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# CONVOLUTION RESULTS FOR SUBCLASSES OF MULTIVALENT MEROMORPHIC FUNCTIONS OF COMPLE ORDER INVOLVING AN INTEGRAL OPERATOR

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Abstract. In the paper, we will define a new integral operator of meromorphic multivalent functions, by using this operator we introduce subclasses of meromorphic multivalent functions. And if we take different values for this operator, we will get new classes of meromorphic multivalent functions. So these classes will contain many special classes. The integral operator $\mathcal{J}_p^m(\mu,\alpha)$ is a specific integral transform used in mathematical analysis, particularly in connection with solving certain differential equations and studying properties of functions. This operator generalizes many operators using different parameter values. The study of multivalent meromorphic functions in connection with the integral operator $\mathcal{J}_p^m(\mu,\alpha)$ involves analyzing how the operator affects the analytic properties of functions, ensuring convergence, and understanding the behavior. These properties are crucial for applications in various branches of mathematics, including differential equations, harmonic analysis, and integral transforms. This paper aims to investigate convolution properties, coefficient estimates and containment properties for these subclasses with the integral operator $\mathcal{J}_p^m(\mu,\alpha)$ within $\mathbb{U}^* = \{\vartheta \in \mathbb{C} : 0 < |\vartheta| < 1\} = \mathbb{U} \setminus \{0\}$. We will establish fundamental properties such as coefficient inequalities, integral means inequalities and subordination results for these subclasses.

## 1 Introduction

Let $\Sigma_p$ be the class of meromorphic functions

$$f(z) = z^{-p} + \sum_{k=1}^{\infty} a_k z^{k-p} \qquad (p \in \mathbb{N} = \{1, 2, 3, ...\}), \tag{1}$$

which are analytic and multivalent in $\mathbb{U}^* = \{z \in \mathbb{C} : |z| < 1\}$.

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For $0 \leq \delta < p$, $f \in \Sigma_p$ is called meromorphically multivalent starlike of order $\delta$ and meromorphically multivalent convex of order $\delta$, respectively, iff

$$\begin{aligned} -\operatorname{Re}\left\{\frac{zf^{'}(z)}{f(z)}\right\} &> \delta, \\ -\operatorname{Re}\left\{1+\frac{zf^{''}(z)}{f^{'}(z)}\right\} &> \delta, \end{aligned} \tag{2}$$

which denoted by $\Sigma_p^*(\delta)$ $(\Sigma_p^c(\delta))$, respectively. Note that the class $\Sigma_p^*(\delta)$ and various other subclasses of $\Sigma_p^*(0)$ have been studied by [7] (see also [3, 4], [9, 10, 11, 12]).

For functions $f(z) \in \sum_p$ given by (1) and $g(z) \in \sum_p$ given by

$$g(z) = z^{-p} + \sum_{k=1}^{\infty} b_k z^{k-p} \qquad (p \in \mathbb{N}), \tag{3}$$

their Hadamard product (or convolution) is

$$(f * g)(z) = z^{-p} + \sum_{k=1}^{\infty} a_k b_k z^{k-p} = (g * f)(z). \tag{4}$$

We define the following operator $\mathcal{J}_p^m(\mu,\alpha)$, for $f \in \sum_p$, $\mu,\ \alpha \geq 0,\ p \in \mathbb{N}$ and $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ by:

$$\begin{aligned} \mathcal{J}_p^0(\mu,\alpha)f(z) &= f(z), \\ \mathcal{J}_p^1(\mu,\alpha)f(z) &= \frac{(p+\alpha)}{\mu} z^{-\left(p+\frac{p+\alpha}{\mu}\right)} \int_0^z z^{p+\frac{p+\alpha}{\mu}-1} f(t)\ dt = \mathcal{J}_p(\mu,\alpha)f(z) \\ &= z^{-p} + \sum_{k=1}^{\infty}\left(\frac{p+\alpha}{p+\mu(\epsilon+p)+\alpha}\right) a_k z^{k-p}, \\ \mathcal{J}_p^2(\mu,\alpha)f(z) &= \frac{(p+\alpha)}{\mu} z^{-\left(p+\frac{p+\alpha}{\mu}\right)} \int_0^z z^{p+\frac{p+\alpha}{\mu}-1} \mathcal{J}_p^1(\mu,\alpha)f(z)\ dt \\ &= z^{-p} + \sum_{k=1}^{\infty}\left(\frac{p+\alpha}{p+\mu(\epsilon+p)+\alpha}\right)^2 a_k z^{k-p}, \end{aligned} \tag{5}$$

and

$$\begin{aligned} \mathcal{J}_p^m(\mu,\alpha)f(z) &= \mathcal{J}_p(\mu,\alpha)f(z)\left(\mathcal{J}_p^{m-1}(\mu,\alpha)f(z)\right) \\ &= z^{-p} + \sum_{k=1}^{\infty}\left(\frac{p+\alpha}{p+\mu(\epsilon+p)+\alpha}\right)^m a_k z^{k-p}. \end{aligned} \tag{6}$$

It follows that

$$z\mu\left(\mathcal{J}_p^{m+1}(\mu,\alpha)f(z)\right)^{'} = (p+\alpha)\mathcal{J}_p^m(\mu,\alpha)f(z) - [\alpha+p(1+\mu)]\,\mathcal{J}_p^{m+1}(\mu,\alpha)f(z),\ \mu \neq 0. \tag{7}$$

Note that:
(i) $\mathcal{J}_1^m(\mu,\alpha)f(z) = \mathcal{J}^m(\mu,\alpha)f(z)$;
(ii) $\mathcal{J}_p^m(\mu,1)f(z) = \mathcal{J}_p^m(\mu)f(z)$;
(iii) $\mathcal{J}_1^m(\mu,1)f(z) = \mathcal{J}^m(\mu)f(z)$;
(iv) $\mathcal{J}_p^m(1,\alpha)f(z) = \mathcal{J}_p^m(\alpha)f(z)$;
(v) $\mathcal{J}_1^m(1,\alpha)f(z) = \mathcal{J}^m(\alpha)f(z)$.

Making use of principle of subordination, we introduce the subclasses $\mathcal{S}_p^*(\mathbb{A},\mathbb{B})$ and $\mathcal{K}_p(\mathbb{A},\mathbb{B})$ as follow:

**Definition 1** [4, at $p=1$], [6]. *Let* $-1\le\mathbb{B}<\mathbb{A}\le 1,\ \tau\in\mathbb{C}^*=\mathbb{C}\backslash\{0\}$. Then

$$\mathcal{S}_p^*(\mathbb{A},\mathbb{B})=\left\{f\in\Sigma_p: 1-\frac{1}{\tau}\left(\frac{zf'(z)}{f(z)}+p\right)\prec\frac{1+\mathbb{A}z}{1+\mathbb{B}z}\right\}, \tag{8}$$

and

$$\mathcal{K}_p(\mathbb{A},\mathbb{B})=\left\{f\in\Sigma_p: 1-\frac{1}{\tau}\left(\frac{zf''(z)}{f'(z)}+p+1\right)\prec\frac{1+\mathbb{A}z}{1+\mathbb{B}z}\right\}, \tag{9}$$

from (8) and (9), we can conclude that

$$f(z)\in\mathcal{K}_p(\mathbb{A},\mathbb{B})\Leftrightarrow -\frac{1}{p}zf'(z)\in\mathcal{S}_p^*(\mathbb{A},\mathbb{B}). \tag{10}$$

**Definition 2** ([5, 8]). If $f$ and $g$, analytic in $\mathbb{U}$; we say that $f(z)$ is subordinate to $g(z)$ in $\mathbb{U}$ written $f(z)\prec g(z)$, if $\exists\ \omega(z)$, analytic in $\mathbb{U}$, with $\omega(0)=0$ and $|\omega(z)|<1\ \ (z\in\mathbb{U})$, such that $f(z)=g(\omega(z))\ \ (z\in\mathbb{U})$. Furthermore, if $g(z)$ is univalent in $\mathbb{U}$, then

$$(\forall z\in\mathbb{U})\ (f(z)\prec g(z))\Leftrightarrow(f(0)=g(0)\ \ \&\ \ f(\mathbb{U})\subset g(\mathbb{U})).$$

By using the operator $\mathcal{J}_p^m(\mu,\alpha)$ defined by (6), we introduce the classes $\mathcal{JS}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$ and $\mathcal{JK}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$ as follow:

**Definition 3.** For $\mu,\ \alpha\ge 0,\ p\in\mathbb{N},\ -1\le\mathbb{B}<\mathbb{A}\le 1,\ m\in\mathbb{N}_0$. *Let*

$$\mathcal{JS}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})=\left\{f\in\Sigma_p:\mathcal{J}_p^m(\mu,\alpha)f(z)\in\mathcal{S}_p^*(\mathbb{A},\mathbb{B})\right\}, \tag{11}$$

and

$$\mathcal{JK}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})=\left\{f\in\Sigma_p:\mathcal{J}_p^m(\mu,\alpha)f(z)\in\mathcal{K}_p(\mathbb{A},\mathbb{B})\right\}, \tag{12}$$

where $\mathcal{S}_p^*(\mathbb{A},\mathbb{B})$ and $\mathcal{K}_p(\mathbb{A},\mathbb{B})$ are given by (8) and (9).

From (11) and (12), we can conclude that

$$f(z)\in\mathcal{JK}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})\Leftrightarrow -\frac{1}{p}zf'(z)\in\mathcal{JS}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B}). \tag{13}$$

## 2 Main results

Unless indicated let
$\mu,\ \alpha\ge 0,\ p\in\mathbb{N},\ \tau\in\mathbb{C}^*,\ -1\le\mathbb{B}<\mathbb{A}\le 1,\ \theta\in[0,2\pi),\ m\in\mathbb{N}_0,\ z\in\mathbb{U}^*$
and $f(z)$ defined by (1).

Using the method for convolution properties developed by [13] and [1, 2], we prove the following theorems.

**Theorem 1.** *The function* $f\in\mathcal{S}_p^*(\mathbb{A},\mathbb{B})$ *if and only if*

$$z^p\left[f(z)*\frac{1+[\zeta(\theta)-1]z}{z^p(1-z)^2}\right]\neq 0\ \ (z\in\mathbb{U}^*), \tag{14}$$

*where*

$$\zeta(\theta)=\frac{(e^{-i\theta}+\mathbb{B})}{(\mathbb{A}-\mathbb{B})\,\tau}. \tag{15}$$

**Proof.** For $f \in \Sigma_p$, we can write

$$\begin{aligned} f(z) &= f(z) * \frac{1}{z^p(1-z)} \; and \\ -\frac{1}{p} z f^{'}(z) &= f(z) * \left[ \frac{1-(1+\frac{1}{p})z}{z^p\,(1-z)^2} \right]. \end{aligned} \tag{16}$$

To prove (14), we write (8) as

$$-\frac{1}{p}\frac{zf^{'}(z)}{f(z)} = \frac{1+\left[\mathbb{B}+\frac{1}{p}\left(\mathbb{A}-\mathbb{B}\right)\tau\right]\omega(z)}{1+\mathbb{B}\omega(z)},$$

hence

$$z^p\left[-zf^{'}(z)\left(1+\mathbb{B}e^{i\theta}\right)-\left(p+\left[p\mathbb{B}+\left(\mathbb{A}-\mathbb{B}\right)\tau\right]e^{i\theta}\right)f(z)\right] \neq 0. \tag{17}$$

Now from (16), we may write (17) as

$$z^p\left\{\left[f(z) * \frac{\left\{1-(1+\frac{1}{p})z\right\}p}{z^p\,(1-z)^2}\right] - \left(\frac{p+\left[p\mathbb{B}+\left(\mathbb{A}-\mathbb{B}\right)\tau\right]e^{i\theta}}{1+\mathbb{B}e^{i\theta}}\right)\left[f(z) * \frac{1}{z^p(1-z)}\right]\right\} \neq 0,$$

which is equivalent to

$$z^p\left[f\left(z\right) * \frac{1+(\frac{e^{-i\theta}+\mathbb{B}}{(\mathbb{A}-\mathbb{B})\tau}-1)z}{z^p\,(1-z)^2}\left[e^{i\theta}(\mathbb{A}-\mathbb{B})\tau\right]\right] \neq 0,$$

or

$$z^p\left[f\left(z\right) * \frac{1+(\frac{e^{-i\theta}+\mathbb{B}}{(\mathbb{A}-\mathbb{B})\tau}-1)z}{z\,(1-z)^2}\right] \neq 0,\;\; z \in \mathbb{U}^*, \tag{18}$$

which represents (14).

Reversely, suppose that $f \in \Sigma_p$ satisfying (14). Since (14) is equivalent to (17), then

$$-\frac{1}{p}\frac{zf^{'}(z)}{f(z)} \neq \frac{1+\left[\mathbb{B}+\frac{1}{p}\left(\mathbb{A}-\mathbb{B}\right)\tau\right]e^{i\theta}}{1+\mathbb{B}e^{i\theta}}. \tag{19}$$

Assume

$$\Omega(z) = -\frac{1}{p}\frac{zf^{'}(z)}{f(z)} \text{ and } \Pi(z) = \frac{1+\left[\mathbb{B}+\frac{1}{p}\left(\mathbb{A}-\mathbb{B}\right)\tau\right]e^{i\theta}}{1+\mathbb{B}e^{i\theta}},$$

the relation (19) shows that $\Omega(\mathbb{U}^*) \cap \Pi(\partial\mathbb{U}^*) = \emptyset$ and thus the simply-connected domain $\Omega(\mathbb{U}^*)$ is included in a connected component of $\mathbb{C}\backslash\Pi(\partial\mathbb{U}^*)$. From here and using the fact $\Omega(0) = \Pi(0)$ together with the univalence of the function $\Pi$, it follows that $\Omega(z) \prec \Pi(z)$, that is $f \in \mathcal{S}_p^*\left(\mathbb{A},\mathbb{B}\right)$.

**Theorem 2.** *Let $f \in \Sigma_p$. Then $f \in \mathcal{K}_p\left(\mathbb{A},\mathbb{B}\right)$ if and only if*

$$z^p\left[f(z) * \frac{1-3z-2\left(\zeta(\theta)-1\right)z^2}{z^p(1-z)^3}\right] \neq 0 \;\; \left(z \in \mathbb{U}^*\right), \tag{20}$$

where $\zeta(\theta)$ is given by (15).

**Proof.** From (10), $f \in \mathcal{K}_p(\mathbb{A},\mathbb{B})$ if and only if $-\frac{1}{p}zf^{'}(z) \in \mathcal{S}_p^*(\mathbb{A},\mathbb{B})$. Then from Theorem 1, $-\frac{1}{p}zf^{'}(z) \in \mathcal{S}_p^*(\mathbb{A},\mathbb{B})$ if and only if

$$z^p\left[-\frac{1}{p}zf^{'}(z) * g(z)\right] \neq 0, \tag{21}$$

where

$$g(z) = \frac{p + p\left(\zeta(\theta) - 1\right)z}{z^p\left(1-z\right)^2},$$

thus

$$g^{'}(z) = \frac{-p + 3pz + 2p(\zeta(\theta)-1)z^2}{z^{p+1}\left(1-z\right)^3},$$

and therefore

$$\frac{1}{p}zg^{'}(z) = \frac{-1 + 3z + 2(\zeta(\theta)-1)z^2}{z^p\left(1-z\right)^3}.$$

Using the above relation and the identity

$$-\frac{1}{p}zf^{'}(z) * g(z) = f(z) * (-\frac{1}{p}zg^{'}(z)),$$

it is simple to check that (21) is identical to (20).

**Theorem 3.** *If* $f \in \mathcal{JS}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$. Then

$$1 + \sum_{k=1}^{\infty}\left[1 + \frac{(e^{-i\theta}+\mathbb{B})k}{(\mathbb{A}-\mathbb{B})\tau}\right]\left(\frac{p+\alpha}{p+\mu(k+p)+\alpha}\right)^m a_k z^k \neq 0 \quad \left(z \in \mathbb{U}^*\right). \tag{22}$$

**Proof.** *If* $f \in \Sigma_p$, *from Theorem 1, we have* $f \in \mathcal{JS}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$ *if and only if*

$$z^p\left[\mathcal{J}_p^m(\mu,\alpha)f(z) * \frac{1+[\zeta(\theta)-1]z}{z^p(1-z)^2}\right] \neq 0 \;\; \left(z \in \mathbb{U}^*\right), \tag{23}$$

*where* $\zeta(\theta)$ is given by (15). Then

$$\frac{1+[\zeta(\theta)-1]z}{z^p(1-z)^2} = z^{-p} + \sum_{k=1}^{\infty}(1+\zeta(\theta)k)z^{k-p}. \tag{24}$$

Now a simple computation shows that (14) is identical to (22). Thus, we have the theorem.

**Theorem 4.** *Let* $f(z) \in \Sigma_p$ *satisfies*

$$\sum_{k=1}^{\infty}\left[k(1+\mathbb{B}) + (\mathbb{A}-\mathbb{B})\left|\tau\right|\right]\left(\frac{p+\alpha}{p+\mu(k+p)+\alpha}\right)^m \left|a_k\right| \leq (\mathbb{A}-\mathbb{B})\left|\tau\right|, \tag{25}$$

*then* $f \in \mathcal{JS}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$.

**Proof.** Since

$$\begin{aligned}
&\left|1 + \sum_{k=1}^{\infty}\left(\frac{k(e^{-i\theta}+\mathbb{B}) + (\mathbb{A}-\mathbb{B})\tau}{(\mathbb{A}-\mathbb{B})\,\tau}\right)\left(\frac{p+\alpha}{p+\mu(k+p)+\alpha}\right)^m a_k z^k\right| \\
\geq\;\; & 1 - \left|\sum_{k=1}^{\infty}\left(\frac{k(e^{-i\theta}+\mathbb{B}) + (\mathbb{A}-\mathbb{B})\tau}{(\mathbb{A}-\mathbb{B})\,\tau}\right)\left(\frac{p+\alpha}{p+\mu(k+p)+\alpha}\right)^m a_k z^k\right| \\
\geq\;\; & 1 - \sum_{k=1}^{\infty}\left(\frac{k\left|(e^{-i\theta}+\mathbb{B})\right| + (\mathbb{A}-\mathbb{B})\tau}{(\mathbb{A}-\mathbb{B})\,\tau}\right)\left(\frac{p+\alpha}{p+\mu(k+p)+\alpha}\right)^m \left|a_k\right| > 0,
\end{aligned}$$

then, (25) holds and our result follows from Theorem 3.

**Theorem 5.** *If* $f \in \mathcal{JK}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$. Then

$$1+\sum_{k=1}^{\infty}\left[\frac{k\left|(e^{-i\theta}+\mathbb{B})\right|+(\mathbb{A}-\mathbb{B})\tau}{(\mathbb{A}-\mathbb{B})\,\tau}\right](1-k)\left(\frac{p+\alpha}{p+\mu(k+p)+\alpha}\right)^m a_k z^k \neq 0 \;\; (z\in\mathbb{U}^*)\,. \tag{26}$$

**Proof.** *From Theorem 2, we have* $f \in \mathcal{JK}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$ *if and only if*

$$z^p\left[\mathcal{J}_p^m(\mu,\alpha)f(z) * \frac{1-3z-2\left(\zeta(\theta)-1\right)z^2}{z^p(1-z)^3}\right]\neq 0 \;\; (z\in\mathbb{U}^*)\,, \tag{27}$$

where $\zeta(\theta)$ is given by (15). Then

$$\frac{1-3z-2(\zeta(\theta)-1)z^2}{z^p(1-z)^3}=z^{-p}+\sum_{k=1}^{\infty}(1+\zeta(\theta)k)(1-k)z^{k-p}. \tag{28}$$

Now a simple computation shows that (27) is identical to (26). Thus, the proof is completed.

Using similar arguments to those in the proof of Theorem 4, we can prove the next result.

**Theorem 6.** *Let* $f(z)\in\Sigma_p$ *satisfies*

$$\sum_{k=1}^{\infty}\left[k\left|(e^{-i\theta}+\mathbb{B})\right|+(\mathbb{A}-\mathbb{B})\tau\right](1-k)\left(\frac{p+\alpha}{p+\mu(k+p)+\alpha}\right)^m |a_k| \leq (\mathbb{A}-\mathbb{B})\,|\tau|\,, \tag{29}$$

*then* $\mathcal{JK}_p^m(\mu,\alpha,\mathbb{A},\mathbb{B})$.

**Remark 1.**

(i) By specializing the parameters $\mu,\alpha$, we obtain various special cases for different operators;

(ii) Putting in the above results $\tau = e^{-i\lambda}\cos\lambda$, where $\lambda\in\mathbb{R}$ with $|\lambda|\leq\frac{\pi}{2}$, $\mathbb{A}=(1-2\gamma)\beta$ and $\mathbb{B}=-\beta$, $0\leq\gamma<1$ and $0<\beta\leq 1$, we obtain analogous results for the classes $\mathcal{S}_p^{*\lambda}(\gamma,\beta)$ and $\mathcal{K}_p^{\lambda}(\gamma,\beta)$.

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**Author Contributions**

A. O. Mostafa and S. M. Madian : Conceptualization, methodology, resources, review and editing, supervision.

Z. M. Saleh, A. O. Mostafa and S. M. Madian: validation, formal analysis, investigation.

Z. M. Saleh : data creation , writing-original draft preparation.

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**Declarations**

*Competing interests*

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