\right] = \int_{q1}^{q2} \pi_1(\theta|\underline{r})d\theta = \tau$$

$$\left(\frac{q1}{q2}\right)^{m+a} e^{-b(q1-q2)} = \frac{\int_c^d \beta^{m+1}\prod_{i=0}^{m} r_i^{\beta-1} e^{-q2 r_m^{\beta}} d\beta}{\int_c^d \beta^{m+1}\prod_{i=0}^{m} r_i^{\beta-1} e^{-q1 r_m^{\beta}} d\beta} \tag{3.14}$$

Numerical technique is needed to solve integration in (3.14). In view of computational difficulties that may arises in the HPD interval one can do the interval estimation with the equal tail. Therefore the equal tail Bayesian estimation bound of two sided interval for the parameter θ, is obtained by solving the following two equations for lower bound L_θ and upper bound U_θ

$$P\left[\theta > L_\theta|\underline{r}\right] = \int_{L_\theta}^{\infty} \pi(\theta|\underline{r})d\theta = \frac{1+\tau}{2} \text{ and } P\left[\theta > U_\theta|\underline{r}\right] = \int_{U_\theta}^{\infty} \pi(\theta|\underline{r})d\theta = \frac{1-\tau}{2} \tag{3.15}$$

When β is known, then based on Gamma prior distribution for the parameter θ, the posterior density function for the parameter θ will be

$$\pi_1(\theta|\underline{r}) = \frac{\left(b + r_m^{\beta}\right)^{m+a+1}}{\Gamma(m+a+1)} \theta^{m+a} e^{-\theta(b + r_m^{\beta})} \quad \theta > 0, \tag{3.16}$$

From (3.16), we conclude that the distribution of the parameter θ follow Gamma distribution with parameters $(m+a+1,(b+r_m^{\beta}))$, then using the fact that $2(b+r_m^{\beta})\theta$ has a chi square distribution with $2(m+a+1)$ degrees of freedom, then with respect to the quadratic loss function the point estimator for the parameter θ (the posterior mean) will be

$$\hat{\hat{\theta}}_2 = \frac{m+a+1}{(b+r_m^{\beta})}. \tag{3.17}$$

and the posterior risk will be

$$Var(\hat{\hat{\theta}}_2) = \frac{2(m+a+1)}{(b+r_m^{\beta})}.$$

For the non informative prior (i.e., if $b=0$) then $\hat{\hat{\theta}}_2 = \frac{m+a+1}{r_m^{\beta}}$, which is the same as the MLE of θ when $a=0$.

From the marginal posterior density function for the parameter θ (3.17) then a $100(1-\alpha)\%$ Bayesian interval estimation for the parameter θ is (L_θ, U_θ) where

$$L_\theta = \frac{\chi^2_{2(m+a+1),(1-\alpha/2)}}{2(b+r_m^{\beta})} \text{ and } U_\theta = \frac{\chi^2_{2(m+a+1),\alpha/2}}{2(b+r_m^{\beta})}. \tag{3.18}$$

Using non informative prior then a $100(1-\alpha)\%$ Bayesian interval estimation for the parameter θ is may be obtained from (3.18) by putting $b=0$.

For exponential distribution, Bayesian point and interval estimator of the parameter θ based on the first (m+1) upper record values will be obtained from (3.17) and (3.18) by putting $\beta=1$. When $\beta=2$ in (3.17) and (3.18) then the Bayesian point and interval estimation may be obtained for Rayleigh distribution

3.2.2 Univariate Posterior Estimation for the Shape Parameter

Integrating (3.5) with respect to θ then the marginal distribution of the shape parameter β will be

$$\pi_2(\beta|\underline{r}) = K_2 \frac{\beta^{m+1}}{\left(b + r_m^{\beta}\right)^{(m+a+1)}} \left[\prod_{i=0}^{m} r_i\right]^{\beta-1}, c < \beta < d \tag{3.19}$$

where

$$K_2 = 1/\int_c^d \frac{\beta^{m+1}\left[\prod_{i=0}^{m} r_i\right]^{\beta-1}}{\left(b+r_m^{\beta}\right)^{m+a+1}} d\beta$$

The Bayesian point estimation (posterior mean) for the shape parameter β which minimize the quadratic loss function will be

$$\hat{\hat{\beta}}_2 = E(\beta|\underline{r}) = \int_c^d \beta\,\pi_2(\beta|\underline{r})\,d\beta$$

$$= K_2 \int_c^d \frac{\beta^{m+2}}{(b+r_m^{\beta})^{m+a+1}} \left[\prod_{i=0}^{m} r_i\right]^{\beta-1} d\beta \qquad (3.20)$$

The second moment for the shape parameter is given by

$$E(\beta^2|\underline{r}) = \int_c^d \beta^2\,\pi_2(\beta|\underline{r})\,d\beta$$

$$= K_2 \int_c^d \frac{\beta^{m+3}}{(b+r_m^{\beta})^{m+a+1}} \left[\prod_{i=0}^{m} r_i\right]^{\beta-1} d\beta \qquad (3.21)$$

Equations (3.20) and (3.21) need numerical methods to solve. Hence, the posterior risk will be obtained from (3.20) and (3.21)

An HPD interval $(h1, h2)$ for the shape parameter β with probability cover τ will be obtained by solving the following equations

$$P\left[h1 < \beta < h2|\underline{r}\right] = \int_{h1}^{h2} \pi_2(\beta|\underline{r})\,d\beta = \tau$$

$$\left(\frac{h1}{h2}\right)^{m+1} \prod_{i=0}^{m} r_i^{h1-h2} = \left(\frac{b+r_m^{h1}}{b+r_m^{h2}}\right)^{(m+a+1)} \qquad (3.22)$$

Also the Bayesian estimation Limits for the shape parameter β with cover τ is given by solving the following two equations

$$P\left[\beta > L_\beta|\underline{r}\right] = \int_{L_\beta}^{d} \pi_2(\beta|\underline{r})d\beta = \frac{1+\tau}{2} \text{ and } P\left[\theta > U_\beta|\underline{r}\right] = \int_{U_\beta}^{d} \pi_2(\beta|\underline{r})d\beta = \frac{1-\tau}{2} \qquad (3.23)$$

Equation (3.22) and (3.23) need numerical technique to solve them to find the Bayesian prediction interval for unknown shape parameter for Weibull distribution based on the first $(m+1)$ upper record values.

4-Numerical Illustration

Using equations (3.8), (3.9) and by using the Mathcad (2001) program and for different prior parameters a and b we evaluate the Bayesian point estimators (posterior mode) for the unknown parameters. Also Using Equation (3.12) and (3.20) point estimators (posterior mean) for unknown parameters are obtained. Bayesian interval estimation will be obtained using equation (3.15) and (3.23). The calculations are carried out according to the following steps:

(1) For given values of the Weibull parameters $\theta = 2$ and $\beta = 3$ generate a random variable X from the Weibull distribution (1.1) and selected the first 10 records which will be
0.306, 0.572, 0.665, 0.855, 1.092, 1.114, 1.132, 1.225, 1.397, 1.439

(3) Using Mathcad (2001) program applying equations (3.10) up to (3.23), to obtain the Bayesian point estimation (posterior mode and posterior mean) for unknown parameters. Also 95% Bayesian interval estimation for unknown parameters for several different values of the prior parameters (a=0,0.5,1,3,5,7) and (b=0,1,..5) will be computed. Tables (1and 2) contained the results which show

(i) the values of the prior parameters a, b, c and d.
(ii) the Bayesian point estimators unknown parameters (posterior mode).
(iii) the Bayesian point estimators for the unknown parameters (posterior mean).
(iv) the 95% Bayesian interval estimation for the parameters.
(v) the lengths of the Bayesian interval estimation.

Using the same data, the maximum likelihood estimators unknown parameters are computed from equation (2.5) and (2.6) to be $\hat{\theta} = 4.62$ and $\hat{\beta} = 2.116$. The corresponding approximated variance covariance matrix is computed to be

$$I^{-1}(\hat{\theta}, \hat{\beta}) = \begin{bmatrix} 3.414 & -0.754 \\ -0.754 & 0.448 \end{bmatrix}$$

The Bayesian estimators (posterior mode) for the unknown parameters θ and β when the prior parameters a and b takes the value 5 and 2 respectively, are $\hat{\theta}_1 = 2.166$ and $\hat{\beta}_1 = 4.111$. While the Bayesian estimators (posterior mean) for the scale and shape parameters when used the same prior parameters a and b, and when $c = 3$ and $d = 4$ will be $\hat{\theta}_2 = 2.803$ and $\hat{\beta}_2 = 3.325$. While the Bayesian point estimators with the minimum risk will be obtained at prior parameter $a = 0$ and $b = 5$ for the scale parameter in this case Bayesian interval estimation computed to be $(0.559, 2.012)$. Also for the shape parameter minimum risk when $a = 20$ and $b = 1$ with Bayesian interval estimation will be $(3.004, 3.507)$.

Table 1

Bayesian Interval and Point Estimation for the Parameter θ for Different Prior Parameters When m=9, c=3 and d=4

Prior parameters		Interval		Length	Point Est.	Point Est.	Bayes Risk
a	b	Lower	Upper		$\hat{\hat{\theta}}_1$	$\hat{\hat{\theta}}_2$	
0	0	1.403	5.243	3.84	1.042	3.005	0.978
0.5	0	1.51	5.462	3.952	2.404	3.168	1.036
0	1	1.071	3.945	2.874	1.057	2.274	0.548
1	1	1.232	4.269	3.037	1.43	2.515	0.61
2	1	1.399	4.59	3.191	0.824	2.757	0.673
3	1	1.569	4.91	3.341	2.235	3.001	0.737
4	1	1.744	5.229	3.485	2.657	3.247	0.801
5	1	1.921	5.545	3.624	3.09	3.494	0.866
6	1	2.102	5.86	3.758	3.50	3.742	0.93
0	2	0.869	3.172	2.303	0.748	1.835	0.351
1	2	0.999	3.428	2.429	1.042	2.027	0.39
2	2	1.132	3.682	2.55	1.311	2.219	0.43
3	2	1.268	3.936	2.668	1.59	2.413	0.47
4	2	1.407	4.188	2.781	1.875	2.608	0.51
5	2	1.548	4.439	2.891	2.166	2.803	0.551
6	2	1.692	4.689	2.997	2.461	3	0.591
0	3	0.732	2.657	1.925	0.63	1.542	0.245
1	3	0.841	2.869	2.028	0.828	1.701	0.272
2	3	0.952	3.08	2.128	1.033	1.861	0.209
3	3	1.065	3.498	2.433	1.245	2.022	0.326
5	3	1.299	3.706	2.407	1.68	2.345	0.382
7	3	1.54	4.19	2.65	2.128	2.671	0.438
10	3	1.913	4.733	2.82	2.816	3.164	0.522
1	4	0.727	2.47	1.743	0.69	1.467	0.201
3	4	0.92	2.829	1.909	1.027	1.742	0.24
5	4	1.12	3.184	2.064	1.378	2.018	0.28
7	4	1.327	3.536	2.209	1.738	2.296	0.321
10	4	1.646	4.059	2.413	4.007	2.717	0.383
0	5	0.559	2.012	1.453	0.458	1.171	0.14
5	5	0.986	2.793	1.807	1.172	1.773	0.215
7	5	1.166	3.1	1.934	1.473	2.016	0.246
10	5	1.445	3.556	2.111	1.933	2.382	0.293
10	7	1.164	2.853	1.689	1.479	1.914	0.187
10	8	1.061	2.597	1.536	1.325	1.744	0.155
10	9	0.975	2.384	1.409	1.201	1.602	0.13
10	10	0.902	2.204	1.302	1.098	1.481	0.111

Table 2

Bayesian Interval and Point Estimation for the Shape Parameter β for Different Prior Parameters When m=9, c=3 and d=4

Prior parameters		Interval		Length	Point Est.	Point Est.	Bayes Risk
a	B	Lower	Upper		$\hat{\hat{\beta}}_1$	$\hat{\hat{\beta}}_2$	
0	0	3.01	3.897	0.887	5.572	3.314	0.062
0.5	0	3.009	3.886	0.877	3.914	3.303	0.059
0	1	3.013	3.933	0.92	5.541	3.368	0.07
1	1	3.012	3.921	0.909	4.919	3.348	0.067
2	1	3.011	3.908	0.897	4.438	3.33	0.064
5	1	3.008	3.855	0.847	3.467	3.28	0.054
7	1	3.007	3.81	0.803	3.04	3.252	0.047
10	1	3.006	3.734	0.728	2.576	3.216	0.038
15	1	3.005	3.609	0.604	2.063	3.172	0.026
20	1	3.004	3.507	0.503	1.725	3.141	0.018
0	2	3.016	3.949	0.933	6.179	3.405	0.075
1	2	3.015	3.942	0.927	5.572	3.388	0.073
2	2	3.014	3.934	0.92	5.095	3.371	0.071
5	2	3.011	3.903	0.892	4.111	3.325	0.063
7	2	3.009	3.875	0.866	3.666	3.298	0.057
10	2	3.008	3.823	0.815	3.171	3.261	0.049
15	2	3.006	3.721	0.715	2.607	3.213	0.036
20	2	3.005	3.619	0.614	2.224	3.177	0.027
0	3	3.019	3.958	0.939	6.66	3.432	0.078
5	3	3.013	3.927	0.914	4.599	3.36	0.069
10	3	3.009	3.875	0.866	3.63	3.299	0.057
15	3	3.007	3.799	0.792	3.035	3.249	0.045
20	3	3.006	3.711	0.705	2.623	3.21	0.036
0	4	3.021	3.963	0.942	7.048	3.453	0.079
5	4	3.015	3.941	0.926	4.994	3.388	0.073
10	4	3.011	3.905	0.894	4.007	3.33	0.063
15	4	3.009	3.851	0.842	3.391	3.281	0.053
20	4	3.007	3.781	0.774	2.958	3.241	0.043

References

Ahsanullah, M. (1994). Record statistic. Nova Science Publishers, Inc.
Arnold, C. B., Balakrishnan, N. & Nagaraja, H. N. (1992). A first course in order statistics. John Wiley & Sons, Inc.
Arnold, C. B., Balakrishnan, N. & Nagaraja, H. N. (1998). Records. John Wiley & Sons, Inc.
Canavos, G. C. and Tsokos, C. P. (1973). Bayesian estimation of life parameters in the Weibull distribution. Operation research, 21, 755-768.
Chandler, K. N. (1952). The distribution and frequency of record values. Journal of Royal statistics society, series B, 14, 220-228.
Dunsmore, I. R. (1983). The future occurrence of records, Annals of the Institute Statistical Mathematics, 51, 267.
El-Qasem, A. A. (1996). Estimation via record values. Journal of Information & Optimization Sciences. 17, 541-548.
Fisher, R. A. (1922). On the mathematical foundations of theoretical statistics, Philosophical transactions of Royal society of London, Series A, 222, 309-368.
Foster, F. G. and Stuart, A. (1954). Distribution free tests in time series based on the braking of records, Journal of Royal statistics society, series B, 16, 1-22.
Galambos, J. (1978). The asymptotic theory of extreme of extreme order statistics, New York: John Wiley & Sons.
Johnson, N. L., Kotz, S., and Balakrishnan, N. (1995). Continuous univariate distribution, Volume 2, Second edition, John Wiley & Sons, New York.
Lawless, J. F. (1982) Statistical models and method of lifetime data. John Wiley, New York.
Nagaraja, H. N. (1988). Record values and related statistics. A review. Communication in Statistics- Theory and Methods,7, 7, 2223-2238.
Resnick, S. L. (1973). Record values and maxima. Annals of Probability, 1, 650-662
Mathcad 2001 professional (1986-2000) Math soft. Inc.