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Reliability of a Series Chain for Time Dependent Stress-Strength Models

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Abstract.

In this article we consider the problem of determining the reliability of a series chain consisting of k identical links. The stress acting on the chain is known a prior, i.e., deterministic. We consider the case of repeated applications of stresses, i.e., cycles of stresses. We also consider the change of the distribution of strengths of the links with time, i.e., (the change of the distribution) during different cycles of stresses.. We find an expression of the reliability function after m cycles of stresses. The strengths of the links of the chain could be random independent, random fixed or deterministic. A two-sided confidence interval for the reliability is obtained. As an application, the cases of exponential and Rayleigh distributions are studied. In order to highlight the results obtained a numerical illustration is performed.

1- Introduction.

In stress-strength models a component fails if at any time the applied stress X, is greater than its strength Y, and there is no failure if $Y > X$. Thus $Pr(Y > X)$ is a measure of the reliability of the component.

The problem of estimating $R = Pr(Y > X)$, has been studied in the literature in both distribution free and parametric frameworks. However in this paper we are concerned with the parametric case.

Church and Harris [2], derived the maximum likelihood estimator (MLE) of R assuming X and Y are independent normal and that the distribution of X is completely known. Downton [4], obtained the MVUE of R in the case of independent normal with parameters of X also unknown. Reiser and Guttman [8], Presented two approximate methods for obtaining confidence intervals and an approximate Bayesian probability interval. Owen et. al., [7], discussed the normal case for equal standard deviation and presented non parametric confidence limits for this problem, in addition to the normal case, the problem has been extensively studied for many other models including exponential, Gamma and Burr distributions, for example see Sathe and Shah [9] for exponential case, Constantine and Karson [3] for the Gamma case, and Awad and Gharraf [1] for the Burr case. Nassar et. al., [6], obtained confidence intervals for $R = Pr(Y > X)$, where Y and X follow Rayleigh and normal distribution respectively.

If the stress and strength change with time, we call it time dependent stress-strength model. Kapur and Lamberson [5], stated that time dependent stress-strength models (SST) are models that consider the repeated application of stresses and also; consider the change of the distribution of strength with time, which may be caused by aging and/or cumulative damage. Such models are frequently observed in practice. Xue and Yang [10], obtained a simple formula for estimating upper and lower bounds for stress-strength interference reliability when X and Y are s-independent normally distributed. However, not too much work is done on time dependent models.

The present paper gives an explicit expression for the reliability function of a series chain consisting of k links after m cycles of stress. The repeated stress is deterministic i.e., its value is known in prior. The reliability is derived under three strength forms of the links of the chain: random-independent, random-fixed and deterministic. As an application, the Rayleigh and exponential distributions are considered. A two-sided confidence intervals for the reliability are then obtained for the case of random-independent and random-fixed strength. Finally a numerical illustration of the results obtained is performed.

2-Assumption and Notation.

1- The system is a series chain consisting of k links.

2- The links are identical and independent.

3- The chain is subjected to cycles of common repeated stresses. These stresses are the main cause to break the chain and are independent of the strength of the links of the chain.

4- The chain will break (fail) if the stress on the chain exceeds the strength of the chain for the first time.

5- The repeated stress acting on the chain is deterministic, i.e., the stress during cycle j is given by x_0, for all j, $j = 1, 2, \ldots, m$, where x_0 is known value.

6- Y_{ij} is the strength of link i during cycle j, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, m$.

7- $E_{k,j}$ event that no failure occurs on j^{th} cycle.

8- $R_{k,m}$ is the reliability of the chain of k links after m cycles.

3- The System Reliability.

We discuss the reliability of the system assuming three different models.

Model I: Random-Independent Strength.

In this model, the strengths Y_{ij}'s, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, m$, are non-identical independent distributed random variables, having c.d.f. $G_j(y)$ and p.d.f. $g_j(y)$.

The strength of the chain on the j^{th} cycle will be;

$$Y_j^* = \min(Y_{1j}, Y_{2j}, \ldots, Y_{kj})$$

with c.d.f.

$$G_{Y_j^*}(y) = 1 - [1 - G_j(y)]^k.$$

Clearly, the system reliability is

$$R_{k,m} = Pr(E_{k,1}, E_{k,2}, \ldots, E_{k,m}),$$

where,

$$E_{k,j} \sim (Y_j^* > x_0).$$

Since the successive values of Y_j^*'s are independent for $j = 1, 2, \ldots, m$,

$$R_{k,m} = \prod_{j=1}^{m} Pr(E_{k,j}), \tag{1}$$

where,

$$\begin{aligned} Pr(E_{k,j}) &= Pr(Y_j^* > x_0) \\ &= [1 - G_j(x_0)]^k. \end{aligned}$$

Hence,

$$R_{k,m} = \prod_{j=1}^{m} [1 - G_j(x_0)]^k. \tag{2}$$

Model II: Random-Fixed Strength.

In this model, the random variable of strength varies in time (during cycle j) in a known manner, i.e., the strength of i^{th} link during the j^{th} cycle Y_{ij} is given by

$$Y_{ij} = Y_{i0} - a_j,$$

where Y_{i0} is the initial random strength of the i^{th} link, and a_j is a known non-decreasing function in j.

Assume that Y_{i0} are i.i.d., $i = 1, 2, \ldots, k$, having c.d.f. $G_0(y)$ and p.d.f. $g_0(y)$. We could easily see that the strength of the chain during the j^{th} cycle is

$$Y_j^* = Y_0^* - a_j, \tag{3}$$

where,

$$Y_0^* = \min(Y_{10}, Y_{20}, \ldots, Y_{k0}),$$

having c.d.f.

$$G_{Y_0^*}(y) = 1 - [1 - G_0(y)]^k.$$

The system reliability is,

$$\begin{aligned} R_{k,m} &= Pr(E_{k,1}, E_{k,2}, \ldots, E_{k,m}) \\ &= Pr(E_{k,1}|E_{k,2}, \ldots, E_{k,m}).Pr(E_{k,2}|E_{k,3}, \ldots, E_{k,m}) \times \ldots \\ &\quad \times Pr(E_{k,m-1}|E_{k,m}).Pr(E_{k,m}). \end{aligned} \tag{4}$$

All but the best term $[Pr(E_{k,m})]$ in the R.H.S. of the Equation (4) are 1's this is due to the restriction on the a_j's that they are non-decreasing which cause the strength Y_j^* to decrease in time (in j). Then,

$$\begin{aligned} R_{k,m} &= Pr(E_{k,m}) \\ &= Pr(Y_m^* > x_0) \\ &= [1 - G_0(x_0 + a_m)]^k. \end{aligned} \tag{5}$$

Model III: Deterministic Strength.

In this model, the strength of the i^{th} link on the j^{th} cycle is deterministic given by y_{ij}, $1 \le i \le k$; $1 \le j \le m$. Since the chain consists of k links connected in series, the strength of the chain on the j^{th} cycle will be

$$y_j^* = \min(y_{1j}, y_{2j}, \ldots, y_{kj})$$

Since,

$$R_{k,m} = Pr(E_{k,1}, E_{k,2}, \ldots, E_{k,m}),$$

where, $E_{k,j}$ is the event that $(y_j^* > x_0)$, we get

$$R_{k,m} = \begin{cases} 0 & \text{if} \quad y_j^* < x_0 \quad \text{for some} \quad j \quad 1 \le j \le m \\ 1 & \text{if} \quad y_j^* > x_0 \quad \text{for all} \quad j \quad 1 \le j \le m \end{cases}$$

Remarks.

1- Taking $k = 1$, we obtain the reliability of an item after m cycles of stress.

2- Taking $a_m = 0$, we obtain the reliability of the system in the static case.

4- Confidence Intervals for System Reliability.

We obtain confidence intervals for system reliability under models I and II, considering Rayleigh and exponential distributions.

4.1- The Rayleigh distribution.

Assume that $G_j(y) = 1 - \exp\{-y^2/\beta_j\}$ for Model I,
and $G_0(y) = 1 - \exp\{-y^2/\beta_0\}$ for Model II.
Using Equations (2) and (5), we obtain,

$$R_{k,m} = \begin{cases} \exp\{-kx_0^2\psi\} & \text{for Model I} \\ \exp\{-k(x_0 + a_m)^2\psi_0\} & \text{for Model II,} \end{cases} \tag{7}$$

where,

$$\psi = \sum_{j=1}^{m}(\frac{1}{\beta_j})$$

and

$$\psi_0 = \frac{1}{\beta_0}.$$

If the parameters β_0 and β_j, $j = 1, 2, \ldots, m$ are known, then by Equation (7), we obtain the exact reliability.

If the parameters β_0 and β_j, $j = 1, 2, \ldots, m$ are unknown, then we can replace these parameters by their MLE's, to get MLE, $\hat{R}_{k,m}$ of $R_{k,m}$ for the two models as follows:

$$\hat{R}_{k,m} = \begin{cases} \exp\{-kx_0^2\hat{\psi}\} & \text{for Model I} \\ \exp\{-k(x_0 + a_m)^2\hat{\psi}_0\} & \text{for Model II,} \end{cases} \tag{8}$$

where,

$$\hat{\psi} = \sum_{j=1}^{m}(\frac{1}{\hat{\beta}_j}) \quad , \quad \hat{\beta}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} y_{ji}^2,$$

and

$$\hat{\psi}_0 = \frac{1}{\hat{\beta}_0} \quad , \quad \hat{\beta}_0 = \frac{1}{n_0}\sum_{i=1}^{n_0} y_{0i}^2.$$

$y_{j1}, y_{j2}, \ldots, y_{jn_j}$, and $y_{01}, y_{02}, \ldots, y_{0n_0}$, are random samples of sizes n_j and n_0 drawn from $G_j(y)$, $j = 1, 2, \ldots, m$, and $G_0(y)$, respectively.

It can be easily shown that $\hat{\beta}_j$, $j = 1, 2, \ldots, m$ and $\hat{\beta}_0$ have Gamma distributions with parameters $(n_j, \beta_j/n_j)$, $j = 1, 2, \ldots, m$ and $(n_0, \beta_0/n_0)$, respectively, or $(2n_j\hat{\beta}_j/\beta_j)$, $j = 1, 2, \ldots, m$, and $2n_0\hat{\beta}_0/\beta_0$, have a Chi-square distribution with n_j and n_0 degrees of freedom respectively.
Clearly,

$$E(\hat{\beta}_j) = \beta_j \quad , \quad var(\hat{\beta}_j) = (\beta_j^2/n_j),$$
$$E(\hat{\beta}_0) = \beta_0 \quad ; \quad var(\hat{\beta}_0) = (\beta_0^2/n_0).$$

Define $W_j = \hat{\beta}_j - \beta_j$, $j = 1, 2, \ldots, m$, and $U = \hat{\beta}_0 - \beta_0$. Clearly W_j, $j = 1, 2, \ldots, m$, and U are asymptotically normally distributed with zero means and variances β_j^2/n_j, $j = 1, 2, \ldots, m$, and β_0^2/n_0, respectively.

Using Taylor's expansion and Equation (8), we obtain

$$\hat{R}_{k,m} = \begin{cases} R_{k,m} + (kx_0^2 R_{k,m}) \sum_{j=1}^{m} \frac{W_j}{\beta_j^2} + \mathbb{R}_1 & \text{for Model I} \\ R_{k,m} + k(\frac{x_0 + a_m}{\beta_0})^2 R_{k,m} U + \mathbb{R}_2 & \text{for Model II,} \end{cases} \tag{9}$$

where, $\mathbb{R}_1$ and $\mathbb{R}_2$ are remainder terms.

We see that for the two models, $\hat{R}_{k,m}$ are asymptotically normal with means $R_{k,m}$ as given by Equation (7), and variances

$$\sigma^2_{\hat{R}_{k,m}} = \begin{cases} (kx_0^2 R_{k,m})^2 \sum_{j=1}^{m} (\frac{1}{n_j \beta_j^2}) & \text{for Model I} \\ \frac{1}{n_0}((\frac{k}{\beta_0})(x_0 + a_m)^2)^2 (R_{k,m})^2 & \text{for Model II.} \end{cases} \tag{10}$$

Consequently $\hat{R}_{k,m}$ in (8), is a consistent estimator of $R_{k,m}$.

Two-sided $(1-\alpha)100\%$ confidence intervals of $R_{k,m}$ for Model I, and Model II, are given by:

for Model I.

$$Pr\{\chi^2(2n_j, 1-\alpha) < \frac{2n_j \hat{\beta}_j}{\beta_j} < \chi^2(2n_j, \alpha)\} = 1 - \alpha$$

$$Pr\{-kx_0^2 \sum_{j=1}^{m} \frac{\chi^2(2n_j, \alpha)}{2n_j \hat{\beta}_j} < -kx_0^2 \sum_{j=1}^{m} \frac{1}{\beta_j} < -kx_0^2 \sum_{j=1}^{m} \frac{\chi^2(2n_j, 1-\alpha)}{2n_j \hat{\beta}_j}\} = 1 - \alpha$$

$$Pr\{\exp(-kx_0^2 \sum_{j=1}^{m} (\frac{\chi^2(2n_j, \alpha)}{2n_j \hat{\beta}_j})) < R_{k,m} < \exp(-kx_0^2 \sum_{j=1}^{m} (\frac{\chi^2(2n_j, 1-\alpha)}{2n_j \hat{\beta}_j}))\} = 1 - \alpha \tag{11}$$

for Model II.

$$Pr\{\chi^2(2n_0, 1-\alpha) < \frac{2n_0 \hat{\beta}_0}{\beta_0} < \chi^2(2n_0, \alpha)\} = 1 - \alpha$$

$$Pr\{-k(x_0+a_m)^2 (\frac{\chi^2(2n_0, \alpha)}{2n_0 \hat{\beta}_0}) < \frac{-k(x_0 + a_m)^2}{\beta_0} < -k(x_0+a_m)^2 (\frac{\chi^2(2n_0, 1-\alpha)}{2n_0 \hat{\beta}_0})\} = 1 - \alpha$$

$$Pr\{\exp(-k(x_0+a_m)^2 (\frac{\chi^2(2n_0, \alpha)}{2n_0 \hat{\beta}_0})) < R_{k,m} < \exp(-k(x_0+a_m)^2 (\frac{\chi^2(2n_0, 1-\alpha)}{2n_0 \hat{\beta}_0}))\} = 1 - \alpha \tag{12}$$

where, $1 - \alpha$ is the confidence coefficient.

4.2- The Exponential distribution.

Assume that for Model I, the distribution of the strength of each link on the j^{th} cycle is exponential with mean β_j, $j = 1, 2, \ldots, m$. For Model II, assume that the initial strength Y_{i0} is exponentially distributed with mean β_0.

Arguing in a similar manner as in subsection 4.1, we obtain,

$$R_{k,m} = \begin{cases} \exp\{-kx_0\psi\} & \text{for Model I} \\ \exp\{-k(x_0 + a_m)\psi_0\} & \text{for Model II,} \end{cases} \tag{13}$$

where,

$$\psi = \sum_{j=1}^{m}(\frac{1}{\beta_j}) \qquad \text{and} \qquad \psi_0 = \frac{1}{\beta_0}.$$

$$\hat{R}_{k,m} = \begin{cases} \exp\{-kx_0\hat{\psi}\} & \text{for Model I} \\ \exp\{-k(x_0 + a_m)\hat{\psi}_0\} & \text{for Model II,} \end{cases} \tag{14}$$

where,

$$\hat{\psi} = \sum_{j=1}^{m}(\frac{1}{\hat{\beta}_j}) \quad , \quad \hat{\beta}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} y_{ji},$$

and

$$\hat{\psi}_0 = \frac{1}{\hat{\beta}_0} \quad , \quad \hat{\beta}_0 = \frac{1}{n_0}\sum_{i=1}^{n_0} y_{0i}.$$

$$\hat{R}_{k,m} = \begin{cases} R_{k,m} + (kx_0R_{k,m}) \sum_{j=1}^{m} \frac{W_j}{\beta_j^2} + \mathbb{R}_1 & \text{for Model I} \\ R_{k,m} + k(\frac{x_0+a_m}{\beta_0})R_{k,m}U + \mathbb{R}_2 & \text{for Model II,} \end{cases} \tag{15}$$

Clearly, for the two models, $\hat{R}_{k,m}$, are asymptotically normal with means $R_{k,m}$ as given by Equation (13), and variances

$$\sigma^2_{\hat{R}_{k,m}} = \begin{cases} (kx_0R_{k,m})^2 \sum_{j=1}^{m}(\frac{1}{n_j\beta_j^2}) & \text{for Model I} \\ \frac{1}{n_0}((\frac{k}{\beta_0})(x_0 + a_m))^2(R_{k,m})^2 & \text{for Model II.} \end{cases} \tag{16}$$

and consequently $\hat{R}_{k,m}$ in (14), is a consistent estimator of $R_{k,m}$.

Two-sided $(1-\alpha)100\%$ confidence intervals of $R_{k,m}$ for the two models are given by:

for Model I.

$$Pr\{\exp(-kx_0\sum_{j=1}^{m}\frac{\chi^2(2n_j,\alpha)}{2n_j\hat{\beta}_j}) < R_{k,m} < \exp(-kx_0\sum_{j=1}^{m}\frac{\chi^2(2n_j,1-\alpha)}{2n_j\hat{\beta}_j})\} = 1-\alpha \tag{17}$$

for Model II.

$$Pr\{\exp(-k(x_0+a_m)(\frac{\chi^2(2n_0,\alpha)}{2n_0\hat{\beta}_0})) < R_{k,m} < \exp(-k(x_0+a_m)(\frac{\chi^2(2n_0,1-\alpha)}{2n_0\hat{\beta}_0}))\} = 1-\alpha \quad (18)$$

5- Spocial Case.

If the strength of link i, $i = 1, 2, \ldots, k$, during repeated cycles of stress are independent but identical random variables, we have in Model I, $G_j(y) = G(y)$ for all j. Then Equation (2), becomes

$$R_{k,m} = [1 - G(x_0)]^{km} \quad (19)$$

(i) Rayleigh distribution.

if

$$G(y) = 1 - \exp\{-y^2/\beta\}. \quad (20)$$

Using Equations (7), (8), we obtain

$$R_{k,m} = \exp\{-kmx_0^2/\beta\}, \quad (21)$$

and

$$\hat{R}_{k,m} = \exp\{-kmx_0^2/\hat{\beta}\}, \quad (22)$$

where, $\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n} y_i^2$, $y_1, y_2, \ldots, y_n$ is a random sample drawn from $G(y)$ in (20).

Also,

$$\sigma^2_{\hat{R}_{k,m}} = \frac{1}{n}(\frac{kmx_0^2}{\beta})^2(R_{k,m})^2. \quad (23)$$

Two-sided $(1-\alpha)100\%$ confidence intervals of $R_{k,m}$ is given by

$$Pr\{\exp(-kmx_0^2(\frac{\chi^2(2n,\alpha)}{2n\hat{\beta}})) < R_{k,m} < \exp(-kmx_0^2(\frac{\chi^2(2n,1-\alpha)}{2n\hat{\beta}}))\} = 1-\alpha \quad (24)$$

(ii) Exponential distribution.

if

$$G(y) = 1 - \exp\{-y/\beta\}. \quad (25)$$

Using Equations (7), (8), we obtain

$$R_{k,m} = \exp\{-kmx_0/\beta\}, \quad (26)$$

and

$$\hat{R}_{k,m} = \exp\{-kmx_0/\hat{\beta}\}, \quad (27)$$

where, $\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $y_1, y_2, \ldots, y_n$ is a random sample drawn from $G(y)$ in (25).

Also,

$$\sigma^2_{\hat{R}_{k,m}} = \frac{1}{n}\left(\frac{kmx_0}{\beta}\right)^2 (R_{k,m})^2. \tag{28}$$

Two-sided $(1-\alpha)100\%$ confidence intervals of $R_{k,m}$ is given by

$$Pr\{\exp(-kmx_0(\frac{\chi^2(2n,\alpha)}{2n\hat{\beta}})) < R_{k,m} < \exp(-kmx_0(\frac{\chi^2(2n,1-\alpha)}{2n\hat{\beta}}))\} = 1-\alpha \tag{29}$$

6- Numerical Example.

A simulation study, is made by taking the average of 500 generated samples drawn from Rayleigh distribution with parameters $\beta_1 = 10000$, $\beta_2 = 8000$, and $\beta_0 = 10000$.
The reliability $R_{k,m}$, the MLE of $R_{k,m}$, variance $\hat{R}_{k,m}$ and two-sided confidence interval for $R_{k,m}$, for Model I and Model II are given tables 1, 2, 3 and 4 respectively.
For simplicity we take $k = 1$, $a_j = a.j$, $j = 1, 2, \ldots, m$, $m = 2$ and $x_0 = 10$.

Non-Identical Random Independent Strength (Rayleigh distribution)
$R_{1,2} = 0.9778$, $n_1 = n_2 = n$.

n	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{R}_{1,2}$	$\hat{\sigma}^2$
5	09821	7839	0.972093	0.000099
15	10073	8107	0.976432	0.000020
25	10004	8064	0.976905	0.000011
30	09941	8003	0.976983	0.000009
50	09922	7981	0.977201	0.000005
75	10099	8007	0.977562	0.000003
100	10097	7994	0.977615	0.000003
300	10030	8020	0.977740	0.000001
500	10017	8028	0.977769	0.000000

Table (1)

Confidence Interval for $R_{1,2}$
Non-Identical Random Independent Strength (Rayleigh distribution)

n \ 1-α	0.90			0.95			0.99		
	L	U	D	L	U	D	L	U	D
5	0.9585	0.9926	0.0341	0.9505	0.9948	0.0443	0.9331	0.9975	0.0643
15	0.9674	0.9874	0.02	0.9636	0.9893	0.0257	0.9556	0.9923	0.0366
25	0.9697	0.9854	0.0157	0.9668	0.987	0.0202	0.9611	0.9897	0.0287
30	0.9702	0.9846	0.0144	0.9676	0.9862	0.0186	0.9625	0.9888	0.0264
50	0.9718	0.9831	0.0113	0.9699	0.9844	0.0145	0.9662	0.9867	0.0205
75	0.9732	0.9823	0.0091	0.9717	0.9834	0.0117	0.9687	0.9853	0.0166
100	0.9738	0.9817	0.0079	0.9725	0.9827	0.0102	0.97	0.9844	0.0144
300	0.9755	0.9801	0.0046	0.9748	0.9807	0.0059	0.9734	0.9818	0.0083
500	0.976	0.9796	0.0036	0.9755	0.98	0.0046	0.9745	0.9809	0.0065

Table (2)

Random- Fixed Strength (Rayleigh distribution)
$R_{1,2} = 0.9900$

n_0	$\hat{\beta}_0$	$\hat{R}_{1,2}$	$\hat{\sigma}^2$
5	9978	0.987784	0.000037
15	10156	0.989501	0.000008
25	9985	0.989568	0.000004
30	10122	0.989786	0.000004
50	10021	0.989812	0.000002
75	10005	0.989877	0.000001
100	9908	0.989821	0.000001
300	10023	0.989997	0.000000
500	9986	0.989977	0.000000

Table (3)

Confidence Interval for $R_{1,2}$
Random-Fixed Strength (Rayleigh distribution)

$1-\alpha$ / n_0	0.90			0.95			0.99		
	L	U	D	L	U	D	L	U	D
5	0.9816	0.9968	0.0152	0.978	0.9977	0.0197	0.9701	0.9989	0.0288
15	0.9854	0.9944	0.009	0.9837	0.9952	0.0116	0.98	0.9966	0.0165
25	0.9863	0.9934	0.0071	0.985	0.9941	0.0092	0.9823	0.9954	0.013
30	0.9868	0.9932	0.0064	0.9856	0.9939	0.0083	0.9833	0.9951	0.0118
50	0.9874	0.9925	0.0051	0.9866	0.9931	0.0065	0.9849	0.9941	0.0092
75	0.9879	0.992	0.0041	0.9872	0.9925	0.0053	0.9859	0.9934	0.0075
100	0.9881	0.9917	0.0036	0.9875	0.9921	0.0047	0.9863	0.9929	0.0066
300	0.989	0.9911	0.0021	0.9887	0.9913	0.0027	0.9881	0.9918	0.0038
500	0.9892	0.9908	0.0016	0.9889	0.991	0.0021	0.9885	0.9914	0.0029

Table (4)

Non-Identical Random Independent Strength (Exponential distribution)
$R_{1,2} = 0.9978$, $n_1 = n_2 = n$.

n	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{R}_{1,2}$	$\hat{\sigma}^2$
5	9821	7839	0.997168	1.06E-06
15	10073	8107	0.997617	2.05E-07
25	10004	8064	0.997666	1.15E-07
30	9941	8003	0.997674	9.4E-08
50	9922	7981	0.997696	5.47E-08
75	10099	8007	0.997733	3.51E-08
100	10097	7994	0.997738	2.61E-08
300	10030	8020	0.997752	8.5E-09
500	10017	8028	0.997754	5.1E-09

Table (5)

Confidence Interval for $R_{1,2}$
Non-Identical Random Independent Strength (Exponential distribution)

n \ 1-α	0.90			0.95			0.99		
	L	U	D	L	U	D	L	U	D
5	0.9958	0.9993	0.0035	0.9949	0.9995	0.0045	0.9931	0.9997	0.0066
15	0.9967	0.9987	0.002	0.9963	0.9989	0.0026	0.9955	0.9992	0.0038
25	0.9969	0.9985	0.0016	0.9966	0.9987	0.0021	0.996	0.999	0.0029
30	0.997	0.9985	0.0015	0.9967	0.9986	0.0019	0.9962	0.9989	0.0027
50	0.9971	0.9983	0.0011	0.997	0.9984	0.0015	0.9966	0.9987	0.0021
75	0.9973	0.9982	0.0009	0.9971	0.9983	0.0012	0.9968	0.9985	0.0017
100	0.9973	0.9982	0.0008	0.9972	0.9983	0.001	0.997	0.9984	0.0015
300	0.9975	0.998	0.0005	0.9974	0.9981	0.0006	0.9973	0.9982	0.0009
500	0.9976	0.9979	0.0004	0.9975	0.998	0.0005	0.9974	0.9981	0.0007

Table (6)

Random- Fixed Strength (Exponential distribution)
$R_{1,2} = 0.9990$

n_0	$\hat{\beta}_0$	$\hat{R}_{1,2}$	$\hat{\sigma}^2$
5	9978	0.998772	3.83E-07
15	10156	0.998947	7.92E-08
25	9985	0.998953	4.57E-08
30	10122	0.998976	3.62E-08
50	10021	0.998979	2.13E-08
75	10005	0.998985	1.39E-08
100	9908	0.998979	1.05E-08
300	10023	0.998997	3.4E-09
500	9986	0.998995	2E-09

Table (7)

Confidence Interval for $R_{1,2}$
Random-Fixed Strength (Exponential distribution)

n \ 1-α	0.90			0.95			0.99		
	L	U	D	L	U	D	L	U	D
5	0.9981	0.9997	0.0015	0.9978	0.9998	0.002	0.997	0.9999	0.0029
15	0.9985	0.9994	0.0009	0.9984	0.9995	0.0012	0.998	0.9997	0.0017
25	0.9986	0.9993	0.0007	0.9985	0.9994	0.0009	0.9982	0.9995	0.0013
30	0.9987	0.9993	0.0006	0.9986	0.9994	0.0008	0.9983	0.9995	0.0012
50	0.9987	0.9992	0.0005	0.9987	0.9993	0.0007	0.9985	0.9994	0.0009
75	0.9988	0.9992	0.0004	0.9987	0.9993	0.0005	0.9986	0.9993	0.0008
100	0.9988	0.9992	0.0004	0.9987	0.9992	0.0005	0.9986	0.9993	0.0007
300	0.9989	0.9991	0.0002	0.9989	0.9991	0.0003	0.9988	0.9992	0.0004
500	0.9989	0.9991	0.0002	0.9989	0.9991	0.0002	0.9988	0.9991	0.0003

Table (8)

Comments.

1- From tables (1) and (3), we see that the variances decrease with sample sizes, showing consistency of the estimator.

2- From tables (2) and (4), we see that our procedure gives a substantially good confidence interval for $R_{k,m}$ taking different confidence coefficients $1-\alpha = 0.90, 0.95, 0.99$. The difference between upper and lower limits of confidence interval is small even for small sample sizes. Obviously, this difference decreases by increasing n.

3- We find that the reliability of the system under Model II is greater than that under Model I. This means that the value of reliability change by changing the type of strength.

4- Similar results are obtained in the cases of the exponential distribution.

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