On Interclass and Intraclass Correlations of Familial Data with Application on Hemoglobin Level from EDHS 2000 Familial Data

Zakaria A. Abd El-Wahed
Faculty of Economics & Political Science, Cairo University

Summary
In the analysis of familial data, the primary aim is to estimate the degree of resemblance between family members. Here we are interested in the special case where one group of family members consists of one individual, as typified in the following mother-sib situation. The degree of mother-sib resemblance is measured by the interclass correlation, and the degree of sib-sib resemblance is measured by the intraclass correlation. We derive the MINQUE estimators of the interclass and the intraclass correlation coefficients, also new three tests are proposed. Based on Egypt[1] sibship size, a Monte Carlo simulation is used to compare several different estimators of the interclass and intraclass correlation coefficients, also to compare several different tests of hypotheses of the interclass correlation

Nationally representative hemoglobin level among ever marred women aged 15 to 49 and their children under the age of 5 were recently determined in conjunction with the Egypt Demographic and Health Survey 2000 (2000 EDHS). The interclass correlation between mothers and their children of hemoglobin level and the intraclass correlation among children are estimated and tested.

1. Introduction

One of the main aims in the analysis of familial data is to estimate the degree of resemblance among family member with respect to some biological and medical attributes. Examples are given by Higgins and Keller (1975), Tager et al. (1976), An et al. (1999), Smeeth and Ng (2002), Wu et al. (2003), Adams et al. (2004), Mularski et al. (2004), Parker et al. (2005) and Witham et al. (2007).

To estimate the interclass correlation, several estimators have been proposed, some of these estimators have been discussed in detail by Rosner et al. (1977) and include, the pairwise estimator where each child in a family is paired with mother of that family, the sib-mean estimator where the mean offspring score from a family is paired with mother of that family, the random-sib estimator where a random offspring is chosen for each family and is paired with the mother of that family, and the ensemble estimator, a variant of the random-sib estimator whereby an 'expected value' for the random-sib estimator is computed over all possible choices of random sibs from each family. For the pairwise, sib-mean and random-sib estimators, an ordinary Pearson correlation is computed form the set of pairs formed over all families in the sample. It was shown by Rosner et al. (1977) that the pairwise estimator and the ensemble estimator are far superior to the sib-mean and the random-sib estimators in terms of mean squared error with the pairwise estimator being superior in the case of low

[1] Almost all the previous Monte Carlo simulations, the sibship sizes were randomly generated based on the distribution of sibship size in U.S. in 1950

intraclass correlation between sibs, and the ensemble estimator being superior when the intraclass correlation is high.

Rosner at al. (1977) derived explicit expressions for the maximum likelihood estimators when the sibship sizes are equal for all families in this case the maximum likelihood estimator of interclass correlation is equivalent to the pairwise estimator. When the sibship sizes are not all equal, Rosner (1979) proposed an algorithm for finding the maximum likelihood estimates, which involves iterative maximization (the technique is iterative and uses standard Newton-Raphson procedures to facilitate convergence) of an implicit function of interclass and intraclass correlations. This algorithm is difficult to implement and as Rosner pointed out, his algorithm may not converge for some sets of data. The work is parallel to the extensive work on the maximum likelihood estimation of the intraclass correlation (Patterson and Thompson, 1971; Hemmerale and Hartley, 1973; Harville, 1977)

Mak and Ng (1981) used a linear model approach of Kempthorne and Tandan (1953) to derive the maximum likelihood estimate of interclass correlation when the families have unequal numbers of offspring. It leads to an algorithm which involves the maximization of an explicit function of a variable for estimating one parameter and direct substitutions for other parameters. This algorithm is believed to be much simpler and more practical than that proposed by Rosner (1979).However; nothing is known about the convergence of the procedure. Another iterative method of finding the maximum likelihood estimate has been given by Smith (1980a). An alternative approach was given by Srivastava (1984) which requires solving only one equation.

Smith (1980 a, b) noted that an iterative method for determining the maximum likelihood estimate may fail to converge. Srivastava and Keen (1988) found that the quasi-Newton method failed to convergent for 18% to 36% of the samples for each combination for the interclass and intraclass correlation coefficients. Therefore a number of noniterative estimators have been proposed in the literature. Srivastava and Keen (1988) showed that some of these estimators associated with special cases of the generalized estimator derived from weighted sums of squares of measurements on parents and offspring.

All estimators except the ML estimator are derived by principles which are ad hoc in nature and do not minimize a clearly defined loss function, all of them are based on estimators of parent offspring covariance and among parent and among offspring variances which also do not exhibit optimal properties. ML estimators have been widely ignored because of computational difficulties. Kleffe (1993) derived simple explicit expressions for C. R. Rao's MINQUE (Minimum Norm Quadratic Unbiased Estimator/ Estimation) for the unknown parameters of the within-family covariance matrix and he used these optimal estimators to improve estimation of interclass and intraclass correlations in case of family data contained father, mother and siblings scores. He showed that in this case the father-sibling and mother-sibling covariances utilize father's and mother's scores. So if we use his method, we should observe both even though interest may focus on mother–sibling or father-sibling correlation only like our interest in this paper. In this paper we derive the MINQUE estimators for familial data contend mother's and her sibling's scores only.

For the intarclass correlation, several estimators have been proposed; some of these estimators have been discussed in detail by Donner and Koval (1980) and include the pairwise estimator, the ANOVA estimator and the maximum likelihood estimator. The pairwise estimator, introduced by Fisher in 1925, can be defined as the Pearson product-moment correlation as computed over all possible pairs of observations that can be constructed within families (each distinct pair is computed twice in this process) this estimator perhaps the oldest measure of intraclass correlation (Fieller and Smith, 1951;

Donner and Koval, 1980; Karlin et al., 1981). As shown by Fieller and Smith (1951), however, the pairwise estimator is an inefficient estimator of intraclass correlation for varying sibship size, since it tends to give too much weight to large-sized samples. For fixed sibship size the maximum likelihood estimator of intraclass correlation reduces to the pairwise estimator (Donner and Koval, 1980 and Rosner et al., 1977). The ANOVA estimator as suggested by Fisher in 1938 is widely accepted as the estimator of choice (see Rosner et al., 1979; Donner and Koval 1980, 1983; Coreil and Searle, 1976a, 1976b, Merwin and Harris, 1998, Donner and Zou, 2002). A question that sometimes arises in the calculation of the ANOVA estimate is whether or not those families having only one member should be included. Some available evidence indicates that they should not. In particular, a study by Swiger et al. (1964) suggests that inclusion of the one-member families in the analysis will tend to increase the standard error of the ANOVA estimator when both the ANOVA estimate and the numbers per family are small. Donner and Koval (1980) suggested that the ANOVA estimator of intraclass correlation coefficient can be computed over those families having two or more siblings only.

As regards significance testing of interclass correlations, four tests have been discussed in detail by Rosner et al. (1979), the classical pairwise test, the conservative pairwise test, the sib-mean test and the adjusted pairwise test. The classical pairwise, conservative pairwise and adjusted pairwise tests are based on the pairwise estimator. The classical pairwise, whereby one degree of freedom is attributed to each pair, the conservative test, whereby one degree of freedom is attributed to family, the adjusted pairwise test where number of degrees of freedom in a family are estimated as a function of the number of siblings in the family and the estimated sib-sib (intraclass) correlation, the aggregate degrees of freedom are then summed up over all families. The sib-mean test is based on the sib-mean estimator and one degree of freedom is attributed to each family in conducting the significance test. Rosner et al. (1979) showed that use classical pairwise test provides overstated type I error yielding estimated significance levels two to five times larger than the nominal levels, and the adjusted pairwise test was shown to compare favorably in power to three (the classical pairwise, conservative and sib-mean) other tests.

Other procedures have since been proposed for testing the significance of the interclass correlation. Konishi (1982) derived the large sample variance of the pairwise estimator, and proposed this expression as a basis for significance testing of interclass correlation. Procedures based on the method of maximum likelihood may also be developed, as discussed by Elston (1975) and Smith (1980a, 1980b). One advantage of the maximum likelihood approach is that it unifies the general problem of estimating and testing interclass correlation. One disadvantage, at least form the practical point of view is that the resulting procedures generally require complex, iterative procedures which are not widely available. A new three tests are presented in this paper based on the large sample variance of the ensemble, the Srivastava and the family-weighted estimators.

2. Definitions and terminology

Suppose we have a sample of measurements from n families and let x_i, y_{i1}, y_{i2}, ….., y_{ik_i} represent measurements from the ith family where x_i is the mother's score (in general, the parent's score) and y_{i1}, y_{i2}, ….., y_{ik_i} are scores of her k_i siblings. Let us assume the following model holds

$$Z_i' = (x_i, y_{i1}, y_{i2}, \ldots\ldots\ldots, y_{ik_i}) = (x_i, Y_i') \sim N(\mu_i, \Sigma_i) \qquad (2.1)$$

where $Y_i' = (y_{i1}, y_{i2}, \ldots\ldots\ldots, y_{ik_i})$, $\mu_i' = (\mu_m, \mu_s, \ldots\ldots\ldots \mu_s)$, $\Sigma_i^{11} = \sigma_m^2$, $\Sigma_i^{1j} = \Sigma_i^{j1} = \rho_{ms}\sigma_m\sigma_s$.
$j = 2, \ldots\ldots\ldots, k_i+1$, $\Sigma_i^{jj} = \sigma_s^2$, $\Sigma_i^{jl} = \rho_{ss}\sigma_s^2$, $j, l = 2, \ldots\ldots\ldots, k_i+1, j \neq l$.

Further the mother-sib resemblance or interclass correlation is denoted by ρ_{ms} and sib-sib or intraclass correlation is denoted by ρ_{ss}, where $\rho_{ss} \geq 0$. We are thus assuming that ρ_{ms} and ρ_{ss} are constant, and in particular, are independent of sibship size. We will assume that the sibship sizes are not necessarily the same in each family, since this the problem most frequently encountered in practice.

In order to estimate the intraclass correlation, a more frequently adopted model in epidemiological research is the components of variance model (Sahai, 1979) which states that the observations y_{ij} can be mathematically described as

$$y_{ij} = \mu + a_i + \varepsilon_{ij}, \tag{2.2}$$

where μ is the grand mean of all scores, the family effects $\{a_i\}$ are identically distributed with mean 0 and variance σ_A^2, the residual errors $\{\varepsilon_{ij}\}$ are identically distributed with mean 0 and variance σ_e^2, and $\{a_i\}$ and $\{\varepsilon_{ij}\}$ are completely independent. The variance of y_{ij} is then given by $\sigma^2 = \sigma_A^2 + \sigma_e^2$, and the intraclass correlation ρ_{ss} is then defined as $\dfrac{\sigma_A^2}{\sigma_A^2 + \sigma_e^2}$.

Equivalently, since

$$\rho_{ss} = E\{(y_{ij} - \mu)(y_{il} - \mu)\}/\sigma^2 = E\{(a_i + \varepsilon_{ij})(a_i + \varepsilon_{il})\}/\sigma^2$$
$$= \frac{E(a_i^2)}{\sigma^2} = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_e^2}$$

The components of variance model (2.2) is more appropriate in the analysis of family data than the common correlation model if there is interest in obtaining separate estimates of the parameters σ_A^2 and σ_e^2, while both model are satisfactory if interest focuses solely on estimating the intraclass correlation.

3. Estimation of Interclass Correlation

3.1. The pairwise estimator

The pairwise estimator is analogous to the method of computing the ordinary product-moment correlation, and is obtained by pairing each mother's score with each of her sibling's scores and considering the collection of all such pairs over all families. In this case the pairwise estimator of interclass correlation is given by:

$$\hat{\rho}_{ms.p} = \frac{\sum_i (x_i - \bar{x}) \sum_j (y_{ij} - \bar{y})}{\left\{\sum_i k_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\sum_i \sum_j (y_{ij} - \bar{y})^2\right\}^{\frac{1}{2}}}, \tag{3.1}$$

where $\bar{x} = \sum_i k_i x_i \big/ K$, $\bar{y} = \sum_i \sum_j y_{ij} \big/ K$, $K = \sum_i k_i$.

Several authors including Higgins and Keller (1975) and Tager et al. (1976), have used this method as an estimator of ρ_{ms} despite the fact that the independence assumption is not appropriate one for two reasons (1) a mother with more than one sibling appears in several different pairs and (2) two siblings in the same family are in general correlated $(\rho_{ss} \geq 0)$

3.2. The family- weighted pairwise estimator

Karlin et al. (1981) suggested weighting the pairs in the pairwise estimator by the inverse of the number of pairs contributed by each family, to reduce the disproportionate effect of large families in the final estimate. In this case the family-weighted pairwise estimator of interclass correlation is given by:

$$\hat{\rho}_{ms.fwp} = \frac{\sum_i (x_i - \bar{x})(1/k_i)\sum_j (y_{ij} - \bar{y})}{\left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\sum_i (1/k_i)\sum_j (y_{ij} - \bar{y})^2\right\}^{\frac{1}{2}}}, \tag{3.2}$$

where $\bar{x} = \frac{1}{n}\sum_i x_i$, $\bar{y}_{i.} = \frac{1}{k_i}\sum_j y_{ij}$, $\bar{y} = \frac{1}{n}\sum_i \bar{y}_{i.}$.

Another formula of the family-weighted pairwise estimator was given by Eliasziw and Donner (1990) which is:

$$\hat{\rho}_{ms.fwp} = \frac{\sum_i (x_i - \bar{x})(\bar{y}_{i.} - \bar{y})}{\left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\left[\sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2 / k_i\right] + \sum_i (\bar{y}_{i.} - \bar{y})^2\right\}^{\frac{1}{2}}} \tag{3.3}$$

3.3. The corrected pairwise estimator

Srivastava and Keen (1988) obtained the corrected pairwise estimator; the difference between the corrected pairwise estimator and the pairwise estimator is that the corrected pairwise estimator uses an unbiased estimator of the sibling variance σ_s^2 in place the biased in the definition of the pairwise estimator. The corrected pairwise estimator of interclass correlation is given by:

$$\hat{\rho}_{ms.cp} = \frac{\sum_i (x_i - \bar{x})\sum_j (y_{ij} - \bar{y})}{\left\{\sum_i k_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\sum_i k_i (\bar{y}_{i.} - \bar{y})^2 + n_k \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2\right\}^{\frac{1}{2}}}, \tag{3.4}$$

where $n_k = \frac{n-1}{K-n}(\bar{k} - \frac{s_k^2}{K} - 1)$, $s_k^2 = \sum_i (k_i - \bar{k})^2 / n - 1$, $\bar{k} = \sum_i k_i / n$.

3. 4. The sib-mean estimator

Falconer in 1960 (Rosner et al., 1977) has recommended paring each mother's score with the mean of her sibling scores for each family and then constructing a product- moment correlation coefficient. The resultant estimator is then given as follows

$$\hat{\rho}_{ms.m} = \frac{\sum_i (x_i - \bar{x})(\bar{y}_{i.} - \bar{y})}{\left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\sum_i (\bar{y}_{i.} - \bar{y})^2\right\}^{\frac{1}{2}}} \tag{3.5}$$

$\hat{\rho}_{ms.m}$ is known to be biased (O'Neill et al., 1987).

3. 5. The modified sib-mean estimator

Konishi (1982) pointed out that the sib-mean estimator is not consistent for the interclass correlation coefficient parameter. The modified sib-mean estimator, obtained by Konishi (1982), adjusts the sib-mean estimator in such a way that the new estimator is asymptotically unbiased and consistent.

$$\hat{\rho}_{ms.mm} = \left[\bar{k}_h^{-1} + (1-\bar{k}_h^{-1})\hat{\rho}_{ss}\right]^{\frac{1}{2}} \hat{\rho}_{ms.m}, \qquad (3.6)$$

where $\bar{k}_h = \left\{\frac{1}{n}\sum_i \frac{1}{k_i}\right\}^{-1}$ is the harmonic mean family size and $\hat{\rho}_{ss}$ may be estimated by any consistent estimator of the intraclass correlation

3.6. The random-sib estimator

Another method which may be used to deal with the lack of independence is to select a random sibling from each family and compute the correlation from the set of pairs of mothers and propositi. The resultant estimator is then given as follows:

$$\hat{\rho}_{ms.r} = \frac{\sum_i (x_i - \bar{x})(y_{ij}^{*} - \bar{y}^{*})}{\left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\sum_i (y_{ij}^{*} - \bar{y}^{*})^2\right\}^{\frac{1}{2}}}, \qquad (3.7)$$

where y_{ij}^{*} denote a random sibling from ith family and $\bar{y}^{*} = \sum_i y_{ij}^{*}/n$.

3.7. The ensemble estimator

The practical problem with the random-sib estimator is the loss of information resulting from considering only one sibling per family. The ensemble estimator was proposed by Rosner et al. (1977), which is an attempt to modify the random-sib estimator. The ensemble estimator is given as follows:

$$\hat{\rho}_{ms.e} = \frac{\sum_i (x_i - \bar{x})(\bar{y}_{i.} - \bar{y})}{\left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\left[\sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2/k_i\right]\left[1 - \frac{1}{n}\right] + \sum_i (\bar{y}_{i.} - \bar{y})^2\right\}^{\frac{1}{2}}}. \qquad (3.8)$$

It is noted that the expression $\sum_i (x_i - \bar{x})(\bar{y}_{i.} - \bar{y}) \Big/ \left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}}$ appears in both $\hat{\rho}_{ms.m}$ (3.5) and $\hat{\rho}_{ms.e}$ (3.8). The estimators differ only in the latter term of their respective denominators and in particular, Rosner et al. (1977) showed that $|\hat{\rho}_{ms.e}| \le |\hat{\rho}_{ms.m}|$. Equality is achieved only in the degenerate case when there is no within-sample variability within any family. Also from (3.3) and (3.8), it is observed that $\hat{\rho}_{ms.fwp}$ and $\hat{\rho}_{ms.e}$ are equivalent since $(1-1/n)$ in $\hat{\rho}_{ms.e}$ tends to unity as n increases.

3. 8. The maximum likelihood estimator

Srivastava (1984) used the transformation $\beta = \sigma_{ms}/\sigma_s^2 = \rho_{ms}(\sigma_s/\sigma_m)$, $\delta = (\sigma_s^2 - \beta^2\sigma_m^2)/\gamma_s^2$, $\gamma_s^2 = \sigma_s^2(1-\rho_{ss})$, $\mu = \mu_s - \beta\mu_m$, $\xi_i = \delta - a_i$, where $a_i = 1 - k_i^{-1}$. Thus, the transformation from the six parameters $(\mu_m, \mu_s, \sigma_m^2, \sigma_s^2, \rho_{ms}, \rho_{ss})$ to $(\mu_m, \mu, \sigma_m^2, \delta, \beta, \gamma_s^2)$ is one to one. Srivastava (1984) obtained the maximum likelihood of the parameters as follows

$$\hat{\mu}_m = \bar{x}\ ,\ \hat{\sigma}_m^2 = \sum_i (x_i - \bar{x})^2 \Big/ n\ ,\ \hat{\mu} = c^{-1}(\sum_i c_i \bar{y}_{i.} - \hat{\beta}\sum_i c_i x_i),$$

$$\hat{\beta} = \{c\sum_i c_i x_i \bar{y}_{i.} - (\sum_i c_i x_i)(\sum_i c_i \bar{y}_{i.})\} \Big/ \{c\sum_i c_i x_i^2 - (\sum_i c_i x_i)^2\},$$

$$\hat{\gamma}_s^2 = K^{-1}\{\sum_i [\sum_j y_{ij}^2 - \frac{1}{k_i}(\sum_j y_{ij})^2] + \sum_i c_i (\bar{y}_{i.} - \hat{\mu} - \hat{\beta}x_i)^2\},$$

where $c_i = \xi_i^{-1}$ and $c = \sum_i c_i$. Also δ can be estimated from the equation

$\sum \hat{\xi}_i^{-1} - \hat{\gamma}_s^2 \sum_i \hat{\xi}_i^{-2} (\bar{y}_{i.} - \hat{\mu} - \hat{\beta}x_i)^2 = 0$, after substitution the values of $\hat{\mu}$, $\hat{\beta}$, $\hat{\gamma}_s^2$ and $\hat{\xi}_i$. This equation will involve only one unknown parameter δ , the solution of which can be obtained iteratively. Having obtained $\hat{\delta}$, $\hat{\mu}$, $\hat{\beta}$, $\hat{\gamma}_s^2$ and $\hat{\sigma}_s^2$ can be obtained. Hence, the maximum likelihood estimate of ρ_{ms} is given by

$$\hat{\rho}_{ms.ML} = \hat{\beta}\hat{\sigma}_m \big/ \hat{\sigma}_s \tag{3.9}$$

3. 9 The Srivastava estimator

Srivastava (1984) obtained a noniterative estimator for interclass correlation which is easier to compute than the maximum likelihood estimator. the Srivastava estimator is given as follows

$$\hat{\rho}_{ms.s} = \frac{\sum_i (x_i - \bar{x})(\bar{y}_{i.} - \bar{y})}{(n-1)^{\frac{1}{2}} \tilde{\sigma}_s \left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}}}, \tag{3.10}$$

where $\tilde{\sigma}_s^2 = (n-1)^{-1}\sum_i (\bar{y}_{i.} - \bar{y})^2 + n^{-1}\tilde{\gamma}_s^2(\sum_i (1 - k_i^{-1})$, $\tilde{\gamma}_s^2 = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2 \Big/ (K - n)$,

substituting $\tilde{\sigma}_s^2$ into (3.10) results in the form of estimator given later by Srivastava and Keen (1988)

$$\hat{\rho}_{ms.s} = \frac{\sum_i (x_i - \bar{x})(\bar{y}_{i.} - \bar{y})}{\left\{\sum_i (x_i - \bar{x})^2\right\}^{\frac{1}{2}} \left\{\sum_i (\bar{y}_{i.} - \bar{y})^2 + k_s \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2\right\}^{\frac{1}{2}}} \tag{3.11}$$

where $k_s = \dfrac{n-1}{K-1}(1 - \bar{k}_h^{-1})$

3. 10 The MINQUE

The MINQUE is obtained by equating certain quadratic forms to their expectations. These quadratic forms are chosen to minimize local variance and depend on a priori information about the variance and covariance parameters. The MINQUE estimators are unbiased, have locally minimum variance given normality distributed data, are strongly consistent for bounded k_i as n tends to infinity and are asymptotically normal if the ratios of different k_i converge. These asymptotic properties also extend to the case of random sampling of families (see Kleffe, 1993)

The first and second moments of Z_i (2.1) can be written as.

$$E(Z_i)=\begin{pmatrix}\mu_m\\ \mu_s 1_{k_i}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1_{k_i}\end{pmatrix}\begin{pmatrix}\mu_m\\ \mu_s\end{pmatrix}=X_i\alpha\ ,\operatorname{cov}(Z_i)=V_i=\begin{pmatrix}\sigma_m^2 & \sigma_{ms}1'_{k_i}\\ \sigma_{ms}1_{k_i} & \sigma_e^2 I_{k_i}+\sigma_A^2 J_{k_i}\end{pmatrix}, \quad (3.12)$$

where σ_e^2 and σ_A^2 are covariance parameters. Here I_{k_i} is the unit matrix of order k_i, 1_{k_i} is a k_i-vector with all elements are ones, $X_i=\begin{pmatrix}1 & 0\\ 0 & 1_{k_i}\end{pmatrix}, \alpha=\begin{pmatrix}\mu_m\\ \mu_s\end{pmatrix}$ and $J_{k_i}=1_{k_i}1'_{k_i}$. The interclass and intraclass correlations can be expressed as

$$\rho_{ms}=\frac{\sigma_{ms}}{\left\{\sigma_m^2(\sigma_A^2+\sigma_e^2)\right\}^{\frac{1}{2}}}\ ,\ \rho_{ss}=\frac{\sigma_A^2}{\sigma_A^2+\sigma_e^2}. \quad (3.13)$$

The MINQUE estimator of σ_m^2 and σ_{ms} are given by $\hat{\sigma}_m^2=\frac{SSR_p}{n-1}$ and $\hat{\sigma}_{ms}=\frac{\hat{\sigma}_m^2}{\sigma_{om}^2}\sigma_{oms}-C_1^{-1}Q$. Also the MINQUE estimator $\hat{\sigma}_e^2$ and $\hat{\sigma}_A^2$ can be obtained from the flowing equations

$$Q_1=C_3\hat{\sigma}_e^2+C_2[\hat{\sigma}_A^2+\frac{\sigma_{oms}}{\sigma_{om}^2}(\frac{\hat{\sigma}_m^2}{\sigma_{om}^2}\sigma_{oms}-2\hat{\sigma}_{ms})] \quad (3.14)$$

$$Q_2=C_4\hat{\sigma}_e^2+C_3[\hat{\sigma}_A^2+\frac{\sigma_{oms}}{\sigma_{om}^2}(\frac{\hat{\sigma}_m^2}{\sigma_{om}^2}\sigma_{oms}-2\hat{\sigma}_{ms})] \quad (3.15)$$

where $SSR_p=\sum_i(x_i-\tilde{x})^2$, $SSR_s=\sum_i\sum_j(y_{ij}-\bar{y}_{i.})^2$, $Q=\sum_i\kappa_i(x_i-\tilde{x})$, $Q_1=\sum_i\kappa_i^2$,

$Q_2=SSR_s+\sum\kappa_i/k_i$, $\kappa_i=L_i\{\sigma_{oms}(x_i-\tilde{x})-(\bar{y}_{i.}-\tilde{y})\}$, $\tilde{x}=\frac{1}{S_{10}}\sum_i\frac{k_i}{1+\lambda k_i}x_i$,

$\tilde{y}=\frac{1}{S_{10}}\sum_i\frac{k_i}{1+\lambda k_i}\bar{y}_{i.}$, $\lambda=\frac{\sigma_{oA}^2-\sigma_{oms}^2\sigma_{om}^{-2}}{\sigma_{oe}^2}$, $L_i=k_i\left\{1+k_i(\sigma_{oA}^2-\sigma_{oms}^2)\right\}^{-1}$, $C_1=S_{10}-S_{20}S_{10}^{-1}$,

$C_2=S_{20}-2S_{30}S_{10}^{-1}+S_{20}^2S_{10}^{-2}$, $C_3=S_{21}-2S_{31}S_{10}^{-1}+S_{21}S_{20}S_{10}^{-2}$,

$C_4=K-n+S_{22}-2S_{32}S_{10}^{-1}+S_{21}^2S_{10}^{-1}$, $S_{hk}=\sum_i\left(\frac{k_i}{1+\lambda k_i}\right)^h\left(\frac{1}{k_i}\right)^k$ (*h=1,2,3, k=0,1,2*) and σ_{om}^2, σ_{oms}, σ_{oe}^2 and σ_{oA}^2 are the priori parameters of σ_m^2, σ_{ms}, σ_e^2 and σ_A^2

The estimator for the σ_m^2 does not depend on a priori information and is the uniformly minimum variance invariant unbiased estimator given normality (Rao, 1971).The estimators for σ_e^2 and σ_A^2 do not depend only on sibling scores but also on mother scores. Chose of $\sigma_{oms}=0$ removes the mother scores from the constants κ_i, and the estimation of σ_e^2 and σ_A^2 reduced to the MINQUE estimators under the well-investigated under unbalanced one-way classification model for the sibling's scores only.

4. Estimation of Intraclass Correlation

4.1. The pairwise estimator

The pairwise estimator of intraclass correlation $\hat{\rho}_{ss.p}$, can be defined as the Pearson product-moment correlation as computed over all possible pairs of observations that can be constructed within families (each distinct pair is computed twice in this process, which eliminates the need to designate one member as X and other as Y). If $k_i = k$ ($i=1, 2, \ldots, n$), the pairwise estimator is given by

$$\hat{\rho}_{ss.p} = \frac{\sum_{i=1}^{n}\sum_{j\neq l}^{k}(y_{ij} - \bar{y})(y_{il} - \bar{y})}{K(k-1)S_y^2}, \tag{4.1}$$

where S_y^2 is the sample variance, computed over all K observations.

As shown by Fieller and Smith (1951), however, $\hat{\rho}_{ss.p}$ is an inefficient estimator of ρ_{ss} for varying k_i, since it tends to give too much weight to large-sized samples. In the special case where $k_i = k$ ($i=1,2,\ldots,n$) the maximum likelihood estimator of intraclass correlation reduces to the pairwise estimator (Donner and Koval,1980a and Rosner et al. , 1977)

4.2. The ANOVA estimator

Using the one way variance component model (2.2), we can estimate the variance components (σ_A^2 and σ_e^2) and then estimate the intraclass correlation, the analysis of variance (ANOVA) method is very frequently used (see Donner and Koval 1980, 1983, Coreil and Searle, 1976a,1976b, Merwin et al., 1998, Donner and Zou, 2002). The ANOVA estimator of the intraclass correlation ρ_{ss} is given by

$$\hat{\rho}_{ss.A} = \frac{(MSA - MSE)}{[MSA + (k_0 - 1)MSE]}, \tag{4.2}$$

where $k_0 = (n-1)^{-1}[K - \frac{\sum_i k_i^2}{K}]$, $MSA = SSA/(n-1)$, $MSE = SSE/(K-n)$, SSA and SSE are the among groups and within groups sum of squares.

A question that sometimes arises in the calculation of $\hat{\rho}_{ss.A}$ is whether or not those families having only one member should be included. Some available evidence indicates that they should not. In particular, a study by Swiger et al. (1964) suggested that inclusion of the one-member families in the analysis will tend to increase the standard error of $\hat{\rho}_{ss.A}$ when both $\hat{\rho}_{ss.A}$ and the numbers per family is small. Donner and Koval (1980) denoted that the analysis of variance estimator of intraclass correlation coefficient as computed over those families having two or more siblings only by $\hat{\rho}_{ss.AD}$.

4.3. The maximum likelihood estimator

A maximum likelihood approach to the estimation of variance components and then estimation of intraclass correlation has some attractive features. The maximum likelihood estimators are functions of every sufficient statistic and are consistent and asymptotically normal and efficient (Miller, 1977). In spite of these properties, the maximum likelihood estimators of variance components take no account of the loss in degrees of freedom resulting from the estimation of the model's fixed effects, also, the maximum likelihood estimators are derived under the assumption of a particular parametric form, generally normal, for the distribution of the data vector. The first of these problems has in effect been eliminated by Patterson and Thompson (1971) through their restricted maximum likelihood approach. With regard to the second problem, Harville (1977) showed that the maximum likelihood estimator derived on the basis of normality may suitable even when the form of the distribution is not specified.

Donner and Koval (1980) showed that the maximum likelihood estimator $\hat{\rho}_{ss.ML}$ of ρ_{ss} is given by the value of ρ_{ss} that minimizes

$$-2\ln L = K(1+\ln\hat{\sigma}_s^2+\ln 2\pi)+(K-n)\ln(1-\rho_{ss})+\sum_i \ln W_i \ , \qquad (4.3)$$

where

$$\hat{\sigma}_s^2 = \left\{\sum_i\left(\frac{W_i-\rho_{ss}}{W_i}\right)\sum_j(y_{ij}-\hat{\mu}_s)^2-\rho_{ss}\sum_{i=1}^{n}\sum_{j=1}^{k_i}\sum_{l\neq j}^{k_i}\frac{(y_{ij}-\hat{\mu}_s)(y_{il}-\hat{\mu}_s)}{W_i}\right\}\Bigg/K(1-\rho_{ss})$$

$$\hat{\mu}_s = \sum_i\frac{k_i\bar{y}_{i.}}{W_i}\Bigg/\sum_i\frac{k_i}{W_i}, \ W_i = 1+(k_i-1)\rho_{ss}.$$

This estimator also corresponds to the maximum likelihood estimator of ρ_{ss} under the random effects model (2.2).

4.4. The restricted maximum likelihood estimator

The restricted maximum likelihood (RML) method consists of maximizing the likelihood, not of all the data, but of a set of selected error contrasts. When the sibship sizes are equal (balanced case) results are identical with those obtained by the method of Neder (1968).

Based on the one way variance components model (2.2), the first and the second moments of $Y' = (y_{11},......,y_{1k_1},y_{21},......,y_{2k_2},..........,y_{n1},......,y_{nk_n})$ can be written as $E(Y) = X\mu = 1_K\mu$ and $\text{cov}(Y) = V = V_1\sigma_A^2+V_2\sigma_e^2$, where $V_1 = Diag(J_{k_i})$ and $V_2 = I_K$. The RMLE of σ_A^2 and σ_e^2 calculated from the following equation.

$$\hat{\sigma}_{RML} = F^{-1}G\ , \qquad (4.4)$$

where $\hat{\sigma}'_{RML} = (\hat{\sigma}^2_{A.RML},\ \hat{\sigma}^2_{e.RML})$, the ij*th* element of F is $tr(RV_iRV_j)$, the *i*th element of G is $Y'RV_iRY$ and $R = V^{-1}-V^{-1}X(X'V^{-1}X)^{-1}X'V^{-1}$

The system of equation (4.4) cannot be solved analytically because the elements of R in contained σ_A^2 and σ_e^2, then an iterative procedure must be adopted. Finally the RMLE of ρ_{ss} is given by

$$\hat{\rho}_{ss.RML} = \frac{\hat{\sigma}^2_{A.RML}}{\hat{\sigma}^2_{A.RML}+\hat{\sigma}^2_{e.RML}} \qquad (4.5)$$

4.5. The MINQUE

The MINQUE estimator of ρ_{ss} is $\hat{\rho}_{ss.MINQUE} = \dfrac{\hat{\sigma}_A^2}{\hat{\sigma}_A^2+\hat{\sigma}_e^2}$ where $\hat{\sigma}_A^2$ and $\hat{\sigma}_e^2$ are the MINQUE estimators of σ_A^2 and σ_e^2 as given in subsection 3.10. As shown in subsection 3.10, chose of $\sigma_{oms}=0$ removes the mother scores from the constants κ_i, and the estimation of σ_e^2 and σ_A^2 reduced to the MINQUE estimators under the one way variance components model (2.2) for the sibling's scores only.

4.6. The unweighted means estimator

A well known alternative to the ANOVA method for estimating ρ_{ss} is an analysis based on the unweighted group means. The unweighted means estimator of the intraclass correlation is given by

$$\hat{\rho}_{ss.u} = \frac{V_B}{[V_B + MSE]}, \qquad (4.6)$$

where $V_B = \frac{\sum_i (\bar{y}_{i.} - \bar{y})^2}{n-1} - \frac{\sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2}{(K-n)\bar{k}_h}$

4.7. The Srivastava estimator

Srivastava (1984) obtained an estimator for intralass correlation; this estimator is given as follows

$$\hat{\rho}_{ss.s} = 1 - \tilde{\gamma}_s^2 / \tilde{\sigma}_s^2 . \qquad (4.7)$$

where $\tilde{\gamma}_s^2$ and $\tilde{\sigma}_s^2$ are given in 3.9

5. Test of hypotheses about interclass correlation

The null hypothesis H_o: $\rho_{ms} = 0$ versus H_A: $\rho_{ms} \neq 0$ is investigated. In this section ten possible tests are presented.

5.1. The likelihood ratio test

The likelihood ratio (LR) test statistic is computed by (1) setting $\rho_{ms} = 0$ in the likelihood function (2) minimizing the resulting expression for $-2\log L_o$ (L_o is the likelihood function under H_o) with respect to all remaining parameters and (3) subtracting this minimum from the minimum value of $-2\log L$ as computed over all parameters in the model. It follows from standard likelihood theory that the resulting test statistic W is approximating chi-square with one degree of freedom under H_o.

$$W = -2\log L - (-2\log L_o) \qquad (5.1)$$

Mak and Ng (1981) discuss a regression approach to testing H_o: $\rho_{ms} = 0$. If $\rho_{ss} = 0$, standard regression method may be used, with the child score as the dependent variable. For the case of unknown ρ_{ss}, they provide an LR test for H_o which , unlike the LR test derived by Donner and Bull (1984) , is valid whether the parent score is assumed to be nonstochastic or random. They also provide an algorithm which may be useful for implementing this test.

5.2. A test based on the large sample variance of the maximum likelihood estimator

Rosner (1982) showed that the large sample variance of $\hat{\rho}_{ms.ML}$, the maximum likelihood of ρ_{ms}, is given under H_o: $\rho_{ms} = 0$ by

$$\mathrm{var}(\hat{\rho}_{ms.ML})_o = \left[\sum_i k_i \Big/ \{1 + (k_i - 1)\rho_{ss}\} \right]^{-1} . \qquad (5.2)$$

Therefore an appropriate large sample test of significance for interclass correlation is given by

$$Z_{ML} = \hat{\rho}_{ms.ML} \Big/ \sqrt{\mathrm{var}(\hat{\rho}_{ms.ML})_o}, \qquad (5.3)$$

where Z_{ML} is referred to tables of the standard normal distribution. Since ρ_{ss} will, in general, be unknown, it is replaced by the maximum likelihood estimator. The resulting test is referred to as the Z_{ML} procedure.

5.3. The classical pairwise test

The most straightforward procedure is to assume that each of the mother-child pairs within a family is independent and thus to assume that one has $K=\sum_{i=1}^{n} k_i$ independent pairs over n families. One then can compute the test statistic

$$t_{class}=(K-2)^{\frac{1}{2}}\hat{\rho}_{ms.p}\Big/\sqrt{(1-\hat{\rho}^2_{ms.p})}, \qquad (5.4)$$

which under H_o would have a t distribution with (K-2) d.f.. H_o would be rejected if $|t_{class}|>t_{K-2,1-\frac{\alpha}{2}}$, where $t_{K-2,1-\frac{\alpha}{2}}$ is $100(1-\frac{\alpha}{2})$percentile of a t distribution with K-2 d.f.

5.4. The conservative pairwise test

If the pairs are not independent, the one effectively has less than K d.f. over all families and one would expect that the true significance level for the classical pairwise test will be larger than the nominal level of α. Indeed, in the extreme case where $\rho_{ss}=1$, one has exactly one d.f. per family or n d.f. over all families, and one could propose a significance test based on the test statistic

$$t_{cons}=(n-2)^{\frac{1}{2}}\hat{\rho}_{ms.p}\Big/\sqrt{(1-\hat{\rho}^2_{ms.p})}, \qquad (5.5)$$

which under H_o would have a t distribution with (n-2) d.f.. H_o would be rejected if $|t_{cons}|>t_{n-2,1-\frac{\alpha}{2}}$. If one applies this test procedure when $0<\rho_{ss}<1$, the test is likely to be conservative and will yield true significance level lower than the nominal level of α, since one is surely understanding the total d.f. over all families.

5.5. The adjusted pairwise test

The problem remains of assessing the true aggregate d.f. over n families. Under the model in (2.1), Rosner et al. (1979) showed that the "effective degrees of freedom" among observations in the ith family is given approximated by

$$d_i=k_i/\{1+(k_i-1)\rho_{ss}\} \qquad i=1,2,\ldots\ldots\ldots,n$$

Since ρ_{ss} is in general unknown, d_i is estimated in practice by

$$d_i^*=k_i/\{1+(k_i-1)\hat{\rho}_{ss.AT}\} \qquad i=1,2,\ldots\ldots\ldots,n$$

where $\hat{\rho}_{ss.AT}$ is the truncated ANOVA estimator of the intraclass correlation ρ_{ss}. Letting $D^*=\sum_{i=1}^{n} d_i^*$, denote the aggregate degrees of freedom over all n families, this result implies that $H_o:\rho_{ms}=0$ may be tested using the statistic

$$t_{adj}=(D^*-2)^{\frac{1}{2}}\hat{\rho}_{ms.p}\Big/\sqrt{(1-\hat{\rho}^2_{ms.p})}. \qquad (5.6)$$

Then approximately under H_o t_A has t-distribution with n-2 d.f, and H_o is rejected if $|t_{adj}|>t_{n-2,1-\frac{\alpha}{2}}$

5.6. A test based on the large sample variance of the pairwise estimator

Konishi (1982) Showed that the large sample variance of $\hat{\rho}_{ms.p}$ under $H_o : \rho_{ms} = 0$ is given by

$$\text{var}(\hat{\rho}_{ms.p})_o = \left\{1 + \rho_{ss}\left(\frac{\sum_{i=1}^{n} k_i^2}{K} - 1\right)\right\}\frac{K}{\sum_{i \neq j}^{n} k_i k_j}. \tag{5.7}$$

Thus $H_o : \rho_{ms} = 0$ may be tested by referring the statistic $Z_p = \hat{\rho}_{ms.p} \Big/ \sqrt{\text{var}(\hat{\rho}_{ms.p})_o}$ to tables of the standard normal distribution. The unknown parameter ρ_{ss} is again replaced by the $\hat{\rho}_{ss.AT}$.

5.7. A test based on the sib-mean estimator

If $k_i = k$ $i=1,\dots\dots,n$ then a test of significance with appropriate type I error is given by the test statistic (see Rosner et al. ,1979)

$$t_m = (n-2)^{\frac{1}{2}} \hat{\rho}_{ms.m} \Big/ \sqrt{1 - \hat{\rho}_{ms.m}^2}, \tag{5.8}$$

which under H_o would have a t distribution with *(n-2)* d.f. H_o would be rejected if $|t_m| > t_{n-2, 1-\frac{\alpha}{2}}$ .Velu and Rao (1990) showed that the distribution of t_m is student's t with *n-2* degrees of freedom under the assumption that $\rho_{ms} = 0$ even where the k_i's are not the same. Thus t_m may be referred to tables of the t-distribution with *n-2* degrees of freedom to provide an approximate p-value for testing H_o.

5.8. A test based on the large sample variance of the ensemble estimator

O'Neill et al. (1987) showed that the asymptotic variance of the ensemble estimator is given by:

$$\text{var}(\hat{\rho}_{ms.e}) = \frac{1}{n}[\bar{k}_h^{-1}(1-\rho_{ms}^2)^2 + (1-\bar{k}_h^{-1})\{(\rho_{ss} - \rho_{ms}^2)(1-\rho_{ms}^2) + \frac{1}{2}\rho_{ms}^2(1-\rho_{ss})^2\}] \tag{5.9}$$

Thus the large sample variance of $\hat{\rho}_{ms.e}$ under $H_o : \rho_{ms} = 0$ is given by

$$\text{var}(\hat{\rho}_{ms.e})_0 = \frac{1}{n}[\bar{k}_h^{-1} + \rho_{ss}(1-\bar{k}_h^{-1})]. \tag{5.10}$$

Therefore $H_o : \rho_{ms} = 0$ may be tested by referring the statistic $Z_e = \hat{\rho}_{ms.e} \Big/ \sqrt{\text{var}(\hat{\rho}_{ms.e})_o}$ to tables of the standard normal distribution. The unknown parameter ρ_{ss} is again replaced by $\hat{\rho}_{ss.AT}$.

5.9. A test based on the large sample variance of the Srivastava estimator

Srivastava and Katapa (1986) showed that the large sample variance of the Srivastava estimator $\hat{\rho}_{ms.s}$ is

$$\text{var}(\hat{\rho}_{ms.s}) = \frac{1}{n}[\rho_{ms}^4 + \rho_{ms}^2\{\frac{1}{2}c^2 - 2\lambda - \frac{1}{2}\} + \lambda] \tag{5.11}$$

where $c^2 = 1 - 2(1-\rho_{ss})(1-\bar{k}_h^{-1}) + (1-\rho_{ss})^2\left\{\frac{1}{2}\sum_i (1-k_i^{-1})^2 + (1-\bar{k}_h^{-1})^2/(\bar{k}-1)\right\}$ and

$\lambda = 1 - (1-\rho_{ss})(1-\bar{k}_h^{-1})$

Thus the large sample variance of $\hat{\rho}_{ms.s}$ under $H_o : \rho_{ms} = 0$ is given by

$$\text{var}(\hat{\rho}_{ms.s})_0 = \frac{1-(1-\rho_{ss})(1-\bar{k}_h^{-1})}{n}. \tag{5.12}$$

Therefore $H_o : \rho_{ms} = 0$ may be tested by referring the statistic $Z_s = \hat{\rho}_{ms.s} \big/ \sqrt{\text{var}(\hat{\rho}_{ms.s})_o}$ to tables of the standard normal distribution.

5.10. A test based on the large sample variance of the family-weighted estimator

Eliasziw and Donner (1990) showed that the large sample variance of family-weighted estimator $\hat{\rho}_{ms.fwp}$ equals the large sample variance of ensemble estimator

Thus the large sample variance of $\hat{\rho}_{ms.fwp}$ under $H_o : \rho_{ms} = 0$ is given by

$$\text{var}(\hat{\rho}_{ms.fwp})_o = \frac{1}{n}[\bar{k}_h^{-1} + \rho_{ss}(1-\bar{k}_h^{-1})] \tag{5.13}$$

Therefore $H_o : \rho_{ms} = 0$ may be tested by referring the statistic $Z_f = \hat{\rho}_{ms.fwp} \big/ \sqrt{\text{var}(\hat{\rho}_{ms.fwp})_o}$ to tables of the standard normal distribution.

6. Test of hypotheses about intraclass correlation

Donner and Koval (1980) showed that the test of significance for $H_o : \rho_{ss} = 0$ is provided by the usual procedure of comparing the calculated value of F ($F = MSA/MSW$) to F_o, the tabulated value of the F-distribution with n-1 and K-n degrees of freedom at the chosen level of significance. A significant value of F implies that $\rho_{ss} > 0$

7. Monte Carlo simulation study

The theoretical properties of the estimation methods and the test procedures are for the most part intractable, then a Monte Carlo study was undertaken to compare the mean square errors of the estimation methods and the powers of the test procedures. Brass in 1958 (Donner and Bull, 1984) has shown that the negative binomial distribution, truncated below one, fits observed distribution of sibship sizes very well in a wide variety of human populations for appropriate choice of the parameters m and p in the probability density. Almost all the previous Monte Carlo simulations, the sibship sizes were randomly generated based on the distribution of sibship size in U.S. in 1950. In this section the sibship sizes were randomly generated based on the distribution of sibship size in Egypt in 2000.

First: Based on the distribution of sibship size in U.S. in 1950. Rosner et al. (1977) showed that the pairwise and ensemble estimators are more efficient that the sib-mean and random-sib estimators in terms of mean squared error. In particular the pairwise estimator was found to be superior in the case of low sib-sib correlation, whereas the ensemble estimator was found to be superior when the sib-sib correlation is high. They observed that the pairwise estimator has a smaller mean squared error than the ensemble estimator whenever the intraclass correlation was less than about 0.3 with not much to distinguish then when ρ_{ss} =0.3, similar results were obtained by Konishi (1982). Rosner (1979) in a further simulation study showed that the pairwise estimator is roughly equivalent in mean squared error to the maximum likelihood estimator for small values of the intraclass correlation, although the former loses efficiency as intraclass correlation increases. By contrast, the mean squared error of the ensemble estimator is approximately equal to that of the maximum likelihood estimator for large values of the intraclass correlation.

Also, O'Neill et al. (1987) showed that the pairwis estimator is better than the sib-mean and ensemble estimator, certainly when the interaclss correlation is less than about 0.3: this "watershed" value has been observed by other workers in simulation studies, and varies from about 0.3 for families with a low mean number of offspring to about 0.1 for those with a high mean number. Both pairwis estimator and ensemble estimator on average underestimate ρ_{sm}.

Compared with the corrected pairwise estimator, the Srivastava estimator was recommended by Srivastava and Keen (1988) when $\rho_{SS} \geq 0.3$ and the corrected pairwise estimator was recommended when $\rho_{SS} < 0.3$. They showed that the proportion of samples for which the quasi-Newton method converged to the ML estimates did not exceed 82%, the maximum likelihood estimator for the interclass correlation cannot be unconditionally recommended.

With respect to the intraclass correlation, Donner and Koval (1980) showed that $\hat{\rho}_{ss.ML}$ is more effective estimator than $\hat{\rho}_{ss.A}$ at the extreme values of ρ_{ss} (0, 0.1, 0.8) while the $\hat{\rho}_{ss.p}$ and $\hat{\rho}_{ss.AD}$ are about equally effective at $\rho_{ss} = 0.3$ and 0.5. Donner and Koval (1983) showed that the method of unweighed means is preferable to the ANOVA method of estimating ρ_{ss} only if $\rho_{ss} > 0.5$.

As regards significance testing of interclass correlations, Rosner et al.(1979) showed that the classical pairwise test of significance gives true significance levels that are two to five times as large as the nominal level of $\alpha = 0.05$. Thus, the significance levels from much of the published work based on this test procedure are likely to be overstated and many reportedly significance result may in fact be non-significance especially for cases in which ρ_{ss} is moderately large. Similarly, the conservative pairwise test gave significance levels that were as little as one over ten of the nominal level of $\alpha = 0.05$ and generally had very low powers which would imply that many reported non-significance results based on this procedure may in fact be significant. Finally, they showed that the adjusted pairwise test and sib-mean test yielded type I errors that were approximately correct for all parameter combinations tested. However they believed that the procedure of choice for assessing statistical significance of interclass correlations is the adjusted pairwise test since it generally had higher power than the sib-mean test.

Donner and Bull (1984) compared the powers of various procedures for testing the statistical significance of the interclass correlation. They compared the power of procedures based maximum likelihood methods to the power of the adjusted pairwise test and the power of the test based on the ratio of the pairwise estimate to its large sample standard error. It is seen from the Monte Carlo that the likelihood ratio test and the test based on the large sample variance of the likelihood estimator procedure, known to be asymptotically equivalent, also have very similar powers in small to moderate-sized samples. Comparing these procedures to the two tests based on pairwise estimator, it is seen that the relative advantage of the likelihood based procedure tends to increase with the underlying value of ρ_{ms}. For example, the average difference in empirical power between the likelihood ratio test and the Z_p test at *n=25* is virtually zero at ρ_{ms} *=0.1, 0.02* at ρ_{ms} *=0.2, 0.06* at ρ_{ms} *=0.3* and *0.08* at ρ_{ms} *=0.4*. In one instance (*n=50;* $\rho_{ms} = \rho_{ss} = 0.1$) the likelihood ratio test is significantly less powerful than the Z_p procedure.

Also Donner and Bull (1984) showed that the test based on the large sample variance of the pairwise estimator may be used to test the statistical significance of interclass correlation in studies of 25 or more families especially for values of ρ_{ms} near to zero. The simplicity of this statistic is quite appealing, since it allows the statistical significance of $\hat{\rho}_{ms.p}$ to be judged almost immediately. Moreover, this procedure compares very well in power to tests based on maximum likelihood theory. The adjusted pairwise test, which may be regarded as a studentized version of the Z_p test is also a reasonable alternative to likelihood- based procedures. However, it tends to be somewhat anti-conservative at high values of ρ_{ms} and not quite as powerful as the Z_p when both ρ_{ms} and ρ_{ss} are close to zero.

Second: Based on the distribution of sibship size in Egypt in 2000, we generated the scores x_i, y_{ij} in (2.1) by implementing the following algorithm for each of the n families (Rosner et al., 1979)

(i) Generate a collection of standard normal deviates $v_{i0}, \ldots\ldots, v_{iki} \sim N(0,1)$

(ii) Set $x_i = v_{i0}$ = mother's score for the ith family, and

(iii) Generate the sibling scores for the ith family $y_{i1},\ldots\ldots\ldots, y_{iki}$ iteratively as follws:

$$y_{ij} = v_{ij}\sigma_j^* + \mu_{ij}^* \qquad j = 1,\ldots\ldots\ldots, k_i$$

Where letting

$$y_j = 1 + (j-2)\rho_{ss} - (j-1)\rho_{ms}^2$$

$$\mu_{ij}^* = \begin{cases} \rho_{ms}(1-\rho_{ss})x_i / y_j & \text{if } j = 1 \\ \left[\rho_{ms}(1-\rho_{ss})x_i + \sum_{l=1}^{j-1}(\rho_{ss}-\rho_{ms}^2)y_{il}\right] \Big/ y_j & \text{if } j > 1 \end{cases} \tag{7.1}$$

$$\sigma_j^{*2} = 1 - [\rho_{ms}^2(1-\rho_{ss}) + (j-1)\rho_{ss}(\rho_{ss}-\rho_{ms}^2)]/y_j \qquad j = 1,\ldots, k_i .$$

In the formula above μ_{ij}^* and σ_j^{*2} are the conditional mean and variance, respectively of y_{ij} given $x_i, y_{i1}\ldots\ldots, y_{ij-1}, j = 1,\ldots\ldots k_i$

The parameters used for this simulation include

(1) The total numbers of families n = (25, 50).

(2) The mother-sibling (interclass) correlation $\rho_{ms} = 0, 0.1, 0.3, 0.5, 0.8$

(3) The sib-sib (intraclass) correlation $\rho_{ss} = 0.1, 0.3, 0.5, 0.8$

Regarding the choice of interclass and intraclass correlation for the simulation, we did not use all combination of (ρ_{sm}, ρ_{ss}), since it is a necessary and sufficient condition that for variance covariance matrix of Z_i to be positive definite for all k_i, we must have $1+(k_i-1)\,\rho_{ss} - \rho_{ms}^2 k_i > 0$, for all $k_i>0$. However, it can be clearly seen that the condition $1+(k_i-1)\,\rho_{ss} - \rho_{ms}^2 k_i > 0$, for all $k_i>0$ is equivalent to the condition that $\rho_{ms}^2 < \rho_{ss}$. This latter condition was satisfied for the pairs (0, 0.1), (0, 0.3), (0, 0.5), (0, 0.8), (0.1, 0.1), (0.1, 0.3), (0.1, 0.5), (0.1, 0.8), (0.3, 0.1), (0.3, 0.3), (0.3, 0.5), (0.3, 0.8), (0.5, 0.3), (0.5, 0.5), (0.5, 0.8), (0.8, 0.8).

In addition, we required an underlying distribution of sibship sizes which we simulated using Monte Carlo method. In this simulation the aim was to use a distribution of sibship size which is typical of that found in Egypt. Brass in 1958 (Donner and Bull, 1984) has shown that the negative binomial distribution, truncated below one, as specified in (7.2) fits the observed distribution of sibship sizes in different countries very well for appropriate selection of the parameters *m, p*.

$$\Pr(r\ offspring) = \frac{(m+r-1)!q^{-m}(\frac{p}{q})^r}{r!(m-1)!} \qquad q=1+p \quad r=1,2,... \qquad (7.2)$$

Based on the distribution of sibship size in Egypt in 2000, $m = 8.94$ and $p = 0.38$ (Based on the distribution of sibship size in U.S. in 1950, $m = 2.84$ and $p = 0.93$). We therefore use this distribution, truncated further above 15, and with $m = 8.94$ and $p = 0.38$. We performed 1000 iterations of the algorithm described in (7.1) to obtain the mean square errors which we utilized to compare the estimators and to obtain powers which we utilized to compared the power of the test procedures.

Tables (7.1) and (7.2) show the mean squared errors of interclass correlation estimators for different combinations of ρ_{ms} and ρ_{ss}, for $n = 25, 50$. For $n = 25$, the pairwise estimator is the superior estimator followed by the maximum likelihood estimator when $\rho_{ss} < 0.3$, the maximum likelihood estimator is the superior estimator followed by the family-weighted pairwise estimator when $0.3 \le \rho_{ss} < 0.5$ and $\rho_{ss} \ge 0.8$ except for $0.3 \le \rho_{ms} \le 0.5$, it followed by the modified sib-mean estimator. But for $0.5 \le \rho_{ss} < 0.8$ the modified sib-mean estimator is the superior estimator followed by the maximum likelihood estimator.

Table 7.1: Mean squared errors of interclass correlation estimators for different combinations of ρ_{ms} and ρ_{ss} ($n = 25$)

ρ_{ms}, ρ_{ss}	$\hat{\rho}_{ms.p}$	$\hat{\rho}_{ms.m}$	$\hat{\rho}_{ms.r}$	$\hat{\rho}_{ms.e}$	$\hat{\rho}_{ms.fwp}$	$\hat{\rho}_{ms.cp}$	$\hat{\rho}_{ms.mm}$
0, 0.1	0.01526	0.03982	0.03891	0.01928	0.01889	0.01576	0.01946
0, 0.3	0.02439	0.03966	0.03879	0.02367	0.02329	0.02495	0.02389
0, 0.5	0.03361	0.03972	0.03890	0.02816	0.02783	0.03405	0.02389
0, 0.8	0.04762	0.03992	0.03935	0.03514	0.03496	0.04756	0.03539
0.1, 0.1	0.01507	0.04042	0.03857	0.01893	0.01855	0.01553	0.01894
0.1, 0.3	0.02419	0.03949	0.03856	0.02333	0.02297	0.02471	0.02339
0.1, 0.5	0.03339	0.03929	0.03870	0.02782	0.02751	0.03380	0.02798
0.1, 0.8	0.04740	0.03949	0.03911	0.03479	0.03462	0.04732	0.03500
0.3, 0.1	0.01184	0.04498	0.03375	0.01559	0.01535	0.01203	0.01418
0.3, 0.3	0.02019	0.03735	0.03410	0.01967	0.01943	0.02044	0.01858
0.3, 0.5	0.02868	0.03466	0.03435	0.02384	0.02362	0.02889	0.02317
0.3, 0.8	0.04172	0.03417	0.03459	0.03029	0.03017	0.04161	0.03016
0.5, 0.3	0.01230	0.03575	0.02419	0.01274	0.01270	0.01214	0.00991
0.5, 0.5	0.01904	0.02647	0.02483	0.01608	0.01602	0.01892	0.01403
0.5, 0.8	0.02981	0.02386	0.02522	0.02143	0.02138	0.02965	0.02071
0.8, 0.8	0.00670	0.00602	0.00641	0.00486	0.00492	0.00653	0.00361

For $n = 50$, the corrected pairwise estimator is the superior estimator followed by the pairwise estimator when $\rho_{ss} < 0.3$ except for $0.3 \le \rho_{ms} < 0.5$, the pairwise estimator is the superior estimator followed by the maximum likelihood estimator. The maximum likelihood estimator is the superior estimator followed by the corrected pairwise estimator when $\rho_{ss} \ge 0.3$ except for $\rho_{ms} \ge 0.3$, it followed by the modified sib-mean estimator when $\rho_{ms} \ge 0.5$, and it followed by the pairwise estimator for combinations $0.3 \le \rho_{ss} < 0.5$ and $0.3 \le \rho_{ms} < 0.5$, but it followed by the family-weighted pairwise estimator for combinations $0.5 \le \rho_{ss} < 0.8$ and $0.3 \le \rho_{ms} < 0.5$.

Table 7.1: continue

ρ_{ms}, ρ_{ss}	$\hat{\rho}_{ms.s}$	$\hat{\rho}_{ms.ML}$	$\hat{\rho}_{ms.MINQ}$
0, 0.1	0.02198	0.01536	0.02269
0, 0.3	0.02622	0.02173	0.02890
0, 0.5	0.03031	0.02716	0.02992
0, 0.8	0.03622	0.03493	0.03515
0.1, 0.1	0.02156	0.01514	0.02280
0.1, 0.3	0.02581	0.02134	0.02475
0.1, 0.5	0.02991	0.02700	0.02892
0.1, 0.8	0.03584	0.03457	0.03579
0.3, 0.1	0.01771	0.01192	0.02976
0.3, 0.3	0.02158	0.01737	0.02061
0.3, 0.5	0.02543	0.02275	0.02495
0.3, 0.8	0.03111	0.03007	0.03165
0.5, 0.3	0.01378	0.00977	0.01826
0.5, 0.5	0.01682	0.01493	0.01510
0.5, 0.8	0.02181	0.02143	0.02270
0.8, 0.8	0.00465	0.00460	0.00474

Table 7.2: Mean squared errors of interclass correlation estimators for different combinations of ρ_{ms} and ρ_{ss} ($n = 50$)

ρ_{ms}, ρ_{ss}	$\hat{\rho}_{ms.p}$	$\hat{\rho}_{ms.m}$	$\hat{\rho}_{ms.r}$	$\hat{\rho}_{ms.e}$	$\hat{\rho}_{ms.fwp}$	$\hat{\rho}_{ms.cp}$	$\hat{\rho}_{ms.mm}$
0, 0.1	0.00715	0.02011	0.01814	0.00969	0.00959	0.00361	0.00983
0, 0.3	0.01167	0.02002	0.01834	0.01194	0.01184	0.00581	0.01206
0, 0.5	0.01639	0.02000	0.01874	0.01419	0.01410	0.00811	0.01431
0, 0.8	0.02385	0.02407	0.01950	0.01760	0.01756	0.01178	0.01769
0.1, 0.1	0.00713	0.02157	0.01809	0.00958	0.00949	0.00454	0.00963
0.1, 0.3	0.01163	0.02063	0.01837	0.01187	0.01178	0.00681	0.01193
0.1, 0.5	0.01633	0.02023	0.01883	0.01416	0.01408	0.00915	0.01425
0.1, 0.8	0.02381	0.02001	0.01963	0.01761	0.01757	0.01288	0.01769
0.3, 0.1	0.00568	0.03120	0.01590	0.00789	0.00783	0.01115	0.00711
0.3, 0.3	0.00974	0.02295	0.01628	0.01008	0.01002	0.01349	0.00948
0.3, 0.5	0.01400	0.01940	0.01680	0.01226	0.01220	0.01584	0.01189
0.3, 0.8	0.02090	0.01778	0.01762	0.01556	0.01552	0.01952	0.01546
0.5, 0.3	0.00587	0.02823	0.01139	0.00637	0.00636	0.02630	0.00475
0.5, 0.5	0.00914	0.01747	0.01188	0.00816	0.00814	0.02862	0.00696
0.5, 0.8	0.01465	0.01270	0.01272	0.01094	0.01093	0.03209	0.01051
0.8, 0.8	0.00295	0.00407	0.00289	0.00223	0.00224	0.06273	0.00158

Its noted that the iterative method for determining the maximum likelihood estimate of interclass correlation failed to convergent for 20% to 35% of the samples for each combination for the interclass and intraclass correlation coefficients

Tables (7.3) and (7.4) show the mean squared errors of intraclass correlation estimators for different values of ρ_{ss} for $n = 25, 50$. For small and large samples ($n = 25, 50$), the pairwise estimator is the superior estimator when $\rho_{ss} < 0.3$, while the maximum likelihood estimator is the superior estimator when $\rho_{ss} \geq 0.3$ followed by the restricted maximum likelihood and the MINQUE estimators. .

Table 7.2: continue

ρ_{ms} , ρ_{ss}	$\hat{\rho}_{ms.s}$	$\hat{\rho}_{ms.ML}$	$\hat{\rho}_{ms.MINQ}$
0, 0.1	0.01113	0.00762	0.01140
0, 0.3	0.01328	0.00496	0.01457
0, 0.5	0.01531	0.00768	0.01510
0, 0.8	0.01814	0.01750	0.01811
0.1, 0.1	0.01105	0.00726	0.01154
0.1, 0.3	0.01322	0.00646	0.01263
0.1, 0.5	0.01528	0.00874	0.01579
0.1, 0.8	0.01816	0.01751	0.01819
0.3, 0.1	0.00933	0.00587	0.01506
0.3, 0.3	0.01127	0.00890	0.01261
0.3, 0.5	0.01320	0.01170	0.01355
0.3, 0.8	0.01601	0.01544	0.01698
0.5, 0.3	0.00733	0.00489	0.00913
0.5, 0.5	0.00874	0.00658	0.00829
0.5, 0.8	0.01117	0.01094	0.01135
0.8, 0.8	0.00219	0.00211	0.00297

Table 7.3: Mean squared errors of intraclass correlation estimators for different values of ρ_{ss} ($n = 25$)

ρ_{ss}	$\hat{\rho}_{ss.p}$	$\hat{\rho}_{ss.AD}$	$\hat{\rho}_{ss.A}$	$\hat{\rho}_{ss.ML}$	$\hat{\rho}_{ss.RML}$	$\hat{\rho}_{ss.MINQ}$	$\hat{\rho}_{ss.u}$	$\hat{\rho}_{ss.s}$
0.0	0.00358	0.00883	0.00924	0.01180	0.00983	0.00993	0.02382	0.02612
0.1	0.00784	0.01195	0.01417	0.00983	0.01215	0.01205	0.02205	0.02457
0.3	0.02254	0.02168	0.03582	0.01259	0.01968	0.01958	0.01940	0.01965
0.5	0.0246٢	0.02149	0.03811	0.01140	0.02049	0.02050	0.01262	0.01178
0.8	0.02490	0.00164	0.00203	0.00082	0.00154	0.00153	0.01321	0.01254

Table 7.4: Mean squared errors of intraclass correlation estimators for different values of ρ_{ss} ($n = 50$)

ρ_{ss}	$\hat{\rho}_{ss.p}$	$\hat{\rho}_{ss.AD}$	$\hat{\rho}_{ss.A}$	$\hat{\rho}_{ss.ML}$	$\hat{\rho}_{ss.RML}$	$\hat{\rho}_{ss.MINQ}$	$\hat{\rho}_{ss.u}$	$\hat{\rho}_{ss.s}$
0.0	0.00193	0.00675	0.00767	0.00960	0.00677	0.00667	0.02672	0.09602
0.1	0.00499	0.01040	0.01052	0.00829	0.01030	0.01020	0.02487	0.09516
0.3	0.02126	0.01979	0.02300	0.01106	0.01879	0.01869	0.01991	0.09331
0.5	0.02550	0.01985	0.02381	0.01053	0.01775	0.01765	0.01249	0.06291
0.8	0.02613	0.00178	0.00182	0.00089	0.00153	0.00150	0.01462	0.00377

We present in tables 7.5 and 7.6 the empirical significance levels corresponding to the nominal level $\alpha = 0.05$ for each of the ten significance testing procedures. The classical pairwise test gives empirical significance levels that are higher than the nominal level of 0.05 for all values of ρ_{ss} for both $n = 25$ and $n = 50$ (two to eight times larger than nominal levels). The adjusted pairwise test gives empirical significance levels that are higher than the nominal level of 0.05 for $\rho_{ss} \geq 0.5$ for both $n = 25$ and $n = 50$, but the conservative pairwise test gives empirical significance levels that are lower than the nominal level of 0.05 for $\rho_{ss} < 0.5$. In general, all other seven significance testing procedures give satisfactory significance levels for all values of ρ_{ss} for both $n = 25$ and $n = 50$.

Table 7.5: Empirical significance levels of interclass correlation tests for different combinations of ρ_{ms} and ρ_{ss} ($n = 25$)

ρ_{ms}, ρ_{ss}	Z_p	t_{adj}	t_{cons}	t_{class}	t_m	Z_e	Z_s	Z_f	Z_{ML}	LR
0, 0.1	0.045	0.048	0.001	0.111	0.042	0.051	0.071	0.051	0.048	0.042
0, 0.3	0.036	0.063	0.009	0.201	0.042	0.046	0.062	0.044	0.042	0.039
0, 0.5	0.041	0.072	0.025	0.270	0.042	0.044	0.056	0.044	0.035	0.043
0, 0.8	0.037	0.087	0.067	0.393	0.047	0.047	0.051	0.046	0.031	0.031

Table 7.6: Empirical significance levels of interclass correlation tests for different combinations of ρ_{ms} and ρ_{ss} ($n = 50$)

ρ_{ms}, ρ_{ss}	Z_p	t_{adj}	t_{cons}	t_{class}	t_m	Z_e	Z_s	Z_f	Z_{ML}	LR
0, 0.1	0.042	0.048	0.002	0.106	0.048	0.046	0.058	0.046	0.037	0.042
0, 0.3	0.042	0.064	0.014	0.106	0.046	0.042	0.054	0.042	0.035	0.037
0, 0.5	0.048	0.082	0.032	0.254	0.056	0.052	0.060	0.052	0.038	0.042
0, 0.8	0.036	0.094	0.074	0.376	0.050	0.048	0.052	0.048	0.032	0.032

Table 7.٧: Empirical powers of interclass correlation tests for different combinations of ρ_{ms} and ρ_{ss} ($n = 25$)

ρ_{ms}, ρ_{ss}	Z_p	t_{adj}	t_{cons}	t_{class}	t_m	Z_e	Z_s	Z_f	Z_{ML}	LR
0.1, 0.1	0.102	0.111	0.003	0.209	0.105	0.111	0.134	0.106	0.095	0.106
0.1, 0.3	0.074	0.091	0.021	0.285	0.090	0.100	0.116	0.097	0.081	0.091
0.1, 0.5	0.059	0.100	0.043	0.354	0.088	0.090	0.104	0.088	0.107	0.114
0.1, 0.8	0.052	0.114	0.086	0.430	0.080	0.080	0.087	0.080	0.133	0.143
0.3, 0.1	0.661	0.668	0.156	0.788	0.616	0.594	0.653	0.580	0.630	0.646
0.3, 0.3	0.430	0.521	0.216	0.741	0.511	0.505	0.555	0.495	0.462	0.463
0.3, 0.5	0.309	0.417	0.260	0.711	0.422	0.422	0.458	0.416	0.456	0.462
0.3, 0.8	0.226	0.373	0.329	0.704	0.351	0.364	0.374	0.361	0.304	0.425
0.5, 0.3	0.925	0.943	0.780	0.990	0.932	0.935	0.959	0.934	0.942	0.942
0.5, 0.5	0.794	0.867	0.746	0.967	0.891	0.890	0.902	0.889	0.895	0.905
0.5, 0.8	0.635	0.752	0.719	0.932	0.812	0.815	0.824	0.814	0.826	0.835
0.8, 0.8	0.998	0.999	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The empirical powers corresponding to the ten significance testing procedures are shown in tables 7.7 and 7.8 . Since the classical pairwise test gives empirical significance levels two to eight times larger than nominal levels then it is ignored from this comparison.