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A Simulation Study of Bayesian Prediction From The Rayleigh Distribution with Future Random Sample Size

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1-Summary:

This paper considers Bayesian prediction of order statistics from a future sample from the Rayleigh distribution when the sample size is a random variable. It has been shown (see Lingappaiah(1986)) that this situation arises in the field as Biology ,Agriculture and in Quality Control. A comparison is made with the case when the sample size is fixed assuming that the random sample sizes have a Poisson and Negative Binomial distribution. These comparison are made under basis of relative efficiency for point estimation and tail probability for interval estimation. The results of Monte Carlo Simulation study are presented.

2-Introduction.

The Rayleigh distribution was originally derived in connection with a problem in the field of acoustics. It arises also as the distribution of the distance between an individual and its nearest neighbor when a spatial pattern is generated by a poisson process. The Rayleigh distribution is appropriate for modeling the lifetimes of unites as it possesses a linearly increasing hazard rate , also it is extremely important in communication engineering (see Johnson ,Kotz & Balakrishnan(1994)) .

Let us denote a Rayleigh random variable by X , then the probability density function of X is given by:

$$f(x)=(\frac{2x}{\theta})\exp(-\frac{x^2}{\theta}), \qquad x>0, \theta>0. \qquad (1)$$

with the following C.D.F:

$$F(x)=1-\exp(-\frac{x^2}{\theta}).$$

Clearly X^2 has a one parameter exponential distribution , also the Rayleigh is the Weibull distribution when the shape parameter equals two.

A very interesting problem in the literature is to predict order statistics in a future sample in terms of those in the old sample using Bayesian and non-Bayesian approaches for different life testing models . Recently prediction problems have been carried out under the assumption that the sample size of the future sample is a random variable [see for example Ligappaiah(1986 ,1988) ,Salem(1987 ,1988) , Salem& El-galad(1992,1993) , Ashour & El-Wakeel(1993a,1993b,1994a,1994b) and Ashour & Rashwam (1995,1996)]. This assumption has been shown to be appropriate for Biological, Agriculture and Quality Control problems.

Using Rayleigh distribution and censored samples, Ragab(1992) discusses predictions using maximum likelihood , the conditional median approach ,and linear methods; a comparison of these various predictors is made through Monte Carlo simulation . For type II censored samples Howleder(1985) constructs a prediction interval for failure times from a "future sample" of items to be placed on a life test using a Bayesian approach . He also derived the 100(1-α)% highest posterior density (two sided) prediction for future observation, he further discusses the prediction of the rth order statistics $Y_{(r)}$ from a future sample.

Using Rayleigh distribution and type II censored for old sample, a Bayesian predictive distribution of the future rth order statistics if the future sample size is both fixed and a random variable will be obtained. For a random sample size ,we shall assume that the future sample size has negative Binomial or Poisson distribution. The main objective of the present work is to investigate the problem of randomness numerically using Monte Carlo simulation to study the discrepancy between these cases . The influence of information from the old sample will be studied for complete samples and informative censored type II samples of different sizes with different censoring proportions ,our conclusions will be based on tail probability and relative efficiency.

2- Prediction For the Future rth Order Statistics Using Fixed Sample Size

Suppose $0 < X_{(1)} < X_{(2)} \ldots\ldots\ldots\ldots < X_{(k)}$ denote the observed lifetimes(in the old sample) of the first k items to fail in a random sample of n items put on a test(this gives a type II censored),when lifetimes have the probability density function (1),the joint likelihood function will be:

$$f_n(x_1, x_2, \ldots x_k \mid \theta) \alpha \theta^{-k} \exp(-S_1/\theta), \qquad \theta > 0 \qquad (2)$$

where $S_1 = \sum_{i=1}^{k} x_{(i)}^2 + (n-k) x_{(k)}^2$

Let the natural conjugate prior distribution of the unknown parameter θ be:

$$f(\theta) \alpha \theta^{-a} \exp(-S_2/\theta), \qquad \theta > 0, a > 0. \qquad (3)$$

In the case of non-informative prior, function (3) reduces to Jeffrey's(1961) non-informative(or vague) prior:

$$f(\theta) \alpha \frac{1}{\sqrt{\theta}}.$$

Combining the likelihood function(2) with the prior distribution(3) ,we obtain the following conditional posterior distribution:

$$f(\theta \mid x) = \frac{S^{A-1}}{\Gamma_{(A-1)}} \theta^{-A} \exp(-S/\theta), \qquad \theta > 0, \qquad (4)$$

Where A=a+k ,$S = S_1 + S_2$.

Let $Y_{(1)}, Y_{(2)}, \ldots\ldots Y_{(m)}$ be a second independent sample of future observations from the distribution(1), the conditional density function of the future rth order statistics out of sample of size m is given by:

$$f_m(y|\theta) = \frac{m!}{\theta(r-1)!(m-r)!} 2y \sum_{j=0}^{r-1} \binom{r-1}{j} (-1)^j \exp(-(m-r+j+1)y^2/\theta) \qquad (5)$$

$$\theta > 0$$

From (1) & (5), the conditional probability density function $f_c(y|x)$ of the Bayes predictive distribution of future $Y_{(r)}$ will be:

$$f_c(y|x) = \frac{m! S^{A-1} 2(A-1)}{(r-1)!(m-r)!} \sum_{j=0}^{r-1} \frac{\binom{r-1}{j}(-1)^j y}{(S+(m-r+j+1)y^2)^A}, \qquad Y>0$$

Further, the sth moment & the critical value $u^2_{\alpha,c}$ of the 100 α% prediction bounds for $Y_{(r)}$ respectively are given by:

$$M'_{s,c} = S^{\frac{s}{2}} \frac{m!(A-1)}{(r-1)!(m-r)!} \sum_{j=0}^{r-1} \frac{\binom{r-1}{j}(-1)^j}{(m-r+j+1)^{s/2+1}} B\left(\frac{s}{2}+1, A-\frac{s}{2}-1\right), s = 1,2,3.., \quad (6)$$

and

$$\alpha = \frac{m! S^{A-1}}{(r-1)!(m-r)!} \sum_{j=0}^{r-1} \frac{\binom{r-1}{j}(-1)^j}{(m-r+j+1)!} (S+(m-r+j+1)u^2_{\alpha,c})^{-(A-1)} \qquad (7)$$

where $B(.,.)$ is the complete beta function of the second kind and α is a fixed probability. Equations (6) & (7) have to be solved numerically.

4- Prediction For the Future rth Order Statistics Using A Random Sample Size

Suppose the sample size m of the future sample is a random variable and has a Negative Binomial distribution with the following probability density function:

$$f(m|p,x) = \binom{m+x-1}{x-1} p^m q^x, m = 0,1,2\ldots$$

Referring to Consoul (1984) & Gupta and Gupta(1984) and using the p.d.f. (1), the predictive distribution $f_n(y|x)$ of $Y_{(r)}$ is given by:

$$f_n(y|x) = \frac{S^{A-1} 2(A-1)}{(r-1)!\, p(m>r)} \sum_{m=r}^{\infty} \frac{m!\binom{m+x-1}{x-1} p^x q^m}{(m-r)!} \sum_{j=0}^{r-1} \frac{\binom{r-1}{j}(-1)^j y}{(S+(m-r+j+1)y^2)^A} \qquad (8)$$

The sth moment and the critical value $u_{\alpha,n}$ of the 100 α% prediction bounds will be respectively:

$$M'_{s,n}=\frac{(\sqrt{S})^{s}(A-1)}{(r-1)!\,p(m>r)}\sum_{m=r}^{\infty}\frac{m!\binom{m+x-1}{x-1}p^{x}q^{m}}{(m-r)!}\sum_{j=0}^{r-1}\frac{\binom{r-1}{j}(-1)^{j}B(\frac{s}{2}+1,A-\frac{s}{2}-1)}{(m-r+j+1)^{\frac{s}{2}+1}},s=1,2,3\ldots,\tag{9}$$

and

$$\alpha=\frac{S^{A-1}}{(r-1)!\,p(m>r)}\sum_{m=r}^{\infty}\frac{m!\binom{m+x-1}{x-1}p^{x}q^{m}}{(m-r)!}\sum_{j=0}^{r-1}\frac{\binom{r-1}{j}(-1)^{j}}{(m-r+j+1)(S+(m-r+j+1)u_{\alpha,n}^{2})^{A-1}}\tag{10}$$

The corresponding results if the distribution of the future sample size is a Poisson with probability density function

$$f(m)=\exp(-\lambda)\lambda^{m}/m!,\qquad \lambda>0,m=0,1,2,3\ldots$$

are given by:

$$f_{p}(y|x)=\frac{S^{A-1}2(A-1)}{(r-1)!\,p(m>r)}\sum_{m=r}^{\infty}\frac{e^{-\lambda}\lambda^{m}}{(m-r)!}\sum_{j=0}^{r-1}\frac{\binom{r-1}{j}(-1)^{j}y}{(S+(m-r+j+1)y^{2})^{A}},\tag{11}$$

$$M'_{s,p}=\frac{(A-1)S^{\frac{s}{2}}}{(r-1)!\,p(m>r)}\sum_{m=r}^{\infty}\frac{e^{-\lambda}\lambda^{m}}{(m-r)!}\sum_{j=0}^{r-1}\frac{\binom{r-1}{j}(-1)^{j}}{(m-r+j+1)^{\frac{s}{2}+1}}B(\frac{s}{2}+1,A-\frac{s}{2}-1),s=1,2,\ldots\tag{12}$$

and

$$\alpha=\frac{S^{A-1}}{(r-1)!\,p(m>r)}\sum_{m=r}^{\infty}\frac{e^{-\lambda}\lambda^{m}}{(m-r)!}\sum_{j=0}^{r-1}\frac{\binom{r-1}{j}(-1)^{j}}{(m-r+j+1)(S+(m-r+j+1)u_{\alpha,p}^{2})^{A-1}}\tag{13}$$

Again equation may not be solved analytically.

5-A Special Case

As a special case for the results of the preceding two sections, analogous results for the future first order statistic may be obtained by putting r=1.The sth moments for the first future order statistics for fixed and random sample size cases may be obtained as follows:

(a)- fixed sample size:

$$M^{\circ}_{s,c}=(\frac{S}{m})^{\frac{s}{2}}\frac{\Gamma(\frac{s}{2}+1)\Gamma(A-\frac{s}{2}-1)}{\Gamma(A-1)},\qquad s=0,1,2,3\ldots$$

(b)-for random Poisson:

$$M^{\circ}_{s,p} = (S)^{\frac{s}{2}} \frac{\Gamma(\frac{S}{2}+1)\Gamma(A-\frac{S}{2}-1)}{(1-Exp(-\lambda))\Gamma(A-1)} \sum_{m=1}^{\infty} \frac{e^{-\lambda}\lambda^{m}}{m!\,m^{\frac{s}{2}}}, \qquad s = 0,1,2,3...$$

(c)-for random Negative Binomial:

$$M^{\circ}_{s,n} = (S)^{\frac{s}{2}} \frac{\Gamma(\frac{S}{2}+1)\Gamma(A-\frac{S}{2}-1)}{(1-p^{x})\Gamma(A-1)} \sum_{m=1}^{\infty} \frac{\binom{m+x-1}{x-1}p^{m}q^{x}}{m^{\frac{s}{2}}}, \qquad s = 0,1,2,3... \quad (14)$$

Similarly the tail probabilities for fixed $u_{\alpha,c}$ for fixed and random cases are giving by:

$$\alpha_c = \frac{S^{A-1}}{(S+mu^2_{\alpha,c})^{A-1}},$$

$$\alpha_p = \frac{S^{A-1}}{(1-Exp(-\lambda))} \sum_{m=1}^{\infty} \frac{e^{-\lambda}\lambda^{m}}{m!(S+mu^2_{\alpha,c})^{A-1}}, \text{ and}$$

$$\alpha_n = \frac{S^{A-1}p^{x}}{(1-p^{x})} \sum_{m=1}^{\infty} \frac{\binom{m+x-1}{x-1}q^{m}}{(S+mu^2_{\alpha,c})^{A-1}}, \qquad (15)$$

where α_c, α_p & α_n are the tail probabilities for fixed , Poisson and negative Binomial cases respectively.

In order to compare the Bayes prediction regions for fixed and random sample sizes the following lemmas are proved:

Lemma(1): **For fixed α and $u_{c,\alpha}$ the tail probability based on random sample size and Poisson distribution is greater then tail probability based on fixed sample size.**

Proof: **from equation (15) ,we have:**

$$\frac{\alpha_p}{\alpha_c} = \left(\frac{1}{(1-Exp(-\lambda))}\right) \sum_{j=1}^{\infty} \frac{e^{-\lambda}\lambda^{j}}{j!} \left(\frac{S+mu^2_{c,\alpha}}{S+ju^2_{c,\alpha}}\right)^{A-1} \qquad (16)$$

$$\frac{\alpha_p}{\alpha_c} = 1 + \left(\frac{1}{(1-Exp(-\lambda))}\right) \sum_{j=1, j\neq m}^{\infty} \frac{e^{-\lambda}\lambda^{j}}{j!} \left(\frac{S+mu^2_{c,\alpha}}{S+ju^2_{c,\alpha}}\right)^{A-1}$$

therefore $\frac{\alpha_p}{\alpha_c} > 1$, i.e. the Bayesian prediction region in the case of random Poisson is shorter than that of fixed case.

Lemma(2): **For fixed α and $u_{c,\alpha}$ the tail probability based on random sample size and Negative Binomial distribution is greater then tail probability based on fixed sample size.**

Proof: **from equation (15) ,we have:**

$$\frac{\alpha_n}{\alpha_c}=\frac{p^x}{(1-p^x)}\sum_{j=1}^{\infty}\frac{\binom{j+r-1}{r-1}q^j(S+mu_{c,\alpha}^2)^{A-1}}{(s+ju_{c,\alpha}^2)^{A-1}}$$

$$\frac{\alpha_n}{\alpha_c}=1+\frac{p^x}{(1-p^x)}\sum_{j=1,j\neq m}^{\infty}\frac{\binom{j+r-1}{r-1}q^j(S+mu_{c,\alpha}^2)^{A-1}}{(S+ju_{c,\alpha}^2)^{A-1}} \quad (17)$$

therefore : $\frac{\alpha_n}{\alpha_c}>1$, i.e. the Bayesian prediction region in the case of random Negative Binomial is shorter than that of fixed case.

From lemma(1)&(2), we conclude that future random sample size(Poisson & Negative Binomial) gives greater tail probabilities. In section (6) we shall compare tail probabilities results from Poisson and Negative Binomial cases numerically.

6-Monte Carlo Simulation

The main objective of this section is to make a numerical comparison between prediction results for the first future order statistics in the case of fixed and random future sample size. Using Monte Carlo simulation a numerical investigation will be carried out to compare prediction of the first future order statistics. In particular we shall be interested in the effects of assuming a random future sample size on prediction and to asses the influence of information for the old sample. Two measures will be used for comparison ,which are standard deviation and tail probability. For fixed sample size case since coefficient of variation of $y_{(1)}$ does not depend upon the future sample size m , therefore we shall use standard deviation for comparisons.

Monte Carlo simulation has been carried out under the following conditions :
(i) Let old sample be n=5,10,25 with type II censoring at 60%,80%,100% (this includes complete case) n=k.
(ii)for the future sample size , m=5,10,25 for fixed case .For the random variable cases , the parameters of the Negative Binomial have been chosen to be p=0.25,0.5 ,0.75 and x=2,4,6,10,and for Poisson distribution λ=3,5,10&15.
(iii)we shall use non-informative prior.

A random sample of size n from Rayliegh distribution (1) with parameter θ=1 were generated , using equation (15)&(16) , we calculated standard deviations and tail probabilities for $y_{(1)}$ using fixed and random sample sizes . This procedure was repeated 1000 times ,and the means for the output tail probabilities & standard deviations are obtained ,and are displayed in table(1) &(2) respectively. For tail probability we use α_c=0.05 to obtain critical value for fixed case $u_{c,\alpha}$ and use it to obtain the corresponding tail probabilities α_p & α_n for Poisson and negative Binomial cases .

From our numerical investigation , we conclude :
(i)If information from old sample increases by increasing n or k ,tail probabilities increased and standard deviations decreased ,this conclusion is valid for all cases .

(ii) For fixed n and k and from table (1),standard deviations decreased by increasing ,the future sample size m for fixed and random cases .
(iii)Results for fixed sample size case is more efficient than random sample size cases except one case when p-0.75 and x=6.
(iv)Comparison between Poisson and Negative Binomial shows that results for Poisson is more efficient except for p=0.75. For Negative Binomial increasing p leads to smaller standard deviation.

From lemma (1)& (2) tail probabilities for fixed case are shorter than that of random case regardless of the type of the distribution of the future sample size. From table(2)a numerical comparison between tail probability for Poisson and negative Binomial have been carried out ,this comparison shows that , if p=0.25 poisson has larger tail probabilities ,while if p> 0.25 negative Binomial has larger tail probabilities. For Negative Binomial ,if p increases ,tail probability increases.

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Table(1)
Tail Probabilities For The Predictive Distribution of The First Future Order Statistics For Random Sample Size and Non-informative Prior

n	k	m	POISSON (λ)			NEGATIVE BINOMIAL P=0.25			NEGATIVE BINOMIAL P=0.5			NEGATIVE BINOMIAL P=0.75		
			3	5	10	x=2	x=4	x=6	x=2	x=4	x=6	x=2	x=4	x=6
5	3	5	0.294	0.154	0.078	0.166	0.052	0.016	0.262	0.163	0.096	0.333	0.282	0.235
		10	0.490	0.306	0.185	0.308	0.123	0.049	0.447	0.312	0.209	0.533	0.473	0.414
		25	0.722	0.556	0.417	0.535	0.296	0.163	0.682	0.551	0.433	0.756	0.707	0.656
	4	5	0.330	0.171	0.086	0.183	0.054	0.015	0.294	0.181	0.104	0.372	0.315	0.263
		10	0.535	0.340	0.206	0.339	0.133	0.505	0.489	0.345	0.231	0.579	0.517	0.455
		25	0.759	0.600	0.460	0.574	0.326	0.180	0.719	0.593	0.473	0.789	0.744	0.695
	5	5	0.351	0.181	0.086	0.194	0.056	0.014	0.312	0.191	0.109	0.395	0.335	0.279
		10	0.559	0.360	0.219	0.356	0.140	0.051	0.512	0.363	0.244	0.604	0.540	0.478
		25	0.777	0.623	0.484	0.595	0.344	0.190	0.739	0.615	0.496	0.806	0.763	0.716
10	6	5	0.364	0.187	0.088	0.201	0.566	0.014	0.324	0.198	0.112	0.410	0.348	0.290
		10	0.475	0.373	0.228	0.360	0.144	0.052	0.526	0.375	0.253	0.619	0.556	0.492
		25	0.788	0.638	0.498	0.608	0.355	0.179	0.750	0.629	0.510	0.816	0.774	0.728
	8	5	0.380	0.196	0.091	0.209	0.058	0.013	0.338	0.207	0.116	0.428	0.363	0.303
		10	0.592	0.388	0.238	0.380	0.150	0.053	0.543	0.389	0.263	0.637	0.573	0.509
		25	0.801	0.654	0.517	0.623	0.368	0.206	0.763	0.645	0.527	0.828	0.787	0.743
	10	5	0.390	0.200	0.092	0.214	0.059	0.013	0.347	0.212	0.119	0.439	0.372	0.311
		10	0.603	0.379	0.244	0.388	0.153	0.053	0.553	0.398	0.270	0.647	0.583	0.519
		25	0.807	0.664	0.527	0.632	0.377	0.211	0.771	0.653	0.537	0.834	0.794	0.751
25	15	5	0.402	0.207	0.095	0.221	0.060	0.013	0.358	0.219	0.122	0.541	0.384	0.321
		10	0.615	0.409	0.253	0.398	0.157	0.054	0.567	0.409	0.278	0.659	0.596	0.532
		25	0.816	0.676	0.541	0.642	0.387	0.218	0.780	0.665	0.549	0.841	0.802	0.761
	20	5	0.408	0.210	0.096	0.224	0.060	0.013	0.363	0.222	0.123	0.459	0.390	0.325
		10	0.621	0.415	0.257	0.403	0.159	0.054	0.571	0.414	0.282	0.665	0.602	0.358
		25	0.820	0.682	0.549	0.648	0.393	0.222	0.785	0.671	0.555	0.845	0.807	0.766
	25	5	0.412	0.212	0.096	0.226	0.061	0.013	0.366	0.224	0.125	0.463	0.394	0.329
		10	0.625	0.418	0.259	0.406	0.160	0.055	0.575	0.418	0.285	0.679	0.606	0.541
		25	0.822	0.685	0.551	0.651	0.396	0.224	0.787	0.679	0.559	0.847	0.809	0.768

* For Fixed Case Tail Probability =0.05

Table(2)
Standard Deviation For The Predictive Distribution of The First Future Order Statistics Random Sample Size and Non-informative Prior.

N	C	FIXED	POISSON(λ)			NEGATIVE BINOMIAL P=0.25			NEGATIVE BINOMIAL P=0.5			NEGATIVE BINOMIAL P=0.75		
			3	5	10	x=2	x=4	x=6	x=2	x=4	X=6	x=2	x=4	x=6
5	3	0.355	0.722	0.563	0.433	0.756	0.714	0.668	0.701	0.590	0.483	0.605	0.394	0.268
	4	0.251	0.605	0.476	0.366	0.632	0.600	0.563	0.591	0.500	0.410	0.516	0.337	0.228
	5	0.159	0.560	0.441	0.339	0.583	0.555	0.522	0.547	0.465	0.382	0.481	0.314	0.212
10	6	0.257	0.530	0.420	0.322	0.552	0.526	0.500	0.519	0.442	0.363	0.457	0.300	0.201
	8	0.182	0.500	0.395	0.303	0.517	0.494	0.466	0.488	0.417	0.343	0.431	0.283	0.190
	10	0.115	0.486	0.387	0.297	0.506	0.483	0.456	0.478	0.409	0.336	0.423	0.278	0.186
25	15	0.224	0.466	0.371	0.285	0.484	0.463	0.438	0.458	0.393	0.323	0.407	0.268	0.179
	20	0.158	0.457	0.364	0.279	0.474	0.453	0.430	0.500	0.386	0.317	0.400	0.262	0.176
	25	0.100	0.454	0.362	0.278	0.471	0.451	0.427	0.446	0.383	0.315	0.398	0.262	0.175