# Cost analysis of two- unit cold standby repairable redundant system involving common-cause failures and preventive maintenance

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**Abstract:**

This paper deals with two-dissimilar unit standby redundant system having three states (good, failed and under preventive maintenance) under two types of failures: hardware failure and failure due to common-cause failure. We derive mean time to system failure (MTSF), system availability, steady state availability; busy period and profit function for the system by using Kolmogorov's forward equations method, and then make comparisons to study the effect of preventive maintenance on the performance of the system theoretically and graphically.

**Key words:**
Cost analysis, mean time to system failure (MTSF), steady-state availability, busy period, profit function, preventive maintenance, common-cause failures, Kolmogorov's forward equations method.

## 1- INTRODUCTION

Two-unit cold standby redundant systems have been widely studied by several authors, such as [1, 2, 3, 4] have studied the reliability of the systems involving Common Cause failures without evaluate the profit function for these systems, also,[5 ,7 ,9] have studied the cost analysis of two-unit cold standby systems ,but without involving Common Cause failures,and [6] has Cost analysis of a system involving common-cause failures and preventive maintenance.

The purpose of this paper is to study the cost analysis of a two-dissimilar unit cold standby repairable redundant system with two types of failures: hardware failure and common cause failures (with, without) preventive maintenance.

"Common cause failure" [8] which can occur at different times because of a design defect o

r a repeated external event.

The standby unit support increases the reliability of the system. Also, the improved maintenance of parts of the system originates better reliability, and performance of the system. Maintainability is defined as the probability that a failed state will restored to operating state in a given period of time.

We analyze the system by using Kolmogorov's forward equations method to obtain The following system characteristics :

i. Mean time to system failure (MTSF) with and without preventive maintenance
ii. Steady state availability with and without preventive maintenance
iii. Busy period, expected frequency of preventive maintenance.

iv. Cost analysis with and without preventive maintenance.

Finally the effect of preventive maintenance on the system performance is shown by tables and graphs.

## 2- Assumptions:

1. The system consists of two dissimilar units, one is main and the other is its standby
2. Initially one unit is operative and the other unit is kept as cold standby.
3. A perfect switch is used to switch-on standby unit and switch-over time is negligible.
4. The system has three states: good, failed, and under preventive maintenance
5. Both units suffer two types of failures hardware failures and common-cause failures
6. Unit failure, common-cause failure and repair rates are constants.
7. Failure rates and repair rates follow exponential distribution.
8. The system is down when both units are fail .i,e. $S_3, S_4, S_5, S_6$
9. Common-cause failure brings the system directly from good states to failed state$S_8$.

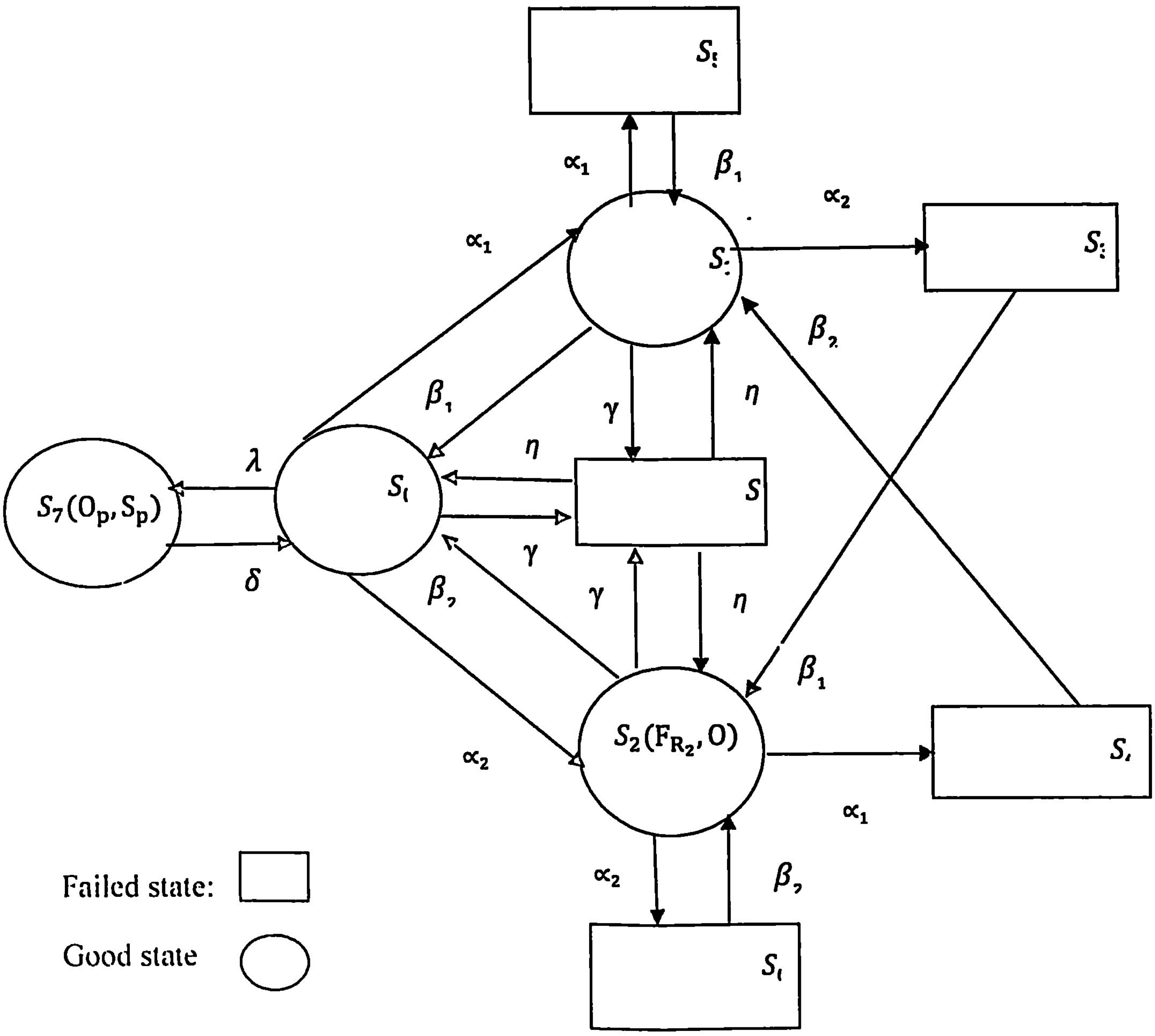


Figure (1): State of the system

**Notations :**

$\alpha_1$ : constant hardware failure rate of type I.
$\alpha_2$ : constant hardware failure rate of type II.
$\beta_1$ : constant repair rate of type I.
$\beta_2$ : constant repair rate of type II.
$\lambda$ : constant rate for taking a unit into preventive maintenance.
$\delta$ : constant rate end of preventive maintenance
$\gamma$ :constant common-cause failure rate
$\eta$ : constant common-cause repair rate
O : the unit is operative.
S : the unit is standby.
$F_{R1}$ : the failed unit is under repair of type I.
$F_{R2}$ : the failed unit is under repair of type II.
$F_{W1}$ : the failed unit is waited for repair of type I.
$F_{W2}$ : the failed unit is waited for repair of type II.
$O_p$ : the operative unit is under preventive maintenance.
$S_p$ : the standby unit is under preventive maintenance.
$P_i(t)$ : Probability that the system is in state $S_i$ at time t, $(t \geq 0)$ , i = [0- 8].

### 3- Reliability Assessment

In this section, by using assumptions and method of linear first order differential equations For Figure. 1, we can transform the system to the following differential equations:

$$
\begin{aligned}
P'_0(t) &= -(\alpha_1+\alpha_2+\gamma+\lambda)P_0(t)+\beta_1P_1(t)+\beta_2P_2(t)+\delta P_7(t)+\eta P_8(t)\\
P'_1(t) &= -(\alpha_1+\alpha_2+\beta_1+\gamma)P_1(t)+\alpha_1P_0(t)+\beta_2P_3(t)+\beta_1P_5(t)+\eta P_8(t)\\
P'_2(t) &= -(\alpha_1+\alpha_2+\beta_2+\gamma)P_2(t)+\alpha_2P_0(t)+\beta_1P_4(t)+\beta_2P_6(t)+\eta P_8(t)\\
P'_3(t) &= -\beta_1P_3(t)+\alpha_2P_1(t)\\
P'_4(t) &= -\beta_2P_4(t)+\alpha_1P_2(t) \qquad (3-1)\\
P'_5(t) &= -\beta_1P_5(t)+\alpha_1P_1(t)\\
P'_6(t) &= -\beta_2P_6(t)+\alpha_2P_2(t)\\
P'_7(t) &= -\delta P_7(t)+\lambda P_0(t)\\
P'_8(t) &= -3\eta P_8(t)+\gamma P_0(t)+\gamma P_1(t)+\gamma P_2(t)
\end{aligned}
$$

With Initial conditions

$$
\begin{aligned}
P(0) &= [P_0(0)\ P_1(0)\ P_2(0)\ P_3(0)\ P_4(0)\ P_5(0)\ P_6(0)\ P_7(0)\ P_8(0)]\\
&= [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]
\end{aligned}
$$

We can put the above system of differential equations in the matrix form as:

$$P^{*} = Q \times P$$

Where,

$$Q=\begin{pmatrix} -(\alpha_1+\alpha_2+\gamma+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & \delta & \eta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1+\gamma) & 0 & 0 & \beta_2 & \beta_1 & 0 & 0 & \eta \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2+\gamma) & \beta_1 & 0 & 0 & \beta_2 & 0 & \eta \\ 0 & \alpha_2 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -\beta_2 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & -\beta_2 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\delta & 0 \\ \gamma & \gamma & \gamma & 0 & 0 & 0 & 0 & 0 & -3\eta \end{pmatrix}$$

- **Mean Time to System Failure (MTSF)**

To calculate the MTSF we take the transpose matrix of Q and delete the rows and columns for the absorbing state (down state), the new matrix is called A. the expected time to reach an absorbing state is calculated from

$$MTSF = P(0)(-A^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \qquad (3-2)$$

Where,

$$A=\begin{bmatrix} -(\alpha_1+\alpha_2+\gamma+\lambda) & \alpha_1 & \alpha_2 & \lambda \\ \beta_1 & -(\alpha_1+\alpha_2+\beta_1) & 0 & 0 \\ \beta_2 & 0 & -(\alpha_1+\alpha_2+\beta_2) & 0 \\ \delta & 0 & 0 & -\delta \end{bmatrix}$$

Then

**The steady state mean Time to System Failure (MTSF) is given by :**

$$\text{MTSF} = \frac{(\gamma+\alpha_1+\alpha_2)(2(\gamma+\alpha_1+\alpha_2)+3\beta_1)+(3(\gamma+\alpha_1+\alpha_2)+4\beta_1)\beta_2}{(\gamma+\alpha_1+\alpha_2)^3+(\gamma+\alpha_2)(\gamma+\alpha_1+\alpha_2)\beta_1+((\gamma+\alpha_1)(\gamma+\alpha_1+\alpha_2)+\gamma\beta_1)\beta_2} \qquad (3-3)$$

## 4-Availability analysis

We solve the differential equations in (3-1) with initial condition by using Kolmogorov's forward equations method:

$$\begin{pmatrix} P_0^{'} \\ P_1^{'} \\ P_2^{'} \\ P_3^{'} \\ P_4^{'} \\ P_5^{'} \\ P_6^{'} \\ P_7^{'} \\ P_8^{'} \end{pmatrix} = \begin{pmatrix} -(\alpha_1+\alpha_2+\gamma+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & \delta & \eta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1+\gamma) & 0 & 0 & \beta_2 & \beta_1 & 0 & 0 & \eta \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2+\gamma) & \beta_1 & 0 & 0 & \beta_2 & 0 & \eta \\ 0 & \alpha_2 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -\beta_2 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & -\beta_2 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\delta & 0 \\ \gamma & \gamma & \gamma & 0 & 0 & 0 & 0 & 0 & -3\eta \end{pmatrix} \begin{pmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \end{pmatrix}$$

In the steady state, the derivatives of the state probabilities become zero, i. e.

$$QP(\infty) = 0 \qquad (4-1)$$

Then the steady state probabilities can be calculated as follows:

$$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_7(\infty) \qquad (4-2)$$

Then the matrix form became:

$$\begin{pmatrix} -(\alpha_1+\alpha_2+\gamma+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & \delta & \eta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1+\gamma) & 0 & 0 & \beta_2 & \beta_1 & 0 & 0 & \eta \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2+\gamma) & \beta_1 & 0 & 0 & \beta_2 & 0 & \eta \\ 0 & \alpha_2 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -\beta_2 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & -\beta_2 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\delta & 0 \\ \gamma & \gamma & \gamma & 0 & 0 & 0 & 0 & 0 & -3\eta \end{pmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

To obtain $P_0(\infty) + P_1(\infty) + P_2(\infty) + P_7(\infty)$, we solve the equation (4 - 1) by using the following normalizing condition:

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_1(\infty) + P_2(\infty) + P_1(\infty) + P_2(\infty) + P_7(\infty) + P_8(\infty) = 1 \qquad (4-3)$$

We substitute the equation (4 - 3) in any one of the redundant rows in equation to (4 - 1) yield.

$$\begin{pmatrix} -(\alpha_1+\alpha_2+\gamma+\lambda) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & \delta & \eta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1+\gamma) & 0 & 0 & \beta_2 & \beta_1 & 0 & 0 & \eta \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2+\gamma) & \beta_1 & 0 & 0 & \beta_2 & 0 & \eta \\ 0 & \alpha_2 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -\beta_2 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & -\beta_2 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\delta & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Then we get the following result:

$$p_0 = \frac{\delta\eta\beta_1\beta_2(\gamma(\gamma + \alpha_2 + 2\beta_1) + \alpha_1(\gamma + 3\beta_1) + (2\gamma + 3\alpha_2 + 3\beta_1)\beta_2)}{M}$$

$$p_1 = \frac{\eta(\gamma + 3\alpha_1\delta\eta(\gamma + 3\alpha_1)\beta_1\beta_2(\gamma + \alpha_1 + \alpha_2 + \beta_2))}{M}$$

$$p_2 = \frac{\delta\eta(\gamma + 3\alpha_2)\beta_1(\gamma + \alpha_1 + \alpha_2 + \beta_1)\beta_2}{M}$$
$$p_3 = \frac{\delta\eta(\gamma + 3\alpha_1)\alpha_2\beta_2(\gamma + \alpha_1 + \alpha_2 + \beta_2)}{M}$$
$$p_4 = \frac{\delta\eta\alpha_1(\gamma + 3\alpha_2)\beta_1(\gamma + \alpha_1 + \alpha_2 + \beta_1)}{M} \qquad (4-4)$$
$$p_5 = \frac{\delta\eta\alpha_1(\gamma + 3\alpha_1)\beta_2(\gamma + \alpha_1 + \alpha_2 + \beta_2)}{M}$$
$$p_6 = \frac{\delta\eta\alpha_2(\gamma + 3\alpha_2)\beta_1(\gamma + \alpha_1 + \alpha_2 + \beta_1)}{M}$$
$$p_7 = \frac{\eta\lambda\beta_1\beta_2(\gamma(\gamma + \alpha_2 + 2\beta_1) + \alpha_1(\gamma + 3\beta_1) + (2\gamma + 3\alpha_2 + 3\beta_1)\beta_2)}{M}$$

where

$$M = \delta\eta(\alpha_1 + \alpha_2)(\gamma + 3\alpha_2)\beta_1(\gamma + \alpha_1 + \alpha_2 + \beta_1) + (\delta\eta(\gamma + 3\alpha_1)(\alpha_1 + \alpha_2)(\gamma + \alpha_1 + \alpha_2) + (\gamma + \alpha_1 + \alpha_2)(\gamma(\gamma\delta + \eta(3\delta + \lambda)) + \delta(\gamma + 3\eta)(\alpha_1 + \alpha_2))\beta_1 + (\gamma(\gamma\delta + 3\delta\eta + 2\eta\lambda) + (\gamma\delta + 3\eta(\delta + \lambda))\alpha_1 + \delta(\gamma + 3\eta)\alpha_2)\beta_1^2)\beta_2 + (\delta\eta(\gamma + 3\alpha_1)(\alpha_1 + \alpha_2) + (\gamma(\gamma\delta + 3\delta\eta + 2\eta\lambda) + \delta(\gamma + 3\eta)\alpha_1 + (\gamma\delta + 3\eta(\delta + \lambda))\alpha_2)\beta_1 + (\gamma\delta + 3\eta(\delta + \lambda))\beta_1^2)\beta_2^2$$

**The steady state availability A(∞) is given by:**

$$A(\infty) = \eta\beta_1\beta_2(3\delta\alpha_1^2 + 3\delta\alpha_2^2 + \gamma(\gamma(3\delta + \lambda) + (3\delta + 2\lambda)\beta_1) + (\gamma(3\delta + 2\lambda) + 3(\delta + \lambda)\beta_1)\beta_2 + \alpha_1(\gamma(6\delta + \lambda) + 6\delta\alpha_2 + 3(\delta + \lambda)\beta_1 + 3\delta\beta_2) + \alpha_2(6\gamma\delta + \gamma\lambda + 3\delta\beta_1 + 3(\delta + \lambda)\beta_2))/M \qquad (4-5)$$

**Busy period analysis:**

By using the initial condition, then the steady state busy period B(∞) is given by:

$$B(\infty) = 1 - \left(P_0(\infty) + P_7(\infty)\right)$$
$$= 1 - \left(\frac{(\eta(\delta+\lambda)\beta_1\beta_2(\gamma(\gamma+\alpha_2+2\beta_1)+\alpha_1(\gamma+3\beta_1)+(2\gamma+3\alpha_2+3\beta_1)\beta_2)}{M}\right) \qquad (4-6)$$

**The expected frequency of preventive maintenance:**

By using the initial condition, then the expected frequency of preventive maintenance per unit time K(∞) is given by

$$K(\infty) = P_7(\infty) = \frac{\eta\lambda\beta_1\beta_2(\gamma(\gamma+\alpha_2+2\beta_1)+\alpha_1(\gamma+3\beta_1)+(2\gamma+3\alpha_2+3\beta_1)\beta_2)}{M} \qquad (4-7)$$

**Cost analysis:**

The expected total profit per unit time incurred to the system in the steady-state is given by:

**Profit = total revenue - total cost**

$PF = RA(\infty) - C_1B(\infty) - C_2K(\infty)$

Where:

PF: is the profit incurred to the system,

R: is the revenue per unit up-time of the system,

$C_1$ : is the cost per unit time which the system is under repair

$C_2$: is the cost per preventive maintenance.

From equations (4 - 5), (4 – 6), (4-7), the expected total profit per unit time incurred to the system in the steady-state is given by:

$$PF = R\left(\frac{\eta\beta_1\beta_2\left(3\delta\alpha_1^2+3\delta\alpha_2^2+\gamma(\gamma(3\delta+\lambda)+(3\delta+2\lambda)\beta_1)+(\gamma(3\delta+2\lambda)+3(\delta+\lambda)\beta_1)\beta_2+\alpha_1(\gamma(6\delta+\lambda)+6\delta\alpha_2+3(\delta+\lambda)\beta_1+3\delta\beta_2)+\alpha_2(6\gamma\delta+\gamma\lambda+3\delta\beta_1+3(\delta+\lambda)\beta_2)\right)}{M}\right) - C_1\left(1-\frac{(\eta(\delta+\lambda)\beta_1\beta_2(\gamma(\gamma+\alpha_2+2\beta_1)+\alpha_1(\gamma+3\beta_1)+(2\gamma+3\alpha_2+3\beta_1)\beta_2)}{M}\right) - C_2\left(\frac{\eta\lambda\beta_1\beta_2(\gamma(\gamma+\alpha_2+2\beta_1)+\alpha_1(\gamma+3\beta_1)+(2\gamma+3\alpha_2+3\beta_1)\beta_2)}{M}\right)$$

(4 – 8)

## 5-Special case

When the preventive maintenance is not available i.e. $\delta=\lambda = 0$, then

$$Q=\begin{pmatrix} -(\alpha_1+\alpha_2+\gamma) & \beta_1 & \beta_2 & 0 & 0 & 0 & 0 & \eta \\ \alpha_1 & -(\alpha_1+\alpha_2+\beta_1+\gamma) & 0 & 0 & \beta_2 & \beta_1 & 0 & \eta \\ \alpha_2 & 0 & -(\alpha_1+\alpha_2+\beta_2+\gamma) & \beta_1 & 0 & 0 & \beta_2 & \eta \\ 0 & \alpha_2 & 0 & -\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & -\beta_1 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & -\beta_2 & 0 \\ \gamma & \gamma & \gamma & 0 & 0 & 0 & 0 & -3\eta \end{pmatrix}$$

By the same way, we can obtain the following:

$$P_0(\infty) = \frac{\eta\beta_1\beta_2(\gamma(\gamma+\alpha_2+2\beta_1)+\alpha_1(\gamma+3\beta_1)+(2\gamma+3\alpha_2+3\beta_1)\beta_2)}{N}$$

Where

$$N = \eta(\alpha_1+\alpha_2)(\gamma+3\alpha_2)\beta_1(\gamma+\alpha_1+\alpha_2+\beta_1) + (\gamma+\alpha_1+\alpha_2)(\eta(\gamma+3\alpha_1)(\alpha_1+\alpha_2) + (\gamma+3\eta)(\gamma + \alpha_1+\alpha_2)\beta_1 + (\gamma+3\eta)\beta_1^2)\beta_2 + (\eta(\gamma+3\alpha_1)(\alpha_1+\alpha_2) + (\gamma+3\eta)(\gamma+\alpha_1+\alpha_2)\beta_1 + (\gamma+3\eta)\beta_1^2)\beta_2^2$$

**The Mean Time to System Failure is given by:**

$$\text{MTSF} = \frac{(\gamma+\alpha_1+\alpha_2)(\gamma+\alpha_1+\alpha_2+2\beta_1)+(2(\gamma+\alpha_1+\alpha_2)+3\beta_1)\beta_2}{(\gamma+\alpha_1+\alpha_2)^3+(\gamma+\alpha_2)(\gamma+\alpha_1+\alpha_2)\beta_1+((\gamma+\alpha_1)(\gamma+\alpha_1+\alpha_2)+\gamma\beta_1)\beta_2} \qquad (5-1)$$

**The steady state availability of the system is given by:**

$$A(\infty) = 3\delta\eta\beta_1(\gamma+\alpha_1+\alpha_2+\beta_1)\beta_2(\gamma+\alpha_1+\alpha_2+\beta_2)/N \qquad (5-2)$$

**The steady state busy period of the system is given by:**

$$B(\infty) = 1 - (P_0(\infty))$$
$$= 1 - (\eta\beta_1\beta_2(\gamma(\gamma+\alpha_2+2\beta_1)+\alpha_1(\gamma+3\beta_1)+(2\gamma+3\alpha_2+3\beta_1)\beta_2)/N) \qquad (5-3)$$

**The expected total profit incurred to the system in the steady-state is given by:**

By substitute from equations (5-2), (5-3), the expected total profit per unit time incurred to the system in the steady-state is given by:

$$PF = RA(\infty) - C_1B(\infty) =$$

$$R\left(\frac{3\eta\beta_1(\gamma+\alpha_1+\alpha_2+\beta_1)\beta_2(\gamma+\alpha_1+\alpha_2+\beta_2)}{N}\right) - C_1\left(1 - \frac{(\eta\beta_1\beta_2(\gamma(\gamma+\alpha_2+2\beta_1)+\alpha_1(\gamma+3\beta_1)+(2\gamma+3\alpha_2+3\beta_1)\beta_2)}{N}\right) \quad (5-4)$$

## 4- Numerical computation

If we put :$\alpha_1$=0.02,$\alpha_2$=0.04,$\beta_1$=0.05,$\beta_2$=0.06,$\lambda$=0.02,$\delta$=0.02,$\gamma$=0.001,$\eta$=0.03 in equations (3 – 3), (4-5),(4 – 6), (4 – 8), (5-1), (5-2) ,(5-3) , (5-4) we get the following:

1. Table (1): Show Relation between failure rate of type I and the MTSF of the system (with and without PM).
2. Table (2): Show Relation between failure rate of type I and availability of the system (with and without PM).
3. Table (3): Show Relation between failure rate of type I and the profit of the system (with and without PM).
4. Fig. 2: Show Relation between the failure rate of type I and the MTSF.
5. Fig. 3: Show Relation between the failure rate of type I and the availability.
6. Fig. 4: Show Relation between the failure rate of type I and the expected total profit.

| $\alpha_1$ | MTSF of the system without human failure | MTSF of the system with human failure |
|---|---|---|
| 0.01 | 124.267 | 83.330 |
| 0.02 | 91.624 | 60.526 |
| 0.03 | 71.277 | 46.452 |
| 0.04 | 57.629 | 37.100 |
| 0.05 | 47.964 | 30.535 |
| 0.06 | 40.827 | 25.727 |
| 0.07 | 35.379 | 22.085 |
| 0.08 | 31.107 | 19.252 |
| 0.09 | 27.683 | 16.996 |
| 0.1 | 24.885 | 15.165 |

Table (1)
Relation between failure rate of type I and the MTSF (with and without PM)

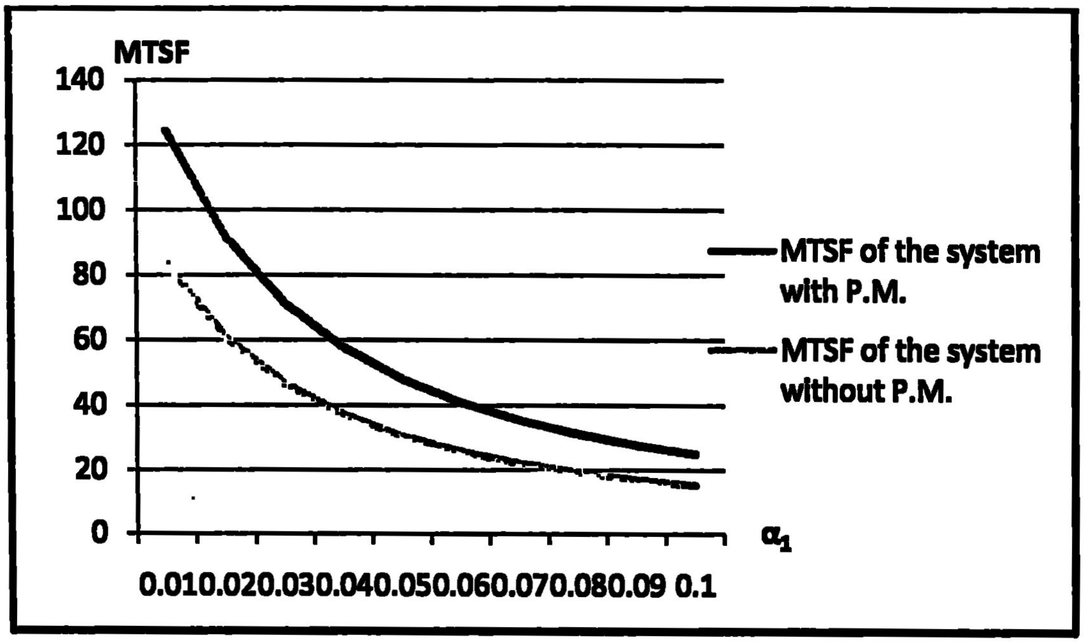


Figure. (2)

Relation between the failure rate of type I and the MTSF

| $\alpha_1$ | The availability of the system without Preventive Maintenance | The Availability of the system with Preventive Maintenance |
|---|---|---|
| 0.01 | 0.78661 | 0.51267 |
| 0.02 | 0.72444 | 0.48834 |
| 0.03 | 0.66649 | 0.46238 |
| 0.04 | 0.61367 | 0.43645 |
| 0.05 | 0.56616 | 0.41148 |
| 0.06 | 0.52369 | 0.38794 |
| 0.07 | 0.48584 | 0.36604 |
| 0.08 | 0.45212 | 0.90518 |
| 0.09 | 0.42203 | 0.32722 |
| 0.1 | 0.39514 | 0.31015 |

Table (2)

Relation between failure rate of type I and availability (with and without PM)

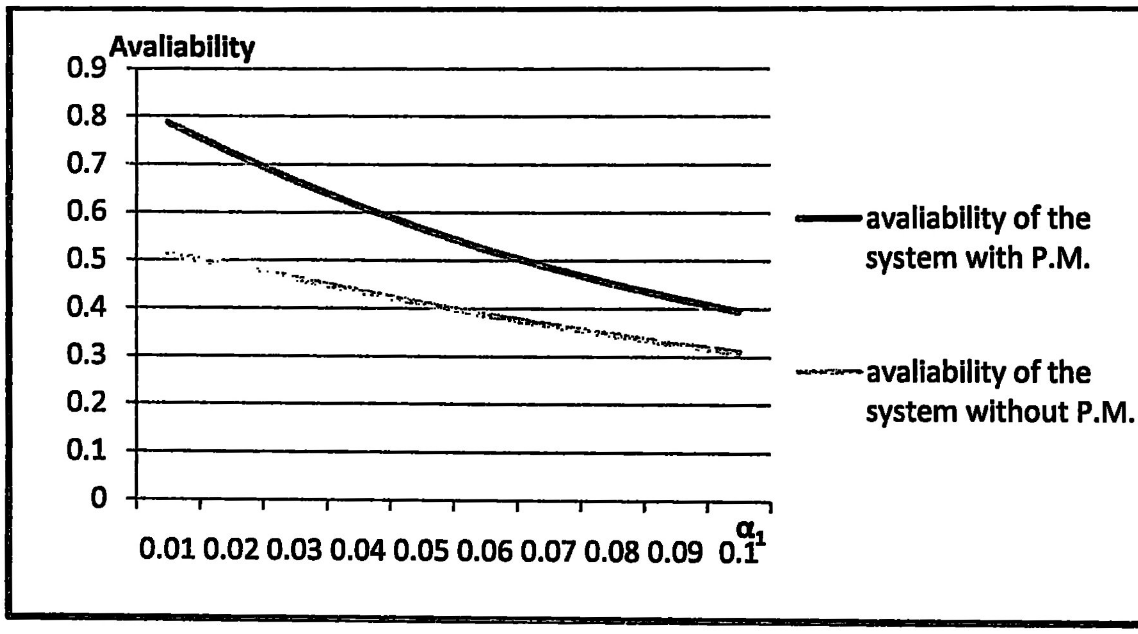


Figure. (3)

Relation between the failure rate of type I and the Availability

| $\alpha_1$ | The profit of the system With Preventive Maintenance | The profit of the system without Preventive Maintenance |
|---|---|---|
| 0.01 | 727.70 | 440.06 |
| 0.02 | 659.86 | 411.95 |
| 0.03 | 597.10 | 382.79 |
| 0.04 | 540.26 | 354.17 |
| 0.05 | 489.36 | 326.94 |
| 0.06 | 444.05 | 301.52 |
| 0.07 | 403.81 | 278.02 |
| 0.08 | 368.06 | 256.45 |
| 0.09 | 336.26 | 236.70 |
| 0.1 | 307.89 | 218.65 |

Table (3)
Show Relation between the failure rate of type I and the profit

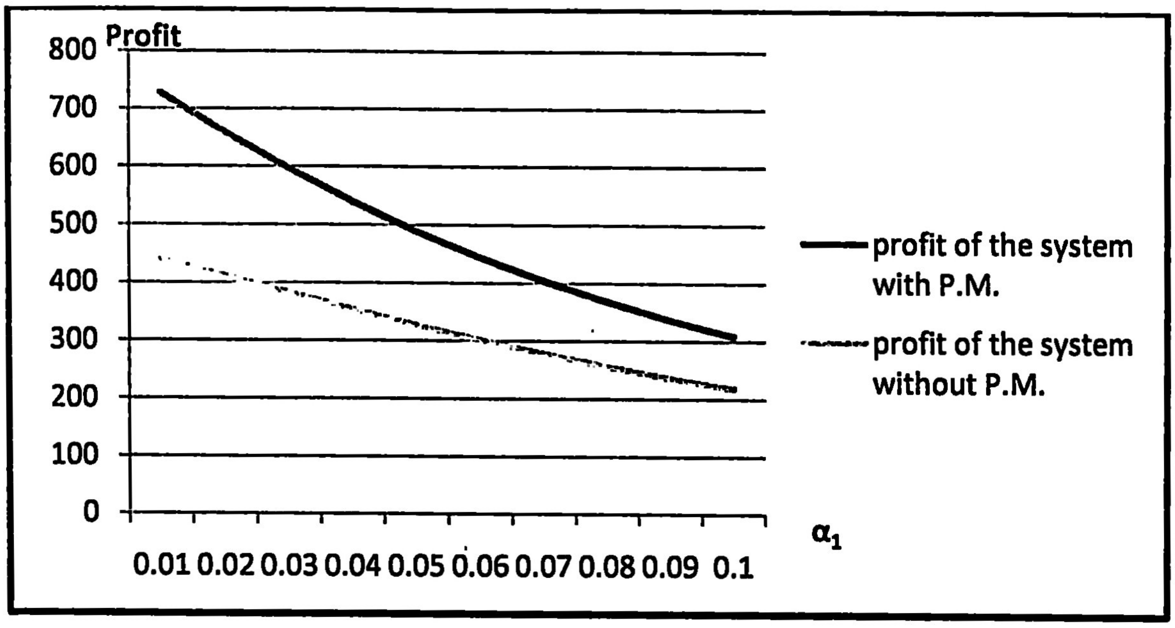


Figure. (4)

Show Relation between the failure rate of type I and the expected total profit.

## CONCLUSION

By comparing the characteristic, MTSF, Availability and the profit function with respect to $\alpha_1$ for both systems with and without preventive maintenance graphically, we observing that the increase of failure rate $\alpha_1$ (at constant $\alpha_2$=0. 04, $\beta_1$=0.05, $\beta_2$=0.06, $\lambda$=0.02, $\delta$=0.02, $\gamma$=0.001, $\eta$=0.04, R =1000, C1= 100, C2 = 50) the MTSF, Availability and the profit function of the system decrease for both systems with and without preventive maintenance, also The system with preventive maintenance is better in the effectiveness than system without preventive maintenance with respect to the MTSF, availability and the profit function, and We conclude that system with preventive maintenance is more effective than system without preventive maintenance.

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