Adaptive Interval Estimation of the Mean in Unbalanced One-Way Random Effects Models

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Abstract

A two-stage sequential procedure is introduced to control the width and coverage (validity) of confidence intervals for the estimation of the mean in unbalanced one-way random effects models. The procedure uses unbalanced pilot sample data to estimate an "optimal" group size and then proceeds to determine the number of groups by a stopping rule. Several asymptotic results concerning the proposed procedure are given along with simulation results to assess its performance in moderate sample size situations, under varying degrees of imbalance. The proposed procedure was found to effectively control the width and probability of coverage of the resulting confidence intervals in all cases. The procedure is illustrated using a real data set.

key Words: confidence intervals, harmonic mean method, sequential estimation, stopping time, two-stage sequential sampling, validity of confidence intervals.

1 Introduction

The unbalanced one-way random effects model can be generalized to more complex designs and has proven useful to practitioners in a variety of fields, where the investigator is often interested in interval estimation of the mean, μ, the between groups variance component, σ_b^2 and/or certain functions of μ and σ_b^2, e.g., El-Bassiouni and Abdelhafez (2000), Hartung and Knapp (2000), Bonett (2002), Krishnamoorthy and Guo (2005) and the references therein.

El-Bassiouni and Zoubeidi (2008) proposed sequential procedures to construct confidence intervals for μ and σ_b^2, such that the width of these intervals is less than or equal to a desired precision d, while the probability of coverage is greater than or equal to a nominal level $1-\alpha$. They considered two-stage sampling plans where the first stage is a pilot sample consisting of n_0 groups of size r_0 each, followed by a second stage consisting of $n - n_0$ groups of size r each, where r is determined by the pilot sample data while n is determined by a stopping rule. The proposed procedures were found to effectively control the width and the probability

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of coverage of the resulting confidence intervals in all cases considered, including moderate sample size situations.

In this paper, we consider unbalanced pilot samples, where each group has size r_i, $i = 1, \ldots, n_0$, and develop a sequential procedure, parallel to that of El-Bassiouni and Zoubeidi (2008), for estimating μ. The performance of the proposed procedure is also assessed under varying degrees of imbalance.

Regarding notation, we use χ^2_ν to denote the chi-square distribution with ν degrees of freedom, t_ν to denote the t distribution with ν degrees of freedom, $N(\mu, \sigma^2)$ to denote the normal distribution with mean μ and variance σ^2 and $z_{1-\gamma}$ to denote the $100 \times \gamma$ upper percentile of the $N(0,1)$ distribution.

This paper is organized as follows: In section 2 some preliminary results are presented. In Section 3 the sequential procedure for estimating μ is described and the theoretical results concerning the width and probability of coverage for the corresponding confidence interval are given. Since the results in Section 3 are asymptotic in nature, the empirical results of a Monte Carlo simulation study are given in Section 4 to assess the performance of the proposed procedures in moderate sample size situations, under varying degrees of imbalance. An example is worked out in Section 5 to illustrate the proposed procedure. The conclusions are provided in Section 6.

2 Preliminaries

Consider the unbalanced one-way random effects model

$$Y_{ij} = \mu + \tau_i + \epsilon_{ij}, \qquad (1)$$

for $j = 1, \ldots, r_i$ and $i = 1, \ldots, n$, where μ is a scalar representing an overall fixed effect, τ_i, $i \geq 1$, are i.i.d. $N(0, \sigma_b^2)$, $\sigma_b^2 \geq 0$, random variables representing the random effects and ϵ_{ij}, $i, j \geq 1$, are i.i.d. $N(0, \sigma_e^2)$, $\sigma_e^2 > 0$, representing the error term. Moreover, τ_i and $\epsilon_{i'j}$ are independent for all i, i', j. Let $N = \sum_{i=1}^n r_i$, denote the total number of observations and define

$$MSE_n = \frac{SSE_n}{N-n}, \quad SSE_n = \sum_{i=1}^n \sum_{j=1}^{r_i} \left(Y_{ij} - \overline{Y}_{i.}\right)^2, \quad \overline{Y}_{i.} = \frac{1}{r_i} \sum_{j=1}^{r_i} Y_{ij},$$

$$S_n^2 = \frac{1}{n-1} \sum_{i=1}^n (\overline{Y}_{i.} - \overline{\overline{Y}}_n)^2, \quad \overline{\overline{Y}}_n = \frac{1}{n} \sum_{i=1}^n \overline{Y}_{i.}.$$

Note that $\overline{\overline{Y}}_n$ and S_n^2 can be recursively computed using the recurrence relations

$$\overline{\overline{Y}}_n = \frac{1}{n}\left[(n-1)\overline{\overline{Y}}_{n-1} + \overline{Y}_{n.}\right] \quad \text{and} \quad S_n^2 = \frac{n-2}{n-1} S_{n-1}^2 + \frac{1}{n}\left[\overline{Y}_{n.} - \overline{\overline{Y}}_{n-1}\right]^2. \qquad (2)$$

Under Model (1), El-Bassiouni and Abdelhafez (2000) showed that

$$\frac{\overline{\overline{Y}}_n - \mu}{S_n/\sqrt{n}} \overset{\text{approx}}{\sim} t_{n-1}. \qquad (3)$$

Hence, one may derive the following asymptotic $(1-\alpha)100\%$ confidence interval for μ

$$\overline{\overline{Y}}_n \pm t_{1-\alpha/2,\, n-1} S_n/\sqrt{n} \tag{4}$$

whose width is given by $\Delta_n = 2t_{1-\alpha/2,\, n-1} S_n/\sqrt{n}$, where $t_{1-\alpha/2,\, n-1}$ represents the $100\times\alpha/2$ upper percentile of the t_{n-1} distribution. From Lemma 1 in El-Bassiouni and Zoubeidi (2008), one can easily show that the expected width of (4) is asymptotically equivalent to

$$\Omega(n,r) = \frac{2z_{1-\alpha/2}}{\sqrt{n}}\sqrt{\sigma_b^2 + \frac{\sigma_e^2}{r}}. \tag{5}$$

Consider a two stage sampling plan where the first stage is a pilot sample consisting of n_0 groups of size r_i each, followed by a second stage consisting of $n - n_0$ groups of size $r_i = r$ each. Let $SSE_0 = SSE_{n_0}$, $SSE_1 = SSE_n - SSE_0$,

$$S^2_{(1)} = \frac{1}{n-n_0-1}\sum_{i=n_0+1}^{n}\left(\overline{Y}_{i.} - \overline{\overline{Y}}_{(1)}\right)^2, \quad \overline{\overline{Y}}_{(1)} = \frac{1}{n-n_0}\sum_{i=n_0+1}^{n}\overline{Y}_{i.}$$

and note that $S^2_{(0)} = S^2_{n_0}$, and $\overline{\overline{Y}}_{(0)} = \overline{\overline{Y}}_{n_0}$.

Remark 1 Given r, it is easily verified under Model (1) that:

$$(n-1)S_n^2 = (n_0-1)S^2_{(0)} + (n-n_0-1)S^2_{(1)} + \frac{n_0(n-n_0)}{n}\left(\overline{\overline{Y}}_{(0)} - \overline{\overline{Y}}_{(1)}\right)^2, \tag{6}$$

$$SSE_0,\quad S^2_{(0)},\quad \overline{\overline{Y}}_{(0)},\quad SSE_1,\quad S^2_{(1)},\quad \text{and}\quad \overline{\overline{Y}}_{(1)}\quad \text{are independent,}$$

$$\frac{(n-n_0-1)S^2_{(1)}}{\sigma_b^2 + \sigma_e^2/r} \sim \chi^2_{n-n_0-1}, \quad \text{and} \quad \frac{n_0(n-n_0)}{n}\frac{\left(\overline{\overline{Y}}_{(0)} - \overline{\overline{Y}}_{(1)}\right)^2}{\sigma_b^2 + \sigma_e^2/\gamma} \sim \chi_1^2,$$

where $\gamma = nr\tilde{r}_{n_0}/[n_0\tilde{r}_{n_0} + (n-n_0)r]$ and $\tilde{r}_{n_0}$ denotes the harmonic mean of $r_1, \ldots, r_{n_0}$.

3 A Two-Stage Sequential Procedure for Estimating μ

A sequential procedure is proposed for estimating μ where n (number of groups) and r (group size in the second stage) are data dependent such that the width of the resulting interval is equal to a desired precision d and its coverage probability is greater than or equal to $1-\alpha$. The proposed procedure is described next.

Proposed Procedure 1

1. For the pilot sample observe Y_{ij}, for $i = 1, \cdots, n_0$ and $j = 1, \cdots, r_i$. Compute MSE_{n_0} and $S^2_{n_0}$.
2. Compute an "optimal" number of replicates per group, r, that depends on the data only through MSE_{n_0} and $S^2_{n_0}$. One such "optimality criterion" is to determine r that minimizes (5) according to a steepest descent approach. To this end, consider the

estimators $\hat{\sigma_b}^2 = \max\{S^2_{n_0} - MSE_{n_0}/\tilde{r}_{n_0}\ ;\ 0\}$, which was proposed by Seely (1979), and $\hat{\sigma_e}^2 = MSE_{n_0}$. Following the argument outlined in El-Bassiouni and Zoubeidi (2008), when $\hat{\sigma_b}^2 > 0$, we sample groups of size r in the second stage, where

$$r = \max\left\{2,\ \left\lceil \frac{1}{2}\left(\sqrt{\frac{\hat{\sigma_e}^4}{\hat{\sigma_b}^4} + 4n_0\frac{\hat{\sigma_e}^2}{\hat{\sigma_b}^2}} - \frac{\hat{\sigma_e}^2}{\hat{\sigma_b}^2}\right)\right\rceil\right\}, \tag{7}$$

and $\lceil x \rceil$ is the smallest integer greater than or equal to x. When $\hat{\sigma_b}^2 = 0$, $r = n_0$.

3. Continue sampling one group at a time (i.e., for the kth group observe Y_{kj}, $j = 1, \ldots, r$) until the stopping time

$$R = \inf\left\{n \geq n_0 :\ 2t_{1-\alpha/2,\,n-1}S_n l_n \leq d\sqrt{n}\right\}, \tag{8}$$

where d is the desired width of the confidence interval, $l_n = 1 + l_0/n$, and l_0 is a pre-specified positive constant. The damping factor l_n is meant to correct for underestimation of the desired confidence level.

4. After stopping, estimate μ using the interval

$$\overline{\overline{Y}}_R \pm d/2. \tag{9}$$

Let $n^*(r)$ denote the solution to $\Omega(n,r) = d$. Then,

$$n^* \equiv n^*(r) = \frac{4z^2_{1-\alpha/2}}{d^2}\left(\sigma_b^2 + \frac{\sigma_e^2}{r}\right). \tag{10}$$

The following proposition shows that, conditional on $S^2_{n_0}$ and MSE_{n_0}, the number of groups, R, is of the same order of magnitude as $n^*(r)$ and that R is asymptotically normally distributed, as $d \to 0$.

PROPOSITION 1 *Let r be a function of $S^2_{n_0}$ and MSE_{n_0}. Then, given $S^2_{n_0}$ and MSE_{n_0}, the stopping time R defined in (8) satisfies*

$$\frac{R}{n^*(r)} \longrightarrow 1 \quad w.p.1 \quad as\ d \to 0$$

and

$$\frac{R - n^*(r)}{\sqrt{n^*(r)}} \xrightarrow{\mathcal{L}} N(0,2), \quad as\ d \to 0.$$

where $n^*(r)$ is given by (10).

Proof of Proposition 1: Using Remark 1, (6) can be re-written as

$$\begin{aligned}(n-1)S_n^2 &= (n_0-1)S^2_{n_0} + \left(\sigma_b^2 + \sigma_e^2/r\right)\left[\frac{(n-n_0-1)S^2_{(1)}}{\sigma_b^2 + \sigma_e^2/r} + \frac{n_0(n-n_0)}{n(\sigma_b^2 + \sigma_e^2/\gamma)}\left(\overline{\overline{Y}}_{(0)} - \overline{\overline{Y}}_{(1)}\right)^2\right] \\ &\quad + \frac{n_0(n-n_0)^2(r-\tilde{r}_{n_0})\sigma_e^2}{r_0 r n^2(\sigma_b^2 + \sigma_e^2/\gamma)}\left(\overline{\overline{Y}}_{(0)} - \overline{\overline{Y}}_{(1)}\right)^2, \\ &= \left(\sigma_b^2 + \sigma_e^2/r\right)(n-n_0)\overline{W}_{n-n_0} + \left(\sigma_b^2 + \sigma_e^2/r\right)(n-1)G_n,\end{aligned} \tag{11}$$

where $\overline{W}_{n-n_0}$ is the average of $n - n_0$ i.i.d χ_1^2 random variables,

$$G_n = \frac{(n_0-1)S_{n_0}^2}{(n-1)(\sigma_b^2+\sigma_e^2/r)} + \frac{(n-n_0)(r-\bar{r}_{n_0})\sigma_e^2}{\bar{r}_{n_0}n(n-1)(r\sigma_b^2+\sigma_e^2)}U_1,$$

and U_1 is a χ_1^2 random variable.

By (8), at stopping $S_n^2 \le d^2n/4l_n^2t_{1-\alpha/2}^2$, which is by (11) equivalent to

$$nl_n^{-2}\left[\frac{n-n_0}{n-1}\overline{W}_{n-n_0}+G_n\right]^{-1} \ge n^*(r). \tag{12}$$

By expanding and collecting the terms on the left hand side, (12) may be re-written as $V_n + \xi_n \ge n^*(r)$ where $V_n = \sum_{i=1}^{n-n_0}(2-W_i)$, $W_1, \ldots, W_{n-n_0}$ are i.i.d. χ_1^2 random variables and ξ_n, $n \ge 1$, are slowly changing random variables, conditional on $S_{n_0}^2$ and MSE_{n_0}. The proof follows from Lemma 10.2 of Woodroofe (1982).

Next we present the main result of this section.

THEOREM 1 *Let r be a function of MSE_{n_0} and $S_{n_0}^2$. Under Procedure 1*

$$\lim_{d\to 0} Pr\left[\overline{\overline{Y}}_R - d/2 \le \mu \le \overline{\overline{Y}}_R + d/2\right] \ge 1-\alpha.$$

Proof of Theorem 1: Observe that the stopping time R depends on the data only through $S_{n_0}^2$, MSE_{n_0} and S_n^2, which are independent of $\overline{\overline{Y}}_n$ for fixed r. Then, given $S_{n_0}^2$ and MSE_{n_0}, $\overline{\overline{Y}}_R$ is independent of R. Therefore, by (3), $\sqrt{R}(\overline{\overline{Y}}_R-\mu)/S_R$ is approximately t_{R-1}, given $S_{n_0}^2$ and MSE_{n_0}. Since, at stopping $d \simeq 2t_{1-\alpha/2,\,R-1}S_Rl_R/\sqrt{R}$, the coverage probability of the proposed confidence interval is

$$Pr\left[\overline{\overline{Y}}_R - 0.5d \le \mu \le \overline{\overline{Y}}_R + 0.5d\right] \simeq Pr\left[-l_Rt_{1-\alpha/2,\,R-1} \le \frac{\sqrt{R}(\overline{\overline{Y}}_R-\mu)}{S_R} \le l_Rt_{1-\alpha/2,\,R-1}\right]$$

$$= \mathrm{E}\left\{Pr\left[-l_Rt_{1-\alpha/2,\,R-1} \le \frac{\sqrt{R}(\overline{\overline{Y}}_R-\mu)}{S_R} \le l_Rt_{1-\alpha/2,\,R-1}\right] \,\middle|\, S_{n_0}^2,\ MSE_{n_0}\right\},$$

which is, by Proposition 1 and the dominated convergence theorem, equivalent to

$$\mathrm{E}\left[\Phi(z_{1-\alpha/2}l_{n^*(r)}) - \Phi(-z_{1-\alpha/2}l_{n^*(r)})\right] \ge 1-\alpha.$$

4 Simulation Results

Since the results of Theorem 1 are asymptotic, a Monte Carlo simulation study was conducted to evaluate the proposed sequential procedures when the sample size ranges from moderate to large, under varying degrees of unbalancedness.

The simulation parameters were set as follows: $\mu = 0$, $\sigma_e^2 = 1$, $\sigma_b^2 = 0.1$, 1 and 9, $n_0 = 6, 10$, $r_0 = 5$, and $\alpha = 0.05$. To introduce unbalancedness in the pilot sample, the number of replicates, r_i $(i = 1, \ldots, n_0)$, was generated from the binomial distribution with parameters r_0 and $1-\delta$, where $\delta = 0.1, 0.2, 0.5$. Following Ahrens and Pincus (1981), we used $\gamma = \tilde{r}_{n_0}/\overline{r}_{n_0}$, where $\overline{r}_{n_0}$ denotes the arithmetic mean of $r_1, \ldots, r_{n_0}$, to measure imbalance in the pilot sample.

We also assumed that $l_0 = 0$, $d =$ (0.10, 0.15, 0.20, 0.30), (0.2, 0.3, 0.5, 0.7) and (0.6, 0.9, 1.2, 1.5) for $\sigma_b^2 = 0.1$, 1 and 9, respectively. Let r^* be given by (7) when the values of σ_b^2 and σ_e^2 are used instead of $\hat{\sigma_b}^2$ and $\hat{\sigma_e}^2$, respectively, and define $N^* = r^* \times n^*(r^*)$. Thus, $N*$ is the sample size based on the steepest descent approach had σ_b^2 and σ_e^2 been known. For the selected values of σ_e^2, σ_b^2, d, n_0 and α we note that r^* ranges from 2 to 7, $n^*(r^*)$ ranges from 42 to 577 and N^* ranges from 96 to 2618. In each simulation setting, r was determined as described in Step 2 of Section 3, then R was sequentially estimated from (8), and the confidence interval was computed from (9) based on R. This process was repeated 1000 times to yield the following statistics: the average group size, $\overline{r}$, the average number of groups, $\overline{R}$, the average sample size, $\overline{N}$, the percentage of waste due to oversampling, %waste $= 100 \times (\overline{N} - N^*)/N^*$, and the estimated probability of coverage $\overline{p}$ (the proportion of times that the interval (9) covers μ), as well as summary statistics concerning γ.

The simulated unbalanced one-way models turned out to represent varying degrees of imbalance, as the values of γ ranged from a minimum of 0.580, for the setting ($n_0 = 6$, $r_0 = 5$, $\delta = 0.5$), to the maximum value of 1. In fact, the average values of γ were 0.97, 0.94, and 0.82 for $\delta = 0.1$, 0.2 and 0.5, respectively, indicating that imbalance increase with δ. Also, there were no violations in the desired confidence level in all settings where l_0 was set to 0, suggesting that damping factors are not needed. Further, the relative error of $\overline{R}$, $\mid \overline{R} - n^* \mid /n^*$, the relative error of $\overline{r}$, $\mid \overline{r} - r^* \mid /r^*$, and %waste were found to slightly increase with δ. To save space, we report only the results for $\delta = 0.5$, which correspond to higher degrees of imbalance (lower values of γ).

Table 1 gives the results of the proposed sequential procedure for the interval estimation of μ in the two settings ($n_0 = 6$, $r_0 = 5$, $\delta = 0.5$, $l_0 = 0$) and ($n_0 = 10$, $r_0 = 5$, $\delta = 0.5$, $l_0 = 0$). It is easily verified that the ratio $\overline{R}/n^*$ converges to 1, as $d \to 0$, in line with Proposition 1, and that the relative error of $\overline{R}$ was small, i.e., $\mid \overline{R} - n^* \mid /n^* \leq 0.12$, except when σ_b^2 is much smaller than σ_e^2 in magnitude and $\delta = 0.5$, in which case the relative error of $\overline{R}$ could be as high as 0.22 for ($n_0 = 6$, $r_0 = 5$, $\delta = 0.5$, $l_0 = 0$) and 0.36 for ($n_0 = 10$, $r_0 = 5$, $\delta = 0.5$, $l_0 = 0$). Further, the relative error of $\overline{r}$ was also small, i.e., $\mid \overline{r} - r^* \mid /r^* \leq 0.09$, except when σ_b^2 is of the same magnitude as σ_e^2, in which case the relative error of $\overline{r}$ could be as high as 0.56 for ($n_0 = 6$, $r_0 = 5$, $\delta = 0.5$, $l_0 = 0$) and 0.33 for ($n_0 = 10$, $r_0 = 5$, $\delta = 0.5$, $l_0 = 0$). Using the normal approximation to the binomial, if the true confidence coefficient is 0.95, there is a 2.5% chance that an estimated confidence coefficient based on 1000 simulations will be less than 0.936. Thus, the estimated probability of coverage $\overline{p}$ is seen to be consistent with Theorem 1. In both settings the waste was rather small except when σ_b^2 is of the same magnitude as σ_e^2, in which case using $n_0 = 10$ produced a smaller waste (12.60 - 22.81%) than $n_0 = 6$ (30.14 - 35.29%). It should be noted that the small negative waste that appears in Table 1 at $\sigma_b^2 = 0.1$ under the setting ($n_0 = 6$, $r_0 = 5$, $\delta = 0.5$, $l_0 = 0$) represents a slight undersampling.

We point out here that the coverage results of Table 1 agree with those of Hall (1981) who found that when n_0 is fixed then as d decreases the coverage probability tends at first to decrease and then to increase, which suggests that the value of n_0 is relatively important for large values of d.

[Insert Table 1 about here]

5 Example

Consider the data set in Table 2 concerning the modulus of elasticity (y) in units of 1000 psi of test pieces of Eastern white pine trees, which was analyzed by Bliss (1967; p. 259) under Model (1). To illustrate let us consider a pilot sample which consists of the six trees numbered 1-4, 7, 8 (each with three replicates) and the four trees numbered 5, 6, 12, 14 (each with four replicates). For such a pilot sample we have $n_0 = 10$, $\gamma = 0.98$, $\hat{\sigma_e}^2 = 8551$, $\hat{\sigma_b}^2 = 18370$. Based on the simulation results, l_0 was set at 0.

[Insert Table 2 about here]

Consider constructing a 95% confidence interval for estimating μ such that the width $d = 160$. Following Procedure 1, it was found from (7) that $r = \max\{2, 1.94\} = 2$. The implementation of the stopping rule (Step 3 of Procedure 1) is illustrated in Table 3, which shows that the procedure stopped at $R = 14$. Table 3 was calculated using Excel to sequentially update the estimates as in (2), where the first update starts with $n = n_0 + 1$. Note that the first two replicates of trees number 9, 10, 11 and 13 were sequentially used to yield the confidence interval (862.22, 1022.22), according to (9).

[Insert Table 3 about here]

6 Conclusions

It has been shown both analytically and numerically that the goal of effectively controlling both the width and validity of confidence intervals for the estimation of the mean in unbalanced one-way random effects models was always met. Furthermore, the simulation results indicated that the proposed sequential procedures performed rather well even for moderate sample sizes and under varying degrees of imbalance. Moreover, it seems that damping factors are not needed. In terms of circumventing unduly waste (oversampling), it is recommended to increase the initial number of groups n_0 (which prevents premature stopping) whenever it is expected that the two variance components (σ_b^2 and σ_e^2) are of comparable magnitudes.

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Table 1: Simulations of 95% confidence intervals for μ, based on Procedure 1, when $\mu = 0$, $\sigma_e^2 = 1$, and the number of replicates is Binomial(5, 0.5)

		$n_0 = 6,\ r_0 = 5,\ \delta = 0.5,\ l_0 = 0$				$n_0 = 10,\ r_0 = 5,\ \delta = 0.5,\ l_0 = 0$			
σ_b^2	d	$\overline{r}$	$\overline{T}$	%waste	$\overline{p}$	$\overline{r}$	$\overline{T}$	%waste	$\overline{p}$
9.0	1.500	2.03	65.58	4.25	0.938	2.03	66.30	7.30	0.943
9.0	1.200	2.04	101.92	3.26	0.955	2.02	102.91	4.31	0.952
9.0	0.900	2.03	180.70	1.97	0.944	2.03	180.16	2.42	0.941
9.0	0.600	2.04	405.14	2.32	0.949	2.03	407.99	2.51	0.937
1.0	0.700	3.08	44.62	30.14	0.944	3.95	43.10	12.60	0.936
1.0	0.500	3.11	85.86	34.80	0.943	3.90	82.67	18.87	0.947
1.0	0.300	3.09	236.33	35.29	0.944	3.98	225.06	22.81	0.947
1.0	0.200	3.04	533.58	35.13	0.950	3.90	505.26	21.72	0.946
0.1	0.300	4.69	63.52	-1.07	0.948	7.22	57.17	10.16	0.950
0.1	0.200	4.60	140.97	-1.48	0.956	7.13	115.61	5.47	0.944
0.1	0.150	4.53	249.63	-2.21	0.956	7.21	194.46	4.62	0.947
0.1	0.100	4.64	542.98	-1.92	0.948	7.16	424.69	2.49	0.948

Table 2: Modulus of elasticity y of test pieces of Eastern white pine trees

Tree	y						r_i	Tree	y						r_i
1	676	738	913				3	10	891	835	905	660	1049	806	6
2	962	872	772				3	11	779	801	795	797	554	736	9
3	779	788	710				3		881	790	839				
4	1002	963	892				3	12	843	1021	968	948			4
7	1002	903	1022				3	13	1123	921	1239	1125	863	789	6
8	952	1184	1233				3	14	1217	1178	1151	1240			4
5	940	893	946	878			4	15	1068	902					2
6	919	1071	1284	1140			4	16	1013	814					2
9	971	834	1144	779	941	1059	8	17	892	1039					2
	803	605													

Table 3: Computation steps of the 95% confidence interval for μ in the example

n	Tree(s)	Pieces	N	$\overline{Y}_{n.}$	$\overline{\overline{Y}}_n$	S_n^2	$t_{0.975,\,n-1}$	$2t_{0.975,\,n-1}S_n/\sqrt{n}$
$n_0 = 10$	1 - 8, 12, 14	all 34	34		961.36	20935.43	2.26	207.01
11	9	first 2	36	902.50	956.01	19156.83	2.23	185.97
12	10	first 2	38	863.00	948.26	18136.16	2.20	171.13
13	11	first 2	40	790.00	936.08	18551.38	2.18	164.61
R=14	13	first 2	42	1022.00	942.22	17651.61	2.16	153.42