UNITED ARAB REPUBLIC

THE INSTITUTE OF NATIONAL PLANNING

Memo. No. 812

The Application of Network Techniques in Planning and Management

By
Dr. Karl-Heinz Horn

December 1967

CAIRO
3, MOHAMED MAZHAR - ZAMALEK

The Application of Network Techniques in Planning and Management

Contents

Introduction :

The Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT) are two modern methods of planning using mathematics. Both methods consist of two main parts, the network as the evident and intuitive part and the mathematical model.

The difference between both methods is, that CPM is a strictly determined model in which all the data are exactly fixed, and PERT is a stochastic model, i.e. the data are not in all cases exactly determined but they are an outcome of the application of probability accounting.

The network techniques can be applied in all those cases, in which we have to plan and to control processes consisting of numerous parts, that means processes which can be subdivided in activities or jobs. In order to find the shortest possible time to implement the entire complex of activities, we have to solve the problem of succession. Therefore we have to know the relations and interdependences between all the activities. We have to recognize wether any an activity is a precondition of other procedures or not, and we must find out which activities can be carried out at the same time, in a parallel way.

One of the most important advantages of network techniques is, that it is possible to calculate a complete time schedule including the exact dates of the finishing respectively of the beginning of each activity.

Both methods have been created by scientist in connection with planning problems, such as the Polaris-Rocket-Project in the United States in the years 1957 and 1958. The targets were to optimize - in this case to minimize - the time needed for the research work, for development, construction, and production of the rockets, to minimize the costs, and to fix all the dates exactly in order to safeguard a frictionless cooperation with all suppliers, etc. As to the Polaris Project: The Americans were able to reduce the estimated time from 5 to 3.5 years by applying PERT. In this case the whole network consisted of 23 part networks containing about 3000 activities.

In the recent years the application of network techniques was rapidly growing, very encouraging experiences have been made all over the world. The construction of industrial and other buildings is one of the most important fields of the application of network techniques, followed by ship building. It is estimated that for instance in the field of the contruction of buildings it is possible to reduce the necessary time by about 20 per cent and the costs by about 5 per cent.

In general it is possible to plan with the aid of networks every process which can be subdivided into several parts, into a number of detail processes. In the Nigerian Republic for instance the whole economic development of the country from 1962 to 1968 was planned by the means of network techniques. Even complicated operations in the field of medicine can be planned with this method.

PERT and CPM are the basic methods.
Besides these some other exist aimed to solve special problems:

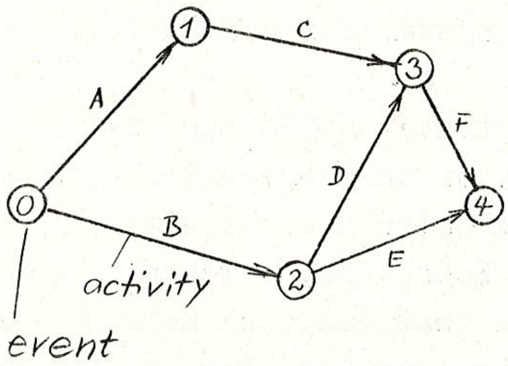
EPS	Critical Path scheduling
LEES	Least Cost Estimating and Scheduling
CPPS	Critical Path Planning and Scheduling
RAMPS	Resource Allocation and Multi-Project Scheduling
JMPACT	Implementation Planning and control Technique.
PEP	Program Evaluation Procedure
PRISM	Program Reliability Information System for Management.

In this Memo we intend to concentrate on CPM and PERT. The examples are partly based upon Part 5 of the GDR-Series "Planning and Management of the Economy", which has the title "Critical Path and PERT," elaborated by Schreiter, Strempèl, and Frotscher, published by the research team "Mathematical Methods in Economy" under the supervision of Prof. Dr. Rudolph, Berlin 1966.

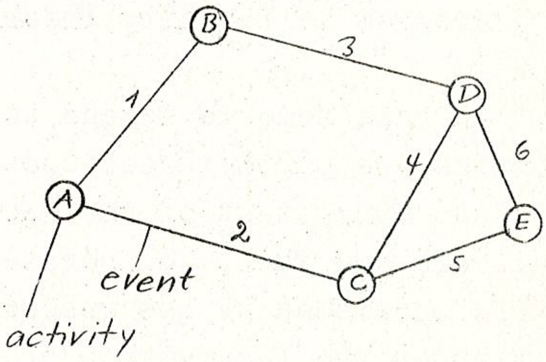
1. Some hints how to construct networks:

The network has the form of an arrow diagram. It consists of events and activities. Every activity starts with and results in an event. We distinguish between networks directed on activities and those which are directed on events:

Graph 1



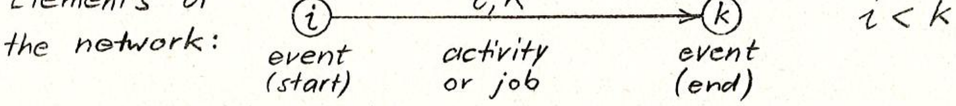
Network directed on activities



Network directed on events

In practice we prefer to use networks directed on activites (or jobs). The activities are the time consuming processes, they can be defined as the dependends between two events, and they are represented by arrows. The arrows point always to the dependent event :

Graph 2



The direct dependence between the events i and k is the activity i, k with $i < k$, that means k depends on i. Any an event i is a start event, if all other events depend on i; any an event k is an end or finishing event, if k depends on all other events of the network.

The procedure to construct a network contains the following steps :

1 - Registration of all activities;

2 - For each of the activities we have to answer the following questions :
Which activity must be finished before the beginning of the activity in question?
Which jobs can be carried out at the same time, side by side?
Which activities we can start only after finishing the definite activity?
The result must be the recognition of the succession respectively the parallelism of the activities.

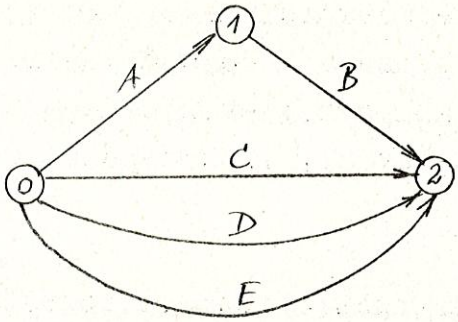
3 - The drawing of the network.

Let us take a simplified example. The construction of a building might consist of the following activities:

A Construction of the walls,
B Construction of the roof,
C Water supply,
D el/ectricity supply, and
E gas supply.

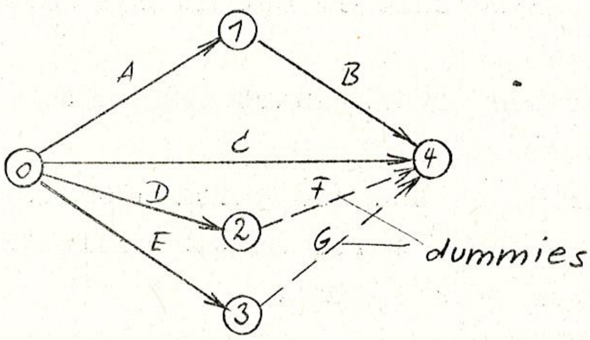
It is obvious that B depends on A, but C, D, and E can be carried out paraclle1 to A and B. The network looks like follows :

Graph 3



In this network C, D, and E have the same indices : 0, 2. This is not very suitable, therefore we introduce socalled dummies, i.e. auxiliary activities, and we obtain a second, a corrected network like follows :

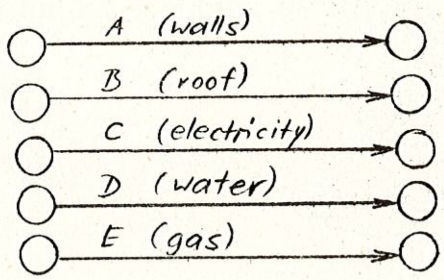
Graph 4



Now each of our activities has its own indices: $A_{0,1}$; $B_{1,4}$; $C_{0,4}$; $D_{0,2}$; $E_{0,3}$; also the dummies can be marked by different indices : $F_{2,4}$ and $G_{3,4}$. The time of the dummies equals zero, but they express real dependences : The event 4 depends on 1 as well as on 2, 3, and 0, the events 1, 2, 3, and 4 depend on 0.

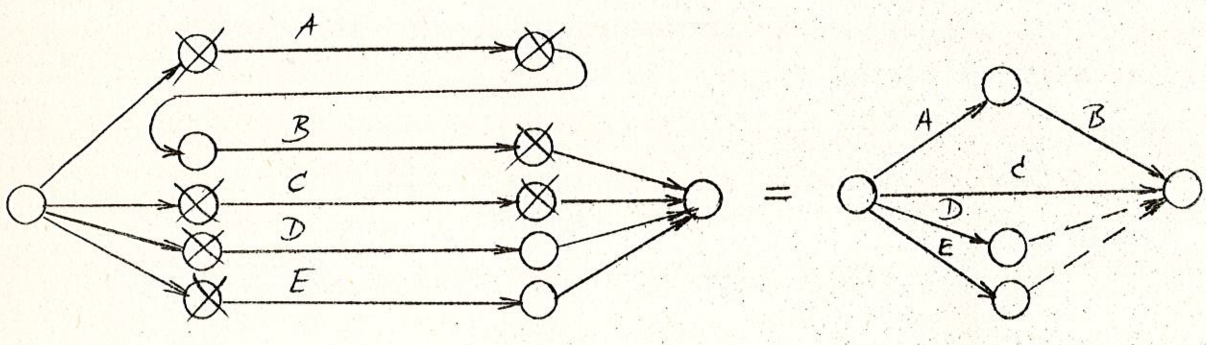
In order to simplify our network construction we can set up our list of activities in such a way, that every activity is written above an arrow :

Graph 5



The next steps will be to add the start and the end-event and to connect the activities in accordance with the dependences. Than we have to decide which of the events respectively activities are necessary and which not. Eleminating all unnecessary events and activities we obtain at last the same picture as our corrected network (graph 4) of this example :

Graph 6

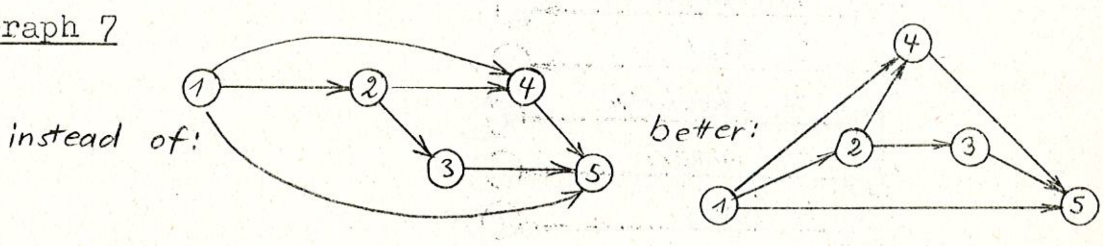


At last some summarized references how to draw networks;

1 - Every network must have one and only one start - as well as stop-event.

2 - So far as possible straight lines should be used :

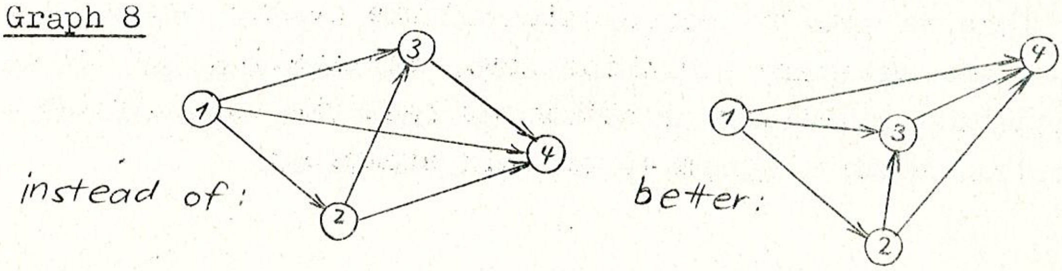
Graph 7



3 - So far as possible auxiliary activities (dummies) should be avoided.

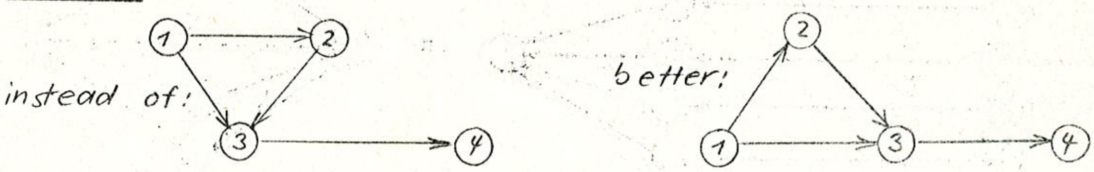
4 - Crossing arrows must be avoided :

Graph 8



5 - So far as possible the arrows should have one direction, from left to right :

Graph 9



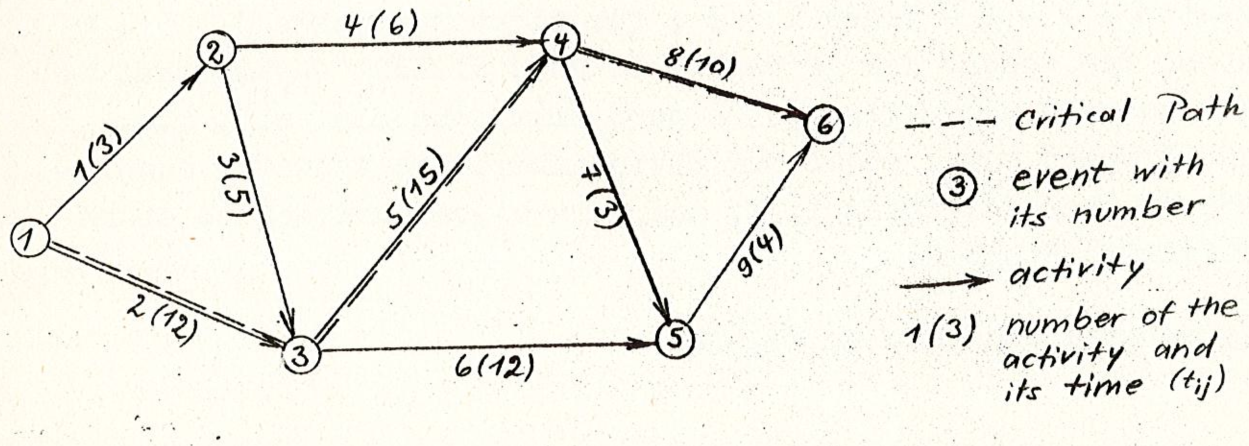
2. The Critical Path Method

Our network is the precondition to find the critical path, i.e. The very succession of activities determining the shortest possible time to implement the process in question. In this sense the network is the starting point of our time scheduling.

To distinguish CPM and PERT we would like to repeat, that the time of each activity must be strictly determined if we want to apply the Critical Path Method. In the following tables $t_{i,j}$ stands for the time of the activity i,j. The $t_{i,j}$ must be given by experiences, scientific based standards, or at last by estimations.

After the elaboration of a table which contains the number of each activity, short descriptions of the activities, and the time periods required to carry out each of them, we can draw the network. We will go through an assumed example which consists of 6 events and 9 activities, as shown in graph 10.[1]

Graph 10 :



1) An example showing the application of network techniques in the U.A.R. is published in Memo. No. 614 (JNP), by Ahmed H.El-Shatoury "Perfect Dredging in Suez Canal", Chapter II and III.

Based on the arrow diagramme we obtain the data corresponding to each activity, such as:

t_{E_i} , the earliest possible time to start activity i,j;

t_{E_j} , the earliest possible time to accomplish activity i,j;

t_{L_i} , the la/test permissible time to start activity i,j;

t_{L_j} , the latest permissible time to accomplish activity i,j,

As to the calculation of these time of figures, it is abvious that the earliest possible time to start the activities 1 and 2, t_{E_1}, must be 0.

Assuming the time units to be weeks, than the activities 3 and 4, starting from event 2, can begin after 3 weeks, because 3 weeks are necessary for the comple/tion of the prior activity 1. Things are little bit more complicated in the case of the activities 5 and 6, because there are two paths leading to event 3 : 1 - 2 - 3 and 1 - 3 , the first way needs 3 + 5 = 8 weeks, the second one 12 weeks. It goes without saying that we have to take the maximum, the most time consuming way, i.e. the 12 weeks path, under the given assumption that the completion of the activities 1, 2, and 3 is a precondition to start activities 5 and 6.

The calculation runs as follows:

$$t_{E_1} = \max [t_{E_o} + t_{o,1}] = \max [0 + 0] = \underline{\underline{0}}$$

$$t_{E_2} = \max [t_{E_1} + t_{1,2}] = \max [0 + 3] = \underline{\underline{3}}$$

$$t_{E_3} = \max [t_{E_1} + t_{1,3} ; t_{E_2} + t_{2,3}] = \max [0 + 12; 3 + 5] = \underline{\underline{12}}$$

$$t_{E_4} = \max [t_{E_2} + t_{2,4} ; t_{E_2} + t_{2,3} + t_{3,4}; t_{E_2} + t_{3,4}] =$$

$$= \max [3 + 6 ; 3 + 5 + 15 ; 12 + 15] = \underline{\underline{27}} \quad \text{etc.}$$

In general: $t_{E_i} = \max [t_{E_{i'}} + t_{i' j}]$,

that means the $t_{E_{i'}} + t_{i' j}$ are those values of the preceding activities which lead to the maximum of t_{E_i}. We list the calculated t_{E_i} in our table:

Table 1 : Values related to the activities

1	2	3	4	5	6	7	8
No.	i - j	$t_{i,j}$	t_{E_i}	t_{E_j}	t_{L_j}	t_{L_i}	total float
1	1-2	3	0	3	7	4	4
2	1-3	12	0	12	12	0	0
3	2-3	5	3	8	12	7	4
4	2-4	6	3	9	27	21	18
5	3-4	15	12	27	27	12	0
6	3-5	12	12	24	33	21	9
7	4-5	3	27	30	33	30	3
8	4-6	10	27	37	37	27	0
9	5-6	4	30	34	37	33	3

To obtain the t_{E_j} values, the earliest possible times to finish each of the activities, we have nothing to do but to add the time needed to carry out the activity i,j to the earliest possible date of its start:

$$t_{E_j} = t_{E_i} + t_{i,j} \quad , \text{ for example:}$$

$$t_{E_2} = t_{E_1} + t_{i,j} = 0 + 3 = \underline{\underline{3}},$$

$$t_{E_3} = t_{E_2} + t_{2,3} = 3 + 5 = \underline{\underline{8}}, \text{ or}$$

$$t_{E_3} = t_{E_1} + t_{1,3} = 0 + 12 = \underline{\underline{12}} \text{ , etc., until}$$

$$\vdots$$

$$t_{E_6} = t_{E_5} + t_{5,6} = 30 + 4 = \underline{\underline{34}}, \text{ respectively}$$

$$t_{E_6} = t_{E_4} + t_{4,6} = 27 + 10 = \underline{\underline{37}}.$$

Using table 1 we obtain these values easily by adding the data of the columns 3 and 4.

From the point of view how to find the critical path the maximum time needed to come to event 6 is the decisive value. In our case e.g. it is impossible to go from event 4 over 5 to 6, which requires 7 weeks, because there would be not enough time to complete activity 8, which goes straight from 4 to 6 requiring 10 weeks. That means the earliest possible time to finish the entire process must be always the maximum value of the t_{E_i}, given in column 5, in our special case 37 weeks.

This is important because we have to use that maximum result as our starting point to calculate the latest permissible time to complete each activity, t_{L_j}. It is evident that the earliest possible time to finish the whole process must be equal to the latest permissible time of completion; keeping in mind that we are going to minimize the required time. We put the 37 weeks figure into the last two rows of column 6 of the table and go on to calculate the latest permissible times to come to the preceding events. We use the formula

$t_{L_j} = \min [t_{L_j'} - t_{ij'}]$ in which the $t_{L_j'}$ and $t_{ij'}$ must be the values which lead to a minimum t_{L_j}. For example:

$$t_{L_5} = \min [t_{L_6} - t_{5,6}] = \min [37 - 4] = \underline{\underline{33}},$$

$$t_{L_4} = \min [t_{L_6} - t_{5,6} - t_{4,5} ; t_{L_6} - t_{4,6}] =$$

$$= \min [37 - 4 - 3 ; 37 - 10] = \underline{\underline{27}}, \text{ etc.}$$

After listing these figures in column 6 of the table we calculate the t_{L_i}, the latest permissible times to start each activity. This again is a simple procedure, we obtain the t_{L_i} by subtracting the required times $t_{i,j}$ from the t_{L_j}, the data of the latest permissible completion:

$$t_{L_i} = t_{L_j} - t_{i,j}.$$

We use the table and subtract the data of column 3 from those of column 6.

At this point it seems to be suitable to recall to our minds the essence of all those formal calculations. The Critical Path Method aims to minimize the time required by any a process, i.e. to minimize costs, labour expenditures, etc. Discussing Table 1 and Graph 10 it becomes obvious how we can use this method as an instrument of time - and manpower-planning as well as a basis of cost planning. So it is e.g. possible to start activity 1, which goes from event 1 to event 2, at the time 0, but this is not necessary . The t_{L_1} - value shows us, that we have the permission to start after 4 weeks. Or as to activity 3: There exists also a certain interval of 4 weeks between the earliest possible and the latest permissible time to start, respectively to finish activity 3. In depends on our decision, on the given circumstances, etc., to fix the most suitable time within the given limits. At any case there will be a slack time, or a float of 4 weeks, either between events 1 and 2 or between 2 and 3. The authorities responsible in the field of planning or management have to take decisions, such as on the utilization of the production elements unused for 4 weeks, to shift the workers during this time to another job considering their mobility. Evidently a lot of conclussions can be drawn out of the diagram and the calculated figures. If we are forced to cut down the time needed to implement the whole procedure than we know , that we have to concentrate our efforts upon those activities which follows each other without any slack time, i.e. the Critical Path. In these cases there are no differences between the earliest possible and the latest permissible dates to start respectively to finish the activities. In Table 1 those cases are accentuated by underlining them. To find the critical path with the aid of the table means to select those activities with no total float, i.e. 2,5, and 8, or the critical Path goes over the events 1 - 3 - 4 - 6, as stressed in the graph.

By the way we can add the formula to calculate the total floats:

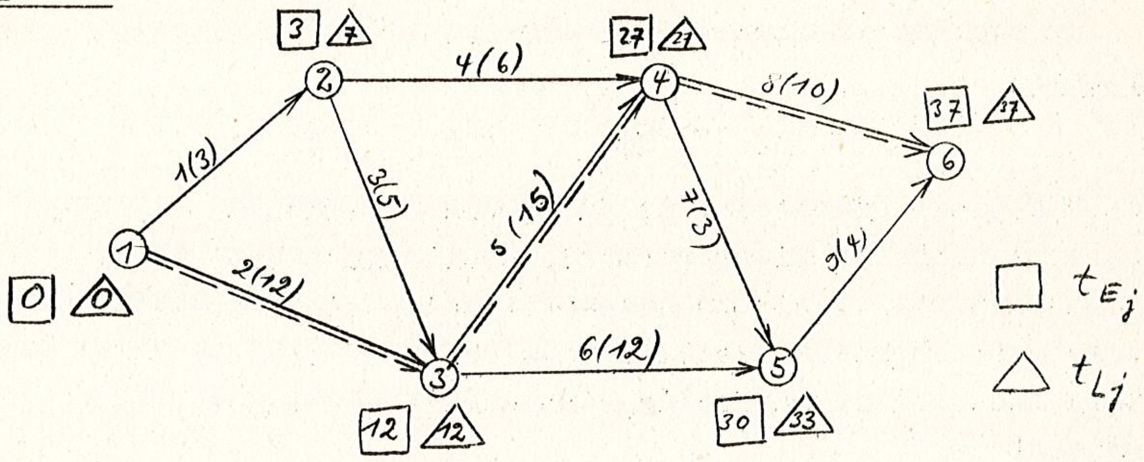
$$F_t = t_{L_j} - t_{E_j} \text{ or } = t_{L_i} - t_{E_i} .$$

In words: The total float is the difference between the latest permissible and the earliest possible completion respectively start of an activity. Only the non-critical activities have floats, so that there can be limited amounts of delay in start or completion time without affecting the completion time of the whole project.

Summing-up all the total floats given in column 8 we get an amount higher than the possible and permissible time of completion of the whole process. This makes evident that only a small part of the total float is free float, i.e. float which is in fact available. For instance the 4 weeks float of activity 3 is only a free float under the assumption, that activity 1 starts and is completed as early as possible, that means without using the float. So the availability of the float is affected by the preceding activity, in other cases by the forthcoming activities. It is also possible that the use of a certain float will not affect any other activity, in this case we speak about scheduled floats. In order to find the critical Path it is not necessary to calculate these additional kinds of floats.

In not very comprehensive networks, such as partical networks, it may be suitable to add the latest permissible and the earliest possible times of completion to the events in the arrow diagram in order to show the Critical Path, as in Graph 11:

Graph 11:



Another method used to determine the Critical Path in the socalled matrix method (Table 2).

Table 2 : Matrix Method to Determine the Critical Path

i \ j	1	2	3	4	5	6	
1	*	3	(12)				0
2			5	6			3
3			*	(15)	12		12
4				*	3	(10)	27
5						4	30
6						*	37
	0	7	12	27	33	37	t_{Ei} / t_{Lj}
	0	4	0	0	3	0	F_t

* 1 — * 3 - * 4 — * 6 critical events (critical path)

(12) + (15) + (10) = 37 times of the critical activities

The matrix doesn't need detailed explanations. The figures inside the square are the $t_{i,j}$, the time units required to come from i to j., e.g. the activity $a_{2,4}$ needs 6 weeks and so we will find a 6 at the crossing point of the second row and the fourth column. The stars on the diagonal are related to identical t_{Ei} and t_{lj} values, in accordance with the definition of the Critical Path saying that there is no delay between the latest permissible finish of the preceding and the earliest possible start of the forthcoming activity. As an additional proof: The total floats are added in the last row and those related to the Critical Path are zero. The $t_{i,j}$ of the Critical Path, connecting the events 1 - 3 - 4 - 6, i.e. $t_{1,3}$, and $t_{4,6}$, are encircled in the table in order to accentuate them.

Now we can summarize the main steps of the procedure to set up a network and to determine the Critical Path:

1. Breakdown of the process in so many activities as suitable
2. Registering of the activities and determination of their succession.
3. Determination of the $t_{i,j}$, the times required to implement the activities. The more the process is subdivided into simple activities so higher the probability that the required time periods are known and fixed in standards. Otherwise we have to use estimations.
4. Drawing of the network, the arrow diagram.
5. Calculation of the values related to the activities- their start, completion, and the floats. Determination of the Critical Path.
6. Though the required time to implement the entire process is determined by the Critical Path, we concentrate our effort on the critical activities in order to cut down the needed time. That means we utilize the network as an instrument of rationalization by redistributing labour forces and other elements of production so far as possible in favour of the critical activities.
7. Analysis of the floats in order to find out how to use the productive factors during those available slack times.

8. During the implementation of the scheduled process we use the network as an instrument of follow - up. Hence the dates of the start and the completion of each activity are fixed we are in a position to control the implementation very exactly. In the cases of any a delay or an overfulfilment of the time targets we have to analyse the reasons and the consequences, i.e. we correct the data permanently.

3. PERT (Program Evaluation and Review Technique)

Though the times t_{ij} needed to carry out each activity are not strictly determined, the PERT - method is a stochastic one. The probability calculus is applied to ascertain three values concerning the time of each activity:

1. optimistic time, a_{ij},
2. probable time, m_{ij}, and
3. pessimistic time, b_{ij}.

Of course this calculation requires additional efforts, but the degree of realism is improved compared with CPM.

The optimistic time is based upon most suitable conditions and elimination of all kinds of disturbances. The contrary extreme, the possimistic time, is fixed considering all possible trouble. With about 99 per cent probability the rael time will be between those two extremes.

After fixing the three time values a,m, and b we determine the data corresponding to each activity and each event. At first we have to calculate the expected time t_e for each activity ij as a mean time under the assumption that the probability distribution corresponds to the socalled beta-function (Graph 12):

5. Network techniques are one of the preconditions to implement the principle of permanent planning, i.e. of permanent correction of the data comparing the planned and the actual figures. This aims to secure the final termination .

6. Arrow diagrams are suitable instruments of follow-up.