Odds-Ratio Distance Measures of Departure from Three Row Effects Models for Two-Way Contingency Tables

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Summary

For two-way contingency tables, Goodman (1979) considered the local row effects (R) model and Agresti (1984, p.125, p.147) considered the logit row effects (LR) model and the global row effects (GR) model. This paper proposes three measures which represent the degrees of departure from the R, LR, and GR models. The proposed measures are expressed as a minimum Euclid distance between (i) the local odds-ratios, (ii) the local-global odds-ratios, and (iii) the global odds-ratios, and the corresponding odds-ratios with the structure of R, LR, and GR, respectively. The measures would be useful for comparing the degrees of departure from each row effects model in several tables. Examples are given. Note that the proposed measures are extensions of Altham's (1970) measure of departure from the null association model, and Tomizawa's (1993a) and Tomizawa, Seo and Fujino's (1995) measures of departure from the local, logit, and global uniform association models.

Key words : Association, Global odds-ratio, Local odds-ratio, Logit, Measure, Row effects model.

1. Introduction

For the $R \times C$ contingency table with nominal row categories and ordered column categories, let p_{ij} denote the probability that an observation will fall in the ith row and jth column of the table ($i = 1, 2, \ldots, R$; $j = 1, 2, \ldots, C$) .

First, for the 2×2 subtables formed from adjacent rows i and $i+1$, and adjacent columns j and $j+1$, let θ_{ij}^{L} denote the corresponding local odds-ratio based on the probabilities, defined by

$$\theta_{ij}^{L} = \frac{p_{ij}\, p_{i+1,j+1}}{p_{i+1,j}\, p_{i,j+1}} \quad \text{for} \quad i = 1, 2, \ldots, R-1; \; j = 1, 2, \ldots, C-1 .$$

The local row effects (R) model (Goodman, 1979) is defined by

$$\theta_{ij}^{L} = \theta_{i}^{L} \quad \text{for} \quad i = 1, 2, \ldots, R-1; \; j = 1, 2, \ldots, C-1 .$$

Special cases of this model obtained by putting $\{\theta_{ij}^{L} = \theta^{L}\}$ and $\{\theta_{ij}^{L} = 1\}$ are the local uniform association (U) model and the null association (i.e., the independence) model, respectively; see Goodman (1979).

Secondly, for the $2 \times C$ subtables formed from adjacent rows i and $i+1$, let θ_{ij}^{LG} denote the local-global odds-ratio based on the row cumulative probabilities, defined by

$$\theta_{ij}^{LG} = \frac{\left(\sum_{t=1}^{j} p_{it}\right)\left(\sum_{t=j+1}^{C} p_{i+1,t}\right)}{\left(\sum_{t=1}^{j} p_{i+1,t}\right)\left(\sum_{t=j+1}^{C} p_{it}\right)}$$

for $i = 1, 2, \ldots, R-1$; $j = 1, 2, \ldots, C-1$. These odds-ratios are local in the row variable but global in the column variable, since all C categories of the column variable are used in each odds-ratio. In terms of these odds-ratios, the logit row effects (LR) model (Agresti, 1984, p.125) is defined by

$$\theta_{ij}^{LG} = \theta_{i}^{LG} \quad \text{for} \quad i = 1, 2, \ldots, R-1; \; j = 1, 2, \ldots, C-1 .$$

Let $L_{j(i)}$ denote the jth cumulative logit within row i; that is,

$$L_{j(i)} = \log\left(\sum_{t=j+1}^{C} p_{it} \Big/ \sum_{t=1}^{j} p_{it}\right).$$

The LR model indicates that for each pair of rows i and $i+1$, the difference in logits

$$L_{j(i+1)} - L_{j(i)} = \xi_i \ \left(= \log\theta_i^{LG}\right)$$

is constant for all $C-1$ logits. Special cases of the LR model obtained by putting $\{\theta_{ij}^{LG} = \theta^{LG}\}$ and $\{\theta_{ij}^{LG} = 1\}$ are the logit uniform association (LU) model and the null association model, respectively; see Agresti (1984, p.122).

Thirdly, for the $R \times C$ table, let θ_{ij}^{G} denote the global odds-ratio based on the cumulative probabilities, defined by

$$\theta_{ij}^{G} = \frac{\left(\sum_{s=1}^{i}\sum_{t=1}^{j} p_{st}\right)\left(\sum_{s=i+1}^{R}\sum_{t=j+1}^{C} p_{st}\right)}{\left(\sum_{s=i+1}^{R}\sum_{t=1}^{j} p_{st}\right)\left(\sum_{s=1}^{i}\sum_{t=j+1}^{C} p_{st}\right)}$$

for $i = 1,2,\ldots,R-1$; $j = 1,2,\ldots,C-1$. These odds-ratios are regular odds-ratios computed for the 2×2 tables corresponding to the $(R-1)(C-1)$ ways of collapsing the row and column classifications into dichotomies. Using these odds-ratios, the global row effects (GR) model (Agresti, 1984, p.147) is defined by

$$\theta_{ij}^{G} = \theta_i^{G} \quad \text{for } i = 1,2,\ldots,R-1;\ j = 1,2,\ldots,C-1.$$

Special cases of this model obtained by putting $\{\theta_{ij}^{G} = \theta^{G}\}$ and $\{\theta_{ij}^{G} = 1\}$ are the global uniform association (GU) model and the null association model, respectively.

Altham (1970) considered a measure which is a function of odds-rations, to represent the degree of departure from the null association model (also see Bishop, Fienberg and

Holland, 1975, p.393). Tomizawa (1993a) considered a measure of departure from the U model, and Tomizawa, Seo and Fujino (1995) considered the measures of departure from the LU and the GU models. The measure proposed by Tomizawa (1993a) is defined by

$$\Psi_U = \min_{\alpha} \left\{ \sum_{i=1}^{R-1} \sum_{j=1}^{C-1} |\log \theta_{ij}^L - \log \alpha|^s \right\}^{1/s} \quad \text{for } s \geq 1 .$$

Especially, when $s = 2$, this would be interesting and useful for describing the degree of departure from the U model. Note that this measure with $s = 2$ is further expressed as

$$\Psi_U = \left\{ \sum_{i=1}^{R-1} \sum_{j=1}^{C-1} (\log \theta_{ij}^L - \log \alpha^*)^2 \right\}^{1/2} ,$$

where

$$\log \alpha^* = \frac{1}{(R-1)(C-1)} \sum_{s=1}^{R-1} \sum_{t=1}^{C-1} \log \theta_{st}^L .$$

This measure expresses the minimum Euclid distance between the log local odds-ratios and the log local odds-ratios with the structure of uniform association. Note that when $s \neq 2$, it is difficult to express the measure as a close form, and thus the choice s different from 2 would not be recommended. The details of measures proposed by Tomizawa et al. (1995) are omitted here.

The purpose of this paper is to propose three measures, which represent the degrees of departure from the R, LR and GR models. The measures proposed would be useful for *comparing* the degrees of departure from each model in several tables.

2. Measures

First, consider a measure of degree of departure from the R model, defined by

$$\begin{aligned}\Psi_R &= \min_{\{\alpha_i\}}\left\{\sum_{i=1}^{R-1}\sum_{j=1}^{C-1}(\log\theta_{ij}^L-\log\alpha_i)^2\right\}^{1/2}\\ &= \left\{\sum_{i=1}^{R-1}\sum_{j=1}^{C-1}(\log\theta_{ij}^L-\log\alpha_i^*)^2\right\}^{1/2},\end{aligned}$$

where

$$\log\alpha_i^* = \frac{1}{C-1}\sum_{t=1}^{C-1}\log\theta_{it}^L .$$

This measure represents the minimum Euclid distance between the log local odds-ratios and the log local odds-ratios with the structure of association of the R model. The range of this measure must be $\Psi_R \geq 0$. Also, $\Psi_R = 0$ if and only if the R model holds. So, the degree of departure from the R model increases as the value of Ψ_R increases.

Secondly, consider a measure of degree of departure from the LR model, defined by

$$\begin{aligned}\Psi_{LR} &= \min_{\{\beta_i\}}\left\{\sum_{i=1}^{R-1}\sum_{j=1}^{C-1}(\log\theta_{ij}^{LG}-\log\beta_i)^2\right\}^{1/2}\\ &= \left\{\sum_{i=1}^{R-1}\sum_{j=1}^{C-1}(\log\theta_{ij}^{LG}-\log\beta_i^*)^2\right\}^{1/2},\end{aligned}$$

where

$$\log\beta_i^* = \frac{1}{C-1}\sum_{t=1}^{C-1}\log\theta_{it}^{LG} .$$

Thirdly, consider a measure of degree of departure from the GR model, defined by

$$\Psi_{GR} = \min_{\{\gamma_i\}} \left\{ \sum_{i=1}^{R-1} \sum_{j=1}^{C-1} (\log \theta_{ij}^{G} - \log \gamma_i)^2 \right\}^{1/2}$$

$$= \left\{ \sum_{i=1}^{R-1} \sum_{j=1}^{C-1} (\log \theta_{ij}^{G} - \log \gamma_i^{*})^2 \right\}^{1/2},$$

where

$$\log \gamma_i^{*} = \frac{1}{C-1} \sum_{t=1}^{C-1} \log \theta_{it}^{G} .$$

The measure Ψ_{LR} (Ψ_{GR}) represents the minimum Euclid distance between the log local-global (global) odds-ratios and the log local-global (global) odds-ratios with the structure of association of the LR (GR) model. The range of this measure must be $\Psi_{LR} \geq 0 (\Psi_{GR} \geq 0)$. Also, $\Psi_{LR} = 0$ ($\Psi_{GR} = 0$) if and only if the LR (GR) model holds. So, the degree of departure from the LR (GR) model increases as the value of Ψ_{LR} (Ψ_{GR}) increases.

3. Approximate confidence intervals for measures

Let n_{ij} denote the observed frequency in the ith row and jth column of the table ($i = 1, 2, \ldots, R$; $j = 1, 2, \ldots, C$) and let $n = \sum\sum n_{ij}$. We assume that all n_{ij} are positive, and a multinomial distribution applies to the $R \times C$ table. We shall consider the approximate standard error and large-sample confidence intervals for Ψ_R, Ψ_{LR} and Ψ_{GR} using delta method, of which descriptions are given by Bishop et al. (1975, Sec.14.6). Also see Appendix for the details of the delta method. For convenience, we denote each of Ψ_R, Ψ_{LR} and Ψ_{GR} by Ψ. The sample version of Ψ, $\hat{\Psi}$, is given by Ψ with $\{p_{ij}\}$ replaced by $\{\hat{p}_{ij}\}$, where $\hat{p}_{ij} = n_{ij}/n$. Using the delta method, $\sqrt{n}(\hat{\Psi} - \Psi)$ has asymptotically (as $n \to \infty$) a normal distribution with mean zero and variance $\sigma^2(\Psi)$. For the measure Ψ_R, we obtain

$$\sigma^2(\Psi_R) = \frac{1}{(\Psi_R)^2} \sum_{i=1}^{R} \sum_{j=1}^{C} \left(\frac{M_{ij}^2}{p_{ij}} \right),$$

where

$$M_{ij} = \log\theta^L_{i-1,j-1} - \log\theta^L_{i-1,j} - \log\theta^L_{i,j-1} + \log\theta^L_{ij} - (\log\alpha^*_i)\Delta^L_{ij},$$

$$\log\theta^L_{s0} = \log\theta^L_{0t} = \log\theta^L_{Rt} = \log\theta^L_{sC} = 0 \ (s = 0, 1, \ldots, R;\ t = 0, 1, \ldots, C),$$

$$\Delta^L_{11} = \Delta^L_{RC} = 1,\ \Delta^L_{1C} = \Delta^L_{R1} = -1,\ \Delta^L_{ij} = 0\,(\text{otherwise}),$$

$$\alpha^*_R = \alpha^*_{R-1},$$

and α^*_i $(i = 1, 2, \ldots, R-1)$ are defined in Section 2. For the measure Ψ_{LR}, we obtain

$$\begin{aligned} \sigma^2(\Psi_{LR}) = \frac{1}{(\Psi_{LR})^2} \sum_{i=1}^{R} \sum_{j=1}^{C} p_{ij} \Bigg\{ & \Delta_{i1}\Delta_{j1} \sum_{v=1}^{j-1} \frac{(\log\theta^{LG}_{i-1,v} - \log\beta^*_i)}{\sum_{t=v+1}^{C} p_{it}} \\ & - \Delta_{i1}\Delta_{jC} \sum_{v=j}^{C-1} \frac{(\log\theta^{LG}_{i-1,v} - \log\beta^*_i)}{\sum_{t=1}^{v} p_{it}} - \Delta_{iR}\Delta_{j1} \sum_{v=1}^{j-1} \frac{(\log\theta^{LG}_{iv} - \log\beta^*_i)}{\sum_{t=v+1}^{C} p_{it}} \\ & + \Delta_{iR}\Delta_{jC} \sum_{v=j}^{C-1} \frac{(\log\theta^{LG}_{iv} - \log\beta^*_i)}{\sum_{t=1}^{v} p_{it}} \Bigg\}, \end{aligned}$$

where

$$\Delta_{km} = \begin{cases} 1 & \text{for } k \neq m, \\ 0 & \text{for } k = m. \end{cases}$$

For the measure Ψ_{GR}, we obtain

$$\sigma^2(\Psi_{GR}) = \frac{1}{(\Psi_{GR})^2}\sum_{i=1}^{R}\sum_{j=1}^{C} p_{ij}\left\{\Delta_{i1}\Delta_{j1}\sum_{u=1}^{i-1}\sum_{v=1}^{j-1}\frac{(\log\theta^G_{uv}-\log\gamma^*_u)}{\sum_{s=u+1}^{R}\sum_{t=v+1}^{C} p_{st}}\right.$$

$$-\Delta_{i1}\Delta_{jC}\sum_{u=1}^{i-1}\sum_{v=j}^{C-1}\frac{(\log\theta^G_{uv}-\log\gamma^*_u)}{\sum_{s=u+1}^{R}\sum_{t=1}^{v} p_{st}} - \Delta_{iR}\Delta_{j1}\sum_{u=i}^{R-1}\sum_{v=1}^{j-1}\frac{(\log\theta^G_{uv}-\log\gamma^*_u)}{\sum_{s=1}^{u}\sum_{t=v+1}^{C} p_{st}}$$

$$\left.+\Delta_{iR}\Delta_{jC}\sum_{u=i}^{R-1}\sum_{v=j}^{C-1}\frac{(\log\theta^G_{uv}-\log\gamma^*_u)}{\sum_{s=1}^{u}\sum_{t=1}^{v} p_{st}}\right\}.$$

We note that each asymptotic distribution is not applicable when $\Psi = 0$. Let $\hat{\sigma}^2(\Psi)$ denote $\sigma^2(\Psi)$ with $\{p_{ij}\}$ replaced by $\{\hat{p}_{ij}\}$. Then $\hat{\sigma}(\Psi)/\sqrt{n}$ is an estimated standard error for $\hat{\Psi}$, and $\hat{\Psi} \pm z_{p/2}\,\hat{\sigma}(\Psi)/\sqrt{n}$ is an approximate $100(1-p)$ percent confidence interval for Ψ, where $z_{p/2}$ is the percentage point from the standard normal distribution corresponding to a two-tail probability equal to p.

4. Examples

Table 1 taken from Agresti (1984, p.87) is the data of political ideology and political party affiliation for a sample of voters taken in the 1976 presidential primary in Wisconsin. Table 2 taken from Andersen (1990, p.358) is the data on the connection between urbanization and social rank in the Danish Welfare Study. Table 3 taken from Stuart (1953) is the data of unaided distance vision of 7477 women aged 30-39 employed in Royal Ordnance factories in Britain from 1943 to 1946 (also see Tomizawa, 1993b).

First, we shall apply the measure Ψ_R to the data in Tables 1, 2, and 3. Table 4 gives the estimated values of local odds-ratios $\{\theta^L_{ij}\}$, applied to these data. We see from Table 5 that the estimated values for the measure Ψ_R are the largest for Table 3, the second largest for Table 2, and the smallest for Table 1. Also, we see from Tables 5 that (i) the values in confidence interval of Ψ_R for Table 3 are greater than those for Tables 1 and 2, and (ii) those for Table 1 overlap with those for Table 2. Therefore, we conclude that (i) the degree of departure from the R model would be stronger in Table 3 than in Tables 1 and 2, and (ii) it may be impossible to compare the degrees of departure from the R model between Tables 1 and 2.

Secondly, we shall apply the measure Ψ_{LR} to the data in Tables 1, 2, and 3. Table 6 gives the estimated values of local-global odds-ratios $\{\theta_{ij}^{LG}\}$, applied to these data. We see from Table 7 that the estimated values for the measure Ψ_{LR} are the largest for Table 3, the second largest for Table 2, and the smallest for Table 1. Also, from the values in confidence interval of of Ψ_{LR}, we conclude that (i) the degree of departure from the LR model would be stronger in Table 3 than in Tables 1 and 2, and (ii) it may be impossible to compare the degrees of departure from the LR model between Tables 1 and 2.

Thirdly, we shall apply the measure Ψ_G to the data in Tables 1, 2, and 3. Table 8 gives the estimated values of global odds-ratios $\{\theta_{ij}^{G}\}$, applied to these data. We see from Table 9 that (i) the values in confidence interval of Ψ_{GR} for Table 3 are greater than those for Tables 1 and 2, and (ii) those for Table 2 are greater than those for Table 1. Therefore, we conclude that the degrees of departure from the GR model are the strongest for Table 3, the second strongest for Table 2, and the weakest for Table 1.

5. Comments

(i) The measure Ψ_R would be useful when one wants to see with a minimum Euclid distance measure how far the local odds-ratios are distant from the local odds-ratios with the structure of association of the R model. In addition, the measure Ψ_R would be useful for *comparing* the degrees of departure from the R model in several tables.

(ii) The measure Ψ_{LR} would be useful when are wants to see with a minimum Euclid distance measure how far the local-global odds-ratios are distant from the local-global odds-ratios with the structure of association of the LR model. In addition, the measure Ψ_{LR} would be useful for *comparing* the degrees of departure from the LR model in several tables.

(iii) The measure Ψ_{GR} would be useful when are wants to see with a minimum Euclid distance measure how far the global odds-ratios are distant from the global odds-ratios with the structure of association of the GR model. In addition, the measure Ψ_{GR} would be useful for *comparing* the degrees of departure from the GR model in several tables.

The reader may be interested in which measure should be used and with what types of data. However, it seems difficult to select one of three measures because three measures are functions of different odds-ratios and these are measuring the degree of departure from different models, as described above.

Finally, we observe that (i) each measure Ψ (i.e., Ψ_R, Ψ_{LR} and Ψ_{GR}) would be useful for describing relative magnitudes rather than absolute magnitudes, (ii) the estimate of degree of departure from each row effects model should be considered in terms of an approximate confidence interval for corresponding measure Ψ and not in terms of $\hat{\Psi}$ itself.

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APPENDIX

The asymptotic distribution of $\sqrt{n}(\hat{\Psi} - \Psi)$ is derived as follows: Let

$$\hat{p} = (\hat{p}_{11}, \hat{p}_{12}, \ldots, \hat{p}_{1C}, \ldots, \hat{p}_{R1}, \hat{p}_{R2}, \ldots, \hat{p}_{RC})^t,$$

where "t" denotes the transpose, and let us define the vector p in terms of p_{ij}'s in the same way as $\hat{p}$. Then $\sqrt{n}(\hat{p} - p)$ is asymptotically distributed as normal distribution $N(0, \Lambda(p))$, where $\Lambda(p) = \mathrm{diag}(p) - pp^t$ and $\mathrm{diag}(p)$ denotes a diagonal matrix with the ith element of p as the ith diagonal element. We also obtain

$$\hat{\Psi} = \Psi + d(p)(\hat{p} - p) + o(||\hat{p} - p||),$$

where $d(p) = \{\partial\Psi/\partial p^t\}$. Using the delta method (see Bishop et al., 1975, Sec.14.6), $\sqrt{n}(\hat{\Psi} - \Psi)$ is asymptotically distributed as normal distribution $N(0, \sigma^2(\Psi))$, where $\sigma^2(\Psi) = d(p)\Lambda(p)d(p)^t$.

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Table 1

Political ideology and political party affiliation for a sample of voters taken in the 1976 presidential primary in Wisconsin.

	Political Ideology			
Party Affiliation	Liberal	Moderate	Conservative	Total
Democrat	143	156	100	399
Independent	119	210	141	470
Republican	15	72	127	214
Total	277	438	368	1083

Table 2

Urbanization and social rank in the Danish Welfare Study.

	Social rank group				
Urbanization	I-II	III	IV	V	Total
Copenhagen	45	64	160	74	343
Copenhagen suburbs	99	107	174	90	470
Three largest cities	57	85	153	103	398
Cities	168	287	415	342	1212
Coutryside	83	346	361	399	1189
Total	452	889	1263	1008	3612

Table 3

Unaided distance vision of 7477 women aged 30-39 employed in Royal Ordnance factories in Britain from 1943 to 1946.

Right eye	Left eye grade				
grade	Best(1)	Second(2)	Third(3)	Worst(4)	Total
Best(1)	1520	266	124	66	1976
Second(2)	234	1512	432	78	2256
Third(3)	117	362	1772	205	2456
Worst(4)	36	82	179	492	789
Total	1907	2222	2507	841	7477

Table 4

The estimated values of local odds-ratios $\{\theta_{ij}^{L}\}$ applied to Tables 1, 2 and 3.

(a) For Table 1

$\hat{\theta}_{ij}^{L}$	j=1	2
i=1	1.62	1.05
2	2.72	2.63

(b) For Table 2

$\hat{\theta}_{ij}^{L}$	j=1	2	3
i=1	0.76	0.65	1.12
2	1.38	1.11	1.30
3	1.15	0.80	1.22
4	2.44	0.72	1.34

(c) For Table 3

$\hat{\theta}_{ij}^{L}$	j=1	2	3
i=1	36.92	0.61	0.34
2	0.48	17.13	0.64
3	0.74	0.45	23.76

Table 5

Estimate of Ψ_R, estimated approximate standard error for $\hat{\Psi}_R$, and approximate 95% confidence interval for Ψ_R, applied to Tables 1, 2 and 3.

Applied data	Estimated measure	Standard error	Confidence interval
Table 1	0.308	0.341	(−0.361, 0.977)
Table 2	1.013	0.158	(0.704, 1.322)
Table 3	5.506	0.133	(5.246, 5.766)

Table 6

The estimated values of local-global odds-ratios $\{\theta_{ij}^{LG}\}$ applied to Tables 1, 2 and 3.

(a) For Table 1

$\hat{\theta}_{ij}^{LG}$	j=1	2
i=1	1.65	1.28
2	4.50	3.41

(b) For Table 2

$\hat{\theta}_{ij}^{LG}$	j=1	2	3
i=1	0.57	0.60	0.86
2	1.60	1.41	1.47
3	1.04	0.92	1.13
4	2.14	1.06	1.28

(c) For Table 3

$\hat{\theta}_{ij}^{LG}$	j=1	2	3
i=1	28.80	2.75	1.04
2	2.31	14.13	2.54
3	1.05	1.38	18.19

Table 7

Estimate of Ψ_{LR}, estimated approximate standard error for $\hat{\Psi}_{LR}$, and approximate 95% confidence interval for Ψ_{LR}, applied to Tables 1, 2 and 3.

Applied data	Estimated measure	Standard error	Confidence interval
Table 1	0.265	0.659	(−1.027, 1.557)
Table 2	0.629	0.399	(−0.154, 1.411)
Table 3	3.589	0.114	(3.365, 3.812)

Table 8

The estimated values of global odds-ratios $\{\theta_{ij}^G\}$ applied to Tables 1, 2 and 3.

(a) For Table 1

$\hat{\theta}_{ij}^G$	j=1	2
i=1	2.29	1.93
2	5.73	3.80

(b) For Table 2

$\hat{\theta}_{ij}^G$	j=1	2	3
i=1	1.06	0.77	1.45
2	1.74	1.09	1.71
3	1.70	1.04	1.58
4	2.39	1.07	1.50

(c) For Table 3

$\hat{\theta}_{ij}^G$	j=1	2	3
i=1	44.05	12.67	4.75
2	14.30	22.38	7.77
3	8.12	8.52	30.09

Table 9

Estimate of Ψ_{GR}, estimated approximate standard error for $\hat{\Psi}_{GR}$, and approximate 95% confidence interval for Ψ_{GR}, applied to Tables 1, 2 and 3.

Applied data	Estimated measure	Standard error	Confidence interval
Table 1	0.314	0.192	(−0.061, 0.690)
Table 2	0.942	0.104	(0.737, 1.146)
Table 3	2.040	0.091	(1.861, 2.219)