Random Coefficient Non-Gaussian First Order Autoregressive Model

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Abstract

This research aims at detecting the behavior of the parameter of first order Autoregressive model. The parameter takes various functions and formulations with the estimates of the parameter through the method of least square.The random errors of the model follow Non-Gaussian distribution (discrete, continuous) with the experiments taking various sample sizes through conducting simulation experiments. The results wold be compared with absolute bias, mean square errors, mean absolute percentage errors and absolute standard bias. The most important results of the study: The values of MSE and MAPE will be decreased wherever the size of the sample is increased for all distributions, and the smallest value for MSE and MAPE is when n=150.This value is exclusively relevant to third model and for all distributions. Through simulation experiments no essential differences have appeared for the results between continuous and discrete distributions.

Key Words: Monte Carlo, Non–Gaussian, Random Coefficient, Autoregressive model.

1-introduction:

Let$X_t\{t = 1,2 \dots, n\}$be a stationary time-series generated from the first order autoregressive model.

$$X_t = \emptyset_1 X_{t-1} + a_t \quad , |\emptyset| < 1 \qquad \dots (1.1)$$

Where $a_t$is aset of mutually independent random variables with Non–Gaussian distribution (continuous,discrete) and with finite moments of orders.

The estimation of the parameter $\emptyset_1$on the basis of a sample($X_1, X_2, \dots, X_n$) is well known.

In recent years the random coefficient autoregressive model of order one {RC AR(1)} namely.

$$X_t = \emptyset_{1t} X_{t-1} + a_t \qquad \dots (1.2)$$

Has been an active area of research, anumber of (new)results have been obtained on specific properties of the model.(Rao1970, Ledolter1981,Nichlls and Quinn 1982,Austad and Theim 1990 and Cook and Broemeling1996).

In this paper we investigate same properties of the estimators of wide expressions for random coefficient first order Non–Gaussian Autoregressive Scheme.

2- Typical Model for RC AR(1):

In the practical applications the parameters, in model (1.2)are unknown and have to be estimated from past data. It is important to consider parsimonious members of this class, in this paper we consider eight cases of such models namely

Model1: Stationary AR(1)

The stationary AR(1) where $\emptyset_{1t} = \emptyset_1$ is constant over time such that $|\emptyset_1| < 1$.

Model2: Linear RC AR(1)

Where $\emptyset_{1t} = \propto_0 + \propto_1 t$ and :

$$X_t = (\propto_0 + \propto_1 t)X_{t-1} + a_t \qquad \text{... (2.1)}$$

Model3: Quadratic RC AR(1)

We relax the assumption of constant coefficient and assume that the coefficient it self change according to astochastic process, i.e,,

$\emptyset_{1t} = \propto_0 + \propto_1 t + \propto_2 t^2$ and :

$$X_t = (\propto_0 + \propto_1 t + \propto_2 t^2)X_{t-1} + a_t \qquad \text{... (2.2)}$$

Model4: Special Exponential RC AR(1)

$\emptyset_{1t} = \lambda e^{-\lambda t}$ and :

$$X_t = (\lambda e^{-\lambda t})X_{t-1} + a_t \qquad \text{... (2.3)}$$

Model5: Exponential Autoregressive

$\emptyset_{1t} = \propto_0 + \propto_1 e^{-rX_{t-1}^2}$ and :

$$X_t = (\propto_0 + \propto_1 e^{-rX_{t-1}^2})X_{t-1} + a_t \qquad \text{... (2.4)}$$

Model6: Threshold Autoregressive

$\emptyset_{1t} = \begin{cases} \rho_1 & , X_{t-1} \le 0 \\ \rho_2 & , X_{t-1} > 0 \end{cases}$ and :

$$X_t = \begin{cases} \rho_1 X_{t-1} + a_t & ,X_{t-1} \le 0 \\ \rho_2 X_{t-1} + a_t & ,X_{t-1} > 0 \end{cases} \quad \dots (2.5)$$

Where : $\rho_k = \frac{n}{n-k} \frac{\sum_{t=k+1}^{n} X_t X_{t-k}}{\sum_{t=1}^{n} X_t^2}$ $\quad ,K = 1,2 \quad \dots (2.6)$

Model7: Correlation Autoregressive

$\phi_{1t} = \theta_1 \phi_{1(t-1)} + \theta_0 \quad ,|\theta_1| < 1$, i.e ;

ϕ_{1t} stationary AR(1) i.e ;

$$X_t = (\theta_1 \phi_{1(t-1)} + \theta_0) X_{t-1} + a_t \quad \dots (2.7)$$

Model8: The Bilinear Autoregressive

Where $\phi_{1t} = \alpha_0 + \alpha_1 a_{t-1}$ and so the non-linear Bilinear RC AR(1) can be written as :

$$X_t = (\alpha_0 + \alpha_1 a_{t-1}) X_{t-1} + a_t \quad \dots (2.8)$$

3- The Least Square Estimators:

The least square estimators for various versions of random coefficient first order Non–Gaussian autoregressive can be obtained by minimizing the sum squares of the error term a_t, as an examples:

3-1 For model1:

The stationary AR(1), where $\phi_{1t} = \phi_1$ is constant over time, the OLS estimator of ϕ_1 based on available data is :

$$\hat{\phi}_1 = \frac{\sum_{t=2}^{n} X_{t-1} X_t}{\sum_{t=2}^{n} X_{t-1}^2} \quad \dots . (3.1)$$

3-2 For model2:

Linear RC AR(1), the OLS estimators of α_0 and α_1 based on available data can be obtained by minimizing:

$S(\alpha) = \sum_{t=2}^{n} (X_t - (\alpha_0 + \alpha_1 t) X_{t-1})^2$, where :

$$\hat{\alpha}_1 = \frac{(\sum_{t=2}^{n} X_{t-1}^2)(\sum_{t=2}^{n} X_{t-1} X_t) - (\sum_{t=2}^{n} X_{t-1} X_t)(\sum_{t=2}^{n} t\, X_{t-1}^2)}{(\sum_{t=2}^{n} X_{t-1}^2)(\sum_{t=2}^{n} t^2 X_{t-1}^2) - (\sum_{t=2}^{n} t^2 X_{t-1}^2)^2} \quad \dots . (3.2)$$

$$\hat{\alpha}_0 = \frac{(\sum_{t=2}^{n} X_{t-1} X_t)}{(\sum_{t=2}^{n} X_{t-1}^2)} - \frac{(\sum_{t=2}^{n} t\, X_{t-1}^2)}{(\sum_{t=2}^{n} X_{t-1}^2)} (\hat{\alpha}_1)$$

3-3 For model 3:

$$S(\alpha) = \sum_{t=2}^{n} (X_t - (\alpha_0 + \alpha_1 t + \alpha_2 t^2) X_{t-1})^2$$

And the estimators are

$$\hat{\alpha}_2 = \frac{t^2 \sum_{t=2}^{n} X_{t-1}X_t + \alpha_0\, t^2 \sum_{t=2}^{n} X_{t-1}^2 + \alpha_1\, t^2 \sum_{t=2}^{n} X_{t-1}^2}{t^4 \sum_{t=2}^{n} X_{t-1}^2}$$

$$\hat{\alpha}_1 = \frac{t \sum_{t=2}^{n} X_{t-1}X_t + \alpha_0\, t \sum_{t=2}^{n} X_{t-1}^2 + \alpha_2\, t^3 \sum_{t=2}^{n} X_{t-1}^2}{t^2 \sum_{t=2}^{n} X_{t-1}^2} \quad \ldots (3.3)$$

$$\hat{\alpha}_0 = \frac{\sum_{t=2}^{n} X_{t-1}X_t - \alpha_1\, t \sum_{t=2}^{n} X_{t-1}^2 + \alpha_2\, t^2 \sum_{t=2}^{n} X_{t-1}^2}{\sum_{t=2}^{n} X_{t-1}^2}$$

3-4 For model 5:

$$S(\alpha) = \sum_{t=2}^{n} (X_t - (\alpha_0 + \alpha_1 e^{-rX_{t-1}^2}) X_{t-1})^2$$

And the estimators are

$$\hat{\alpha}_1 = \frac{(\sum_{t=2}^{n} e^{-rX_{t-1}^2} X_{t-1}X_t)(\sum_{t=2}^{n} X_{t-1}X_t) - (\sum_{t=2}^{n} X_{t-1}X_t)(\sum_{t=2}^{n} e^{-rX_{t-1}^2} X_{t-1}^2)}{(\sum_{t=2}^{n} e^{-2rX_{t-1}^2} X_{t-1}^2)(\sum_{t=2}^{n} X_{t-1}^2) - (\sum_{t=2}^{n} e^{-rX_{t-1}^2} X_{t-1}^2)^2} \quad \ldots (3.4)$$

$$\hat{\alpha}_0 = \frac{(\sum_{t=2}^{n} X_{t-1}X_t)}{(\sum_{t=2}^{n} X_{t-1}^2)} - \frac{(\sum_{t=2}^{n} e^{-rX_{t-1}^2} X_{t-1}^2)}{(\sum_{t=2}^{n} X_{t-1}^2)} (\hat{\alpha}_1)$$

4- Simulation Study:

To investigate whether the expression for time varying coefficient leads to in improvement in estimation accuracy, simulation study is preformed, where 5 experiments were carried out to investigate the properties of least square estimators for various versions of first order Non–Gaussian autoregressive scheme .In each experiment,random error agenerated as random variables following acertain distribution(see table 1), This experiment is repeated 5000 for sample size (25,75,150).we using the following comparison tools:(Absolute bias |bias|, Mean square error MSE, Mean absolute percentage error MAPE, Absolute standard bias |Bias/SE|).

Table(1)

General formula for generating variables follows continuous and discrete distributions:

Distribution	Formula
Binomial (n,p)	$a_t = \begin{cases} 1 & if \quad 0 < u \le p \\ 0 & if \quad p < u \le 1 \end{cases}$
Exponential(λ)	$a_t = -\lambda \ln(1-u)$
Weibull(α, β)	$a_t = \alpha - \beta[\ln(1-u)]^{\frac{1}{\alpha}}$
Geometric(P)	$a_t = \ln U / \ln(1-P)$
Laplace(α, β)	$a_t = \alpha - \beta \ln[2(1-u)]$

Table(2)

Initial Values for models coefficient.

Model	Initial Values
1	$\emptyset_1 = 0.5$
2	$\alpha_0 = 0.05$, $\alpha_1 = 0.01$
3	$\alpha_0 = 0.05$, $\alpha_1 = 0.01$, $\alpha_1 = 0.00005$
4	$\lambda = 0.5$
5	$\alpha_0 = 0.8$, $\alpha_1 = -1.1$, $r = 0.5$
6	$\rho_1 = -0.3$, $\rho_2 = 0.8$
7	$\emptyset_0 = 0.01$, $\emptyset_1 = 0.05$
8	$\alpha_0 = 0.03$, $\alpha_1 = 0.35$

Table(3)

Values of (Bias, MSE, MAPE, St. Bias) when the random error Binomial distribution .

n	Model	|Bias|	MSE	MAPE	|Bias / SE|
25	1	2.2210 e-15	0.0028	4.8741	1.5871 e-16
	2	5.1958 e-17	0.0040	5.2258	2.5721 e-18
	3	5.7758 e-16	0.1948	13.1417	5.9286 e-19
	4	6.9729 e-16	0.0197	47.1984	7.0718 e-18
	5	8.7889 e-15	30.3081	167.6656	5.7997 e-20
	6	1.2215 e-15	3.6186 e-04	1.4896	6.7512 e-16
	7	3.2438 e-15	0.0199	45.3267	3.2575 e-17
	8	1.5196 e-15	0.0127	15.5289	2.3964 e-17
75	1	2.8487 e-15	7.2645 e-04	2.4584	7.8429 e-16
	2	9.3798 e-16	4.4424 e-05	0.5076	4.2231 e-15
	3	3.3214 e-15	0.0036	1.3072	1.8693 e-16
	4	3.3394 e-16	0.0068	14.3928	9.7677 e-18
	5	4.0637 e-15	9.6921	69.8509	8.3856 e-20
	6	2.4965 e-15	4.0073 e-05	0.5091	1.2460 e-14
	7	5.2414 e-17	0.0067	14.0313	1.5605 e-18
	8	4.2851 e-16	0.0040	7.8547	2.1279 e-17
150	1	1.8473 e-15	3.2050 e-04	1.6382	1.1528 e-15
	2	2.5575 e-16	1.8847 e-15	2.2266 e-06	2.7139 e-05
	3	4.2588 e-13	1.8138 e-25	4.6955 e-12	4.6961 e+08
	4	1.9544 e-16	0.0032	9.3662	1.2084 e-17
	5	2.8452 e-15	4.6882	60.1416	1.2138 e-19
	6	6.8687 e-16	1.3741 e-05	0.2998	9.9975 e-15
	7	1.3442 e-15	0.0032	9.0856	8.5138 e-17
	8	1.1099 e-15	0.0019	5.2651	1.1828 e-16

Table(4)

Values of (Bias, MSE, MAPE, St. Bias) when the random error Geometric distribution .

n	Model	|Bias|	MSE	MAPE	|Bias / SE|
25	1	2.7994e-15	0.0020	3.9778	2.7348 e-16
	2	3.4408e-15	0.0020	3.5781	3.3857 e-16
	3	2.7674 e-15	0.2247	12.4991	2.4630 e-18
	4	2.6858 e-15	0.0148	21.4730	3.6403 e-17
	5	3.2342 e-15	0.0012	1.5457	5.5828 e-16
	6	2.2399 e-15	0.00035	1.3626	1.8761 e-15
	7	9.9328e-16	0.0150	21.6861	1.3275 e-17
	8	8.5545e-16	0.0086	9.1516	2.0007 e-17
75	1	8.3438 e-16	6.2193e-04	2.2382	2.6832 e-16
	2	6.2814 e-16	4.6318e-05	0.5007	2.7123 e-15
	3	5.0394 e-15	0.0031	1.2098	3.2223 e-16
	4	1.6599 e-15	0.0058	12.7088	5.7272 e-17
	5	1.9198 e-16	3.4621e-05	0.3448	1.1090 e-15
	6	4.9316 e-17	4.2088e-05	0.4934	2.3468 e-16
	7	4.1943 e-16	0.0057	12.4124	1.4720 e-17
	8	1.3409 e-16	0.0032	5.7466	8.3454 e-18
150	1	2.7776 e-15	3.0734e-04	1.5960	1.8076 e-15
	2	1.8721 e-15	1.8039e-15	2.0791e-06	2.0755 e-04
	3	4.2585 e-13	1.8135e-25	4.6952e-12	4.6964 e+08
	4	1.332 e-15	0.0031	9.0531	8.6265 e-17
	5	2.3301 e-15	5.4806e-06	0.1235	8.5032 e-14
	6	8.3822 e-16	1.4150e-05	0.2965	1.1848 e-14
	7	1.3393 e-15	0.0030	8.7844	8.8862 e-17
	8	1.2070 e-15	0.0018	4.3164	1.3257 e-16

Table(5)

Values of (Bias, MSE, MAPE, St. Bias) when the random error Exponential distribution.

n	Model	|Bias|	MSE	MAPE	|Bias / SE|
25	1	2.5922 e-15	0.0022	4.0289	2.3974 e-16
	2	3.2293 e-15	0.0021	3.6695	3.0210 e-16
	3	1.0537 e-14	0.2338	12.6046	9.0152 e-18
	4	2.1047 e-15	0.0151	21.6628	2.7903 e-17
	5	4.6401 e-15	41.7966	96.5408	2.2203 e-20
	6	2.1137 e-15	3.8682 e-04	1.4081	1.0929 e-15
	7	1.4424 e-15	0.0153	21.8281	1.884 e-17
	8	4.1706 e-15	0.0102	12.6926	8.1420 e-18
75	1	7.5473 e-16	6.3208 e-04	2.2611	2.3881 e-16
	2	1.5841 e-15	4.7642 e-05	0.5020	6.6501 e-15
	3	6.4322 e-16	0.0031	1.2105	4.0977 e-17
	4	6.2320 e-16	0.0059	12.8166	2.1267 e-17
	5	2.1204 e-15	0.2488	20.0834	1.7047 e-18
	6	9.0872 e-16	4.1242 e-05	0.4926	4.4067 e-15
	7	9.2388 e-16	0.0058	12.5381	3.1981 e-17
	8	1.1109 e-15	0.0037	7.4869	5.9753 e-17
150	1	2.9092 e-16	3.0911 e-04	1.5949	1.8823 e-16
	2	2.7773 e-15	2.0001 e-15	2.1343e-06	2.7772 e-04
	3	4.2589 e-13	1.8138 e-25	4.6956e-12	4.6960 e+08
	4	2.7310 e-16	0.0031	9.1002	1.7059 e-17
	5	1.1912 e-15	0.0327	8.3342	7.2861 e-18
	6	1.1071 e-16	1.4401 e-05	0.2967	1.5375 e-15
	7	6.9303 e-16	0.0030	8.8315	4.5478 e-17
	8	7.8826 e-17	0.0020	5.3341	8.0525 e-18

Table(6)

Values of (Bias, MSE, MAPE, St. Bias) when the random error Laplace distribution .

n	Model	|Bias|	MSE	MAPE	|Bias / SE|
25	1	2.5128e-15	0.0034	5.5167	1.4896e-16
	2	3.2956 e-16	0.0067	7.4340	9.8524 e-18
	3	1.3247 e-15	0.2933	16.2670	9.0319 e-19
	4	4.1129 e-16	0.0186	38.7145	4.4130 e-18
	5	2.8632 e-15	0.0013	2.2638	4.4277 e-16
	6	2.0108 e-15	5.5135 e-04	1.7295	7.2940 e-16
	7	1.0638 e-15	0.0188	38.5794	1.1307 e-17
	8	1.0016 e-15	0.0231	17.1902	8.6683 e-18
75	1	3.5845 e-16	0.0011	3.1363	6.7282 e-17
	2	2.7624 e-15	2.1062 e-04	1.0834	2.6231 e-15
	3	1.4206 e-15	0.0045	1.4412	6.2688 e-17
	4	3.5361 e-16	0.0073	19.1246	9.6899 e-18
	5	9.8550 e-16	1.1529 e-04	0.5045	1.7096 e-15
	6	1.3201 e-15	7.2489 e-05	0.6638	3.6423 e-15
	7	1.3948 e-16	0.0072	18.5994	3.8624 e-18
	8	4.9344 e-16	0.0089	11.2773	1.1140 e-17
150	1	1.3747 e-15	5.1569 e-04	2.1892	5.3315 e-16
	2	2.7748 e-15	3.0781 e-15	2.6437e-06	1.8029 e-04
	3	4.2587 e-13	1.8137 e-25	4.6954 e-12	4.6962 e+08
	4	3.1624 e-16	0.0038	13.0911	1.6853 e-17
	5	1.4.678 e-15	1.4506 e-05	0.1575	2.0237 e-14
	6	1.4402 e-15	2.6544 e-05	0.4106	1.0852 e-14
	7	1.4270 e-15	0.0037	12.6224	7.7616 e-17
	8	1.5261 e-15	0.0064	9.0006	4.7962 e-17

Table(7)

Values of (Bias, MSE, MAPE, St. Bias) when the random error Weibulldistribution .

n	Model	|Bias|	MSE	MAPE	|Bias / SE|
25	1	2.3093 e-15	8.9594 e-04	2.3772	5.0551 e-16
	2	3.1637 e-15	3.4690 e-04	1.3589	1.8240 e-15
	3	3.4738 e-15	0.1350	8.4214	5.1460 e-18
	4	5.3052 e-16	0.0090	11.2124	1.1779 e-17
	5	1.9023 e-15	5.9628 e-04	1.8381	6.3804 e-16
	6	4.3541 e-16	1.5963 e-04	0.9195	5.4551 e-16
	7	3.8597 e-16	0.0092	11.3292	8.4198 e-18
	8	2.9934 e-16	0.0265	8.2956	2.2621 e-18
75	1	3.5991 e-16	2.3449 e-04	1.2651	3.0697 e-16
	2	1.3932 e-15	2.5838 e-05	0.3544	1.0784 e-14
	3	1.1434 e-14	0.0014	0.8307	1.5909 e-15
	4	1.4866 e-15	0.0035	6.8327	8.5855 e-17
	5	1.1623 e-15	8.3301 e-05	0.6973	2.7907 e-15
	6	1.5753 e-15	1.6372 e-05	0.3052	1.9244 e-14
	7	1.2863 e-15	0.0034	6.7093	7.5957 e-17
	8	2.1090 e-16	0.0087	4.9116	4.8702 e-18
150	1	4.9014 e-16	1.1687 e-04	0.9079	8.3879 e-16
	2	2.1069 e-15	7.7398 e-16	1.3643 e-06	5.4444 e-04
	3	4.2586 e-13	1.8136 e-25	4.6954 e-12	4.6963 e+08
	4	1.5205 e-15	0.0019	5.0854	1.5899 e-16
	5	5.3555 e-16	3.2098 e-05	0.4310	3.3370 e-15
	6	1.7355 e-15	5.0464 e-06	0.1729	6.8782 e-14
	7	4.1189 e-17	0.0018	4.9510	4.4564 e-18
	8	3.3631 e-16	0.0046	4.1208	1.4653 e-17

5- Conclusions:

a- The values of MSE and MAPE will be decreased wherever the size of the sample is increased for all distributions.

b- The largest value for MSE and MAPE is substantiated by the fifth model (Exponential Autoregressive) when n=25 for all distributions.

c- The largest value for |Bias| and |Bias / SE| is when n=150, this value applis to the third model (Quadratic Model) and for all distributions.

d- The smallest value for MSE and MAPE is when n=50, this value is exclusively relevant to third model (Quadratic Model) and for all distributions.

e- Through simulation experiments no essential differences have appeared for the results between continuous and discrete distributions.

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المعاملات العشوائية لأنموذج الانحدار الذاتي غير الطبيعي من الدرجة الاولى.

الاستاذ الدكتور	الاستاذ المساعد	المدرس
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جامعة بغداد	كلية المنصور الجامعة	الجامعة المستنصرية

المستخلص:

يهدف هذا البحث الى تتبع سلوك المعلمة لأنموذج الانحدار الذاتي من الدرجة الاولى وان المعلمة تأخذ دوالا وصيغ مختلفة وتقديرها بطريقة المربعات الصغرى، والاخطاء العشوائية للأنموذج تتبع توزيعا غير طبيعيا (متقطع، مستمر) ولأحجام عينات مختلفة من خلال اجراء تجارب محاكاة ومقارنة النتائج باستخدام (التحيز المطلق ، متوسط مربعات الخطأ، متوسط الخطأ النسبي المعياري، التحيز المعياري المطلق) وأن أهم النتائج التي توصل اليها الباحثون هي: تقل قيم MSE و MAPE كلما ازداد حجم العينة لجميع التوزيعات، وان اقل قيمة لمتوسط مربعات الخطاء MSE ومتوسط الخطاء النسبي المطلق MAPE هي عند (n=150) وكانت للأنموذج الثالث ولجميع التوزيعات ، ومن خلال تجارب المحاكاة لم تظهر اختلافات جوهرية للنتائج بين التوزيعات المتقطعة والمستمرة.