

Estimation of Lorenz, Schotz and Champernowne-Pareto Measures of income Inequality Relative to The poverty Gab-Ratio for Achieving Social Equality

By
Zeinab yousef mahmoud

Abstract:

The fundamental goal of the paper is to introduce an adequate mathematical solution for achieving social equality through analyzing the relation between income distribution curve and poverty line of a society. This relation would determine how many incomes of population groups fall below poverty line and identify their position among low and medium income classes. Hence some relative measures of income inequality derived from three functional curves of income graduation, namely Lorenz, Schotz and Champernowne-Pareto have been analyzed. Meantime three relative indices of deviations of income disparities from poverty line are estimated. Then an adequate treatment to shift these gaps, are obtained to achieve the social and income equality. Both of the two groups of income and poverty measures are estimated from the Egyptian family data 2008/2009 by fitting the three income curves as well as the poverty line to the household income and expenditure. Hence, some specific statements about poverty rate and the income percentages needed to be transformed from the rich to the poor people have been determined.

Introduction:

Analyzing the distribution of income among different groups in a society is one of the interesting issues of any developing (or developed) country. This is because there are ethical grounds for a version to inequality and there is good reason to worry about it. Economic inequality is the main disparity that permits one to have certain material choices while denying another from those very same individuals. Disparity leads to a high degree of poverty which in turn affects the economic development. Also economic inequality is a slippery concept and is linked to other concepts such as political freedoms, personal capabilities and lifetimes. Usually at the beginning stage of development the growth would be in favor of rich people which leads to big divergence between the income classes and consequently to high degree of poverty. But in advanced stages the economic equality would be realized. However for the sake of prevailing this equality the relation between income and poverty must be examined as well as studied and analyzed in the paper. Income inequalities are important not because they stand for all differences in the society but because they represent an important component of these differences. Three functional forms of income distribution are presented to describe the family income graduation from which some measures of income inequality are derived. These are Lorenz curve as one basic characteristic of the scch square distribution, Schotz ogive curve and Champernowne-Pareto distribution function. Then two poverty lines based on nutrition caloric cost and minimal standard of living cost were constructed with three indices for measuring poverty. Hence a poverty rate, poverty and income gab rate, and maximum equalization percentage have been formulated and experimented empirically using the Egyptian family data 2008/2009. Analysing the consumption and income information through connecting some comparable relations between income inequality measures and poverty indices could finally define some descriptive formulae for economic equality. Then it would be possible to determine the amount of income needed to be shifted among the graduated income segments. Consequently a useful tool for achieving social and income equality will be available.

The work in this paper is carried out in six sections. Section 2 is devoted to discuss the three functions of income distribution and their corresponding inequality measures. Section 3 seeks the problem of constructing the poverty lines and poverty indices. Section 4 deals with analyzing the relation between income inequality and poverty gaps resulting in an adequate solution to get rid of poverty and realizes social and income equality. Section 5 presents a comparative study with other researches about income inequality and poverty. Finally section 6 contains the conclusions and recommendations of the paper.

2- Measurement of income inequality:

Existence of great deal of disparity among people incomes would result in a high degree of poverty which may in turn adversely affect the economic development. Income inequality can be measured by two indicators. One is a table or chart that shows a comparison between the different parts of income distribution. Second is an overall index that gives a summery measure of income inequality as a whole. So many methods have been devised for estimating these two indicators from which two descriptive and one functional method are chosen for this paper. Lorenz curves and Schotz ogive curve of income graduation, together with Gini and variability indices of inequality as analyzed here can well express the descriptive measurement. While Pareto-Champernowne curves of income distribution are investigated as a functional relation from which the inequality measure could be estimated from their parameters. Each of the three methods will be presented and fitted empirically to the income expenditure and consumption (IEC) data of the Egyptian family sample research, 2008/2009.

2.1- Lorenz curve (LC)

Lorenz curve represents a relation between the cumulative proportion of income $F(x)$ and the cumulative proportion of recipients $F_1(x)$ having the given income when they are arranged in ascending order of their income intervals x_j . However LC can be measured by the following ratio:

$$LC = \frac{F(x)}{F_1(x)} \quad (1)$$

While the concentrations curve which represents an even distribution of income is defined by the equality:

$$\frac{F(x)}{F_1(x)} = 1$$

From LC an old measure of income inequality, known as Gini index (GI). GI can be derived from the double area below LC subtracted from 1, that is:

$$GI = 1 - 2 \frac{F(x)}{F_1(x)} \quad (2)$$

Then GI may computationally be estimated from the formula:

$$GI = \frac{1}{2n^2\mu} \sqrt{\sum_{j=1}^m \sum_{k=1}^m n_j n_k |x_j - x_k|} \quad (3)$$

Where:

$F_1(x)$: is the cumulative relative distribution of households who receipt the income x , i.e. which represents the proportion of income receivers, having income less than or equal x .

$F(x)$: is the cumulative relative distribution of income x , i.e. the proportion of total income received by income units having income less than or equal x .

x_j : average income of group j $j=1, \dots, m$.

n_j : number of households who receipt the income x_j .

m : number of income groups.

$$\mu = \frac{\sum_{j=1}^m x_j n_j}{\sum_{j=1}^m n_j}, \text{ the total income mean.}$$

Using the IEC Egyptian family data of 2008/2009, the required information for drawing the Lorenz curve could be described as given in table (2.1):

Table (2.1): The cumulative relative distribution of the per household income and their recipients according to 20 income intervals :

Annual income per household x_i	Cumulative relative dist. of households $F_1(x)$ %	Cumulative relative dist. of income , $F(x)$ %
<2000	0.1	0.01
<3000	0.3	0.03
<4000	0.9	0.13
<5000	1.8	0.33
<6000	2.9	0.63
<7000	4.5	1.14
<8000	7.1	2.09
<9000	10.4	3.46
<10000	14.4	5.32
<11500	21.9	9.27
<13000	30	14.13
<15000	41.6	22.08
<17000	51.8	30.07
<20000	64.6	41.66
<25000	78.9	57.41
<30000	86.9	68.18
<50000	96.7	87.37
<75000	98.9	94.1
<100000	99.5	96.67
<200000	100	100

Source: The IEC family data of 2008/2009, CAPMAS, Egypt

Then depicting the cumulative percentages of relative income $F(x)$ on the vertical axis and the cumulative percentage of relative numbers of families $F_1(x)$ on the horizontal axis we can get the Lorenz curve as shown in figure 2.1:

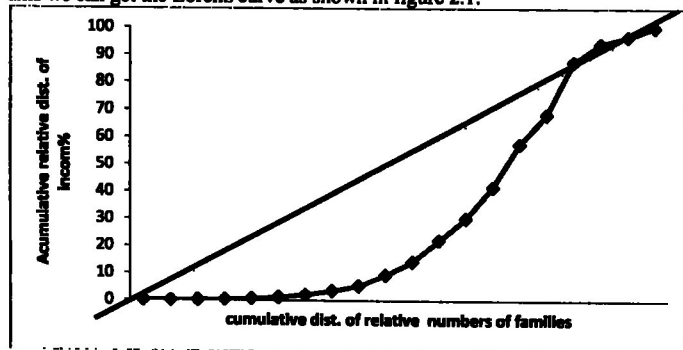


Figure (2.1): Lorenz curve for Egyptian family income distribution in 2008/2009.

Usually Lorenz curve begins and ends on the 45° line. When a society enjoys an even equality state the Lorenz curve will then coincide with that line. With increasing inequality, Lorenz curve starts to fall below the diagram in a loop that is always bowed out to the right of the diagram. The slope of the curve at any point is the contribution of the person to the cumulative share of national income. From table (2.1), GI, was estimated using formula (3) (using STATA 9 package) to be:
 $GI = 0.5719113$

2.2 Schutz Measures:

Schutz suggested two indicators that are used to measure income inequality. As a first indicator, Schutz suggested to plot the proportion of average income \bar{x}_i to the overall income mean \bar{x} , i.e. \bar{x}_i/\bar{x} against the proportion of income recipients having the given income or less, along the income scale arranged in ascending order. It is known as the graduation or ogive curve. From the IEC family data of 2008/2009- Egypt, averages \bar{x}_i of household's income and their proportions to \bar{x} are estimated for the 20 intervals of annual family incomes. The corresponding proportional numbers of households having these incomes or less are also calculated as given in table (2.2).

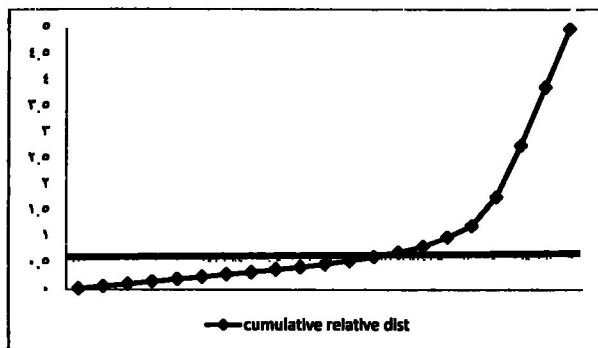
Table(2.2):the relative incomes distribution according to the cumulative distribution of households:

Income intervals	Income averages \bar{x}_i	Relative income \bar{x}_i/\bar{x}	cumulative Relative dist. of the household
<1000	500	0.02	0
<2000	1500	0.06645	0.1
<3000	2500	0.11	0.3
<4000	3500	0.155	0.9
<5000	4500	0.1994	1.8
<6000	5000	0.2437	2.9
<7000	6500	0.288	4.5
<8000	7500	0.3323	7.1
<9000	8500	0.377	10.4
<10000	9500	0.42	14.4
<11500	10750	0.476	21.9
<13000	12250	0.543	30
<15000	14000	0.62	41.6
<17000	16000	0.71	51.8
<20000	18500	0.8196	64.6
<25000	22500	0.9968	78.9
<30000	27500	1.218	86.9
<50000	40000	1.772	96.7
<75000	62500	2.769	98.9
<100000	87500	3.8766	99.5
<200000	112500	4.984	100

Source: the IEC family data of 2008/2009, CAPMAS, Egypt

Where $\bar{x} = 23675$ LE

Representing the relative incomes units $\frac{x_i}{\bar{x}}, i=1, \dots, m$ on the vertical axis while on the horizontal axis depicting the cumulative percentage distribution of households, Schotz curve of income graduation can be obtained as shown in figure(2.2). If the vertical deviations from unity, which mainly equal $(\frac{x_i}{\bar{x}} - 1), i = 1, \dots, m$, where m is the number of income intervals, are shaded on the curve, it will give the disparities in income distribution overall. It means that disparities of income can be measured at each part of income scale as will be seen in table(2.3).



Figure(2.2) Schotz curve of income graduation

The second indicator of income inequality as suggested by Schutz is an index of variability which can be calculated from the absolute value of the relative mean deviations as represented by the shaded area on the ogive curve. This variability index (VI) can be estimated by the formula:

$$VI = \frac{1}{m} \sum \frac{|x_i - \bar{x}|}{\bar{x}}, \quad 0 < VI < 1 \quad (4)$$

Where:

\bar{x}_i : average income of group i .

$\bar{x} = \sum \bar{x}_i / m$, total income mean,

m : number of income groups.

However the VI expresses inequality in income distribution in one rational number ranged between (0,1) .where it indicates wide inequality whenever it approaches 1.

The following table is prepared to calculate the VI:

Table(2.3):Relative deviations of household's income in 2008/2009 data of Egypt:

Intervals of Income	Income Averages	Deviations from income mean $(\bar{x}_i - \bar{x})$	Relative deviations $\frac{ \bar{x}_i - \bar{x} }{\bar{x}}$	relative distribution of households number
-2000	1500	22175	0.94	0.1
2000-	2500	21175	0.89	0.2
3000-	3500	20175	0.85	0.6
4000-	4500	19175	0.81	0.9
5000-	5500	18175	0.77	1.1
6000-	6500	17175	0.73	1.6
7000-	7500	16175	0.68	2.6
8000-	8500	15175	0.64	3.3
9000-	9500	14175	0.60	4
10000-	10750	12925	0.55	7.5
11500-	12250	11425	0.48	8.1
13000-	14000	9675	0.41	11.5
15000-	16000	7675	0.32	10.2
17000-	18500	5175	0.22	12.8
20000-	22500	1175	0.05	14.3
25000-	27500	3825	0.16	8
30000-	40000	16325	0.69	9.8
50000-	62500	38825	1.64	2.2
75000-	87500	63825	2.70	0.6
100000+	112500	88825	3.75	0.6
Total	473500		17.88	%100

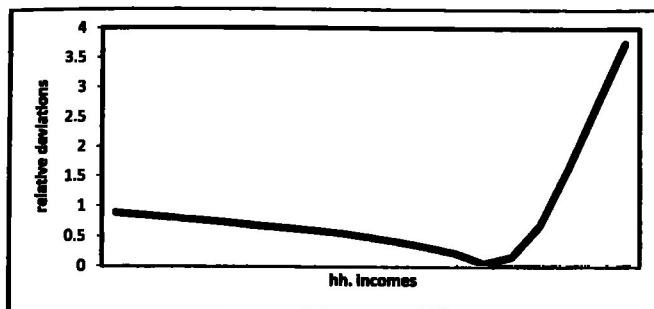
Source : IEC family data ,CAPMAS , Egypt, 2008/2009.

Using equation (4) along with the information of table (2.3) the estimated variability index (VI) will be:

$$VI = \frac{17.88}{20} = 0.894$$

Obviously the value of VI reflects big disparities among the Egyptian family income in 2008-2009 .Moreover it is noted from table (2.3) that the widest disparity as it denoted by the value 3.75 appears before the income class 100000.

Another property about income inequality may be noted from table (2.3) .That is in accordance with the inverted U-curve of the relation between income and inequality levels as suggested by Thirlwall and others, we similarly charted this relation using columns 2,4.Thus Schutz ratios of variations are plotted against the per hh incomes as shown in figure(2.3) below:



Figure(2.3): Schutz ratios of variations according to the per hh income across 20 income groups

From figure(2.3) the inequality to income relation curve does not agree with the Thirlwall hypothesis. A diversely U-curve, where the inequality is steady decreasing within a very short scale of low incomes change (4500-22500)LE is noticed while it broadly varies among the high income classes(22500-100000)LE.

2.3 Champernowne- Pareto curves of income distribution and inequality measures:

Champernowne assumed a family of curves for the distribution of income density functions as follows:

$$y = \frac{A}{\cosh(BX - C) - D} \quad (5)$$

Where:

A, B, C, D are the distribution parameters.

$X = \log t$

t: is the personal income

y: is the number of persons in interval(x , x+dx).

But he stated a symmetrical function that is most convenient be a general density for persons' distribution along X scale with median x_0 as:

$$Q(x) = \frac{n}{\cosh[\alpha\gamma(x - x_0) + \lambda]} \quad (6)$$

Where $n, \alpha, x_0 = \log t_0$ and λ are parameters and $\gamma = \frac{1}{\log e} = 2.30258$,

Hence the corresponding density function of persons' distribution along the income scale (t) is as follows:

$$f(t) = Q(x) \frac{dx}{dt} = \frac{n}{t^t \left\{ \frac{1}{2} \left(\frac{t}{t_0} \right)^{-\alpha} + \lambda + \frac{1}{2} \left(\frac{t}{t_0} \right)^{\alpha} \right\}} \quad (7)$$

Where:

t_0 : is the median income.

n, α, γ , and λ are the distribution parameters. λ takes either one of the values $\lambda = -1$, $\lambda < 1$ or $\lambda = 1$. Then integrating (7) for each value of λ , to find the number of persons with income exceeding t , will give the distribution function $F(t)$ in the three cases of λ 's values.

2.3.1: Champernowne income distribution curve:

From the family of Champernowne income distribution curves he found that the majority of observed income distributions were fitted to that formula which has $\lambda < 1$ as given in this case as the following form:

$$F(t) = \int_t^{\infty} f(u) du = \frac{N}{\theta} \tan^{-1} \left\{ \frac{\sin \theta}{\cos \theta + \left(\frac{t}{t_0}\right)^{\alpha}} \right\}, \quad |\lambda| < 1, 0 \leq t \leq \infty \quad (8)$$

Where :

$$0 < \theta < \pi, \lambda = \cos \theta, \\ N = \frac{2n\theta}{\alpha \gamma \sin \theta} \quad (9)$$

However the distribution function $F(t)$ gives the number of persons with income exceeding (t) . The three parameters N, t_0, α , have some economic interpretation where N is the total number of persons with any income, t_0 is the median income and the parameter α may equal the slope of the high-income asymptote to the Pareto curve $y = F(t)$ when it is plotted on a double logarithmic scale. Hence α is simply the value of Pareto's constant and can be interpreted as a measure of inequality of high incomes. The fourth parameter θ has no economic interpretation but it enables the distribution curve (8) to be asymptote to the Pareto line. This asymptote may be regarded as the Pareto line for high incomes and can take the following formula:

$$\log y = \beta - \alpha \log t \quad (10)$$

Besides giving the parameters N, t_0 , therefore θ must be related to N, t_0, β and α by the following formula:

$$\log \frac{\theta}{\sin \theta} = \log N + \alpha \log t_0 - \beta \quad (11)$$

Accordingly some tables has been prepared in Champernowne's paper to give the value of $\log \frac{\theta}{\sin \theta}$ for various θ

However the corresponding density distribution function for persons along the income scale can be obtained by differentiating (8) with respect to t to find:

$$f(t) = \frac{\alpha N \sin \theta}{\theta t \{ (\frac{t}{t_0})^{\alpha} + 2 \cos \theta + (\frac{t_0}{t})^{\alpha} \}} \quad (12)$$

Moreover some other useful properties of the distribution curve (12) could be derived such as:

$$1. \text{ The arithmetic mean of income} = \frac{t_0}{\theta} \cdot \frac{180 \frac{\sin \theta}{\alpha}}{\sin \frac{180}{\alpha}} \quad (13)$$

2. The modal income t_{mod} is:

$$t_{\text{mod}} = t_0 \left\{ \frac{\sqrt{\alpha^2 + \sin^2 \theta} - \cos \theta}{\alpha + 1} \right\}^{\frac{1}{\alpha}} \quad (14)$$

3. The density function distribution of income flow is:

$$Y = t f(t) = g(t) \quad (15)$$

Where it has its mode at the median income t_0 as may be verified from (12).

4. The maximum density of income flow can be seen as follows:

$$G_{\text{max}} = g(t_0) = \frac{\alpha N}{2\theta} \tan \frac{\theta}{2} \quad (16)$$

2.3.2 Fitting the Pareto line of income:

Since Pareto curve fits income distribution at extremities of the income range as studied by Fisk & others. Then the observed data of 2008-2009 of table (2.4) below can be used to estimate equation (10). Hence the logarithm of the cumulative population receiving income t or more across the logarithm of the cumulative income have been calculated as in table (2.4), according to 20 intervals of family income.

Table(2.4): Accumulated distribution of the households number according to their income classes:

Table 2.4. Estimated distribution of the households number according to their size				
		Log t_i	Log F_i	
>1000	46857	3	4.67077448	46848.61
>2000	46856.87	3.3	4.670773275	46681.13
>3000	46763.15	3.4771	4.669903795	46339.17
>4000	46482.01	3.6	4.667284918	45791.19
>5000	46060.3	3.7	4.663326753	45009.6
>6000	45544.87	3.778	4.658439467	43971.41
>7000	44795.16	3.845	4.651231092	42659.03
>8000	43576.88	3.9	4.639256143	41055.27
>9000	42030.6	3.9542	4.623565559	39176.17
>10000	40156.32	4	4.603753928	37026.55
>11500	36642.05	4.06	4.563979727	33394.42
>13000	32846.63	4.1134	4.516490818	29440.09
>15000	27441.22	4.176	4.437928365	24246.91
>17000	22631.81	4.23	4.354719288	19805.46
>20000	16634.12	4.3	4.220999752	14149.99
>25000	9933.566	4.4	3.997105182	8550.162
>30000	6185.066	4.477	3.791344338	5458.578
>50000	1593.08	4.7	3.202237585	1620.183
>75000	562.288	4.875	2.749958815	608.6556
>100000	281.144	5	2.44892882	304.3278
>200000	0	5.3		55.35868

Source : IEC family data ,CAPMAS , Egypt, 2008/2009.

The relation between t_i and F_i can be illustrated in figure(2.3) below.

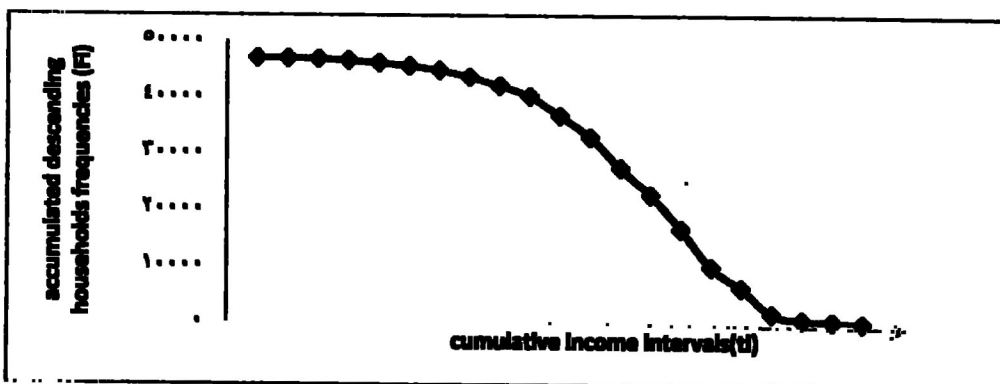


Figure (2.3):distribution of hh according to family incomes in 2008/2009

Then using the indirect least squares (ILS) method the Chambernowne –Pareto functions (8,12) could be estimated from the data in table (2.4).The resulting estimators of the parameters of these functions were obtained according to the estimation equation(10-15) as follows:

The OLS predicted equation (10) is estimated by:

$$\text{Log } y = \beta - \alpha \log t$$

$$Y = 14.55 - 2.415 Z$$

Therefore:

$$Y = \text{Log } y$$

$$Z = \text{Log } t$$

$$\beta = 14.55$$

$$\alpha = 2.415$$

$$N = 46857$$

$$t_0 = 15333.8$$

where t_0 is calculated by rough interpolation.

Hence:

$$\begin{aligned} \log \frac{\theta}{\sin \theta} &= \log N + \alpha \log t_0 - \beta \\ &= \log 46857 + (2.415) \log (15333.8) - 14.55 \\ &= 0.2291 \end{aligned}$$

Therefore

$$\theta = 97^\circ$$

Hence the fitted distribution function $\hat{F}(t)$ is given from equation (8) as

$$\hat{F}(t) = 483.062 \tan^{-1} \left[\frac{\sin (97)}{\cos (97) + \left(\frac{t_i}{15333.8} \right)^{2.415}} \right] \quad (17)$$

And the corresponding density function of income (equation 12) is as follows:

$$f(t) = \frac{112134.892/97}{97 t \left((t/15333.8)^{2.415} - 1.8646 + (15333.8/t)^{2.415} \right)} \quad (18)$$

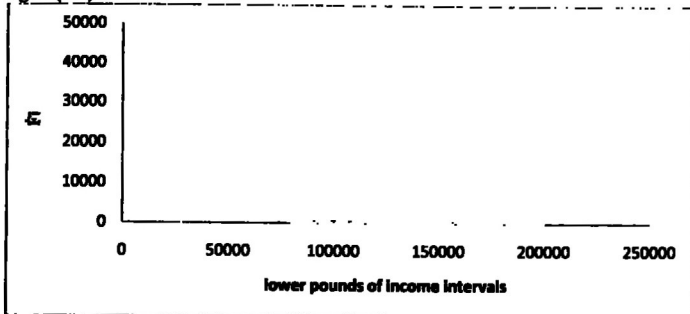
with the distributions mean and mode as

$$t^* = 12358.2 \text{ LE and}$$

$$t^*_{\text{med}} = 5872.69 \text{ LE}$$

and the maximum density of income per pound is given from (16) as:

However by calculating equation (16) as presented in table (2.3) column 5 the Champernowne –Pareto fitted curve of income distribution(8) could be illustrated in figure(2.4) below:



Figure(2.4): the Champernowne –Pareto predictive curve of income distribution

Comparing the observed and the predicted income distribution curves (2.3, 2.4) we can notice that the fitted points are approximately all coincide with the observed ones. Hence from the two figures it obviously seems that the Champernowne-Pareto function of income distribution fits well the observed family data.

2.3.3: Champernowne-Pareto measures of inequality:

Champernowne-Pareto curve presented three measures of inequality indicated by α, I, J such that:

1. The parameter of Pareto curve, α , is considered as a measure of the degree of income inequality between the rich and the very rich people. That is it relates only to the high income classes. Then Champernowne suggested two alternative measures of income inequality indicated by I and J . Both of which are derived from the parameters of its density function of eq.(8) and have the same importance as of α .

2. The indicator measure I is a measure of inequality overall the distribution curve of income, where it is given by the form:

$$I = 1 - \left[\frac{\theta \sin^{\alpha/\theta}}{\pi \sin^{\theta/\alpha}} \right] \quad (19)$$

It can roughly be estimated from the following formula:

$$I = 1 - t_0/t^*, \quad (20)$$

where t^*, t_0 denote the arithmetic mean and median incomes respectively.

3. The inequality indicator, J , as proposed by Champernowne, reflects the inequality among the medium and the upper income classes and given by the following formula:

$$J = \left(1 + 2 \cos \frac{\theta}{2} \right)^{\frac{1}{\alpha}} \quad (21)$$

It can be estimated from the following formula:

$$J = \frac{\text{the highest 25\% income}}{\text{median income}} = \frac{q_3}{t_0} \quad (22)$$

Where

q_3 : is the third quartile of income

From the 2008/2009 family income –expenditure data, the estimators of the inequality measures of income are given by:

1. Pareto's parameter, $\hat{\alpha} = 2.415$

2. The overall measure of inequality, $\hat{I} = 0.354$

3. The between the medium and the rich income class inequality measure,

$$\hat{J} = \frac{21328.8}{15333.4} = 1.4$$

3-Measurment of Poverty

Poverty is the inability to attain a minimal standard of living (SL) and it is concerned with the absolute SL of a part of society (mainly poor). It is required to distinguish between poverty and inequality where both are of intrinsic significance. But inequality refers to relative SL across the whole society. A common fact of the relation between them is that at maximum inequality where the person has everything, poverty is high (all are poor). While minimum inequality where all are equal is possible with zero poverty. Hence both of the SL and the minimal SL must be measured and determined. Income and expenditure per household is often considered as an adequate measure for the SL. But it doesn't reflect any element of welfare as life expectancy, literacy, access to public goods services or common property. Because of these drawbacks current consumption is the better measure than income as it reflects household ability to buffer their SL through saving and borrowing. This measure must be judged by some norm or criteria to distinguish poor from nonpoor. This norm should represent the minimal SL and usually called poverty line. Therefore some concepts had determined poverty line as comprising two elements, the expenditure necessary to buy a minimum standard of nutrition and other basic necessities and a further expenditure that varies from country to another reflecting the cost of participation in everyday life.

Hence one basic measure of poverty is the poverty line and particularly, according to literature review, the consumption –based poverty line. This line may be illustrated by the minimal SL against consumption as a measure of SL. A further summery measure of the degree of poverty is also plausible and more indicative is a poverty index. One of these indices of poverty measures is the head count ratio (HCR), which provides just a crude ratio of poor population to the whole society. Others as the poverty gap ratio (PGR) and the income gap ratio (IGR) are also of high significance in this respect. Mainly poverty gap ratio measures the transfer that would bring the income of every poor person exactly up to poverty line. Using the IEC family data of the sample research 2008/2009, these measures of poverty could be experimented and estimated as explained below.

3.1 Poverty Line

In this paper two consumption –based poverty lines are derived, one is concerned with a calorie nutrition –base where the minimal SL is measured by the cost of human needs of food across the income classes scale. The second is measuring the minimal SL by the expenditure on food and on other necessities as

health, education and shelter, on the regional scale. Choosing of these indicators for measuring the minimal SL was obligated by the shortage of available information about an index of SL as well as by the time inadequacy of estimating it here. So the consumption data were also substituted by the annual total family expenditures which is actually reflecting the different levels of living per family with respect to their incomes. The two poverty lines are explained below.

3.1.1: Nutrition Calorie –based poverty line

Nutrition Calorie –based poverty line permits cross income-comparison within the households sector. Based on the IEC family data of 2008/2009, the total consumption and the annual expenditure on food per household for 20 income classes were calculated, for the Urban and Rural sectors. The two cross data are presented for each Urban and Rural sectors as shown in tables (3.1) below.

Table (3.1): The average annual total consumption and expenditure on food per household, of the Egyptian urban and rural sectors in 2008/2009 (in LE).

Intervals of Income	urban averages of		rural averages of		total averages of Egyptian hh sector	
	consumption per household (x)	expenditure on food (y)	consumption per household (x)	expenditure on food (y)	consumption per household (x)	expenditure on food (y)
-2000	2932.9	1597.5	2635.5	1579.8	2784.2	1588.7
2000-	3399.9	1841.1	3488.9	1968.2	3444.4	1904.7
3000-	4788.5	2391.8	4178.5	2314.8	4483.5	2353.2
4000-	4909.6	2465.9	4880.7	2630.8	4895.15	2548.4
5000-	6127.5	2967.1	5869	3124.3	5998.25	3045.7
6000-	6844	3423.9	6733.9	3603.9	6788.95	3513.9
7000-	7779.3	3726.5	7684.4	4039.6	7731.85	3883.1
8000-	8713.7	4257.1	8623.2	4488.7	8668.45	4372.9
9000-	9649.3	4522.5	9569.6	4991.3	9609.45	4756.9
10000-	10843.2	5044.8	10801.6	5579	10822.4	5311.9
11500-	12263.8	5689.6	12201.7	6298.5	12232.75	5994.1
13000-	14021.4	6367.1	13924.5	7135.9	13972.95	6751.5
15000-	15964.3	7146.6	15822	7985.8	15893.15	7566.2
17000-	18320.8	8068.6	18166.1	9073.9	18243.45	8571.3
20000-	22036.4	9310.9	21589.7	10525.1	21813.05	9918
25000-	26907.9	10795.2	26060.2	12151	26484.05	11473.1
30000-	36257.3	12944.9	33318.3	14615	34787.8	13780
50000-	57829.1	16524.6	46321.4	18212.4	52075.25	17368.5
75000-	83144.1	18870	72077.3	25828.7	77610.7	22349.4
100000+	136141.4	23337.8	126726.9	43741.5	131434.2	28734.9

Source: The IEC family data of 2008/2009, CAPMAS, Egypt

Then let the family consumption be represented on the horizontal x axis and the expenditure on food ,as a minimal nutrition cost required for human life , be plotted on the vertical y axis, a nutrition calorie based poverty line is illustrated for each of the urban and rural sectors as shown on charts (3.1),(3.2).

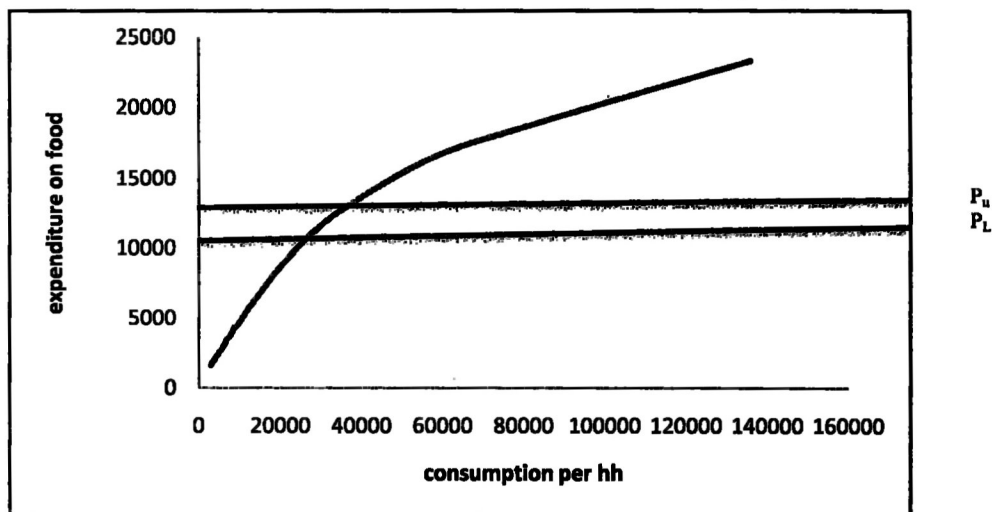


Figure 3.1 Nutritrin-Calorie poverty line for Urban governrates.

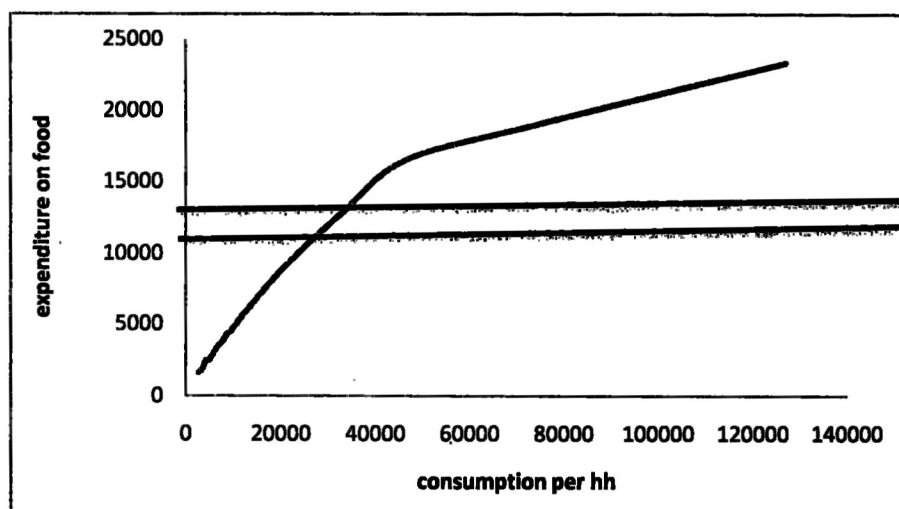


Figure 3.2 Nutritron Calorie- poverty line for Rural governrates.

Where $P_U=16000$ and $P_L=14000$

From charts (3.1),(3.2),it is obvious that the expenditure on food per household is often increasing for the low income classes.But the variation within the high classes is rather stable and slightly changed as consumption expenditure increases.

3.1.2: living cost-based poverty line

The family expenditure share on the living necessities such as food, health, education and accommodation, is regarded as the cost of the minimal SL to represent the poverty line for each of the 21 Egyptian governorates. Then the values of thresholds of these elements along with the total consumption are calculated per household (hh) from the IEC data of urban and rural as shown in table (3.2) below.

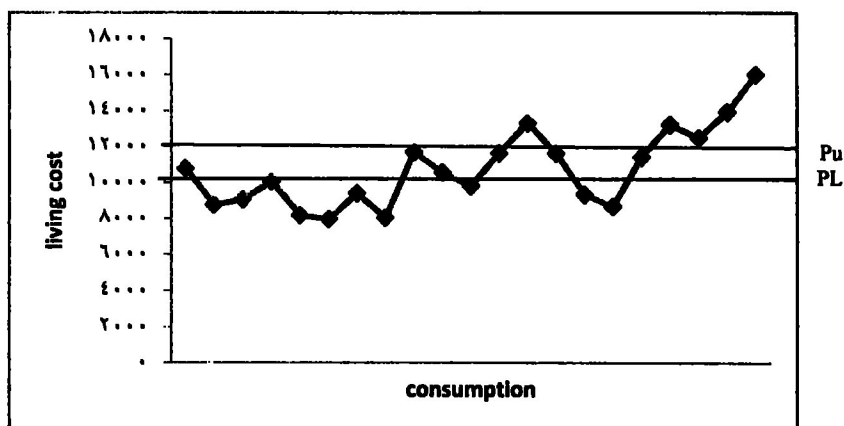
Table (3.2): averages of consumption and living cost per hh of urban and rural Egyptian governorates in 2008/2009 (LE):

governorate	urban expenditure		Rural expenditure	
	consumption(x)	living cost(y)	consumption(x)	living cost(y)
Giza	20137.1	10779.7	13696	7833.6
Banyuef	14857.9	8766.6	13054.8	8301.2
Fayoon	16484.5	9083	15209.2	9176.9
Elmenia	17897.4	10055.3	15827.2	10005.7
Assiout	14182	8192.7	10896.6	7096.5
Sohag	13080.6	7978.2	11542	7487.6
Kena	15398.8	9416.7	13960.1	9394.2
Asswan	14391.8	8050.8	11993.9	7368.2
Domiatia	20020.5	11699	17137.4	10960
Dakahlia	17840	10575.5	15190.7	9782.6
Sharkia	15368.9	9814.7	13462.9	9302
Kalyoubia	19867.9	11640.1	16451.7	9817.8
Kafi-Elshiekh	20599.1	13324.1	20506.4	14026.9
Gharbbia	20671.8	11626.2	17215.9	10566.9
Menouffia	17390.8	9353.6	15355.7	8890.1
Behiera	13997.1	8697.5	14308.1	9388.3
Essmalia	19829.1	11407.9	15950.5	10260.9
Cairo	24548.8	13222.1		
Alexandria	20901.8	12499.7		
Port said	25328.4	13937		
Suez	27444.3	16006.6		

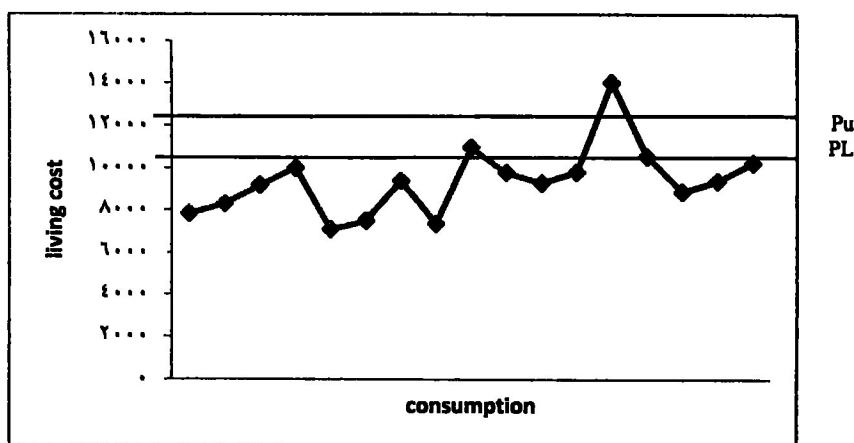
Source: The IEC family data of 2008/2009, CAPMAS, Egypt

A poverty line could hence be charted for Egyptian society from table (3.2) by plotting the living cost on the y axis against the consumption per household on the x-axis for the Egyptian governorates of urban and rural sectors as illustrated in figures (3.3), (3.4). However the living cost of minimal participation in economic life may be best representative to the general poverty line. Another upper and lower bounds of the poverty line are also evaluated as recommended by the world bank

studies, to distinguish which of governrates falls out of these bounds. These bounds were subjectively estimated, where the upper poverty line P_U is chosen to be 12500 LE and the lower poverty line P_L is chosen to be 10500 LE.



Figure(3.3):poverty line for urban governrates



Figure(3.4): poverty line for rural governrates

As shown in figures (3.3),(3.4) it is obvious in general that as governrates become wealthier their perception on the acceptable minimum level of consumption (poverty line) changes. Moreover it appears in the urban chart that about 10 from 21 governrates fall below P_L and about 7 governrates are between P_L and P_U .

3.2 Poverty Indices:

As determining the consumption-based poverty line, either represented by nutrition calorie cost or by living cost, as a minimal SL required for an every-day life participation, it is now possible to evaluate the poverty in any society by one summery indicator denoting the extent to which the perception of poor people is

spread out. In this paper two measures of poverty are presented one is known as head count ratio (HCR) which provides just a crude ratio of the poor population to the whole society. The second is an index of social equality measure which estimate the rational gap of poverty (PGR) line in relation to low income levels.

3.2.1 Head Count Ratio (HCR)

One of the simplest method to estimate a poverty index is to count the proportional number of poor individuals whose incomes are less than the poverty line in relation to the total number of population. This is named head count ratio (HCR). In order to distinguish the poor from the non poor people we must represent the poverty line with one value for the nutrition-cost suit all income classes. Owing to the wide range of per hh income from 1500-100000 LE and the increasing expenditure on food accordingly, we arbitrary choose lower bound P_L and upper bound P_U for the nutrition poverty line which must satisfy the food cost for a moderate level of living scale, regarding implicitly their prices. Therefore we supposed the two bounds of poverty line to be $P_L = 14000$ LE and $P_U = 16000$ LE with average poverty line $P = 15000$ LE. Hence considering those families whose income averages are less than P_L and P_U as the poor and severe poor and it would be possible now to count their numbers (HC) as detailed in table (3.3). Then if the individual income is y_i and p denotes the poverty line which represents the amount of money required to attain an acceptable calorie threshold. Then HCR is given by:

$$HCR = \frac{HC}{n}$$

Where:

HC: the number of households with income $y_i < P$, i.e HC is the number of poor households.

y_i : the average income of class i , $i = 1, 2, \dots, m$.

$P = 15000$: the amount of money required to attain an acceptable calorie threshold (poverty line).

$n = \sum_{i=1}^m n_i$: total number of households.

m : number of income classes.

n_i : number of households in income class i .

Table 3.3: Average Consumption per household and income shortage across the household income classes for the IEC family data of 2008, Egypt.

households		average income per household (LE) y_i	average expenditure on food (LE) P_i	income shortage from			average total consumption per household (LE) (x_i)	consumption to income ratio x_i/y_i	poverty line to average income ratio P/y_i	nutrition cost to income ratio $r=P_i/y_i$
number n_i	percent age $(n_i/n)\%$			$(P_u - y_i)$	$(P_L - y_i)$	$(P - y_i)$				
46.857	0.1	1500	1588.7	14500	12500	13500	2784.2	1.856133	10	1.059133
93.714	0.2	2500	1904.7	13500	11500	12500	3444.4	1.37776	6	0.76188
281.142	0.6	3500	2353.2	12500	10500	11500	4483.5	1.281	4.285714	0.672343
421.713	0.9	4500	2548.4	11500	9500	10500	4895.2	1.087822	3.333333	0.566311
515.427	1.1	5500	3045.7	10500	8500	9500	5998.3	1.0906	2.727273	0.553764
749.712	1.6	6500	3513.9	9500	7500	8500	6789	1.044462	2.307692	0.5406
1218.282	2.6	7500	3883.1	8500	6500	7500	7731.9	1.03092	2	0.517747
1546.281	3.3	8500	4372.9	7500	5500	6500	8668.4	1.019812	1.764706	0.514459
1874.28	4.0	9500	4756.9	6500	4500	5500	9609.5	1.011526	1.578947	0.500726
3514.275	7.5	10750	5311.9	5250	3250	4250	10322.4	0.960223	1.395349	0.49413
3795.417	8.1	12250	5994.1	3750	1750	1750	12232.8	0.998596	1.22449	0.489314
5435.412	11.6	14000	6751.5	2000	1000	1000	13973	0.998071	1.071429	0.48225
4779.414	10.2	16000	7566.2	-	-	-	15893.2	0.993325	0.9375	0.472888
5997.696	12.8	18500	8571.3	-	-	-	18243.5	0.986135	0.810811	0.463314
6700.551	14.3	22500	9918	-	-	-	21863.2	0.971698	0.666667	0.4408
3748.56	8.0	27500	11473.1	-	-	-	26484.2	0.963062	0.545455	0.417204
4591.986	9.8	40000	13780	-	-	-	34787.8	0.869695	0.375	0.3445
1030.854	2.2	62500	17368.5	-	-	-	52075.8	0.833213	0.24	0.277896
281.142	0.6	87500	22349.4	-	-	-	77610.2	0.886974	0.171429	0.255422
281.142	0.6	112500	28734.9	-	-	-	131434.2	1.168304	0.133333	0.255421
46857	%100									

Source: The IEC family data of 2008/2009, CAPMAS, Egypt

Where:

$H_c = 19492.5$ hh from which 14057hh are below the lower poverty line, with percentage 30% of the total number of hh's in 2008/2009 sample data.

$P_u = 16000$ LE

$P_L = 14000$ LE

$P = 15000$ LE

$\mu = 23675$ LE, the income mean

$n = 46857$ hh

$PHC = 292.388$ million LE

$n\mu = 1109.339$ million LE, the total income.

Then from columns 1,7 of table (3.3), the poor ratio of Egyptian society in 2008/2009 is estimated as follows :

The number of poor families in 2008/2009 sample, i.e. $H_c = 19492.5$ hh and $n = 46857$ hh, then:

$$HCR = \frac{19492.5}{46857} = 41.6\%$$

3.2.2 Poverty and income gap ratio (PGR), (IGR):

The most adequate index of poverty is that which may show the range of income deviations from poverty line. This is called the poverty gap ratio (PGR) or alternatively the income gap ratio (IGR) and both are given by the following formulae:

$$PGR = \frac{\sum y_{i < P}(P - y_i)}{n\mu}$$

and the income gap ratio (IGR):

$$IGR = \frac{\sum y_{i < P}(P - y_i)}{PHC}$$

Where:

μ : is the mean income, i.e. $\frac{\sum_{i=1}^m y_i n_i}{n}$,

P : is the poverty line

PHC: is the amount of money required to attain an acceptable calorie threshold for all the poor people (HC). It can be estimated by saying that minimal cost required for living in the everyday life as equal to P times HC.

Mostly the PGR seems to be an underestimated measure of poverty so that the IGR may be preferred for measuring the acuteness of poverty particularly relative to the total income needed to remove poverty. From table (3.3) PGR, IGR are evaluated to be:

PGR=8.4%, and

IGR=31.6%

4-Achieving social and income equality:

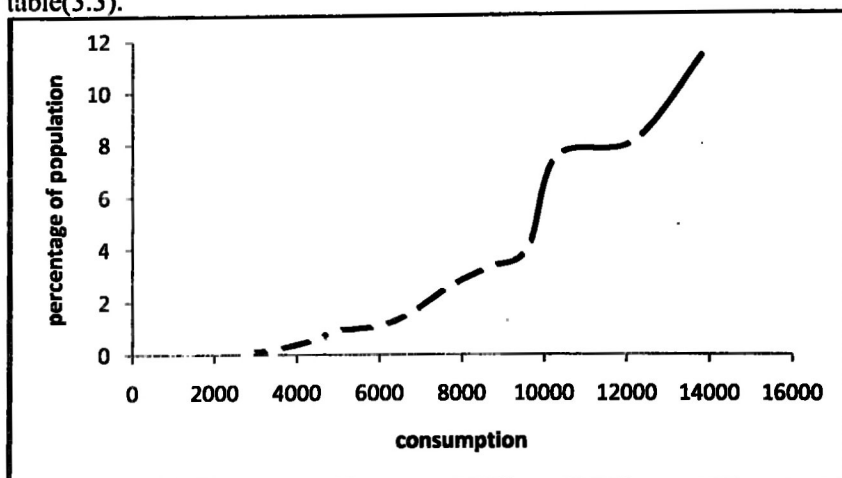
4.1 Social Equality

Economic equality is a broad concept which may relate the economic development with social and income equality .One functional form of this relation is the inverted – U curve as derived by Thirdwall and others.[1].But when this relation as represented by the Schotz inequality and income as was charted through the graduated levels of income classes shown in figure (2.3), it reflected a different shape rather such as a U curve.

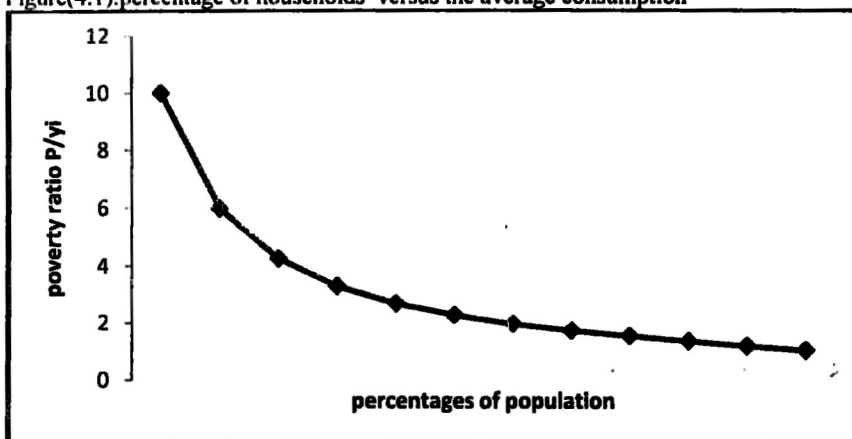
Then since the poor have been distinguished as a proportion of the total population as given by HCR index, it is now possible to measure how to realize social equality in a society. By sorting those poor people from the household income classes, the amount of income needed to remove poverty as indicated by PHC is equal 292.4 million LE, may be used in some sense as a measure of social equality. Moreover the total shortfall of poor incomes from the poverty line as estimated by PGR index has determined the resources required to eradicate poverty as 8.5% of the total income. However the headcount index as calculated from table (3.3) gives about 41.6% poor of the total population. Comparing the poor's incomes per hh with the global upper and lower poverty line (P_u, P_L), it is found that about 52.8% of population have income less than or equal $P_u=16000$ LE ,from which there are about 40% of the total population with incomes below $P_L=14000$ LE ,where they live in severe poverty.

Then an illustration of the poorest 12-income classes is presented by the following charts using table (3.3).From columns (2),(8) the percentages of the households (HCR) below poverty line P ,which represents the lower 12 income groups are plotted against the average consumption per household x_i as illustrated in figure (4.1).While Figure(4.2) plots the poverty-income ratio $r = (\frac{P}{y_i})$ across the population

percentages using columns (2),(10).Finally from columns (8),(11) the food calories to income ratio $r_i = \left(\frac{P_i}{y_i}\right)$ are plotted against the average consumption as illustrated using table(3.3).



Figure(4.1):percentage of households versus the average consumption



Figure(4.2):income shortage to poverty line

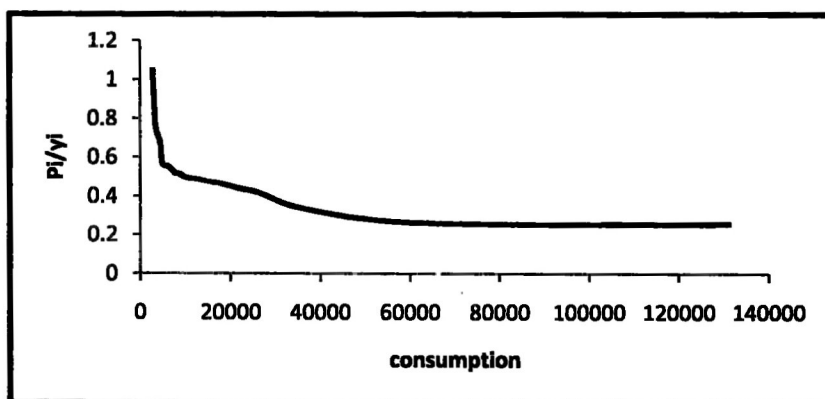


Figure (4.3):income shortage to food calories.

Figure(4.1) shows that the percentages sum of poorest hh are 14.4% as concentrated at the low levels of consumption<10000, then it continues to go upwards to the left with level<14000 .On figure(4.2) the ratio of the income classes who live below the poverty line is counted to be 12 of 20 classes with percentage 60%.Their income shortage from poverty line (P/y_i) constitute the required incomes for those lower 12 classes to attain an acceptable living [intakes].That is the lowest income classes needs about 10 times their current incomes (1500LE) ,while the 12th poor class needs only 1.1 times their incomes (14000LE) to cover their persistence needs. On the other hand those poor classes in average spend more than 50% of their incomes only on food as is shown in figure (4.3).Accordingly the incomes sum needed to raise everybody exactly up to the poverty line as measured by the PGR index represents 8.4% of the total income and about 31.6% of the poor incomes as estimated by the IGR index. However the PGR index can be regarded as a measure of income resource required to realize social equality. It means that the total averages of extra consumption or income needed to get all people above the poverty line are the sum of all gaps $(P-y_i)>1$ or $(P-x_i)>1$ (table(3.3),column 9) .That is if this amount of shortfall incomes is adequately planned to be buffered ,for the poor classes, the poverty would go away and the social equality be achieved.

4.2 Income Equality:

The case of inequality and poverty both are of effective significance. Obviously the income equality is a broader concept than poverty in that it is defined over the entire population and doesn't focus only on the poor. But there may exist a relation between them. If poverty is extremely visible in a society, then the income disparity becomes of intensive consideration to be measured and hence targeted in economic policies to be reduced. The income inequality in a simple meaning may show such a proportion of total income that is absorbed in compensating for the loss of aggregate satisfaction due to inequality. Besides the poverty-income relation, as represented by the two indices PGR, IGR, there are three types of income inequality measures, derived from the three curves of income distribution, that would be used to validate this relation and contribute in treating income equality problem as discussed in section (2.1).

First is the maximum equalization percentage (MEP) as suggested by Kuznets, who derived it directly from Lorenz curve by sorting the population from poorest to richest and showing the percentages of income attributable to each. A summarized table has been augmented from table (2.1) to show the fifth, tenth and quintiles of the population and their income shares as given in table (4.1) below:

Table(4.1):population percentages of the poorest and richest income quintiles for the 2008/2009 data:

household percentages %	Income percentages % of the		Equalization percentages
	poorest	richest	
3	0.6	12	20
5	3.1	18.5	6
10	3.4	32.1	9.4
20	9	42.6	4.7
30	14	60.2	4.2

From table (4.1) the poorest 20th quintile accounts for about 9% of the income while the richest top quintile for 42.6%. The poorest 10% accounts for 3.4% of all incomes and the richest for 32.1%. The poorest 3% accounts for 0.6 % with average income=2500 LE per hh annually, while the richest accounts for 12% with average income 131250 LE, which is about 52.5 times the corresponding average of poorest 3%.

Dividing the richest over the poorest income percentages as shown in column 4 of table (4.1) will give the equalization ratios from which the maximum of all ratios MEP can explain a simple measure of inequality. Hence the MEP=20 may represent the percentage of income which must be transferred from the richest to the poorest quintiles of income classes in order to achieve an equal distribution of income.

The second is the one- number measure of income inequality as represented by the two indices, Gini index (GI) and Schotz index of variability (VI). Both are estimated using income distribution of Lorenz and Schotz graduation curves respectively and lie in the range (0,1) from perfect equality to perfect inequality. The most commonly used is the GI, where data of 2008/2009 of Egypt supplied an estimation of GI=0.59, while VI of Schotz =0.89. Hence using the three estimators of inequality measures (0.59,0.89,0.35) we may conclude that the inequality estimator, in average, equal 0.6.

The third type is the inequality measure derived from the parameters estimators of the fitted density and distribution functions of Champnowne-Pareto functions in section(3.3). The fitted Pareto model to the 2008/2009 data gave three measures of inequality such as:

1. $\hat{\alpha} = 2.14$ (as a measure of the degree of income inequality between the rich and the very rich people. That is it relates only to the high income distribution.
2. $\hat{f} = 1.4$, as a measure of the degree of income inequality between medium and rich people.
3. $\hat{I} = 0.354$, Champnowne considered the measure I as denoting to the proportion of total income that is absorbed in compensating for the loss of aggregate satisfaction due to inequality, and hence can measure the degree of income inequality among the most poor classes in the whole society, where:

However from the above estimators it is seen that there is a big disparity among rich classes as given by $\hat{\alpha}$ and less disparities among medium classes as given by \hat{f} , while the low income classes are nearly similar circumstances that they lived in so low degree of inequality as may be represented by the average indicator of inequality I. Hence using the three estimators of income inequality measures (0.59, 0.89, 0.35) we may conclude that the inequality estimator is in average 0.61. Accordingly income equality needs about 60% of total family income to be transferred among high to poor classes to compensate the material loss of poor people.

5- A comparative study on poverty and income inequality:

5.1 Other studies on poverty reduction and income inequality measures:

Since 1990 till now the economic growth studies have been transformed to poverty reduction in developing countries. The world bank has a major role in measuring the income disparities by the old coefficient "Gini index" and the GNI per capita and the poverty line by the minimum nutrition cost and the poverty level by the head count(HC) ratio, across a number of countries data. Meantime an increasing studies of the development authors had introduced the recent evidence on the transformation of growth to poverty reduction (where they showed that on average income growth has been the driving determinants to the change in poverty level through the change in the inequality factor. Those authors, e.g., Adam 2004, Bouguignon 2003, Easterly 2002, Epaulard 2003, Fosu 2008, Kalwij and Verchoor 1997, have estimated the income and growth elasticities of poverty by applying their suggested mathematical models of the three component relation; poverty, growth and inequality to the growth-countries data. They documented that inequality plays a crucial role in poverty reduction and decomposed the changes in poverty into growth and income distribution components. Ferreira, 2010 has presented the recent global evidence on the transformation of economic growth to poverty reduction in developing countries, with emphasis on the role of income inequality and focused on the period since the 1990s when growth in these countries has generally surpassed that of the advanced economies. In brief we shall present 3 of these models to illustrate their contributions as follows:

Model(1): Ravallion 2003:

Ravallion showed that the high inequality would hinder poverty reduction and assumed the corrected rate of average income growth as an important determinant of poverty change. He found that one percent increase in income level could lead to a poverty reduction of as much as 4.3 percent for very low inequality countries, or as little as 0.6 for very high inequality countries.

Model(2): Bourguignon, 2004:

Bourguignon has decomposed the change in poverty reduction into 2 effects, growth and income redistribution as applied empirically by Datt and Ravallion (1992) and Kakawani (1993). He found that poverty level may be affected due to change in income mean relative to poverty line or due to a change in the relative inequalities.

Model(3): Fosu, 2010:

Fosu had proposed an income- poverty relation assuming a lognormal distribution for the income. He illustrated the effect of income growth transformation to poverty reduction following Bouguignon (2004), Epaulard (2003), Kalwij (2007) and Fosu (2009).

5.2- Comparison between the present research and other researches:

It is possible now to clarify the differences between the present research and other recent poverty-income inequality researches through the following notes:

1. Our main goal is to introduce ,statistically based measures of inequality and poverty indices using three best-fit curves for income distribution for estimating them, empirically from the expenditure-income data in the period 2008-2009. These measures were to be analyzed to follow their changes simultaneously to discover their relation over the different income classes. In order to achieve social equality for the Egyptian community. While the other concerning studies as summerized in the preceding subsection were absolutely assuming an arithmetic causal relation between them where the inequality improvement attributed to economic growth is behind the poverty reduction.
2. All empirical researches of the world bank about the inequality and poverty measures as well as of those studies of growth transformance to poverty reduction are based on cross-country data. There are some exceptions of those studies which had been carried out by the Egyptian experts such as El-Laithy(1997) who studied income distribution and poverty properties. Also, El-Mahdy(1996) studied the structural adoption and meeting poverty. Fergani(1996) studied poverty phenomena and development strategies in Egypt.
3. However our research was allocated to one specific country mainly Egypt and consequently the poverty line, the inequality and poverty measures would reflect real own properties of the its economy. While the same measures as estimated from cross-international countries data are providing more general and indetermined characteristics of poverty line and income disparities measures which are rarely telling certain properties about any individual country.
4. The research has introduced many measures of inequality as estimated from the income distribution functions employed in the study. Also several measures of poverty level and two different poverty lines were calculated. While the three component transformance studies were consulting Gini index based on Lorenz curve and GNI for measuring inequality or using HC coefficient for measuring poverty measurement .
5. The research has estimated the poverty-income ratios across the income classes to record the relative change in poverty attributable to income classes. While in the concerning studies the income and growth elasticities were estimated to evaluate these relative changes in both components owing to the different unit-measures of the developing countries data.

6- Conclutions, Drawbacks & Recommendations:

6.1: Conclutions:

The work of this paper as discussed in the preceding sections is dealing with the analysis of the most applicable and best fitted three functional curves of personal income distribution ,Lorenz, Schotz and Champernowne-pareto .From these curves three types of income inequality measures are derived known as Gini

index,variability index and Champernowne-pareto's parameters.Meanwhile a complementary analysis of poverty based on the world bank's studies through the last 2 decates is included by deriving two poverty lines.Accordingly three indices of poverty measures,as proposed by Kusnets,Thirlwall,Depraj,Jonathan and others ,are formulated and experimented on the Egyptian 2008/2009 family data .Then an analytical combination of the relation between the two groups of income and poverty measures has provided two rational measures for achieving social and income equality.These are the ratio of total income required to get rid of poverty and the percentage of transferred income from high to low income classes needed to realize an even income distribution.The consequences that can be obtained from the work may be summerized in four comparisons as follows:

1-Comparing the three measures of income inequality ,we found the Schotz VI estimator (where $VI=0.89$) reflected more widen disparities among the Egyptian family income than that of the Gini measure $GI=0.59$.In respect to Champernowne-Pareto measures the inequality estimator gave a value 0.35 which indicates an average moderate level of disparities over all the income distribution scale.In the meaning of income inequality measure the income ratio to be transferred among classes to realize an even income distribution may fall between 0.35 and 0.89 of the total family income as obtained from Champernowne-Pareto and Schotz estimators.

On the other hand considering the relation between the inequality and the household income according to Kuznet's hypothesis which was described in figure (2.3) .It was found that the Schotz curve reflected rather a U-curve for the inequality change.It means that Kuznet's relation showed initially low monotonically decreasing income inequality(0.95-0.25) as incomes increase(10000-16000).Then it has a vast increasing variation within high income classes.Concerning the change of Champernowne-Pareto's measures the inequality they seemed to track an adverse direction so that inequality starts with low value (0.35) for poor income classes,tending to increase to 1.4 within the medium classes then it raises up to 2.5 within the rich classes.In both cases the two curves reflect one remarkable note that is the poor classes posses low inequality degree while the rich's disparities look to have high degree with wide variation.

2- Comparison between the poverty measures as assigned by the indices HCR,PGR and IGR ,and the income inequality measures which would provide two evaluation values. First is a determination of the percentage of income needed for the poor classes to get out of the poverty line. However as $HCR=41\%$ gives the percentage of poor people,then $PGR=8.5\%$ gives the percentage of total income needed to make them at or above the poverty line.The second value determines the transferred income ratio from high to low income classes to realize an even income distribution overall the whole community which is ranged between 0.35 to 0.89 as given above.We may then conclude that the society needs about 8.5% of the total personal income to raise the poor above the poverty line and needs approximately about 60% of the total personal income to realize an income equality.

3- The comparison between poverty measures and income inequality as represented by the descriptive relation between the ratio $\left(\frac{P}{Y_i}\right)$ and consumption (figure(4.2)) showed that the income shortage required to cover the nutrition poverty line changed from 10 to 1.1 times the income/hh. It counted in average to 5.6 times the mean income over the 12 poor classes. Hence the poor income mean must be raised up about 6 times in order to satisfy their needs for food.

Likewise from figure (4.3) it is found that the food threshold $\left(\frac{P_i}{Y_i}\right)$ absorbed a tremendous ratio of the income/hh specially across the 12 poor classes. It reaches from 1.06 times hh the lowest income-class to 0.48 of the per hh highest poor income-class with average 0.77. On the other hand the consumption/hh exhausted from $\left(\frac{X_i}{Y_i}\right) = 1.86$ to 1 times the income/hh of the same two classes (table(3.3)). Thus the income shortage needed to satisfy the consumption/hh is in average 1.43 times the average poor income from which the food cost will absorb alone 1.1 times the poor income/hh. But from the view point of the maximum equalization percentage as estimated from the Egyptian data to be $MEP=20\%$ (table(4.1)). We may in general conclude as the MEP's interpretation given before that there must be 20% transference of the total income to be shifted among the different income classes to redistribute the society income in favour of the poor people.

4- Comparison between regional disparities as analyzed by the two nutrition and living cost-consumption poverty lines for rural and urban people. Usually poverty may be common in areas with low average of income but the link as charted in figures(3.1,3.2) showed no significant differences between rural and urban people. That is in case of nutrition calorie poverty line we notice, in urban sector, that there are 17 from 20 income groups live below the upper and the lower bound of the poverty line. Also in rural sector there are 17 from 20 income groups live below the upper and the lower bound of the poverty line except one lies above the lower bound. Also in case of living cost poverty line as charted in figures(3.3,3.4) we found, in urban sector, there are 7 from 21 governates lie below the upper and 10 below the lower bound of the poverty line. While in rural sector there are 2 from 17 governates lie below the upper and 14 below the lower bound of the poverty line.

However, poverty as measured by low incomes and represented by the first points of the four curves, reflect the worst case in both rural and urban areas. Although the cost of living must substantially differ from town to countryside owing to malnutrition, lack of education and health services and substandard housing in rural, the data do not reflect this problem. Obviously the reason is that all the measures were estimated from the (IEC) data which are not surveyed specially for poverty research.

6.2 Drowbacks and recommendations:

First defect is the application of the work to the family expenditure –income data 2008/2009 so that the regional differences, did not give acceptable results to the current economic state. But the fitting curves and measures 's as studied and derived in the paper, would externally benefit in estimating the poverty rate and social and income inequality measures for recent years when the convient data are available.

Second problem is the choice of the two different bounds of the lower poverty line P_L and the upper poverty line P_U corresponding to the nutrition calorie and the living cost based poverty line..That is for the first line we choose $P_L=14000$ LE and $P_U=16000$ LE from the higher expenditure on food in rural and urban sectors(excluding the last 3 highest income classes).For the second line we chose $P_L=10500$ LE and $P_U=12500$ LE from the lowest cost of minimal living in the four urban governates(Cairo,Alex.,Port Said,Suez) and highest cost of minimal living in the rural governates(excluding Kafr-Elshiekh)(table(3.2)).

However this problem does not consider as a drawback since the choice of these bounds must vary according to the time and the level of echonomic development, the level of consumer prices,the kind of data available and the nature of the poverty line as in our case.That is the nutrition poverty line was constructed from the cross income data while the minimal living cost line was formulated from the regional governates scale data.

Finally we recommend to apply the concepts and methods presented here to get and update the estimators for the poverty and income indicators.Moreover since the derived measures formulae have the ability to be generalized on the national scale,it is possible then to enlarge the recent estimators to get an accurate evaluation of the regarded indicators and can hence introduce a valid pictorial state about the economic equality to the policy maker.

References:

- 1- Bourguignon,F.C.2000,The Pace of Economic Growth and Poverty Reduction,Working Paper,World Bank,washington DC.
- 2- Champernowne,D.G,1952,The graduation of income distributions, *Econometrica* , vol 29,No 2.
- 3- Easterly,W,Kremer,Mand Lawrence,H,1993,Good Policy or Good Luck? Country Growth Performance and Temporary Shocks,*J.of Monetary Economics*,vol 32,No.3.
- 4- Economic Research Forum,2011, Measuring Inequality of Outcome Along Economic Dimentions: Income,Wealth and Expenditures, Training Workshop on Measurments of Inequality of Opportunity and Inequality of Outcomes, Egypt, Cairo.
- 5- El-Laithy,H,1997, Structural Adjustment and Poverty in Alia Elmahdy,A Aspects of Structural Adjustment in Africa and Egypt ,Cairo university,Center for Developing Countries Studies.
- 6- Fela,A.M,1998,Poverty Phenomena and Developing Strategies in Egypt:Comparitive Study,MS.c thesis Commerial Faculty ,Ein-Shams University.
- 7- Feldstein,Martin, 1998, Income Inequality and Poverty, Working Paper,National Bureau of Economic Research Program.
- 8- Ferreira,f,2010,Distributions in Motion:Economic Growth Inequality and Poverty Dynamics ,*ECINEQ*,Vol18.
- 9- Fergani,N,1998,The Growth of Poverty in Egypt ,Cairo Almishkkat,Internet.

- 10- Fish, Peter, 1961, The Graduation of Income Distributions, *Econometrica*, vol. 29, No.2.
- 11- Hartley, Michael, J and Nagesh S. Revankar, 1974, On The Estimation of The Pareto Law From Under- Reported data, *Journal of Econometrics*, vol 2, No 6.
- 12- Fosu, A.K, 2010, Cross Inequality and Poverty Reduction in Developing Countries: Recent Global Evidence. OECD, USA.
- 13- Haughton Jonathan, 2009, Handbook on Poverty and Inequality, The World Bank, Washington, DC.
- 14- Hayami Yugtiro, 2009, Development Economics: From The Poverty To The Wealth of Nations 3rd Edition.
- 15- Kakwani, N.C, 1977, Application of Lorenz curves in economic analysis, *Econometrica*, vol 45, No3.
- 16- Kakwani, N.C, 1993, Poverty and Economic Growth With Application to Cote d'Ivoire, *Review of Income and Wealth*, vol 39, No.2.
- 17- Kondor, Yaakov, 1971, An old-new Measure of Economic Inequality, *Econometrica*, vol. 39, No.6.
- 18- Rosenbluth, G, 1951, Note on Mr Schotz's measurement of income inequality, *Communications, American Economic Review*, vol 41.
- 19- Roy, Depraj Roy : Development Economics, Inequality & Poverty and Under Nutrition, Princeton University press book, 1998.
- 20- Rashdan, A, 2012r, The Poverty, Inequality And Growth Nexus: A Theoretical Review, PH.D program, Economics Department, Faculty Of Economics and Political science.
- 21- Tachi, Minoru, 1964, Regional Income Disparity and Internal Migration of Population in Japan, *Economic and Development and Culture Change*, vol. X11, No.2.
- 22- The World Bank, 1995, Development Studies: an Introduction Through Selected Readings, ch.4, what do we know about the poor, Edited by Ron Ayres, Development Studies: An Introduction Through Selected Readings, ch.6.
- 23- The World Bank, 2004, Analyzing of Urban Poverty: A Summary Methods and Approaches
- 24- The World Bank, what Do We Know About The Poor, Edited by Ron Ayres, 1995, Development Studies: An Introduction Through Selected Readings, ch.6.
- 25- Thirlwall, Anthony P, 1995, The distribution of income, growth and distribution, Edited by Ron Ayres, 1995, Development Studies: An Introduction Through Selected Readings, ch.4.