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Computer Applications To The Solution of Inventory Models

I. Deterministic Models

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Contents

Introduction:

The purpose of this note is to illustrate the use of the computers in solving inventory models. Specifically FORTRAN language is used to code these models for use with the IBM 1620 - computer. The theory of the models as well as the methods for using programs are given so that the user will find it convenient to "plug" the data in the appropriate model in order to obtain the results of the program.

Definition of the Inventory problem:

The inventory problem exists in situations where it is necessary to decide upon the amount of certain commodities or goods which should be stocked to meet a certain demand during a specified period of time. The objective is to determine the optimum level of the stock which minimizes the total costs associated with the situation.

Two types of costs are associated with any inventory situation:

1) holding costs.
2) Penalty or set - up costs.

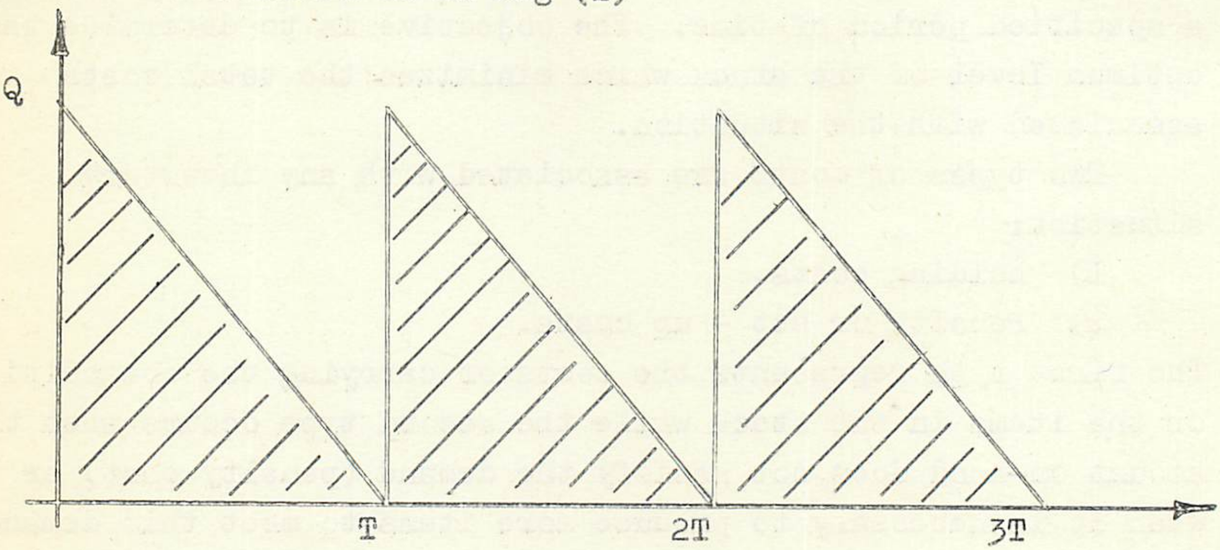
The first type represents the costs of carrying the commodities or the items in the stock while the second type occurs when the amount on-hand does not satisfy the demand (penalty cost) or when it is necessary to produce more items to meet this demand in which case a set-up cost for resetting the production system is alleviated. The results of the analysis should give the optimum level of inventory to be stocked which minimizes the sum of the above-mentioned costs.

The demand for the commodity could be either i) deterministic (i.e, the demand for the given period is known before hand), or ii) probabilistic (i.e., the demand cannot be determined in a definite manner except by a probabilistic distribution function). In this not we shall only consider same of the deterministic models. In a second paper a treatgnent of different probabilistic situation will be given.

DETERMINISTIC MODELS

MODEL I :

The model is theoritical in nature but it gives a basis for introducing more complex models with useful practical applications. Consider the case where the demand occurs at the rate of (D) items / unit time. It is necessary to determine the level of the inventory (Q) for the period of time (T) which will meet this demand. Assume furthes that the production time to produce (Q) items is negligible so that the amount (Q) can be ordered instantaneously. The inventory problem can then be illustrated as in Fig (1)



Let C_1 = holding cost of one unit / unit time

C_2 = set-up cost / run.

Total cost/ unit time=Holding cost + set-up costs

$$C = C_1 \frac{Q}{2} + \frac{C_2}{T} \quad \ldots\ldots\ldots \quad (1)$$

$$= C_1 \frac{DT}{2} + \frac{C_2}{T} \quad \ldots\ldots\ldots \quad (2)$$

Differentiaty (2) w.r.t T and equating to zero gives :

$$\frac{dC}{dT} = C_1 \frac{D}{2} - \frac{C_2}{T^2} = 0$$

$$\text{or } T = \sqrt{\frac{2C_2}{C_1 D}} \qquad \ldots\ldots\ldots \qquad (3)$$

$$\text{or } Q = DT$$

$$= \sqrt{\frac{2C_2 D}{C_1}} \qquad \ldots\ldots\ldots \qquad (4)$$

The resulting corresponding minimum costs is given by :

$$C_{min} = C_1 \times \frac{D}{2} \sqrt{\frac{2C_2}{C_1 D}} + C_2 \times \sqrt{\frac{C_1 D}{CC_2}}$$

$$= \sqrt{2\, C_1\, C_2\, D} \qquad \ldots\ldots\ldots\ldots \qquad (5)$$

In what follows we shall give the necessary program in FORTRAN Language to be prepared for running

into the I.B.M. 1620 computer.

```
C       PRØGRAM FØR INVENTØRY MODEL 1
    1   FØRMAT(3F4. 0)
    2   FØRMAT(/////30HDETERMINISTIC DEMAND - MØDEL 1)
    3   FØRMAT(/22HHØLDING CØST/UNIT C1 = , E11.4)
    4   FØRMAT(20HSET-UP CØST/RUN C2 =, E11.4)
    5   FØRMAT(15HDEMAND RATE D =, E11.4)
    6   FØRMAT(///11HMINIMUM Q =, E11.4)
    7   FØRMAT(16HMINIMUM CØST C =,E11.4)
    8   FØRMAT(//7HPRØBLEM, I3)
        I=1
   10   READ1,C1,C2,D
        TMIN=SQRTF(2.*C2/(C1*D))
        QMIN=D*TMIN
        CMIN=SQRTF(2.*C1*C2*D)
        PRINT2
```

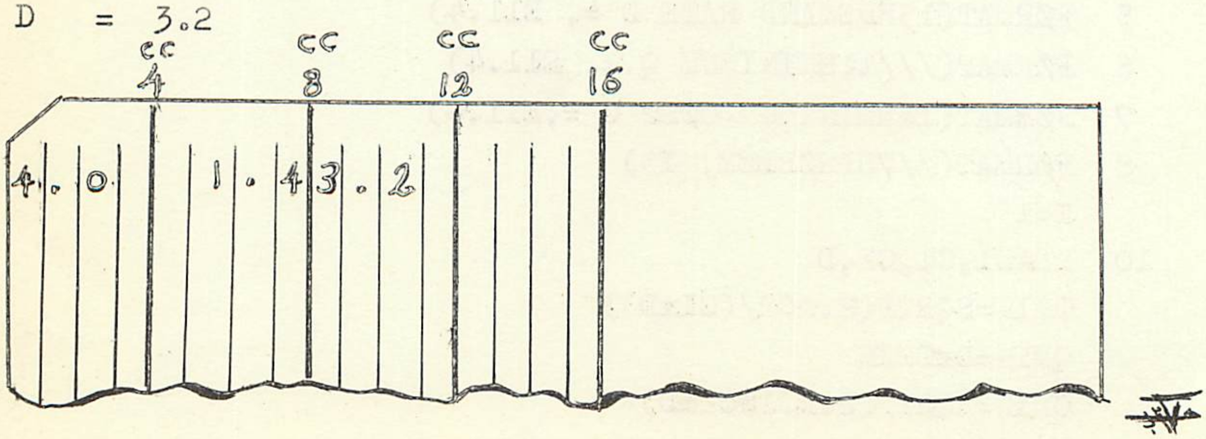
```
PRINT8,I
PRINT3,C1
PRINT4,C2
PRINT5,D
PRINT6,QMIN
PRINT7,CMIN
I=I+1
GØ TØ 10
END
```

Preparation of Input Data for Model I.

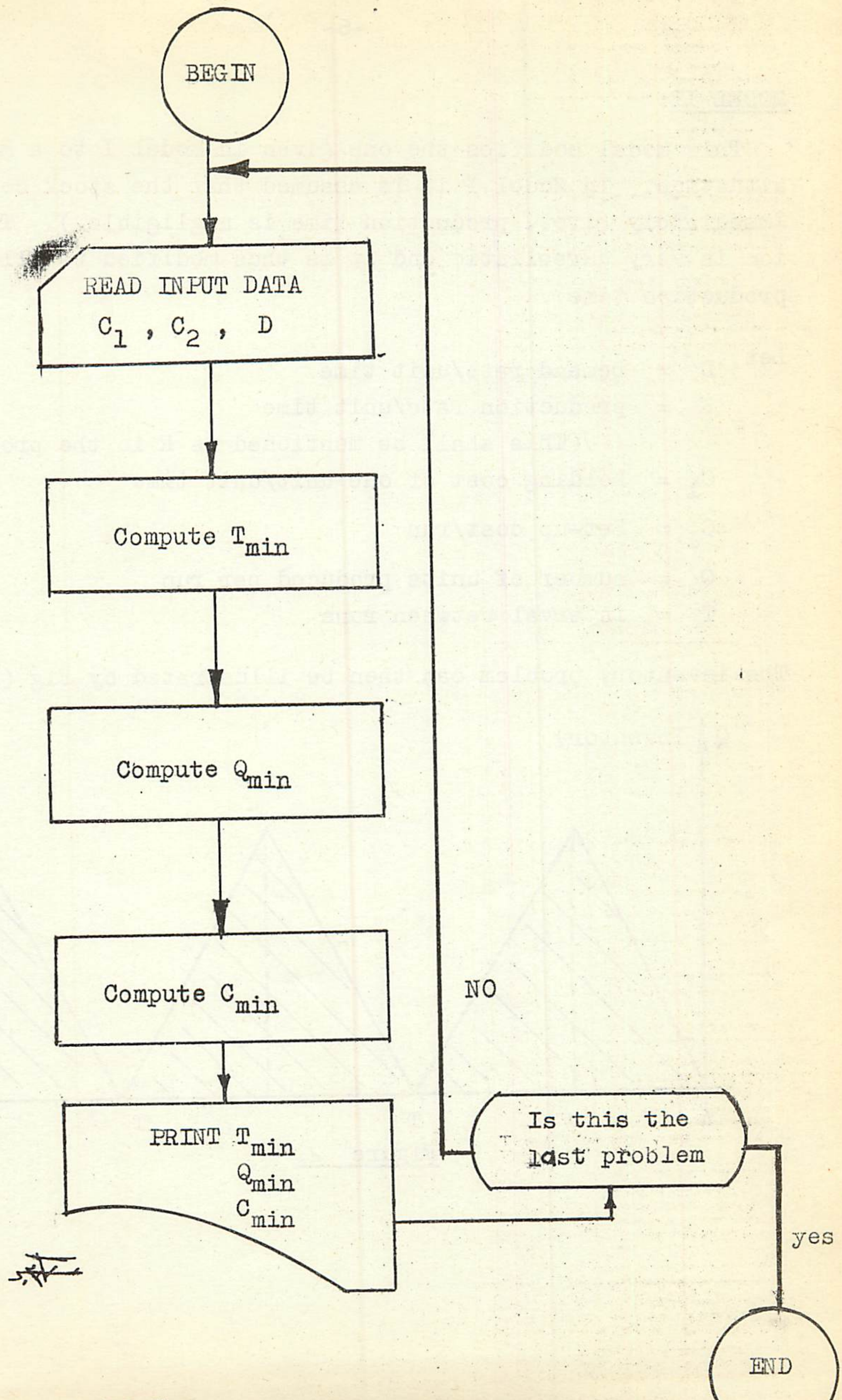
This program can handle more than one problem in the same run. For every problem punch the input data on one card in the following order C1, C2, D according to the FORMAT (3F 4.0). This means that the three input parameters C1, C2 and D will each occupy a field of 4 columns. These columns must come in consequitive order.

Example:

C1 = 4.0
C2 = 1.4
D = 3.2



Example of the computer input data punched on an IBM card.



Flow Chart for Model I.

MODEL II:

This model modifies the one given in Model I to a more practical situation. In Model I it is assumed that the stock could be filled immediately (i.e., production time is negligible.). This assumption is very unrealistic and it is thus modified to allow for the production time.

Let
- D = Demand rate/unit time
- K = production rate/unit time
  (This shall be mentioned as R in the program.)
- C_1 = holding cost of one unit/unit time
- C_2 = Set-up cost/run
- Q = number of units produced per run
- T = interval between runs

The inventory problem can then be illustrated by Fig (2).

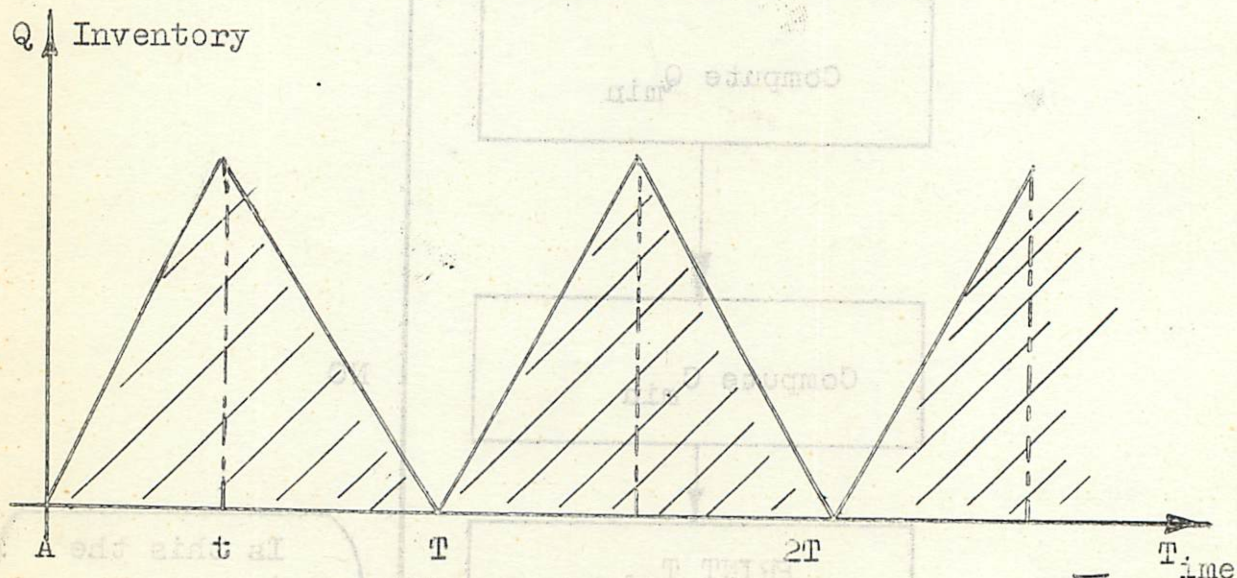


Figure 2.

Before presenting the cost equation, it is noted from Figure (2) that the slope of the line AB is given by the difference between the rate of production and the rate of consumption, i.e. (K-D). The slope of the line BT is also equal to (-D) by assumption. From Figure (2), production continues for a time $t = \frac{Q}{K}$ and the period of the entire inventory cycle is given by $T = \frac{Q}{D}$. Hence:

Total Holding costs/cycle

$$= T. \frac{t\,(K - D)}{2} \quad C_1$$

$$= \tfrac{1}{2}\, C_1 \; tT(K - D)$$

$$= \tfrac{1}{2}\, C_1 \; \frac{DT^2}{K} \; (K - D)$$

$$= \tfrac{1}{2}\, C_1 \; TQ \left(1 - \frac{D}{K}\right) \quad \ldots \qquad (6)$$

Total cost/unit time = Holding cost/unit time + set cost/Run

$$C = \tfrac{1}{2}\, C_1 \; Q \,(1 - D/K) \; + \; C_2/T$$

$$= \tfrac{1}{2}\, C_1 \; Q \left(1 - \frac{D}{K}\right) \; + \; C_2 \frac{D}{Q} \quad \ldots. \qquad (7)$$

Differentiating w.r.t. (Q) and equating to zero, gives:

$$\frac{dC}{dQ} = \frac{C_1}{2} \left(1 - \frac{D}{K} \right) - \frac{C_2 \;\; D}{Q_2} = 0$$

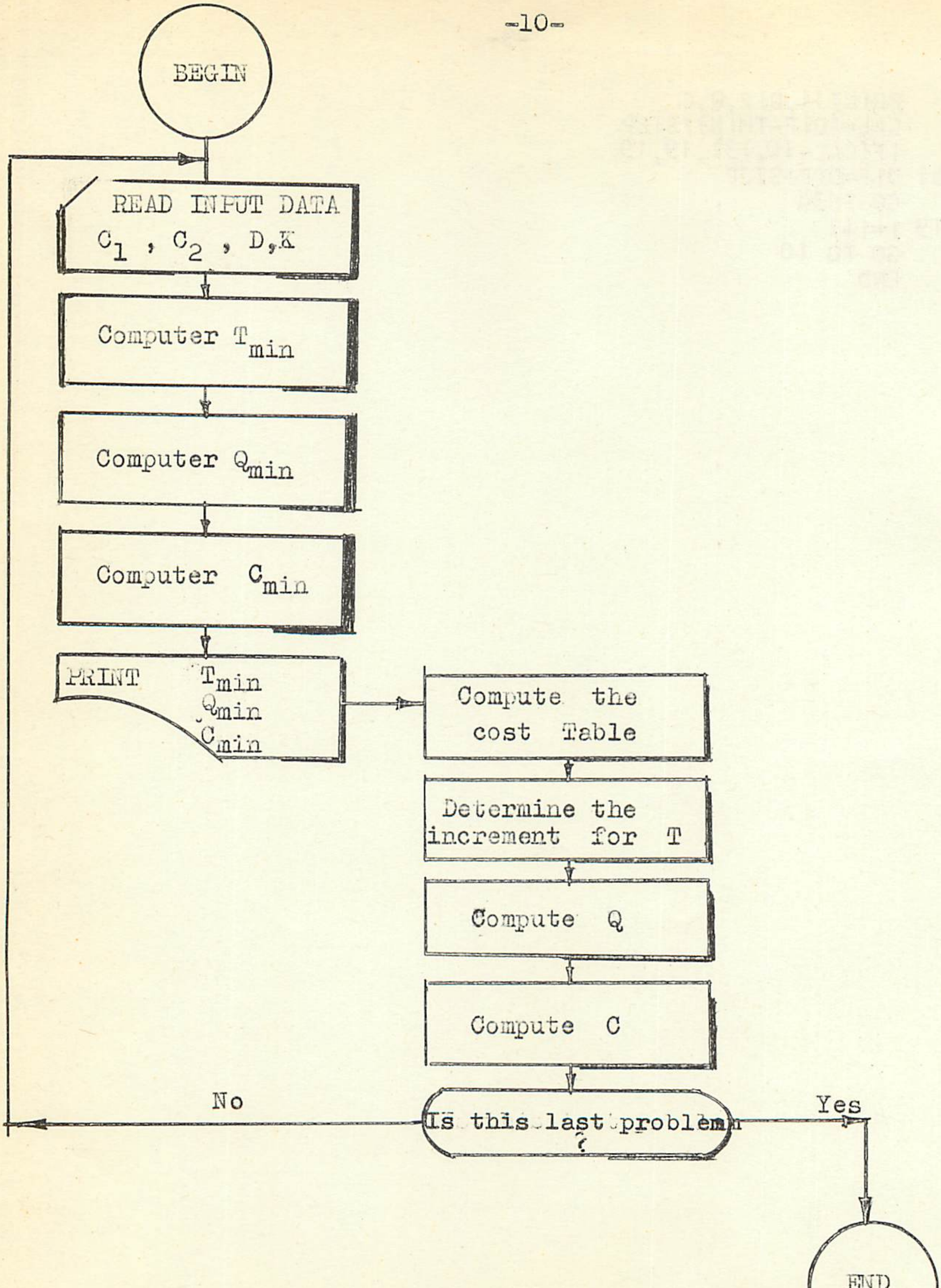
$$Q = \sqrt{\frac{2\, C_2 \;\; D}{C_1(1 - D/K)}} \quad \ldots\ldots. \qquad (8)$$

and $T = Q/D$

$$= \sqrt{\frac{2C_2}{D\, C_1(1 - D/K)}} \quad \ldots\ldots. \qquad (9)$$

```
C      PROGRAM FOR INVENTORY MODEL 2
  1 FORMAT(4F4.0)
  2 FORMAT(/////31HDETERMINISTIC DEMAND - MODEL 2.)
  3 FORMAT(/22HHOLDING COST/UNIT C1 =,E11.4)
  4 FORMAT(20HSET-UP COST/RUN C2 =,E11.4)
  5 FORMAT(15HDEMAND RATE D =,E11.4)
  9 FORMAT(19HPRODUCTION RATE K =,E11.4)
  6 FORMAT(///11HMINIMUM Q =,E11.4)
  7 FORMAT(16HMINIMUM COST C =,E11.4)
  8 FORMAT(//7HPROBLEM,I3)
    I=1
 10 READ1,C1,C2,D,R
    QMIN=SQRTF(2.*C2*D/(C1*(1.-(D/R))))
    TMIN=QMIN/D
    CMIN=.5*C1*QMIN*(1.-(D/R))+C2*D/QMIN
    PRINT2
    PRINT8,I
    PRINT3,C1
    PRINT4,C2
    PRINT5,D
    PRINT9,R
    PRINT6,QMIN
    PRINT7,CMIN
 11 FORMAT(3E20.6)
 12 FORMAT(//7X1HT,20X1HQ,20X1HC)
 13 FORMAT(20X10HCOST TABLE)
    IF(TMIN-5.)20,20,21
 20 IF(TMIN-4.)22,22,23
 22 IF(TMIN-3.)24,24,25
 24 IF(TMIN-2.)26,26,27
 26 IF(TMIN-1.)29,29,27
 21 STEP=1.
    DIF=TMIN-5.
    GO TO 30
 23 STEP=1.
    DIF=TMIN-4.
    GO TO 30
 25 STEP=.5
    DIF=TMIN-3.
    GO TO 30
 27 STEP=.25
    DIF=TMIN-2.
    GO TO 30
 29 STEP=.125
    DIF=TMIN-1.
 30 PRINT13
    PRINT12
 34 Q=DIF*D
    C=.5*C1*Q*(1.-(D/R))+C2*D/Q
```

```
      PRINT11,DIF,Q,C
      CAL=(DIF-TMIN)/STEP
      IF(CAL-10.)31,19,19
   31 DIF=DIF+STEP
      GO TO34
   19 I=I+1
      GO TO 10
      END
EN
```



Flow Chart for Model II

Illustrative Solved Example

To illustrate how the computer gives the final results of the problem, we give here an example. Two different sets of values for C_1 , C_2 & K are used:

DETERMINISTIC DEMAND - MODEL 2.

PROBLEM 1

HOLDING COST/UNIT C1 = 1.0000E-00
SET-UP COST/RUN C2 = 3.0000E-00
DEMAND RATE D = 4.0000E-00
PRODUCTION RATE K = 5.0000E-00

MINIMUM Q = 1.0954E+01
MINIMUM COST C = 2.1908E-00

COST TABLE

T	Q	C
73.861270E-02	29.544508E-01	43.571135E-01
98.861270E-02	39.544508E-01	34.300003E-01
12.386127E-01	49.544508E-01	29.175096E-01
14.886127E-01	59.544508E-01	26.107442E-01
17.386127E-01	69.544508E-01	24.209586E-01
19.886127E-01	79.544508E-01	23.040343E-01
22.386127E-01	89.544508E-01	22.355606E-01
24.886127E-01	99.544508E-01	22.009359E-01
27.386127E-01	10.954450E+00	21.908902E-01
29.886127E-01	11.954450E+00	21.992552E-01
32.386127E-01	12.954450E+00	22.217676E-01
34.886127E-01	13.954450E+00	22.553857E-01
37.386127E-01	14.954450E+00	22.978817E-01
39.886127E-01	15.954450E+00	23.475862E-01
42.386127E-01	16.954450E+00	24.032237E-01
44.886127E-01	17.954450E+00	24.638029E-01
47.386127E-01	18.954450E+00	25.285417E-01
49.886127E-01	19.954450E+00	25.968146E-01
52.386127E-01	20.954450E+00	26.681157E-01

DETERMINISTIC DEMAND - MODEL 2.

PROBLEM 2

HOLDING COST/UNIT C1 = 2.0000E-00
SET-UP COST/RUN C2 = 4.0000E-00
DEMAND RATE D = 5.0000E-00
PRODUCTION RATE K = 7.0000E-00

MINIMUM Q = 8.3666E-00
MINIMUM COST C = 4.7809E-00

COST TABLE

T	Q	C
-32.668000E-02	-16.334000E-01	-12.711083E+00
-76.680000E-03	-38.340000E-02	-52.274382E+00
17.332000E-02	86.660000E-02	23.326298E+00
42.332000E-02	21.166000E-01	10.053859E+00
67.332000E-02	33.666000E-01	69.025973E-01
92.332000E-02	46.166000E-01	56.512211E-01
11.733200E-01	58.666000E-01	50.853011E-01
14.233200E-01	71.166000E-01	48.436450E-01
16.733200E-01	83.666000E-01	47.809144E-01
19.233200E-01	96.166000E-01	48.273372E-01
21.733200E-01	10.866600E+00	49.452450E-01
24.233200E-01	12.116600E+00	51.125138E-01
26.733200E-01	13.366600E+00	53.152955E-01
29.233200E-01	14.616600E+00	55.444788E-01
31.733200E-01	15.866600E+00	57.938239E-01
34.233200E-01	17.116600E+00	60.589136E-01
36.733200E-01	18.366600E+00	63.365333E-01
39.233200E-01	19.616600E+00	66.242877E-01
41.733200E-01	20.866600E+00	69.203555E-01