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# SOME TYPES OF GENERALIZED CLOSED AND GENERALIZED STAR CLOSED SETS IN TOPOLOGICAL ORDERED SPACES

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**ABSTRACT.** In the present work our intention is to establish relationship between new types of closed sets namely g*b-closed sets (resp.gb-closed) and g*i-closed sets(resp.gi-closed) and g*b-closed sets(resp.gb-closed) and g*d-closed sets(resp.gd-closed). We also established the independency between the notions g*i-closedness (resp.gi-closedness) and g*d-closedness (resp.gd-closedness).

**KEYWORDS:** Topological ordered space**;** Increasing set; Decreasing set; Balanced set; gi-closed set; gd-closed set; gb-closed set; g*i-closed set; g*d-closed set; g*b-closed set.

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## 1. INTRODUCTION

The notion of topological ordered space was first studied by L. Nachbin [**9**]. A triple (X, τ, ≤) where X is a non-empty set, τ is a topology and ≤ is a partial order on X called as a topological ordered space. A subset A of topological ordered space (X, τ, ≤) is said to be an increasing set if A = i(A) and is a decreasing set if A = d(A) where $i(A) = \bigcup_{a \in A} [a, \rightarrow]$ and $d(A) = \bigcup_{a \in A} [\leftarrow, a]$.The sets [x, →] = {y∈X / x ≤ y} and [ ←, x] = {y∈X / y ≤ x} are defined for any x∈X. The complement of an increasing set is a decreasing set and vice versa. A subset of a topological ordered space (X, τ, ≤) is a balanced set if it is both increasing and decreasing set.

The study of Increasing closed set, Decreasing closed set and Balanced closed set(briefly i-closed, d-closed and b-closed) in topological ordered spaces was initialized by M. K. R. S. Veerakumar [12]. The notion of generalized closed set (briefly g-closed set) was introduced by N. Levin [7]. Later Bhattacharya and Lahiri [7] introduced and studied semi generalized closed sets (briefly sg-closed sets) in topological spaces. Also, generalized star closed sets (briefly g*-closed sets) were introduced by Veerakumar [13]. In the later years some Authors [11] introduced and studied g*i-closed sets, g*d-closed sets and g*b-closed sets in topological ordered spaces.

In the present work, we established that every g*b-closed (resp.gb-closed) set is both g*i-closed (resp. gi-closed) set and g*d-closed (resp. gd-closed) set. We also provided examples for the independency of the notions namely g*i-closedness and g*d-closedness.

## 2. PRELIMINARIES

Unless otherwise mentioned, (X, τ) represent non-empty topological space on which no separation axioms are assumed. The usual notations, cl(A), int (A) and C(A) denote the closure, the interior of A and the complement of A respectively for a subset A.

We recall the following definitions which are useful in the sequel.

### DEFINITION 2.1.

A subset A of a topological space (X, τ) is called

1. a generalized closed set (briefly g-closed [8]) if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ). The compliment of a g-closed is a g-open set.
2. a g*-closed [13] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X,τ).

### DEFINITION 2.2. [11]

In a topological ordered space (X, τ, ≤), a subset A is a g*i-closed (resp. gi-closed) set if A is both increasing and g*-closed (resp. g-closed) set.

**DEFINITION 2.3. [11]**

In a topological ordered space (X, τ, ≤), a subset A is a g*d-closed (resp.gd-closed) set if A is both decreasing and g*-closed (resp. g-closed) set.

**DEFINITION 2.4. [11]**

In a topological ordered space (X, τ, ≤), a subset A is g*b-closed (resp. gb-closed) set if A is both balanced and g*-closed (resp. g-closed) set.

## 3. SOME APPLICATIONS

**THEOREM 3.1.**

Every g*b-closed set is a g*i-closed set.

**PROOF:**

Let A be a g*b-closed set in the TOS (X, τ, ≤). Then, A is an increasing set and is a g*-closed set. Thus, A is a g*i-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

**EXAMPLE 3.2.**

Let X = {a, b, c}, $\tau_6$ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and ordering $\leq_9$ = {(a, a), (b, b), (c, c), (a, c)}. Then, (X, $\tau_6$, $\leq_9$) is a topological ordered space. In this space, the g*i-closed sets are ϕ, X, {b}, {c}, {a, c}, {b, c} and the g*b-closed sets are ϕ, X, {b}, {a, c}. Then, A = {b, c} is a g*i-closed set but it is not a g*b-closed set.

**THEOREM 3.3.**

Every g*b-closed set is a g*d-closed set.

**PROOF:**

Let A be a g*b-closed set in the topological ordered space (X, τ, ≤). Then A is a decreasing set and is a g*-closed set. Thus, A is g*d-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

**EXAMPLE 3.3.**

Let X = {a, b, c}, $\tau_6$ = {ϕ, X, {a}, {b}, {a, b}, {a, c}} and the ordering $\leq_{10}$ = {(a, a), (b, b), (c, c), (b, a), (b, c), (c, a),}. Then, (X, $\tau_6$, $\leq_{10}$) is a topological ordered space. In this space the g*d-closed sets are ϕ, X, {b}, {b, c} and the g*b-closed sets are ϕ, X. Then, A = {b, c} is a g*d-closed set but it is not a g*b-closed set.

**THEOREM 3.4.**

Every gb-closed set is a gi-closed set.

**PROOF:**

Let A be a gb-closed set in the topological ordered space (X, τ, ≤). Then, A is an increasing set and is a g-closed set. Thus, A is a gi-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

**EXAMPLE 3.5.**

Let X = {a, b, c}, $\tau_6$ = {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} and the ordering $\leq_9$ = {(a, a), (b, b), (c, c), (a, c)}. Then, (X, $\tau_6$, $\leq_9$) is a topological ordered space. In this space, the gi-closed sets are ϕ, X, {b}, {c}, {a, c}, {b, c} and the gb-closed sets are ϕ, X, {b},{a, c}. Then, the subset A = {b, c} is a gi-closed set but it is not a gb-closed set.

**THEOREM 3.6.**

Every gb-closed set is a gd-closed set.

**PROOF:**

Let A be a gb-closed set in the topological ordered space (X, τ, ≤). Then A is a decreasing set and is a g-closed set. Thus, A is gd-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

**EXAMPLE 3.7.**

Let X = {a, b, c}, $\tau_6$ = {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} and the ordering $\leq_{10}$ = {(a, a), (b, b), (c, c), (b, c), (c, a), (b, a)}.Then, (X, $\tau_6$, $\leq_{10}$) is a topological ordered space. In this space, the gd-closed sets are ϕ, X, {b}, {b, c} and the gb-closed sets are ϕ, X. Then, the subset A = {b, c}is a gd-closed set but it is not a gb-closed set.

## 4. INDEPENDENT NOTIONS

**THEOREM 4.1.**

The notions g*i-closedness and g*d-closedness are independent.

**PROOF:**

Follows form the following examples.

**EXAMPLE 4.2.**

Let X = {a, b, c}, $\tau_1$ = {ϕ, X, {a}, {b}, {a, b}} and the ordering $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Then, (X, $\tau_1$, $\leq_2$) is a topological ordered space. In this space the g*i-closed sets are ϕ, X, {b, c} and the g*d-closed sets are ϕ, X, {c}, {a, c}. Then, the subset A = {b, c} is a g*i-closed set but it is not a g*d-closed set. On the other hand, the subset B = {a, c} is a g*d-closed set but it is not a g*i-closed set.

**THEOREM 4.3.**

The notions gi-closedness and gd-closedness are independent.

**PROOF:**

Follows form the following examples.

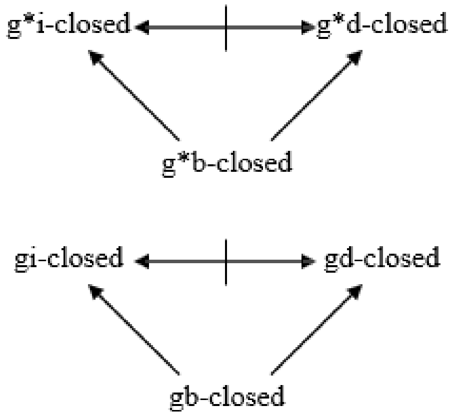
**EXAMPLE 4.4.**

Let X = {a, b, c}, $\tau_1$ = {ϕ, X, {a}, {b}, {a, b}} and the ordering $\leq_2$ = {(a, a), (b, b), (c, c), (a, b), (c, b)}. Then, (X, $\tau_1$, $\leq_2$) is a topological ordered space. In this space the gi-closed sets are ϕ, X, {b, c} and the gd-closed sets are ϕ, X, {c}, {a, c}. Then, the subset A = {b, c} is a gi-closed set but it is not a gd-closed set. On the other hand, the subset B = {a, c} is a gd-closed set but not a gi-closed set.

The following diagram shows the relationships established in the present work.

Here, $A \to B$ means A implies B but not conversely and $A \nleftrightarrow B$ denote A and B are independent notions.

### DIAGRAM




## 5. CONCLUSIONS

In the present work, we established some relationships between g-closed type sets and g*-closed type sets in topological ordered spaces. We also provided examples for the independency of the two types of closedness. As a further study we will focus on the relationships of semi generalized closed type sets with other types of closed sets in topological ordered spaces.

## REFERENCES

[1] M. E. Abd El-Monsef, S. N. El Deeb and R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ., 12(1983),77-90.

[2] D. Andrijević, Semi-pre open sets, Mat. Vesnik, 38(1) 1986, 24-32.

[3] [3]. P. Bhattacharya and B. K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29(3)(1987), 375-382.

[4] K. Krishna Rao, Some concepts in topological ordered spaces using semi-open sets, pre-open sets, $\alpha$-open sets and β-open sets, Ph. D. Thesis, Acharya Nagarjuna University, October, 2012.

[5] K. Krishna Rao and R. Chudamani, $\alpha$-homeomorphisms in topological ordered spaces, International Journal of Mathematics, Technology and Humanities, 52(2012), 541-560.

[6] K. Krishna rao and R. Chudamani, β-Homeomorphisms in topological ordered spaces, international e-Journal of Mathematics and Engineering, 182(2012),

[7] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41

[8] N. Levine, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2) (1970), 89-96.

[9] L. Nachbin, Topology and order, D. Van Nostrand Inc., Princeton, New Jersey, (1965).

[10] O. Njåstad, On some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.

[11] V.V.S. Ramachnadram, B. Sankara Rao and M.K.R.S. Veera Kumar, g- closed type, g*-closed type and sg-closed type sets in topological ordered spaces, Diophantus J. Math., 4(1)(2015), 1-9.

[12] M.K.R.S. Veera Kumar, Homeomorphisms in topological ordered spaces, Acta Ciencia Indica, XXVIII(M)(1)2002, 67-76.

[13] M.K.R.S. Veera Kumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 21(2000), 1-19.