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PROGRAMMING TECHNIQUES AND ECONOMIC
POLICY MODELS

By

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Programming Techniques and Economic Policy Models.

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The anther wishes to express his gratitudes to the I.N.P. for giving this paper the chance to see light. Dr. El-Houdari gave beimdly of his valuable time for the reviesion, he deserves lots of appreciation and acknoweledgment. However, all criticism must be forward to the writer.

PROGRAMMING TECHNIQUES
AND
ECONOMIC POLICY MODELS

Contents

Chapter I

> "By 'economic policy' certain arts of economic behavior are indicated. In its broadest sense, therefore, the phase includes the whole subjectmatter of economic theory."
>
> J. Tinbergen

Introduction

In this paper the potentiality of the utilization of programming techniques in economic policy models is the core of the analysis. A brief review of the theory of economic policy and the difference, in methodology and logic, between it and the general economic theory is under focus. Then the use of programming technique in policy models will be studied along with the limitations on its usage. A modification in the policy problem might be done such that a programming technique can be appropriate, is the main emphasis in this paper. Some arbitrary examples are viewed and different types of programming are utilized.

The author believes that if programming is utilized in solving, for the optimum policy, significant economic interpretations can be drawn in viewing the policy problem when stated in programming formulations. Besides, the determination of the solution and its sensitivity will be more specific and accurate.

Chapter II

On The Theory of Economic Policy

In this chapter two concepts will be under focus. First, an introduction to the theory of economic policy as specified by J. Tinbergen. Secondly, a distinction between the theory of economic policy and the general economic theory is drawn.

A. Introduction to the theory of economic policy:

Three major analytical framework specifications can be specified as the base for the theory of economic policy:

a) Characterization of the policy problem

Specification of the preference function, the quantitive model, and the constraints or boundary conditions.

b) Selection problem

Classification of variables by their properties, like randomness and controllability and time dependence, etc.

c) Steering problem

These are the derivation of optimum decision rules in a static and dynamic sense, flexibility of optimal decisionmaking under changing conditions due to risk, and uncertainty and the sequence of new information.

Tinbergen's set up distinguishes between two types of policies:

a. fixed targets policy

b. flexible targets policy

The first deals with policies where targets are specified at a given level which is fixed. The second deals with problems where the target has to be optimized (i.e. either maximized or minimized).

In either of the two analytical framework, four major variables should be classified and specified clearly at the beginning; these variables are:

a. Exogenous - (i) Instrument Variables
(ii) Other Data

b. Endogenous -(iii) Target Variables
(iv) Irrelevant Variables

Among these variables are considered as "given" the "target variables" and the "other data"; as unknowns, the instrument variables" and the irrelevant variables"

For the fixed targets policy models there are some characteristics which have to be under consideration. For this point to be clear, let us assume this simple linear system, specified in matrix notations:

(1) $Ay = BZ + \Gamma U$

(2) $BZ = \Gamma U - Ay = \gamma_o$

(3) $\therefore Z = B^{-1} \gamma_o$

where; B, A, and Γ are matrices of coefficients, and of appropriate order.

$\mathcal{Z}$ = instrument variables vector,

y = target variables vector,

U = data variables vector, and

γ_o = (Γu - Ay); by definition.

This solution will only be determinate if the border matrix (B γ_o) has a rank of sufficient height. That is:

(4)
$$\begin{vmatrix} B_{11} & B_{12} & \dots B_{1k} & \gamma_{o1} \\ B_{21} & B_{22} & \dots B_{2k} & \gamma_{o2} \\ \vdots & \vdots & \vdots & \vdots \\ B_{k1} & B_{k2} & \dots B_{kk} & \gamma_{ok} \end{vmatrix} \neq 0 \quad \dots (4)$$

(5) and $\rho\,(B\ \gamma_o)$ = "k", where ρ = rank

If this requirement is not fulfilled, then we have inconsistency.

If the rank of (B γ_o) = k - r, then the number of instruments that may be chosen freely amounts to r.

"This rigidity is loosened in the case of flexible target policy; since the number of relations adapts itself to the number of unknowns'.*

Again, let us assume the same previous example as:

$$Ay = B\mathcal{Z} + \Gamma U$$

* Tinbergen (18), P.7.

Here we would like to optimize Ω (e.g., maximize Ω) which is a "social welfare function", and assumed to depend on both targets and instruments:

(6) i-e $\Omega = f_{,}\ (y_j, z_k)$

where y = vector of targets

z = vector of instruments

$j = 1, \ldots, J$

$k = 1, \ldots, K$

There are different approaches for the solution. One of them might be as follows:

a. express y in terms of z and substitute in Ω, thus, we will have $\bar{\Omega} = f_2(z)$.

b. by taking the derivatives of $\bar{\Omega}$ with respect to z's and equate to zero, the optimum can be solved for.

(7) $$\frac{\partial \Omega}{\partial z_k} = 0; \quad (k = 1, \ldots, K)$$

or from (6);(8) $$\sum_{j=1}^{J} \frac{\partial \Omega}{\partial y_j} \frac{\partial y_j}{\partial z_k} + \frac{\partial \Omega}{\partial z_k} = 0; \quad (k=1, \ldots, K)$$

Now as Tinbergen implies the number of optimum (max. or min.) conditions will simply be equal to the number of z's. If there are few z's, a less advantageous situation can be reached. Graphically this can be illustrated as follows:

Then the boundary conditions play a vital role in the determination of the solution. For Ω to be, say, maximized, then the positive coefficient x's and positive coefficient y's will be chosen as large as possible and the ones corresponding with negative coefficients as small as possible. The word "possible" here is mainly function of the boundary conditions.

An explicit and direct programming technique can be used here. However, programming gives more flexibility with respect to the number of targets and instruments. The most important criterion is the objectivity of seting up the problem.

B. <u>Some Differences in Analytical Approach Between Theory of Economic Policy and the General Economic Theory</u>

The analytical framework of the theory of economic policy does differ from that of the general economic theory. Some of these difference-in logic, methodology and set up of the problem are under focus here:

(1) The knowns and the unknowns variables are not the same in both types of study. While targets are known and a solution for the optimum instrument's levels is looked for in the theory of economic policy, it is not the same thing in the general economic theory. This is a crucial difference, since it leads to different methodology and different set up of the analytical framework in both.

(2) As mentioned above the "steering problem" deals with the specification of the optimum solution and the sensitivity analysis approach and is a distinct feature of the theory of economic policy, which

distinguishes it from the usual forecasting and ordinary economitric models in the general economic theory set up.

(3) The reliability of the optimum policy, the flexibility of the suboptimal policy, ... etc. are points of interest, where theories of statistical decision functions, process control and information sequences are used intensively in the theory of economic policy.

(4) Another distinctive feature of the theory of economic policy is the study of the problem of decomposition of an overall model into a set of sub-models. The concept of recursiveness plays a very significant role in this scheme of decomposition.

From the foregoing treatments, a general outline of the theory of economic policy as well as some of the differences in logic and analytical approach between the theory of economic policy and the general economic theory were specified.

It is, however, clear that the set up of the policy problem implies a welfare function to be optimized subject to some specified relation-ships constructing what is known as the model.

The aim of this study is to try to use the programming techniques as a methodology tool in solving some of the economic policy problems. Not only the optimum solution determination is looked for but also the sensitivity analysis, the variable targets analysis, shadow price and recursiveness criteria are all of great interest in this paper.

Chapter III

PROGRAMMING AND ECONOMIC POLICY MODELS

Three major points are under consideration in this chapter. The usage of programming techniques in economic policy, the limitations and its validities, and some of the recent techniques which are useful for quantitative policy models.

(i) Programming usage in economic policy models

(1) The first possible explicit usage of programming in economic policy models is through the boundary conditions. This point is well explained in the chapter to come.

(2) The second important use of programming is in the area of sensitivity analysis . The extent of the response of the optimum solution to a slight variation in the coefficients of the problem. The levels of the boundary conditions on the targets may be treated as variable. And, hence a set of optimum solutions at different levels of targets can be solved for. Also a variation in the weights of the objective function can be analysied too. Sensitivity analysis can be quantitativly studied through programming.

(3) A third use of programming in a policy model is that it permits the specification of the shadow prices. A shadow price indicates how the objective function is affected, as one unit change takes place in a scarce resource.

(ii) <u>Some limitations and its validites</u>

In the literature on economic policy analysis, some limitations in the use of programming techniques for solving a policy problem are indicated. These limitations are viewed and analyzed as follows:

(1) "The optimal solution cannot in general be analytically stated but only numerically." However, the auther does not belive in the validity of this argument. Each problem in economic policy is a special case, and, hence, it has a special formulation and a special solution. Thus, as long as the solution procedure is known, the problem can be solved. Besides in the programming techniques each term has a significant economic meaning. For instance, in the simplex tablue we can see marginal rates of substitution, net revenues, marginal value productivities, ... , etc. Thus, from the analytical viewpoint, programming may help in visulizing some economic relationships and interpretation for many variables and interactions.

(2) "The methods of programming other than the linear a are complicated." This statement is valid for some extent, however today many complicated procedures are used in economic policy problems (e.g., Pontyriagan principle).

(3) Linearity of linear programming is a bottleneck. This is a real limitation especially for policy models involving economic development and planning in developing nations.

(4) Chincry[1] has emphasized: "The principle obstacle to the adoption of the programming solution is the lack of information and the high cost of increasing it." Fox, Thorbick, and Singupta suggested in this respect that this obstacle exaggerates the need perhaps for combining programming techniques with other tools like partial models or even individual project-mix analysis, in an overall policy question like development planning, which is essentially dynamic in its characteristics at different phases. They added that, moreover, there are:

(iii) Some recent techniques which are useful for quantitative policy models as:

(1) General method of sensitivity analysis, which ranges from stochastic linear programming to the formulation of the stability criteria for the set of optimal solution.

(2) Recursive and multistage programming, which allows variations of optimum policy at different stages.

(3) Methods which characterize, although not completely optimizing situations where instead of a scalar objective function a vector function is optimized.

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[1] Chincray (25), P. 11-27

Chapter IV.

ECONOMIC POLICY MODELS IN PROGRAMMING SET UP

> "The central question of economic policy is the question of the effectiveness of its various instruments. In fact the controversial issues in practical and scientific discussion all centers around that problem".
>
> J. Tinbergen

In the previous analysis the role to be played by programming techniques as methodological tool in solving a policy model has been emphasized. How can an economic policy model be modified in a way such that programming techniques can be used in finding (a) the optimum solution and (b) some sensitivity criteria of it?

The author feels that there is no standard answer to the above question. Each policy problem can be considered as a special case. However, if the policy maker has a sufficient knowledge about the dimensions of the problem on one hand and the essence of programming techniques on the other hand he is able to make to appropriate modification.

In this chapter the following points are of interest: (1) an arbitrary example is shown where simple linear programming procedure is used, (2) the implication of some parametric programming in economic policy models are under focus, (3) recursive programming and policy models, and (4) an introduction to stochastic linear programming and its implication in sensitivity analysis.

(1) <u>An Agricultural Model in Programming Set Up</u>

"An arbitrary Example"

Three sectors are specified in this hypothetical analysis. They are: Grain Sector, Fiber Sector, and Livertock Sector.

Six targets are specfied in this study. A minimum boundary condition is set on each:

1. Grain production
2. Fiber production
3. Livestock production
4. Income from Grain
5. Income from Fiber
6. Income from Livestock

Six major instruments are indicated which are:

1. Fertilizer production
2. Fertilizer imports
3. Mechinary production
4. Machinery imports
5. Land Reclamation
6. Land irrigation

A set of activities and restrictions are specified, such that the optimum feasible solution gives indirectly the levels of the instruments. The programming procedure is designed such that the targets are implicily satisfied if the solution is optimum and feasible. This was easily done by introducing the targets as some of the restrictions in the simplex tablue.

A hypothetical input-output tablue, for the given below set of activities and restrictions, is given in the appendix.

The restirictions and the activities specified are:

Restrictions

1. Capital (cap.) b_1
2. Agricultrual labor (Agr. L.), b_2
3. Industrial labor (I. L.), b_3
4. Original land (O.L.) , b_4
5. New land (N. L.) , b_5
6. New irrigated land (N. I.), b_6
7. Fertilizer production (F. P.), b_7
8. Fertilizer imports (F. I.), b_8
9. Machinery productions (M.P.), b_9
10. Machinery imports (M. I.) , b_{10}
11. Minimum grain production (at least Gp) (Min. Gp.) b_{11}
12. Minimum fiber production (at least Rp)(Min.Rp.) b_{12}
13. Minimum livestock production (at least Vp)(Min.Vp) b_{13}
14. Minimum income in grain sector (at least G_I)(Min. G_I). b_{14}
15. Minimum income in fiber sector (at least R_I)(Min. R_I), b_{15}
16. Minimum income in livertock sector (at least V_I)(Min.V_I)b_{16}
17. Fertilizer maximum production (Max. F. P.) b_{17}
18. Fertilizer maximum imports (Max. F. I.) b_{18}
19. Machinery maximum products (Max. M. P.) b_{19}
20. Machinery maximum imports (Max. M. I.) b_{20}
21. Fertilizer fransformation (F. T.) b_{21}
22. Machinery transformation (M.T.) b_{22}
23. Maximum new land (Max. N.L.) b_{23}

Activities

*1. Gp[1] with fertilizer on original land
2. Gp with fertilizer on new land
3. Gp with fertilizer on irrigated land
4. Gp without fertilizer on original land
5. Gp without fertilizer on new land
6. Gp without fertilizer on irrigated land
7. Rp[2] with fertilizer on original land
8. Rp with fertilizer on new land
9. Rp with fertilizer on irrigated land
10. Rp without fertilizer on original land
11. Rp without fertilizer on new land
12. Rp without fertilizer on irrigated land
13. Gp using moder machinery on original land
14. Gp using modern machinery on new land
15. Gp using modern machinery on irrigated land
16. Gp without modern machinery on original land
17. Gp without modern machinery on new land
18. Gp without modern machinery on irrigated land
19. Rp using modern machinery on original land
20. Rp using modern machinery on new land
21. Rp using modern machinery on irrigated land
22. Rp without modern machinery on original land
23. Rp without modern machinery on new land
24. Rp without modern machinery on irrigated land.

---
(1) Gp. Grain produced , (2) Rp: Fiber produced

25. Vp[1] using modern machinery on original land

26. Vp using modern machinery on new land

27. Vp without modern machinery on original land

28. Vp without modern machinery on new land

29. Fertilizer production

30. Fertilizer imports

31. Machinery production

32. Machinery imports

33. New land development

34. New land irrigation

35. Fertilizer transformation

36. Machinery transformation

The objective function is set up such that the above system will be all connected and workable. Let the objective function be one of minimizing the government expenditure subject to the restrictions specified above.

Thus:

The objective function is:

$$\text{Minimize:} \quad = \sum_{j=1}^{36} S_j X_j$$

where $j = 1, \ldots , 36$

Given table (1) in the appendix, which shows the input-output relationship, the following restriction functions will be appropriate: i.e. optimize the objective function subject to:

---

(1) Vp: Livestock produced

1. $\sum_{j=1}^{34} \alpha_{1,j} \; x_j \leq b_1$

2. $\sum_{j=1}^{28} \alpha_{2,j} X_j + \sum_{j=33}^{34} \alpha_{2,j} \; x_j = b_2$

3. $\alpha_{3,29} X_{29} + \alpha_{3,31} X_{31} + \sum_{j=33}^{34} \alpha_{3,j} X_j = b_3$

4. $\alpha_{4,1} X_1 + \alpha_{4,4} X_4 + \alpha_{4,7} X_7 + \alpha_{4,10} X_{10}$
$+ \alpha_{4,13} X_{13} + \alpha_{4,16} X_{16} + \alpha_{4,19} X_{19} + \alpha_{4,22} X_{22}$
$+ \alpha_{4,25} X_{25} + \alpha_{4,27} X_{27} = b_4$

5. $\alpha_{5,2} X_2 + \alpha_{5,5} X_5 + \alpha_{5,8} X_8 + \alpha_{5,11} X_{11} + \alpha_{5,14} X_{14}$
$+ \alpha_{5,17} X_{17} + \alpha_{5,20} X_{20} + \alpha_{5,23} X_{23} + \alpha_{5,26} X_{26}$
$+ \alpha_{5,28} X_{28} - \alpha_{5,33} X_{33} = 0$

6. $\alpha_{6,3} X_3 + \alpha_{6,6} X_6 + \alpha_{6,9} X_9 + \alpha_{6,12} X_{12} + \alpha_{6,15} X_{15}$
$+ \alpha_{6,18} X_{18} + \alpha_{6,21} X_{21} + \alpha_{6,24} X_{24} - \alpha_{6,34} X_{34} = 0$

7. $\alpha_{7,35} X_{35} - \alpha_{7,29} X_{29} = 0$

8. $\alpha_{8.35} X_{35} - \alpha_{8,30} X_{30} = 0$

9. $\alpha_{9,36} X_{36} - \alpha_{9,31} X_{31} = 0$

10. $\alpha_{10,36} X_{36} - \alpha_{10,32} X_{32} = 0$

11. $\sum_{j=1}^{6} \alpha_{11,j} X_j + \sum_{j=13}^{18} \alpha_{11,j} X_j \geq b_{11}$

12. $\sum_{j=7}^{12} \alpha_{12,j} X_j + \sum_{j=19}^{24} \alpha_{12,j} X_j \geq b_{12}$

13. $\sum_{j=25}^{28} \alpha_{13,j} X_j \geq b_{13}$

14. $\sum_{j=1}^{6} \alpha_{14,j} X_j + \sum_{j=13}^{18} \alpha_{14,j} X_j \geq b_{14}$

15. $\sum_{j=7}^{12} \alpha_{15,j} X_j + \sum_{j=19}^{24} \alpha_{15,j} X_j \geq b_{15}$

16. $\sum_{j=25}^{28} \alpha_{16,j} X_j \geq b_{16}$

17. $\alpha_{17,29} X_{29} \leq b_{17}$

18. $\alpha_{18,30} X_{30} \leq b_{18}$

19. $\alpha_{19,31} X_{31} \leq b_{19}$

20. $\alpha_{20,32} X_{32} \leq b_{20}$

21. $\sum_{j=1}^{3} \alpha_{21,j} X_j + \sum_{j=6}^{8} \alpha_{21,j} X_j - \alpha_{21,35} X_{35} = 0$

22. $\sum_{j=13}^{15} \alpha_{22,j} X_j + \sum_{j=19}^{21} \alpha_{22,j} X_j + \sum_{j=33}^{34} \alpha_{22,j} X_j$
$+ \sum_{j=25}^{26} \alpha_{22,j} X_j - \alpha_{22,36} X_{36} = 0$

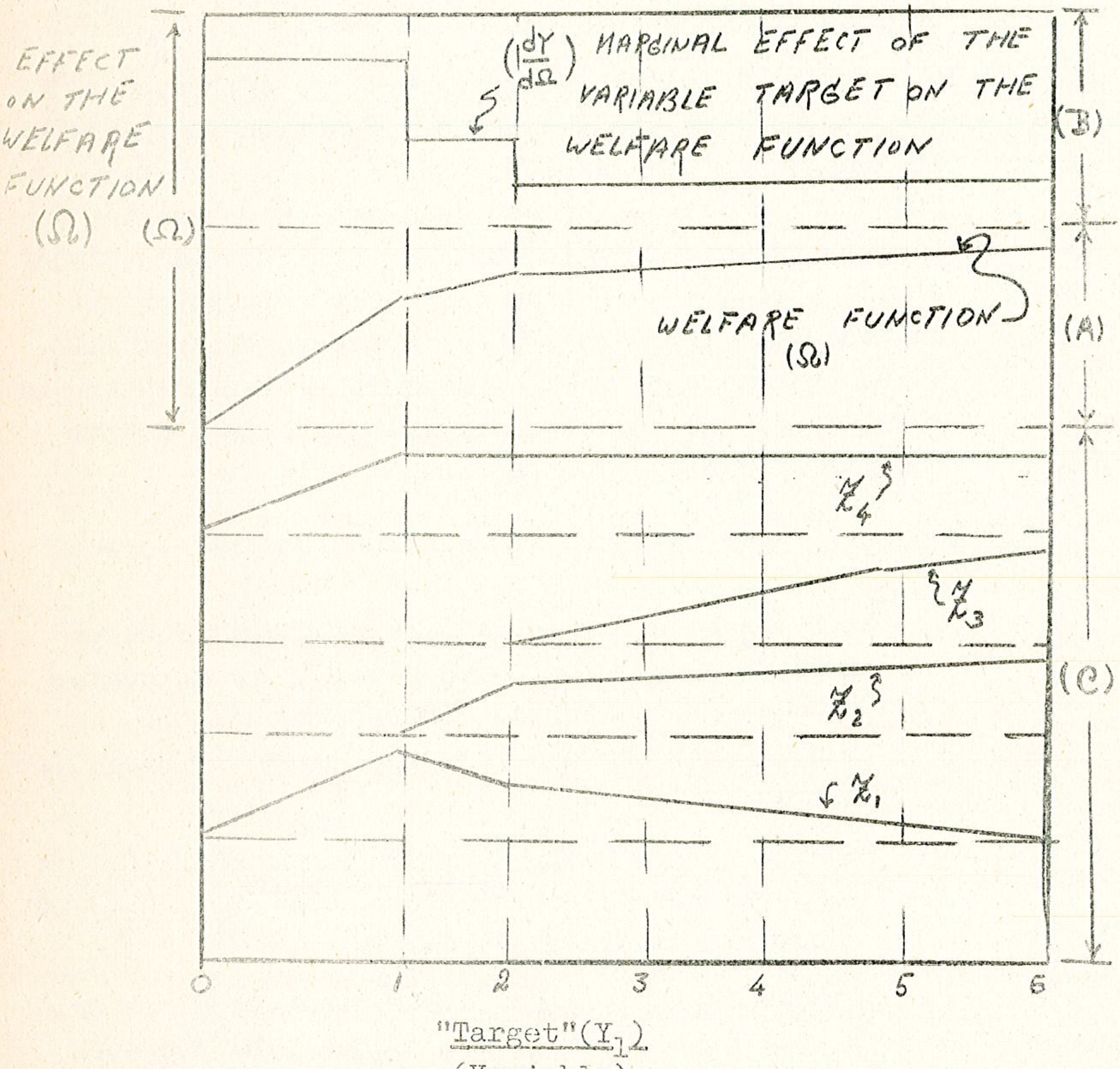
23. $\sum_{j=33}^{34} \alpha_{23,j} X_j \leq b_{23}$

* and $X_j \geq 0$, where $(j = 1, \ldots, 36)$

In the set of the above problems an implicit assumption of a planned economy was drawn. In such planned economy a specified employment level is set up. This appears in the restrictions (2,3) Also a full use of land is a restriction. The rest of the productions and the activities are chosen such that the problem becomes realistic (as much a s possible).

Now, given the objective function, the input-output coefficients, the weight coefficients in the objective function, the slack and artificial variables, the simplex procedure can be used to find the optimum solution. Forcing the system to use the labor force and the land, then by minimizing the government expenditure an optimum solution can be achieved. In this optimum solution we will achieve <u>at least</u> the levels of the restrictions values are positive indicating that an optimum plan has been determined, and increasing the variable factor by any amount after this level will not contribute in optimizing the objective function.

A GRAPHIC PRESENTATION:



Figure(2): Continous set of optimum plans within the given range of a variable target.

Discussion:

(1) From such a diagram, an optimum plan for any target level within the specified range can be determined. That is, in a sense, it is a continous set of optimum plans within the given range.

(2) In section "A" of the above graph, the effect of changing the target level on the welfare function is specified. It happens in this example that the slope of this function is decreasing as the level of the variable target increases. This implies that marginal effect of the variable target in the welfare function is decreasing. This fact too is indicated in part (B) of the same diagram. The economic interpretation of this example is that this production system folows the low of diminsihing returns.

(3) Economic interpretation of the "instruments" curves (section C):

a. Between (o) and (1) (these are arbitrary figures implies only that 1 > 0) both Z_1 and Z_4 curves have positive slopes. Therefore, the two instruments are complementary over the specified target range.

b. Between (1 and 2) Z_4 has zero slope while Z_2 has positive slope, the two "programs"(Z_4 , Z_2) are supplementary in this range.

c. Beyond (2), Z_1 , curve has a negative slope while Z_3 has a positive slope. Hence, the two are competitive.

d. (Z_1 and Z_3) being competitive, the ratio of their slopes defines the marginal rates of substitution between these instruments over "target" ranges. i.e.,
$$\left\{ MRS \text{ of } Z_1 \text{ for } Z_2 = \frac{\Delta Z_1}{\Delta Z_2} = \frac{\text{Slope of } Z_1 \text{ Curve}}{\text{" " } Z_2 \text{ "}} \right\}$$

(b) <u>Prgoramming With Variable Weights of the Objective Function:</u>

* This method allows us to determine the weight range over which a particular plan is optimum and stable. The optimum plan for a "given objective function weights" is determined, then we ask what weights change in necessary to cause another plan to be optimum?

To answer this question the following derivations are essential: We have:

$$(Z_j - C_j) = \sum_{i=1}^{m} c_i r_{ij} \underset{m-1}{-} c_j$$
$$(Z_j - C_j) = c_h r_{hj} + \sum_{i=1} c_i r_{ij} \underset{m-1}{-} c_j$$
$$(Z_j - C_j)' = (c_h + \Delta c_h) r_{hj} \underset{m}{+} \sum_{i=1} c_i r_{ij} - c_j$$
$$(Z_j - C_j)' = \Delta c_h r_{hj} + \sum_{i=1}^{m} c_i r_{ij} - c_j$$
$$(Z_j - C_j)' = \Delta c_h r_{hj} + (Z_j - C_j)$$

For $(Z_j - C_j)$ to be non-negative:
$$(Z_j - C_j)' = \Delta c_h r_{hj} + (Z_j - C_j) \leq 0$$
$$\therefore \left[\Delta c_h \geq (Z_j - C_j) / - r_{hj} \right] \quad \ldots (13)$$

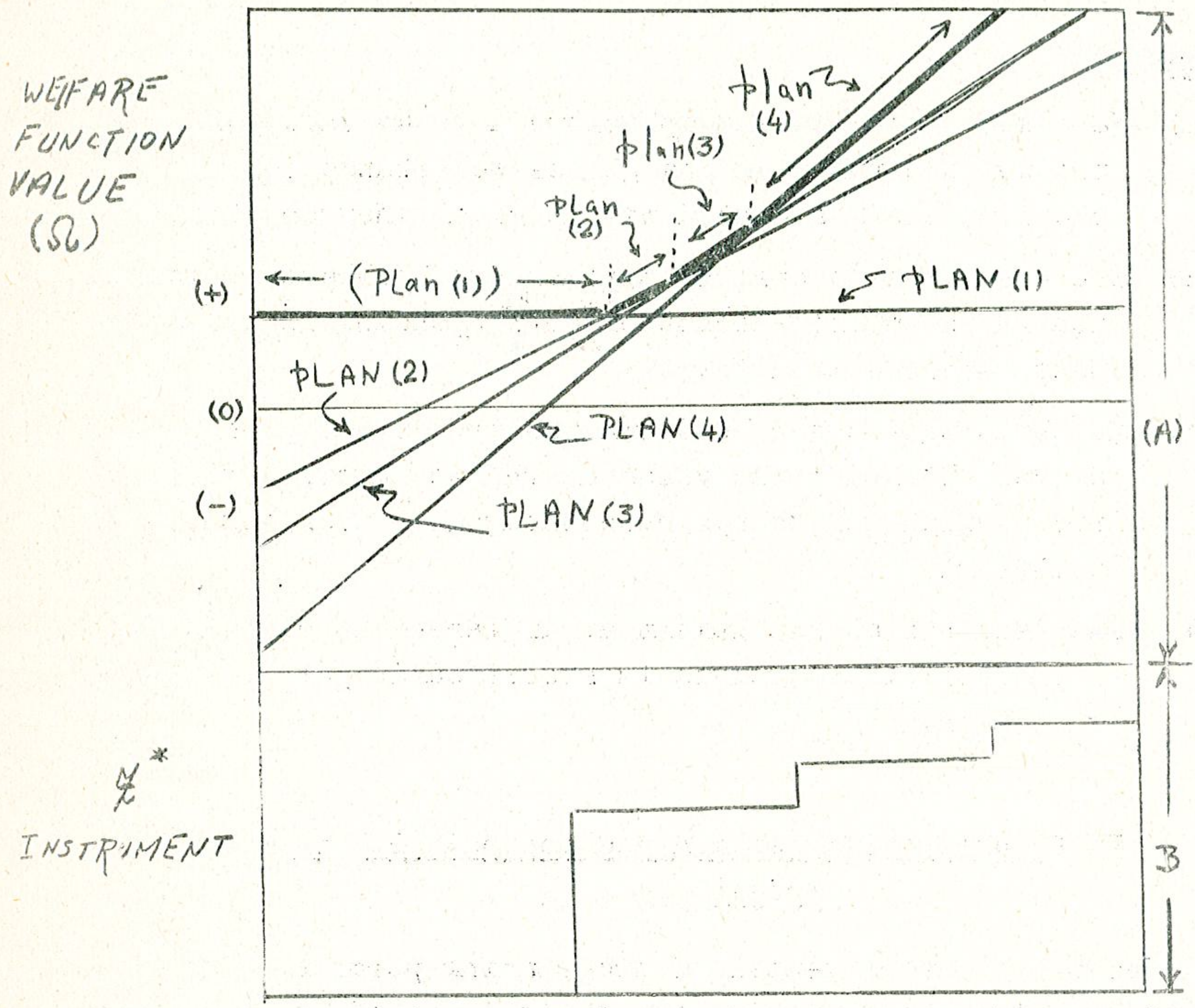
where: Δ = change; r = input-output coefficient

C = weight in the objective function

Z-C = usual criterion function in the simplex tablue.

And the answer now is: the variable weight in the objective function (C_h) can be changed by the quantity (13) without letting one of the coefficient in the $(Z-C)$ row becoming negative, that is without letting the optimum plan to be suboptimum. This quantity is indicated in "Δc_h". The smallest "ΔC_h" gives the range of change in the variable objective function weight to keep the value of the objective function constant. A new plan can be solved for where the value of the variable weight will be the original $+\Delta C_h$. A new ΔC_h can be calculated and a new optimum plan can be solved for at the new weight and so on. Hence, a <u>set</u> of optimum solution are established at different weights.

A GRAPHIC PRESENTATION



"Variable Welfare Function Weight"

(Figure 3): A hypothetical example of a set of optimum plans, at different weights of the objective function.

Discussion:

(1) Part A. of the above graph shows how the value of the welfare function changes as we change the coefficients associated with a given instrument.

(2) The heavy line shows the maximum attainable contribution to the welfare function for the corresponding instrument variable weights.

(3) An obvious fact can be drawn too from these plans curves which is: the value of the welfare function from a plan can change even though the plan remains unaltered.

(4) The need for an instrument at different levels of its coefficients values is specified in part B of the graph.

3. Recursive Programming and Economic Policy Models

* The recursive framework of programming permits a wide flexability in the selection of the optimal mix of instruments from one stage to another in a given sequence.

* It emphasizes the need for orienting optimal policy - making to the characteristics of each stage in relation to proceeding stage.

* Method of Solution:

The problem can be stated in matrix notations as

$$\max W_t = C' X_t$$

(1) Subject to: $A X_t \leq b_t$

and $X_t \geq 0$

Where C = vector of the weights of the objective functions, X_t = vector of activities at the (t) time, and (A) = matrix of input-output coefficients. All the vector and matrices are of appropriate order. Some of the elements of X_t may be instruments others may be targets.

Using the <u>difference</u> equations technique, the recourse b_t might be expressed as related to past expectation and performance as:

(2) $$b_t = \lambda A X^{*}_{t-1} + U_t$$

$$\begin{bmatrix} b_{1t} \\ b_{2t} \\ \vdots \\ b_{nt} \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \emptyset \\ & & \ddots & \\ \emptyset & & & \lambda_n \end{bmatrix} A \begin{bmatrix} X^{*}_{1}(t-1) \\ X^{*}_{2}(t-2) \\ \vdots \\ X^{*}_{m}(t-n) \end{bmatrix} + \begin{bmatrix} U_{1t} \\ U_{2t} \\ \vdots \\ U_{mt} \end{bmatrix}$$

dependent Variable	Initial Conditions	Optimal Conditions	Constant

Now if X_o which is the optimum plan at t = o is known, b_1 can be computed from equation "2". Given B_1, by using the usual programming method, we can solve for X_1. Knowing X^{*}_{1}, through equation (2) we can compute b2, and from b2 we can find X_2^{*}
b_3 ——— X_3 ——— .... etc...

This is one possibility in using recursive programming which is through the "b" vector. However, the method is quite general and can be adopted to other variables in the problem, as long as the problem specification is logical and realistic.

(4) Introduction to Stochastic Linear Programming And Its Implications in Sensitivity Analysis

When randomess and hence error concepts are introduced into the programming problems, it is the stochastic linear programming techniques which is relevant. Here no longer we are only interested in the determination of optimum solution, but also some other statistical criterion (e.g., minimization of the variance) is introduced into the picture. Two main approaches are commonly specified in stochastic programming techniques namely (a) passive approach where we decide on a plan and wish to know how variable the value of the objective function will be, and (b) active approach where we select a plan with a lower but more stable value of the objective function. The active approach will be under focus in this part of the study.

The E.V. Indifference System:

If an indifference map of a society is specified where the maximization of the value of the objective functions drawn against the stability of the solution, it is known as E.V. indifference map (Figure 4). In such a map satisfaction increase in the specified direction of the arrows. In the active approach we find the feasible set of plans (Figure 5). Each plan will determine a value for E (Ω) (expected value of Ω) and a level of σ_Ω^2 (Variauce). A map for such feasible plans can be drawn as in graphs (4, 5). The upper boundary AB implies the most preferable plans. Given the society E.V. map then the optimum policy can be determined. Here again, it is optimum with respect to the maximization of E (Ω) and simultanously minimization of σ_{Ω}^2.

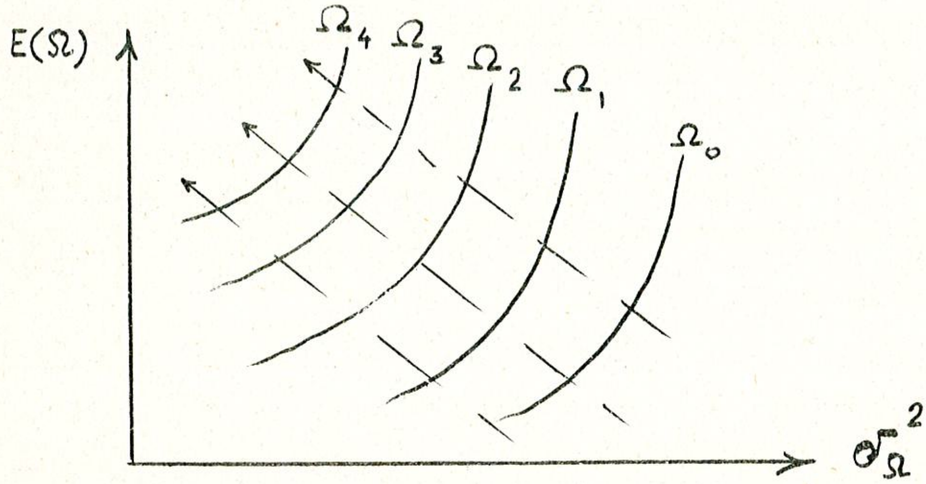


Figure (4): A hypothetical graph of a society indifference map between optimality and stability of the solutions.

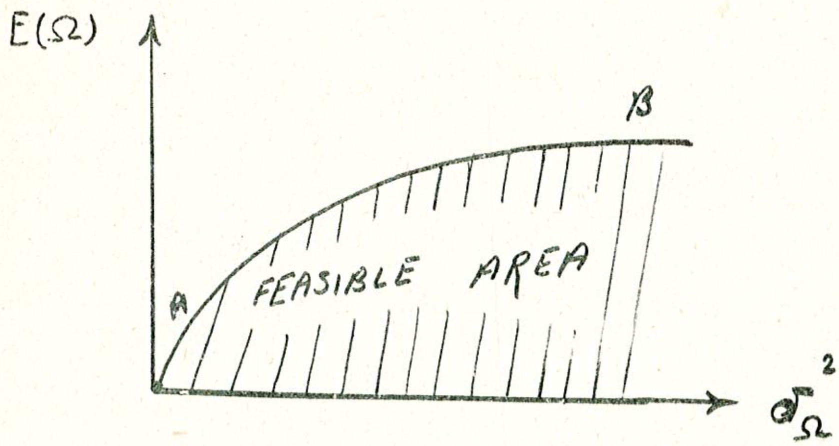


Figure (5): A map of a set of feasible plans.

Optimality and Stability Decision Values:

For the determination of the previously specified active approach idea the following methodology can be indicated as follows:-

If there is random element involved, the problem is one of the following formulations:

$$\text{Maximize: } \Omega = \sum_{j=1}^{n} (c_j + \gamma_j)\, x_j$$

$$\text{Subject to: } \sum_{j=1}^{n} (a_{ij} + \alpha_{ij})\, x_j = (b_i + \beta_i)$$

and $x_j \geq 0$

where , $(\alpha_{ij}, \gamma_j, \beta_i)$ are random elements.

Assumptions:

(i) There is a joint known probability distribution of the random variables.

(ii) We have sample data from the triplet:

$$(\bar{A}, \bar{b}, \bar{C})_1, (\bar{A}, \bar{B}, \bar{C})_2, \ldots, (\bar{A}, \bar{b}, \bar{C})_k, \ldots, (\bar{A}, \bar{b}, \bar{C})_K$$

where

$$(k = 1, \ldots, K);\ A = (a_{ij} + \alpha_{ij});\ b = (b_i + \beta_i)\ C = (C_j + \gamma_j)$$

The total number of the samples are K. For a sample (k) to be admissible the following conditions have to be fulfilled:

(i) The sample space generated by $(\bar{A}, \bar{b}, \bar{C})_k$ is such that it contains more than one feasible solution.

(ii) The sample space generated by $(\bar{A}, \bar{b}, \bar{C})_k$ is such that it satisfied the conditions of an ordinary non-stochastic linear programming problem for any fixed sample "k"

i.e. a programming problem which has a non-empty set of basic solution, all bounded and finite and which is not degenerate. The problem now after introducing the slack variables is:

$$\text{max:} \quad \Omega = \sum_{j=1}^{n+m} \bar{c}_j x_j$$

$$\text{Subject to:} \quad \sum_{j=1}^{n+m} \bar{a}_{ij} x_j \leqq \bar{b}_i$$

$$\text{and} : \quad x_j \geqslant o; \; (j = 1, \ldots, (m+n))$$

$$(i = 1, \ldots, m)$$

$$\text{where} : \quad \bar{C}_j = (C_j + \gamma_j); \; \bar{a}_{ij} = (a_{ij} + \alpha_{ij})$$

$$\bar{b}_i = (b_i + \beta_i)$$

$$x_j \text{ for } j = n+1, \ldots, n+m = \text{slack variables}$$

Now for a fixed "k" we consider "m" linearly independent columns of the augmented matrix $(\bar{a}_{ij})_k$, which can be chosen in a $\binom{m+n}{m}$ way, of these we reject those which do not satisfy the non-negativity condition.

Then we will have for each sample space (sample spaces are 1, ... k, ... , K) the following values of the objective function determined by the set of basic feasible solutions as:

$$\Omega_{1k} = \max_{r}. \{ \Omega_{rk} \mid r = 1, \ldots, R \}$$
$$\Omega_{2k} = \max_{r}. \{ \Omega_{rk} \mid r = 2, \ldots, R \}$$
$$\vdots$$
$$\Omega_{Nk} = \max_{r}. \{ \Omega_{rk} \mid r = R \}$$

This implies that:

$$\{ \Omega_{1k} > \Omega_{2k} > \cdots > \Omega_{Nk} > 0 \}$$

If the maximum solution (Ω_{1k}) for all admissible K is such that it has a very high variability as measured by variance or other index, compared to the second best solution (Ω_{2k}) for all admissible K, ... etc.... , then the maximum solution will have instability in a certain sense. That means stability and optimality characteristics may be competive or complementary for different basic solution.

i.e. If: $\sigma^2_{\Omega_{1k}} < \sigma^2_{\Omega_{2k}} < \cdots$

The optimal value Ω_{1k} is said to be stable. If, however it turns out that

$$\sigma^2_{\Omega_{1k}} > \sigma^2_{\Omega_{2k}} > \cdots$$

and this difference in variance is far outweighs the difference in expected values, then it <u>might</u> be more reasonable to accept the second best solution Ω_{2k}. In any case a value judgment about the stability optimality (if they are competitive) marginal rates of substitutions has to be set up.

Example

Plan	z_1 1	z_2 2	z_3 3	z_4 4	$E(\Omega)$ (UNIT) 100	σ_Ω^2 (UNIT) 1000,000
1	0	0	0	0	0	0
2,3	0	18.29	12.51	0	30.80	.17
4	7.67	32.67	2.06	18.96	61.31	2.44
5	11.12	25.11	.45	49.90	86.57	9.21
6	22.14	0	11.62	57.55	91.31	11.86

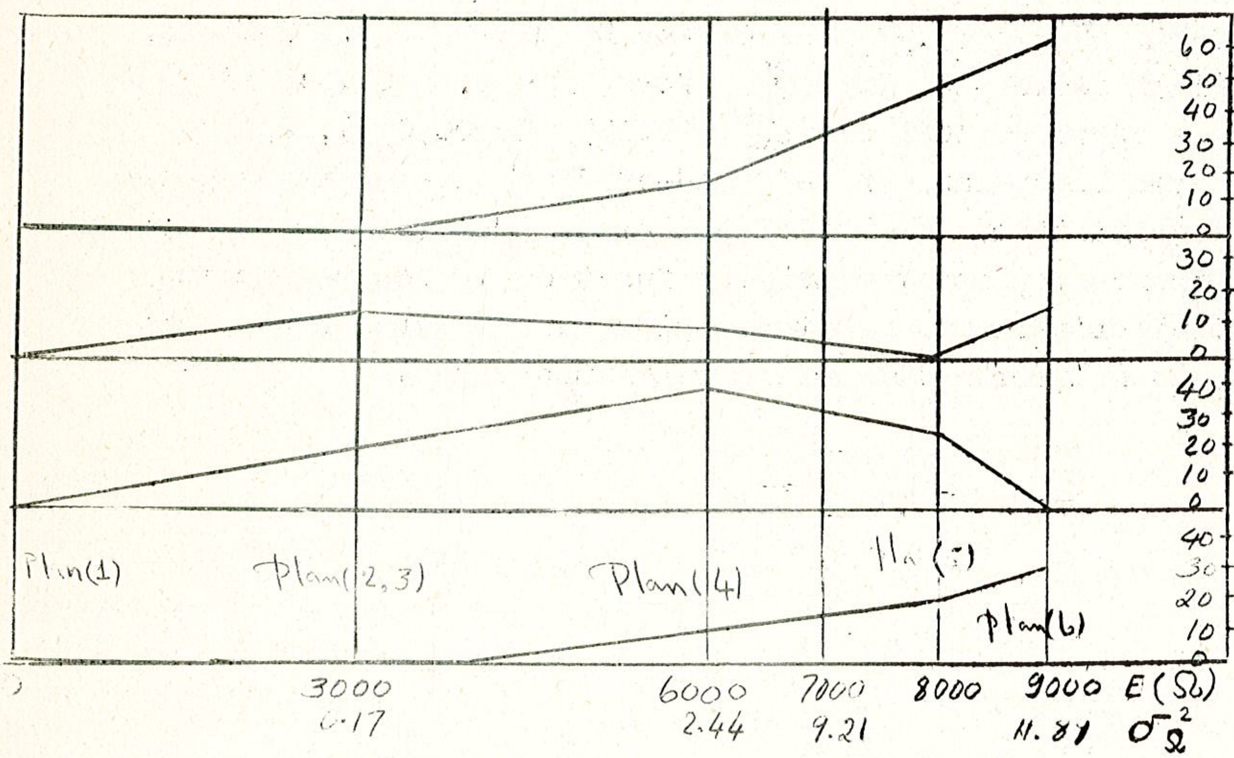


Figure (6): A hypothetical set of optimum plans at a corresponding set of expect values of

Discussion:

This figure shows the preferred plans as expected objective function values increases from zero to the maximum attainable.

Different plans (in this example there are six) are indicated by the vertical lines. e.g. plan (1) shows the use of $\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4 = 0$ for $E(\Omega) = 0$. However, plan (2,3) shows the use of $\mathcal{Z}_i$ for ($i = 1, \ldots, 4$) if $E(\Omega)$ is to be equal to 3080 and $\sigma_\Omega^2 = (0.17)(1{,}000{,}000)$, and so on. An intermediate plan can be "read-off" by picking the appropriate quantity on the horizontal axis. Thus, given the society indifference map for the walfare function values versus the stability criterion, an optimum plan can be determined. And hence, such information is of great importance.

CHAPTER V: CONCLUSIONS

* Economic policy problems are problems which can be solved in a manner such that programming techniques (from the simple linear programming methods to the most advanced ones) can be introduced very efficiently.

* Using programming in economic policy models helps not only in finding optimum solutions, but also many meaningful economic interpretations can be drawn out of it.

* The importance of using programming in policy models gets more and more significant results if more intensive research is directed in this direction.

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Appendix: Input-Output table of the hypothetical example (Page 15)

	B	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}
1	b_1	$\alpha_{1,1}$	$\alpha_{1,2}$	$\alpha_{1,3}$	$\alpha_{1,4}$	$\alpha_{1,5}$	$\alpha_{1,6}$	$\alpha_{1,7}$	$\alpha_{1,8}$	$\alpha_{1,9}$	$\alpha_{1,10}$	$\alpha_{1,11}$	$\alpha_{1,12}$	$\alpha_{1,13}$	$\alpha_{1,14}$	$\alpha_{1,15}$	$\alpha_{1,16}$	$\alpha_{1,17}$	$\alpha_{1,18}$
2	b_2	$\alpha_{2,1}$	$\alpha_{2,2}$	$\alpha_{2,3}$	$\alpha_{2,4}$	$\alpha_{2,5}$	$\alpha_{2,6}$	$\alpha_{2,7}$	$\alpha_{2,8}$	$\alpha_{2,9}$	$\alpha_{2,10}$	$\alpha_{2,11}$	$\alpha_{2,12}$	$\alpha_{2,13}$	$\alpha_{2,14}$	$\alpha_{2,15}$	$\alpha_{2,16}$	$\alpha_{2,17}$	$\alpha_{2,18}$
3	b_3	α	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	b_4	$\alpha_{4,1}$	0	0	$\alpha_{4,4}$	0		$\alpha_{4,7}$	0	0	$\alpha_{4,10}$	0	0	$\alpha_{4,13}$	0	0	$\alpha_{4,16}$	0	0
5	0	0	$\alpha_{5,2}$	0	0	$\alpha_{5,5}$	0		$\alpha_{5,8}$	0	0	$\alpha_{5,11}$	0	0	$\alpha_{5,14}$	0	0	$\alpha_{5,17}$	0
6	0	0	0	$\alpha_{6,3}$	0	0	$\alpha_{6,6}$	0	0	$\alpha_{6,9}$	0	0	$\alpha_{6,12}$	0	0	$\alpha_{6,15}$	0	0	$\alpha_{6,18}$
7	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	b_{11}	$\alpha_{11,1}$	$\alpha_{11,2}$	$\alpha_{11,3}$	$\alpha_{11,4}$	$\alpha_{11,5}$	$\alpha_{11,6}$	0	0	0	0	0	0	$\alpha_{11,13}$	$\alpha_{11,14}$	$\alpha_{11,15}$	$\alpha_{11,16}$	$\alpha_{11,17}$	$\alpha_{11,18}$
12	b_{12}	0	0	0	0	0	0	$\alpha_{12,7}$	$\alpha_{12,8}$	$\alpha_{12,9}$	$\alpha_{12,10}$	$\alpha_{12,11}$	$\alpha_{12,12}$	0	0	0	0	0	0
13	b_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	b_{14}	$\alpha_{14,1}$	$\alpha_{14,2}$	$\alpha_{14,3}$	$\alpha_{14,4}$	$\alpha_{14,5}$	$\alpha_{14,6}$	0	0	0	0	0	0	$\alpha_{14,13}$	$\alpha_{14,14}$	$\alpha_{14,15}$	$\alpha_{14,16}$	$\alpha_{14,17}$	$\alpha_{14,18}$
15	b_{15}	0	0	0	0	0	0	$\alpha_{15,7}$	$\alpha_{15,8}$	$\alpha_{15,9}$	$\alpha_{15,10}$	$\alpha_{15,11}$	$\alpha_{15,12}$	0	0	0	0	0	0
16	b_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	b_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	b_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	b_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	b_{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	$\alpha_{21,1}$	$\alpha_{21,2}$	$\alpha_{21,3}$	0	0	0	$\alpha_{21,7}$	$\alpha_{21,8}$	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	$\alpha_{22,13}$	$\alpha_{22,14}$	$\alpha_{22,15}$	0	0	0
23	b_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	X_{30}	X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	X_{36}
1	$\alpha_{1,19}$	$\alpha_{1,20}$	$\alpha_{1,21}$	$\alpha_{1,22}$	$\alpha_{1,23}$	$\alpha_{1,24}$	$\alpha_{1,25}$	$\alpha_{1,26}$	$\alpha_{1,27}$	$\alpha_{1,28}$	$\alpha_{1,29}$	$\alpha_{1,30}$	$\alpha_{1,31}$	$\alpha_{1,32}$	$\alpha_{1,33}$	$\alpha_{1,33}$	0	0
2	$\alpha_{1,19}$	$\alpha_{2,20}$	$\alpha_{2,21}$	$\alpha_{2,22}$	$\alpha_{2,23}$	$\alpha_{2,24}$	$\alpha_{2,25}$	$\alpha_{2,26}$	$\alpha_{2,27}$	$\alpha_{2,28}$	0	0	0	0	$\alpha_{2,33}$	$\alpha_{2,34}$	0	0
3	0	0	0	0	0	0	0	0	0	0	$\alpha_{3,29}$	0	$\alpha_{3,31}$	$\alpha_{3,32}$	$\alpha_{3,33}$	0	0	0
4	$\alpha_{4,19}$	0	0	$\alpha_{4,22}$	0	0	$\alpha_{4,25}$	0	$\alpha_{2,27}$	0	0	0	0	0	0	0	0	0
5	0	$\alpha_{5,20}$	$\alpha_{5,21}$	0	$\alpha_{5,23}$	0	0	$\alpha_{5,26}$	0	$\alpha_{5,28}$	0	0	0	0	$-\alpha_{5,33}$	0	0	0
6	0	0	$\alpha_{6,21}$	0	0	$\alpha_{6,24}$	0	0	0	0	0	0	0	0	0	$-\alpha_{6,34}$	0	0
7	0	0	0	0	0	0	0	0	0	0	$-\alpha_{7,29}$	0	0	0	0	0	$\alpha_{7,35}$	0
8	0	0	0	0	0	0	0	0	0	0	0	$-\alpha_{8,30}$	0	0	0	0	$\alpha_{8,35}$	0
9	0	0	0	0	0		0	0	0	0	0	0	$-\alpha_{9,31}$	0	0	0	0	$\alpha_{9,36}$
10	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha_{10,32}$	0	0	0	$\alpha_{10,36}$
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	$\alpha_{12,19}$	$\alpha_{12,20}$	$\alpha_{12,21}$	$\alpha_{12,22}$	$\alpha_{12,23}$	$\alpha_{12,24}$	0	0	0	0	0	0	0	0	0	0	0	0
13	-	0	0	0	0	0	$\alpha_{13,25}$	$\alpha_{13,26}$	$\alpha_{13,27}$	$\alpha_{13,28}$	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	$\alpha_{15,19}$	$\alpha_{15,20}$	$\alpha_{15,21}$	$\alpha_{15,22}$	$\alpha_{15,23}$	$\alpha_{15,24}$	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0		0	$\alpha_{16,25}$	$\alpha_{16,26}$	$\alpha_{16,27}$	$\alpha_{16,28}$	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	$\alpha_{17,29}$	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	$\alpha_{18,30}$	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	$\alpha_{19,31}$	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0		$\alpha_{20,32}$	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-\alpha_{21,35}$	0
22	$\alpha_{22,19}$	$\alpha_{22,20}$	$\alpha_{22,21}$	0	0	0	$\alpha_{22,25}$	$\alpha_{22,26}$	0	0	0	0	0	0	$\alpha_{22,33}$	$\alpha_{22,34}$	0	$-\alpha_{22,36}$
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\alpha_{23,33}$		0	0

E.Z.