DEMOGRAPHIC MODELS FOR CHANGING MARRIAGE FREQUENCIES

S. M. FARID

1. INTRODUCTION

The annual number of first marriages in a population is primarily determined by such factors as the quantity of nuptiality, the pattern of age at marriage, and the number and age distribution of the never-married population. The measurement of change in the first-marriage frequencies and the way in which change is defined are of theoretical interest as well as of practical importance in the study of nuptiality and fertility patterns. Any approach to the study of change in nuptiality must involve two or more measurements of the same variable at different times in order to provide a basis for inferring that change has or has not taken place.

For a given time period, first-marriage frequencies may be visualized as a two-way table showing the numbers of persons marrying for the first time, where each horizontal line shows the age distribution of spinsters marrying bachelors of a given age, and each vertical column shows the age distribution of bachelors marrying spinsters of a given age.

The graduation of such a table may be done by one of three diflerent methods :

(a) First, by operating on the marginal totals. In this case nuptiality is related to either males or females which means that either sex is considered completely dominant in determining its nuptiality conditions.

(b) Secondly, by taking into consideration the nuptiality of both sexes applying the assumption known as «male dominence» which implies that the first-marriage frequencies for both males and females depend only on male nuptiality conditions.

(c) Thirdly by operating on the individual cell frequencies and thus taking the nuptiality of both sexes into consideration.

The first approach will be referred to as the «unisexual nuptiality analysis», the second as the «joint nuptiality analysis» and the third as the «bisexual nuptiality analysis».

This paper is centered upon three models developed to investigate the bahaviour of the change in the first-marriage frequencies between two different dates using the three approaches mentioned above.

2. STATEMENT OF THE PROBLEM

If the number of first marriages be symbolized by M, the marriage rate for the ith group by a(i) and the single population in the ith group by n(i), then M is identical with the sum of the products [a(i). n(i)] over all ages.

This suggests an analytic approach to the study of nuptiality, that is : changes in the first-marriage frequencies between two different dates arise from three factors : (i) changes in the intensity of first marriage ; (ii) changes in the age-composition of the single population; and (iii) interaction between nuptiality schedule and age-composition.

Accordingly, letting D denote the difference between the number of first marriages in a given time period, t say, and that in a base time period, o say, the problem is to ascertain «how much of D is due to change in nuptiality schedule, how much to change in age composition of the nevermarried population and how much to interaction between nuptiality and age composition» ?

To this end, we shall resort to decomposition techniques developed by Kitagawa (1955) and Keyfitz (1968). It should be noted that there is always an element of arbitrariness in the decomposition of a difference between two rates into a sum of contributions, for it is, to some extent, a matter of definition. Perhaps, the main virtue of the decomposition techniques presented in this paper is that they demonstrate that the contributions of the factors involved are sufficiently important to merit separate identification and evaluation.

3. NOTATION

To define the complete system of first-marriage frequencies, the following notation will be used :

n(i) = Nuber of single males in the ith group.

n (j) = Number os single females in the jth group.

m(i) = Number of bachelors marrying from the ith group.

m(j)	=	Number of spinsters marrying from the jth group.
m(i,j)	=	Number of marriages between the bachelors of ith group and the spinsters of jth group.
a(i)	=	First-marriage rate for males in the ith group.
a(i,j)	=	Rate of contracting marriage between a bachelor of age i and a spinster of age j.
T	=	Number of all males in the nuptial span.
S	=	Proportion of single males in the nuptial span.
h(i)	=	$n(i) / \Sigma n(i)$ = Percentage of single males in the ith group.
$p(f_j/m_i)$	=	Conditional probability that the bride is from the jth group, given that the bridegroom is from the ith group.
R(i,j)	=	Sex ratio at marriage, expressed as the ratio between the number of single males in the ith group and the number of single females in the jth group.

4. THE UNISEXUAL NUPTIALITY ANALYSIS

4.1. *Introduction.*

The model presented in this section is a simple extension of decomposition techniques developed by Kitagawa (1955) and Keyfitz (1968). Our discussion will refer to the components of the change in the first-marriage frequencies of males between two different dates.

4.2. *The Total Dfference.*

The total number of first marriages at time t is given as

$$M_t = \Sigma\, a_t\,(i)\,.\,n_t\,(i) \qquad (1)$$

The difference between the number of first marriages at time t and that at time o can be written as

$$D = \sum_i \left[a_t(i)\,.\,n_t(i) - a_o(i)\,.\,n_o(i) \right] \qquad (2)$$

4.3 *The Nuptiality Component.*

The measurement of the effect of changes in age-specific first-marriage rates-between two different dates-on the first-marriage frequencies, can be based on the technique of direct standardization which by definition «permits the determination of an interaction effect but obscures the effect of differing age-compositions» (Fennessey, 1968). Thus, the product of $[a_t(i).\, n_o(i)]$ would give the age distribution of first marriages which would have occurred if the never-married population at time t had had the same age distribution as the base population.

The difference

$$D(1) = \sum_i [a_t(i) - a_o(i)] . n_o(i), \tag{3}$$

would thus indicate how much of D is due to changes in marriage rates independent of changes in age composition.

4.4 *The Age Composition Component.*

The measurement of the effects of changes in the age distribution the never-married population on the first-marriage frequencies may be done by employing the technique of indirect standardization which «obscures any possible interactive effects but permits the determination of the effect of differing age compositions» (Fennessey, 1968).

The difference

$$D(2) = \sum_i a_o(i) . [n_t(i) - n_o(i)], \tag{4}$$

would thus indicate how much of D is due to changes in age composition independent of changes in nuptiality schedule.

The change in the age composition of the never-married population between two different dates may be considered to have been caused by one or more of the following factors :

(i) a change in the general level of celibacy ;

(ii) a change in the total number of persons (single + never married) at the marriageable ages ;

(iii) changes in the proportional age distribution of the single population.

It can be shown that (Farid, 1969)

$$n_t(i) - n_o(i) = C_1 . n_t(i) + (1-C_1) . C_2 . n_t(i) + (1-C_1) . (1-C_2) . n_t(i) . C_3(i). \tag{5}$$

where

$$C_1 = 1 - s_o/s_t$$

$$C_2 = 1 - T_o/T_t$$

$$C_3(i) = 1 - h_o(i)/h_t(i)$$

Thus the age composition component may be further divided into effects of the three factors mentioned above.

4.5 *The Interaction Component.*

The interaction between changes in the nuptiality schedule and changes in age composition may be obtained by subtracting the sum of the nuptiality and age composition components from the total change,

$$D(3) = \sum_i [\bar{a}_t(i) - a_o(i)] [\bar{n}_t(i) - n_o(i)] \quad (6)$$

4.6 *The Complete Formula.*

The complete formula for the components of the change in the first-marriage frequencies between two different dates may be expressed by the following identity :

$$D = D(1) + D(2.1) + D(2.2) + D(2.3) + D(3)$$

$$\sum_i [\bar{a}_t(i).n_t(i) - a_o(i).n_o(i)]$$

$$= \sum_i [\bar{a}_t(i) - a_o(i)].n_o(i)$$

$$+ C_1 . \sum_i a_o(i).n_t(i)$$

$$+ (1 - C_1).C_2.\sum_i a_o(i).n_t(i)$$

$$+ (1 - C_1) (1 - C_2). \sum_i a_o(i).n_t(i).C_3(i)$$

$$+ \sum_i [\bar{a}_t(i) - a_o(i)] [\bar{n}_t(i) - n_o(i)]$$

Formula (7) decomposes the change in the first-marriage frequencies between two different dates into five components :

(i) The first component, D (1), measures the contribution of changes in the nuptiality schedule.

(ii) The second component, D (2·1) measures that part of the total change which is due to a change in the proportion of the population that consists of the single persons in the nuptial span.

(iii) The third component, D(2·2), measures the contribution of the change in the size of the population at the marriageable ages.

(iv) The fourth component, D(2.3), measures that part of the total change which is due to a change in the degree to which the never-married population is concentrated at different ages.

(v) The fifth component, D(3), which is a residual term, may be interpreted mathematically as the contribution of the interaction between nuptiality and age-composition. The interaction is described by the difference between the effect of changing nuptiality at time t level of age-composition and the effect of changing nuptiality at time o level of age-composition (Keyfitz, 1968).

5. THE JOINT NUPTIALITY ANALYSIS

5. 1 *Introduction.*

In the unisexual analysis, nuptiality is related to either males or females which means that either sex is considered completely dominant in determining its nuptiality conditions. Our interest in the preceding section was in the changing marriage frequencies of either sex indep-dent of changes in the nuptiality of the other sex. However, the one-sex approach is unrealistic because the first-marriage frequencies in any given period is influenced by the size and age distribution of both sexes Rather than analysing changes in marriage frequencies uni-sexually an approach taking into consideration the nuptiality of both sexes simultaneously seems, therefore, preferable (Hoem, 1969). In this section an attempt is made to decompose changes in the first-marriage frequencies between two different dates using the assumption known as «male dominance».

5. 2. *The Total Difference.*

The total number of first mariages at time t is given as (Shah and Giesbrecht, 1969).

$$M_t = \sum_i \sum_j m_t(i,j) = \sum_i \sum_j a_t(i).n_t(i).p_t(f_j/m_i).$$

Thus the difference between the number of first marriages at time t and that at time o can be written as

$$D = \sum_i \sum_j [a_t(i).n_t(i).p_t(f_j/m_i) - a_o(i).n_o(i)p_o(f_j/m_i)] \qquad (9)$$

5. 3. *The Nuptiality Component.*

The product of $[a_t(i).n_o(i).p_o(f_j/m_i)]$ would result in the bivariate age distribution of first marriages which would have occurred at time t if the matrix of conditional probabilities of marrying at time t was as that at time o, and if the population of single males at time t had the same age distribution as that at time o while retaining its observed marriage rates.

The difference

$$D(1) = \sum_i \sum_j [a_t(i) - a_o(i)].n_o(i).p_o(f_j/m_i) \quad (10)$$

wonld thus indicate how much of D is due to changes in the male age-specific first marriage rates independent of changes in the age composition of single males and in the coinditional probabilities of marrying.

5.4. *The Age Composition Component*

The application of the bachelor mariage rates and the conditional probabilities of marrying observed at time o to the age distribution of single males at time t would result in the bivariate age distribution of first marriage which would have occurred at time t with its age distribution of single males if both its bachelor marriage rates and its conditional probabilities of marrying had been exactly the same as for the base period.

The difference

$$D(2) = \sum_i \sum_j a_o(i).p_o(f_j/m_i).[n_t(i) - n_o(i)], \quad (11)$$

would thus indicate how of D is due to changes in the age distribution of single males.

5·5. *The Conditional Probabilities of Marrying Component*

Similarly the conditional probabilities of marrying component can be expressed as

$$D(3) = \sum_i \sum_j a_o(i).n_o(i).[p_t(f_j/m_i) - p_o(f_j/m_i)]$$

Eq. (12) indicates how much of D is due to changes in the matrix of the conditional probabilities of marrying between the two time periods considered independent of changes in the bachelor marriage rates and in the age distribution of single males.

It should be noted that the sum of the entries in the matrix of the conditional probabilities of marrying component along any horizontal line is equal to zero, as the sum of the conditional probabilities of marrying along any row is equal to unity. This means that D (3) will always be equal to zero. But the sum of the entries along any vertical line can be positive or negative. Thus, the matrix of the conditional probabilities of marrying component will mainly show the changes in the bivariate age distribution of first marriages which are due to shifts in the pattern of age at marriage.

5.6. *The Ineraction Component*

The interaction component may be obtained by subtracting the sum of the nuptiality, age composition and conditional probabilities of marrying components from the total change.

5.7. *The Complete Formula*

The complete formula for components of the change in first-marriage frequencies between two different dates-based on the joint nuptiality analysis-may be expressed by the following identity :

$$D = D\ (1) + D\ (2) + D\ (3) + \text{residual}$$

$$\sum_i \sum_j [a_t(i).n_t(i).p_t(f_j/m_i) - a_o(i).n_o(i).p_o(f_j/m_i)]$$

$$= \sum_i \sum_j [a_t(i) - a_o(i)].n_o(i).p_o(f_j/m_i)$$

$$+ \sum_i \sum_j a_o(i).p_o(f_j/m_i).[n_t(i) - n_o(i)]$$

$$+ \sum_i \sum_j a_o(i).n_o(i).[p_t(f_j/m_i) - p_o(f_j/m_i)]$$

$$+ \text{residual} \qquad (13)$$

6. THE BISEXUAL NUPTIALITY ANALYSIS

6.1. *Introduction*

In the preceding section an attempt was made to incorporate simultaneously the nuptiality of both sexes into our marriage system. In formulating the joint nuptiality model the two sexes, however, still did not enter symmetrically because the matrix of the conditional probailities of marrying was considered determined by the number of marriages of i-aged males. In this section an attempt is made to relax the assumption of male dominance.

6.2. *The Problem of Sex Dominance*

We may compute the bisexual first-marriage rate for the age group combination (i, j,), i.e. the rate of contracting marriage between a bachelor of age i and a spinster of age j, as the ratio between the number of first marriages occurring in the (i, j)-cell and the number of persons exposed to the risk of first marriage in the (i, j)-cell (Yntema, 1954). The latter quantity will be referred to as n (i, j). We can, therefore, write a (i, j) as

$$a(i, j) = \frac{m(i, j)}{n(i, j)} \tag{14}$$

or, alternatively, as

$$m(i, j) = a(i, j).\, n(i, j). \tag{15}$$

The problem is to specify a set of values for n (i, j) for the various (i, j) cells. We shall consider marriage as a contract established in a «marriage market» in which there are marriageable men and women. The actual number of marriages and the pattern of age at marriage depend upon the size and age distribution of the marriageable male and female populations (Henry, 1972). From (15) it is evident that n (i, j), as a multiplier attached to the bisexual first-marriage rate a (i, j), serves as the effective marriageable population for the calculation of m (i, j), the number of first marriages in the (i, j)-cell.

Several authors suggested that the bisexual population in the (i, j)-cell can be computed as a weighted average of i-aged males and j-aged females. However, the use of a weighted average is unrealistic because whenever $n(i) \neq n(j)$, a weighted average will always lie somewhere between n (i) and n (j). It is obvious that the maximum number of possible marriages is primarily determined by the number of the less numerous sex. Thus, if the number of single males aged 25 is 100 and the number of single females aged 22 is 80, the maximum number of marriages that could be contracted between 25 years bachelors an 22 years spinsters is 80. We can, therefore, say that the bisexual marriageable population in the (i, j)-cell is determined by the number of the less numerous sex,

$$n(i,j) = \min[n(i),\, n(j)] \tag{16}$$

It is obvious that n (i, j) = O, when either n (i) or n (j) is zero. This will ensure that no marriage is possible in the absence of one of the sexes When n (i) = n (j), n (i, j) reduces to the mean value of n (i) and n (j), as it should be.

6.3. The Total Difference·

The total number of first marriages at time t is given as

$$M_t = \sum_i \sum_j m_t(i,j) = \sum_i \sum_j a_t(i,j).n_t(i,j) \tag{17}$$

Thus the difference between the number of first marriages at time t and that at time O can be written as

$$D = \sum_i \sum_j [a_t(i,j).n_t(i,j) - a_o(i,j).n_o(i,j)] \tag{18}$$

6.4. The Nuptiality Component.

The product of $[a_t(i,j).n_o(i,j)]$ would give the bivariate age distribution of first marriages which have occurred at time t if the bisexual marriageable population at time t had had the same age distribution as the base population while retaining its observed marriage rates.

The difference

$$D(1) = \sum_i \sum_j [a_t(i,j) - a_o(i,j)].n_o(i,j), \tag{19}$$

would thus indicate how of the total change D is due to changes in the bisexual first-marriage rates independent of changes in the bisexual marriageable population.

6.5. The Age Composition Component.

The application of the bisexual first-marriage rates at time t to the bisexual mariageable population at time o, would result in the bivariate age distribution of first marriages which would have occurred at time t with its age composition, if its bisexual marriage rates had been exactly the same as for the base population.

The difference

$$D(2) = \sum_i \sum_j a_o(i,j) \cdot [n_t(i,j) - n_o(i,j)], \qquad (20)$$

would indicate how much of D is due to changes in the age distribution of the bisexual marriageable population independent of changes in nuptiality.

The eligible male and female populations will change between any two different dates either by the same amount or there will be inequality in the growth of the two sexes. We have, therefore, cases for the change in the age-distribution of the bisexual marriageable population. In the first case, the sex ratio at marriage does not undergo any change between the two time periods considered and the less numerous sex at time o is also the less numerous sex at time t. Thus, the change in the bisexual marriageable population as measured by the difference $[n_t(i,j) - n_o(i,j)]$ will entirely be due to equal growth of the sexes. In the second case, there are two possible outcomes. The sex ratio at marriage may change in such a way that does not alter the sex determining the bisexual marriageable population in the (i, j)-cell. Or the sex ratio at marriage does change in a such a way that the less numerous sex at time o becomes the more numerous sex at time t.

Therefore, the change in the age distribution of the bisexual marriageable population between two different dates may be further divided into effects of the sex-ratio at marriage and equal growth of the sexes. This may be expressed by the following identity :

$$n_t(i,j) - n_o(i,j) = [n_t(i,j) - k.n(x)] + [k.n(x) - n_o(i,j)]. \qquad (21)$$

where

$$K = R_o(i,j) \quad \text{if} \quad n_o(i) < n_o(j)$$
$$= 1/R_o(i,j) \quad \text{if} \quad n_o(i) > n_o(j)$$

and

$$n(x) = n_t(i) \quad \text{if} \quad R_o(i,j) > 1$$
$$= n_t(j) \quad \text{if} \quad R_o(i,j) < 1$$

6.6. The Complete Formula.

The complete formula for components of the change in first-marriage frequencies between two different dates-based on the bisexual nuptiality analysis-may be expressed by the following identity :

$$D = D(1) + D(2.1) + D(2.2) + D(3).$$

$$\sum_i \sum_j a_t(i,j).n_t(i,j) - \sum_i \sum_j a_o(i,j).n_o(i,j)$$

$$= \sum_i \sum_j [a_t(i,j) - a_o(i,j)].n_o(i,j)$$

$$+ \sum_i \sum_j a_o(i,j).[n_t(i,j) - k.n(x)]$$

$$+ \sum_i \sum_j a_o(i,j).[k.n(x) - n_o(i,j)]$$

$$+ \sum_i \sum_j [a_t(i,j) - a_o(i,j)].[n_t(i,j) - n_o(i,j)].$$

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