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Macroeconomic Production Function in Czechoslovakia

A new methodological tool for working out the projections of economic growth, which is used more and more at present is the MPF (Macroeconomic Production Function) may be regarded as a pivotal part of the aggregate growth models. They may be used especially for long-term prognoses (for 10, 15 even more years) because of the lack of structural models which are not so suitable and were not verified for such a long period. The aggregate growth models in spite of their limited possibilities may be used as an insight into the future.

From the MPF we may obtain useful information for a long-term orientation of economic policy such as the selection of effective alternatives or types of future growth. This information is necessary for creating material and organisational conditions for future economic development. These conditions are the first target of macroeconomic planning. With the help of MPF we may find the effective growth trajectories and among them the most suitable strategy of economic growth. The results of the MPF may be used as an input information for the formulation of the tactics of growth, which may be better examined by the instruments of structural models for short- and medium-term planning.

This paper deals with the experience gained by the study of the possible utilization of the MPF in modelling the process of growth of the Czechoslovak economy. Firstly we shall describe the questions of the construction of the MPF (selection of dependent and independent variables), the problems of indentification of a certain type of function, the problems arissing from the estimations of the MPF parameters and finally the results achieved in numerous calculations.

The choice of production indicators and corresponding production factors modelled on a certain type of production function is determined by the obtainable statistical information of the past development. On the macroeconomic level we have a limited number of global economic indicators.

From the indicators of the production in Czechoslovakia we may use the volume of the gross social product in industry net value added at constant prices. The productive factors on this level are characterized by fixed assets (divided between building and machinery and equipment at constant prices), consumption of material and energy and the number of persons employed (working hours). From these principal volume indicators a set of analytical indicators may be derived. By means of these indicators the efficiency of given factors may be treated.

The modelling of the supply side of the reproduction process is connected mainly with the determination of a suitable analytical type of the production function /PF/. This PF has to enable the determination of the degree of intensity of the influence of production factors and their groups on a certain volume of production.

We know from the theory of the PF, that the complementary or the substitutional type of the PF's according to the substitutions among the production factors may be used for modelling purposes. We consider one or more factors PF according to the number of production factors. If we consider more production factors in a PF, then the problem of estimation of the intensity of the influence of each factor has to be solved.

In the one factored PF the volume of the production, as a rule, is a function of one variable - capital, working capital or their increment/investment/. The parameter of the independent variable in the PF of this type is the output-capital ratio or its invers the capital-output ratio or the incremental capital-output ratio:

(1) $Y(t) = u\, K(t)$, where

$Y(t)$ stands for volume of production
$K(t)$ for volume of capital
u for output-capital ratio
t for time

or (2) $Y(t) = \frac{1}{n} K(t)$, where n stands for capital-output ratio.

Up to the present a more known relation was used in Marxian literature:

(3) $Y(t) = w\, L(t)$, where

w stands for labour productivity
$L(t)$ stand for a number of workers (units of working hours).

In pronciple it is a certain type of complementary PF. We may express it also by the help of the parameter of labouriousness (inverse of the parameter of the labour productivity):

(4) $Y(t) = \frac{1}{p} L(t)$, where   stands for labouriousness.

The problems with the practical use of these types of PF are not so connected with the parameter's estimations, as with the unrealistic assumption of its constancy. It may be clearly seen from eq. (3) and (4) where labour productivity and labouriousness are dynamically changing variables. They cannot be used as constants and at least have to be considered as functions of time; $w(t)$ and $p(t)$.

A slightly better result of application of PF given in eq. (2) and (3) is due to the relatively small variability of the capital-output ratio. Here a direct interrelation with the hypothesis of so-called neutral technical progress (constant capital-output ratio assumption) may be considered. But empirical results show that this hypothesis was not generally proved to hold valid for all countries. The PF's given in eq. (2) and (3) are therefore mainly of a theoretical aspect. For empirical application in statistical data they may be used only exceptionally. For an appropriate explanation of real changes in production we have to dynamize these parameters. The same result may be obtained more elegantly by the use of other analytical types of PF.

On the level of the whole economy we may count with a certain elasticity of substitution among the production factors. Therefore, for the purpose of demonstration of the supply side of the production, we have used the PF which is familir to the Cobb-Douglas type. In spite of an unrealistic assumption of the Cobb-Douglas PF (CDPF) (the elasticity of substitution between independent variables is one) this function is appropriate especially for the economic interpretation of the contents of its parameters. This type of PF is not limited from above, all production factors are necessary for a certain volume of production. It is a monotonously increasing function with decreasing productivity of particular extra added production factor. These properties may be empirically verified in the course of growth of every developed national economy.

In this respect I would like to stress two different economic interpretations of CDPF's parameters estimated from cross section and time series analysis. Analytical type CDPF is in both cases identical

(5) $Y = b K^{\alpha} L^{1-\alpha}$

but the economic contents, absolute magnitude, bilateral relations of both parameters α and $/1-\alpha/$ and the properties of this function are quite different. The same case we find in a more general type of CDPF, which does not consider the homogenity of degree one. The homogenity of CDPF is of degree ($\alpha + \beta$) and may be less or greater than one:

(6) $$Y = b\,K^{\alpha}\,L^{\beta}.$$

The degree of homogenity in PF of this ty[illegible] determines the decreasing $[(\alpha+\beta) < 1]$, con[stant] $[(\alpha+\beta) = 1]$ or increasing $[(\alpha+\beta) > 1]$ returns to scale.

From the marginal rate of substitution of labour for capital (ratio of first partial derivates of cross-section CDPF) follows

(7) $$s' = \frac{\partial Y/\partial L}{\partial Y/\partial F} = -\frac{\beta}{\alpha}\,\frac{K}{L}.$$

For a constant volume of production at a given capital-labour ratio (K/L) it is necessary to provide more units of capital of the ratio β/α for an economy of one unit of labour L. In the case of the cross-section CDPF, the parameters α, β describe the technological structure of production. (From the point of view of bilateral relationship between capital and labour.)

It was proved in numerous empirical studies, that the static CDPF parameters estimated on the basis of econometric methods (f. e. method of least squares) are in their magnitude similiar to the parameters estimated by the so called factor-share method, often used by non-marxian econometricians. This factor-share method is in coincidence with the cross-section estimate of the CDPF. Parameter α is estimated as the share of profits and parameter $1-\alpha$ as the share of wages in the national income. In cases where the marginalistic theory of income distribution are not fullilled (perfect competition) these "share" parameters are biassed. According to L. R. Klein these parameters should be corrected by long special formula:

If we treat the question of the positive residual more carefully, then with the help of the generalization of Cauchy's inequality by Holder, we may conclude, that such a fact must be observed each time. The first part of the residual is the result of inserting individual or aggregated variables into the CDPF. This is connected with the well known relationship between the arithmetical and geometrical means (the sum of logarithms of positive numbers is always greater than the logarithm of the sum of these numbers). The second part of the positive residual is due to the random variable. This way we obtain in each case the space for inserting additional independent variable at the assumeption that the number of degrees of freedom is greater than one.

The necessity of a combined approach to the parameters estimation should be stressed. The parameters α, β are to be determined from the cross-section analysis; parameters "r" and the constant "b" from the time series.

On the other hand when all parameters are estimated by the least squares method (LSM) directly from the time series, they are of a different magnitude and economic contents and aduittedly they are often biassed. The proof of this may be derived from the following construction of the "pure" dynamic CDPF.

The dynamic CDPF may be derived from the assumption that in time there exists a constant relationship between the relative labour-productivity increase and the relative capital-labour increase. This assumption may be described by the first-order differential equation:

$$(10) \qquad \alpha' = \frac{\dfrac{d\,w\,(t)}{d\,t} : w\,(o)}{\dfrac{d\,v\,(t)}{d\,t} : v\,(o)}$$

where $dw(t)$ stands for the labour-productivity increase in time "t";

$w(o)$ stands for the absolute level of the labour productivity in the basic period;

$dv(t)$ stands for the capital-labour ratio increase in time "t";

$v(o)$ stands for the absolute level of the capital-labour ratio in the basic period;

α' stands for the coefficient of relative labour-productivity increase per unit of relative capital-labour ratio increase.

Solving this equation according to "t" after simple arithmetic we get

(11) $w(t) = b \cdot v(t)^{\alpha'}$,

where b is the initial integration constant which we may gained from w (o) and v (o).

If we assume that this dynamic PF is a homogeneus function of degree one, we may substitute eq. (11) for eq. (3) and we obtain

(12) $Y(t) = b\, K(t)^{\alpha'} \quad L(t)^{1-\alpha'}$

This is formally identical with the cross-section type CDPF.

Parameter α' has its above defined content. Vice versa may be proved, that the parameter $(1-\alpha')$ is not only a formal complement to one; it is also an elasticity coefficient of capital output ratio to capital-labour ratio. In the magnitude of both factors even the technical and organizational progress is embodied which is seen in increasing (decreasing) productivity of capital and in the increasing (decreasing) parameter α'. The same naturally concerns the parameter $(1-\alpha')$. here stress that the technical and organizational progress is embodied in estimated parameters and is not dealt with separately.

If $\alpha' < 1$, the technical and organizational progress is capital absorbing; if $\alpha' = 1$ it is neutral and if $\alpha' > 1$ it is capital saving if $\alpha' > 1$, then $(1 - \alpha')$ is negative.

In the last case $(1 - \alpha < 0)$ the PF may be transformed to

(13) $$Y = b\, v^{\alpha'} L$$

For long-term projections it is not necessary to estimate separatelly the changes of parameters α, β and the proper influence of so called "technical" progress.

The estimation of the parameters of this CDPF may be effected in two ways.

Firstly it is possible to estimate exogeneusly both parameters as true elasticity coefficients immediately from time series, that is from the relative/average/ increase of labour productivity and capital-labour ratio in the modelled period. With the help of those parameters and independent variables we then "purify" the production volume, and by LSM we estimate the initial constant b. This procedure is methodologically (formally) identical with the multi-stage method of least squares.

Secondly it is possible to use some econometric method of estimation. Usually we use LSM after a logarithmic transformation of variables and construction of a system of simultaneous equations. In this system it is possible to estimate all parameters, the initial constant b and both parameters α', β', $(1 - \alpha)$ resp.

With the help of a computer we made large scale computations for both possible approaches. Moreover we have considered dynamic PF also with a new variable e^{rt}. This is a characteristic of changes in development of labour productivity and capital-labour ratio trends. It follows from the original differential equation (12) that this PF has a constant ratio of these trends.

The extended CDPF is as follows

(14) $Y(t) = b\, K(t)^{\alpha'} \; L(t)^{1-\alpha'} \; e^{r't}$

We have calculated the estimation of parameters of these PF for the period 1950 - 1961 and subperiods 1950 - 1955, 1955 - 1961. For dependent variables we have taken into account alternatively the gross product and national income for the whole economy and industry. We selected alternatively fixed assets, machinery and equipment, electric power consumption for the whole economy and for industry as the first independent variable we took into account the number of persons employed in the same division.

For these dependent and independent variables six types of dynamic CDPF were estimated. The methods of estimation of their parameters were as follows:

Type of dynamic CDPF	Methods of estimation of parameters			
	b	α'	β'	r'
1	LSM	EC	$1-\alpha'$	X
2	LSM	EC	$1-\alpha'$	LSM
3	LSM	EC	LSM	LSM
4	LSM	LSM	$1-\alpha'$	LSM
5	LSM	LSM	LSM	LSM
6	LSM	LSM	LSM	x

where LSM ... the least squares method
EC .... the elasticity coefficient
$1-\alpha'$... the complement of the elasticity coefficient to one
x ..... was not estimated

If I evalute, very briefly, the results obtained on unusually extensive data, may offer the following conclusions: The estimates of the parameters of the dynamic CDPF carried out on the basis of the LSM gave a very close fit of observed theoretical and empirical values of production, which is due to a relatively small number of degrees of freedom. But the particular parameters had a broad confidential intervals at 5% significance level. In many cases particular parameters and initial constants were biassed. I May give some examples, where both parameters are negative and the initial constant is greater than 10^{20} or vice versa the value of both parameters is high and positive and the initial constant is small (less than 10^{-17}). Even in these examples the fit of empirical values is very good. From this follows, that a compensation of bias is done among the parameters and the initial constant. This bias is due to the principal lack of primal information.

This lack is explained by multicollinearity, that we often meet in the estimation of parameters of analytical functions from time series by LSM. It is due to the fact, that the considered "independent" variables of time observations of capital and employment are not independent in the statistical sense of the word. Among those variables a statistically significant relation was measured. An ill - conditioned estimating matrix for regressive estimates results from this. Using the known procedures for significance tests of estimated parameters /t - test/, we get a very broad (an even in many cases a multiple of estimated values) confidential intervals. The estimation matrix is ill-conditioned and therefore the element of its inverse on the main-diagonal are relatively large. These elements and the average residual deviation define the lower and upper limits of confidential intervals for estimated parameters.

In comparison with this procedure a method based on exogeneusly estimated elasticity coefficients provided more favourable results. This method may be considered - as mentioned above - as a multi-stage estimation. The reality of the parameters α' and $(1 - \alpha')$ values was verified on the basis of productivity function (11) for all variants of given indicators

$$w(t) = b\, v^{\alpha'}(t)$$

and/or $w(t) = b\, v^{\alpha'}(t)\, e^{r't}$ /11a/

These calculations proved that the parameters estimated from the productivity function by the LSM are practically identical with the elasticity coefficients estimated independently from time series observations as a ratio of the relative increase of labour productivity and the capital-labour ratio. This procedure served to prove a coincidence of empirical results with deductive theoretical assumptions. By this process the approach to the construction of true dynamic CDPF mentioned above in eq. (12) was proved.

The high degree of the fit theoretical and empirical values of production and high coefficient of determination $R^2 > 0,99$ was achieved by the type of CDPF number 2, practically for all alternatives of investigated indicators. The types of CDPF 3 and 4 as a combination of both procedures, were not more succefull in comparison with the type number 5 (LSM). This was due to the fact that the third independent variable e^{rt} which was a statistically significant relationship with the unexplained residual of production, was considered.

If we take into account the alternative choice of independent variables, the best results are in most cases achieved by inserting the factor machinery and equipment and in rate cases alternatively even the electric power consumption. If we measure the accuracy of modelling the production indicators, then the obtained results are better for the gross product than for national income (as the dependent variable) in the sphere of material production and in industry.

The CDPF type 2. for modelling the gross-product on the basis of machinery one equipment and average number of persons employed in the years 1950 - 1961 is as follows

(15) $$Y(t) = 7,571\, K(t)^{0,657}\, L(t)^{0,343}\, e^{0,0072\, t}$$

If we substitute factor machinery and equipment for the factor electric power consumption we get

$$(16) \quad Y(t) = 25{,}51 \; E(t)^{0{,}593} \; L(t)^{0{,}407} \; e^{0{,}0107 \; t}$$

For national income and for the factors machinery and equipment and average number of persons employed the CDPF is in the same time period as follows

$$(17) \quad Y(t) = 0{,}118 \; K(t)^{0{,}55} \; L(t)^{0{,}45} \; e^{0{,}008 \; t}$$

The CDPF for the gross output in industry and for the factors machinery and equipment and average number of persons employ is for 1950 - 1961 as follows:

$$(18) \quad Y(t) = 6{,}746 \; K(t)^{0{,}738} \; L(t)^{0{,}262} \; e^{0{,}0025 \; t}$$

In the same way the parameters of other types of CDPF for every variant of indicators were estimated.

These results prove that it is possible, to avoid the difficulties, that arise, when carrying out the regression estimates of parameters of the macroeconmic substitutional CDPF. It is an advantage, that the parameters estimated independently as a elasticity coefficients directly from time series data may be used. The initial constant and/or the time trend (the addition of third variable e^{rt}) may be computed on the basis of LSM. In this way we achieve more accurate results for the aggregata growth model, which we have construted and applied on the basis of the mentioned dynamic CDPF in our Research Institute for Economic Planning.

A Long-term Projection of Conditions for Economic Growth of the Czechoslovak Economy Based on an Aggregate Growth Model

Aggregated growth models are beginning to play a specific role in the complex of methodological instruments, which are being created and applied in CSSR to the working out of long-term forecasts and on their basis even of long-term plans. These models are important mainly because they make possible the study some of the very long-term macro-economic aspects of economic growth, especially the growth of the national income, of the gross product, investments, fixed assets, consumption, employment, capital accumulation, etc.

In the course of the preparations for the working out of a long-term forecast of the growth of Czechoslovakia's economy the aggregated growth model was applied also by the Research Institute for Economic Planning. The aim of the application of aggregate model was to as certain the objective relations or, as they are sometimes called, the technico-economical relations, which are the inner content of a certain type of economic growth characterized by the volume and rate of growth of the national income and of the consumption as well as by further economic indicators.

The aggregate growth model was therefore able to give us important data for disaggregated structural model based on input-output methodology.

At the present level of knowledge of various economic growth models it is quite obvious that there are and for some time still will be many aggregate growth models, each reflecting more or less at least the basic relations of the actual economy. Of course they differ also by the number of basic relations, which they reflect.

In spite of that, however, we can say today that there are certain types of models suitable to solve specific types of problems. Within the scope of a particular type of models a process of integration or at least of the expression of mutual interrelations may be observed, offering thus an opportunity to evaluate their ability to analyse or predict.

After extensive computations decided to use a very simple aggregate model based on the mentioned dynamized CDPF. This model, which we have later disaggregated into the branch structure, is not so elegant from the mathmatical point of view. But it is computable, it is not only theoretical model and this is its great advantage.

$$Y_{(t)} = aK^{\alpha}_{(t)} \cdot L^{1-\alpha}_{(t)} \qquad (1)$$

where $Y_{(t)}$ = net national income (Marxist method) ($Y' = Y$ + depreciation, i.e. gross national income)

$K_{(t)}$ = volume of fixed assets

$L_{(t)}$ = labour force, for example number of employees

α = function parameter (according to out conception, as explained further, the coefficient of elasticity of labour productivity to the capital-labour ratio);

I would like to stress the economic meaning of the parameters α and $1 - \alpha$ as distinct from the common interpretation.

The CD production function was dynamized by special approach to the estimation of its parameters.

Parameter α can be ascertained mathematically and concretely numerically as the elasticity coefficient of the productivity of labour to the capital-labour ratio:

$$\alpha = \frac{dw}{w} : \frac{dv}{v} \qquad (2)$$

where w = productivity of labour (dw - its absolute increment)

v = capital-labour ratio (dv - its absolute increment);

and parameter $(1 - \alpha)$ either as a supplement of α to one or separately as a elasticity coefficient of the capital-output ratio (m) to the capital-labour ratio (v) [x)]

$$1 - \alpha = \frac{dm}{m} : \frac{dv}{v} \qquad (3)$$

The economic significance of both elasticity coefficients is very clear. The parametr α indicates by how many percent the productivity of labour will increase if the capital-labour ratio will grow by 1%. The parameter $(1-\alpha)$ indicates by how many percent the capital-output ratio will increase (or decrease), if the capital-labour ratio will also grow by 1%. The increment of the national income also corresponds to these relations, as can be seen from the original substitutional production function (4) because simultaneously it is true that the parameter α is at the same time the elasticity coefficient of the national income to the capital assets and parameter $(1-\alpha)$ the elasticity coifficient of the national income to the labour force. It is obvious that the technical progres in the magnitude of this parameters is reflected.

There is a possible method of determining both parameters by some of the econometric methods, for example by the method of the least squares from the data of the time series on the national income, capital assets and the labour force. As shown by extensive calculations, this method in contradiction to the preceding method, usually does not yield satisfactory results.[x)] This may be caused by frequent multicolinearity of time series data characterizing economic development.

I shall now explain briefly the procedure, which we have used in the first step of determining the rate of growth of the national income and of the relations and proportions connected with it.

Let us start from the production function defined by the mentioned dynamized CDPF

$$\frac{dY}{Y_{(t)}} = \alpha \frac{dK}{K_{(t)}} + (1-\alpha) \frac{dL}{L_{(t)}} \qquad (4)$$

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x) cf. quoted Paper p. 884

The relative increment of the national income depends on the relative increase of fixed assets weighted by parameter α and on the relative increment of employment weighted by parameter $(1 - \alpha)$.x)

From this relation (4) we can also derive the corresponding share of the increment of the fixed assets in the national income, which we further shall treat as the share of net investments:

$$\frac{dK}{Y(t)} = \frac{1}{\alpha} \quad \frac{K(o)}{Y(o) \quad (1+y)^t} \left[\frac{dY}{Y(o)} - (1 - \alpha) \frac{dL}{L(o)} \right] \qquad (5)$$

The share of the net increment of the fixed assets in the national income is inversely proportional to the parameter "α" and the rate of growth of national income and directly proportional to the capital-output ratio and to rate of growth of the national income decreased by the rate of growth of employment weighted by parameter $(1 - \alpha)$.

If we don't take into account the increase of employment in the production sphere, which is an assumption acceptable with a high degree of probability for the perspective of the Czechoslovak economy, then the relation expressing the relative increment of the national income will simplify as follows:

$$\frac{dY}{Y(o)} = \alpha \frac{dK}{K(o)} \qquad (6)$$

After a certain modification we can get the production function for the growth of national income in the form

$$Y_{(t)} = Y_o \quad e^{\alpha kt} \qquad (7)$$

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x) In relation (4) a simplification has been made with respect to the assumption about the continuity of the volume of fixed assets $K_{(t)}$ with respect to time eventhough current statistions record their volume in discreet time intervals.

where k is the average annual rate of growth of the fixed assets, or in the form

$$Y_t = Y_o \ (1 + \alpha k)^t \qquad (7a)$$

Due to the fact that changes of employment are not foressen, the relative increment of fixed assets is in this case identical with the relative increment of the capital-labour ratio. This simultaneously formulates a demand on the technical progress. The new fixed assets have to have proportionately lower demands on labour. In other words it determines the qualitative requirements on the level of the capital-labour ratio no matter whether it concerns fixed assets for replacement or for development.

The net capital accumulation can be further adjusted with regard to (7) as follows:x)

$$i \ (t)^c = n_o k \quad \frac{(1 + k)^{t-1}}{(1 + \alpha k)^t} \qquad (8)$$

Provided we known the ratio of the gross investments $I^{(h)}_{(t)}$ xx) to the net increment of the fixed assets $dK_{(t)}$:

$$\frac{I^{(h)}_{(t)}}{dK_{(t)}} = h \qquad (9)$$

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x) And due to the fact that $K_{(t)} = K_{(o)} \ (1 + k)^t$

xx) $h = 1 + \frac{1}{\varrho}$ where ϱ is the so-called reproduction ratio, i.s. the ratio of expansion to replacement investments. Cf. Politicka ekonomie, No. 1 (1966), p. 36.

It is possible to demonstrate under the condition of a constant labour force that this rate of growth (k) dependends on the quantity k vor ϱ under the following conditions: $k = kv\varrho$ or $k = \frac{1}{h-1}$, where (kv) is a percentage of fixed assets replacement.

the share of the gross investments in the national income is equal to:

$$i^{(h)}_{(t)} = n_o \cdot kh \frac{(1+k)^{t-1}}{(1+ \alpha k)^{t}} \qquad (10)$$

The symbol h is influenced partly by the degree of retirement of fixed assets, partly by the volume of capital construction in progress. Three and five year moving averages of the variable h for the total productive fixed assets of the production sphere demonstrate a relatively constant ratio of abour 1,2.

Further from the share of the investments in the national income we can derive the absolute investment volume.

The following relation holds true for net investments:x)

$$I^{(c)}_{(t)} = K_o k \ (1+k)^{t-1} \qquad (11)$$

The relation for gross investments can be written as follows:

$$I^{(h)}_{(t)} = K_o kh \ (1+k)^{t-1} \qquad (12)$$

Since we assume in the first step a division of the national income only into two components:

$$Y_{(t)} = I^{(c)}_{(t)} + C_t \qquad (13)$$

we can ascertain the volume of consumption as:

$$C_{(t)} = Y_{(t)} - I^{(c)}_{(t)} \qquad (14)$$

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x) Provided that the increment of the fixed assets corresponds to the volume of net investments. As far as the dynamics is concerned, we can to a certain extent, introduce the assumption of an invariable time lag between the investments and the increment of the fixed assets. Since we operate with the increment of the fixed assets and since we define the net investments on its basis, the time lag between the expediture of the investment funds and the increment of the national income will become quite negligible.

Besides the usual concept of the national income as a sum of the volume of net investments and consumpttion we can use as a basis for the projections the so-called gross national income ($Y^{(h)}_{(t)}$)

$$Y^{(h)}_{(t)} = I^{(h)}_{(t)} - C_{(t)} \qquad (15)$$

On the basis of the gross national income it is possible to ascertain the share of gross or net investments, which is more comparable with the highly developed capitalist economies.

For the verification calculations, however, we have used a variant of the model, which is based on the active, decisive component of the fixed assets - on machinery and equipment - instead of the total fixed assets.

The reason for this procedure is above all a closer link between the development of the capital-labour ratio and thus of the capital output ratio and the productivity of labour.

In this case the parameter α' has to be ascertained in relation only to the machinery and equipment capital assets - labour ratio. In Czechoslovakia in the years 1950 - 1961 this parameter displays a considerably lower variability then the parameter ascertained on the basis of the total fixed assets and with a tendency to an absolute descrease at that.

Preliminary calculations based on the machinery and equipment only have demonstrated that this model reflects a more realistic growth of the volume of productive investments and their share in the national income than a model based on the total fixed assets. The reason for this difference is above all the higher stability of parameter α.

The projection based on machinery and equipment fixed assets stipulates "harder" conditions for the attaining of a certain rate of growth of the national income in the future in our conditions due to the level of parameter α', than the model based on the total fixed assets. In addition to this it seems to be more advantageous to base the projection on fixed assets of the machinery and equipment also in view of the development trends mentioned earlier. The change of the share of fixed assets of a non-machinery character (buildinge, structures) influances the parameter α but does not have such a close relation to the growth of the productivity of labour as in connection with fixed assets of the machinery and equipment class.

However, it is necessary to stress another aspect of the long-term development, which is very important for the Czechoslovak economy. Besides the improvement of the system of planning and management of the national economy at all levels, this is evidently the most important prerequisite not only for our effective economic growth but of any growth in general.

We have shown a preference for the substitutional type of model yet the problems of the substitution of labour by fixed assets dont disappear by the assumption of constant employment, which appears to have eliminated the labour forces from the model. The substitutional character of the model is maintained even in this case. As we have already shown it is partly included in the parameter α', even though it can be interpreted in that case as an elasticity coefficient of production and fixed assets. It is also, however indirectly included in the choice of the rate of growth of the fixed assets.

In order to evaluate correctly the possibilities of our further economic development it is necessary to realize that the rate of growth of the fixed assets cannot be an arbitrarily planned quantity. <u>It follows from the substitution principles in the economy and from the model that the rate of growth of the fixed assets in the production sphere can be only as high as the of growth of the capital-labour ratio.</u>[x)]

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x) It is perhaps clear that it cannot be just a matter of a simple mechanical commensurability of the volume of fixed assets and of the number of employees. Objectively determined technological and organisational relations always exist.

The rate of growth of the fixed assets is, from a methodological point of view, exogenous variable in the original model, but it is from the point of view of lack of the labour force in CSSR a limited quantity.

The rate of growth of the fixed assets will be effective under the condition that its productive utilization by labour can be insured. Generally speaking the labour savings achieved by the liquidation and reconstruction of the existing capacities has to meet the needs not only of the reconstructed fixed assets but of the new fixed assets as well. The logical conclusion is that the rate of growth, i.e. the net increment of the fixed assets will have to be curtailed within the acope of their gross increment in favour of an increase in the retirement of obsolete fixed assets. On a high technical level the resulting decrease of the rate of growth of the national income should be compensated by an increase of the parameter α.

In the revised enlarged version of the model we use therefore the rate of growth of fixed assets as an endogeneous variable. It depends on the ratio of the scrapped fixed assets $k^{(t)}$ and on the so called reproduction ratio ϱ. (which is the ratio of net increase of fixed assets to the volume of the scrapped fixed assets).

The reproduction ratio should not be greater than the capital-labour ratio of new fixed assets to the capital-labour ratio of scrapped assets. Under these conditions and under the presumption of constant level of employment, the new fixed assets could be sufficiently supplied by labour force. The admisible rate of growth of fixed assets gives us the relation

$$k = k^{(t)} \quad .$$

The parameter α can be expressed also as an endogenous one. It may be proved that parameter α is a function of the rate of growth of the output-capital ratio (u) and capital-labour ratio (v)

$$\alpha = \frac{u + v}{v} , \qquad \text{or}$$

$$\alpha = 1 + \frac{u}{v} .$$

The production function in the revised enlarged version of the aggregated model (under conditions of constant level of employment in material sphera of production) could therefor be written as follows:

$$Y_{(t)} = \left[1 + k_{(t)}^{(1,s)} \quad \varrho_{(t)}^{(s)} + u_{(t)} \right] \quad Y_{(t - 1)}$$

The rate of growth of national income as

$$y = k^{(1,s)} \, . \, \varrho^{(s)} + u^{(s)}$$

which is identical with

$$y = \alpha k$$

from the first version of the model.

The enlarged aggregated model considers ten exogeneous parameters and ten equations for the endogenous variables; the initial values for them are to be known.

The exogeneous parameters are as follows:

a) the share of scrapped fixed assets, both for machinery and equipment and for buildings
b) the reproduction ratio even for both groups of fixed assets
c) the average duration of construction for both groups of fixed assets
d) the rate of growth of capital-output ratio
e) the ratio of amortization of productive fixed assets
f) parameter of the increment of circulating assets and stocks (ratio to the increment of fixed assets)
g) the parameter for the value of overhaules

The last three parameters are relevant for the projection of the structure of the national income.

The endogeneous variables are as follows:

a) the national income
b) fixed assets (machinery, equipment, buildings)
c) the volume of net investments for machinery and equipment and buildings
d) the increment of fixed assets in construction both for machinery and buildings
e) the increment of circulating assets and stocks
f) the ammortization fund
g) the value of overhaules

There are also definition equations, such as for gross national income, capital accumulation, consumption, gross investment, etc. There was a possibility to calculate a great number of such indicators like labour productivity, capital-labour ratio, capital-consumption ratio etc. All these calculations were accomplished step by step for every year of the projection.

The determination of the objective possibilities for the rate of growth of the national income and even for the rate of growth of the fixed assets supposes the working out of a separate analysis of the problem of the reproduction of fixed assets in a longterm development. This has not been available so far and we have therefore started in this stage of the work to investigate alternative variants of the rate of growth of the fixed assets and variants of the changes of parameter α or $\underline{u}$ respectively.

We consider, however, the method based on various alternative analyses at the present stage of our investigations as an advantage. The approach towards a prognosis of the long-term growth cannot be based, in our opinion, on an endeavour to "guess" as exactly as possible the future. Today we can only indicate the conditions leading towards the growth, which we are striving to attain. In this case it is rather a project than a prognosis of the future development.

The first approach is however a simple case of extrapolation. This extrapolation, though usually a priori rejected can have a certain importance. In connection with an analysis of the development of economic relations and proportions, which can also be deduced from aggegated model, a mechanical extrapolation can show the objective possibility or on the contrary the impossibility of a continuation of the past trends of the growth. In the case of a national economy whose growth is governed by a plan this means implicitly that the mechanical extrapolation can detect in time the harmfulness of a certain trends in the national economy, for instance in the investment policy, which objectively leads to such a development.

The second approach consists in using the aggregated growth model to analyse the future growth while keeping certain initial conditions. In stipulating these conditions, or better said, the limits of the future development, by using the "mechanism" of the production model, we will not be entirely dependent only on "pure" extrapolation as would appear at first glanse. We shall apply the the methodic principles of dynamic programming and we shall consider every admissible value of the parameters to be one of the possible variants of the future growth. In contradiction to simple extrapolation this method requires unusually exacting calculations, but it permits a direct confrontation of economic contemplations with their impact on the main indicators of the national economy.

The model should therefore provide the answer to the question "what should be done" and from that we can derive the answer to the question "what will happen" or more accurately "what can come about under certain specific conditions". Only after we will know what can be attained under certain specific conditions, will it be possible to try and find out from the given set of possible and admissible solutions an optimum solution for the economy.

However, even in this case it will not be possible to accept a deterministic solution but a probability interval solution.

The most significant results of the application of the growth model

It is quite obvious from the preceding discussion that, as long as certain specific conditions exist, the model reliably reflects the long-term rate of growth of the national incom, consumption, the net and gross increment of fixed assets and the corresponding volume of productive investments as well as the qualitative indicators derived from them such as the share of the net and gross productive investments in the national income, the capital and investments output ratios the productivity of labour, the capital-labour ratio etc.

In the first stage of the calculation of the individual projection variants we have assumed average parameters and exogenous variables of the model for the entire investigated period. It would have just as well been possible to assume their rise or decrease in individual intervals of the entire period of time. However, the method applied was in conformity with the questions to which an answer was sought.

The first question was, what are the long-term requirements imposed by a certain rate of growth of the national income, employment remaining constant, on investments, on the rate of growth of the fixed assets and thus also on the investment and capital output ratios, on the capital accumulation, characterized in this case by the share of the net or gross productive investments in the national income and on the consumption.

The next questions was, how should the character of the future type of technical, or better said of economical progress (i.e. also in relation to the non-investment factors) be changed in comparison with the present development in order to increase the effectiveness of accumulation and thus decrease the invesment intensity of the growth or, on the contrary, to increase the rate of growth of the national income at a given investment and capital.

The first question can of course also be formulated in such a way as to ask: what is the maximum long-term rate of growth of the national income attainable under the condition of not exceeding the share of the gross productive investments in the national income, for example in the selected range of 14% - 21%. How high should in that case be the rate of growth of the fixed assets and the volume of the gross productive investments, what values should parameter α, i.e. the elasticity coefficient of labour productivity, to the capital-labour ratio attain. All this of course under the condition that the ascertained rates of growth of the national income would not be limited by a foreign trade barrier, including an acceptable degree of its effectiveness.

An analysis of past development by means of the model is not defficult. In the course of such an analysis the model has tested its own ability to give evidence. The correctness of the picture of the real relations in the long-term average is quite natural because it is derived, as has already been shown, from the way the model has been design. The search for variants of the future relations is of course more difficult because that supposes a knowledge of at least some of the quantities constituting the model.

I have already said that there is no sense in trying to "guess" complexely the future economic development, that the realistic method is the ascertaining of the conditions and prerequisites required to achieve a certain type of future development. In this sense it is quite expedient to use the analyses of the consequences of past development as a springboard because a certain continuity can never be quite denied and on the other hand above all the prerequisites, conditions and consequences of deviations from the past development, as long as they can contribute more positive results.

The application of a growth model for the mechanical extrapolation of the development due to the apparently favourable economic conditions of the period from 1955 to 1961 proves that from a long-term point of view it would not only be a very ineffective, but even an economically unrealizable development.

It is true that during the above mentioned period relatively high rates of growth of the national income exceeding 7% on the average were achieved. To this development corresponded also a high rate of growth of the capital-labour ratio in the production sphere, on the average almost 10% annually. At the same time, however, this development was also accompanied by a high rate of growth of machinery and equipment, which exceeded 10% and a substantially lower rate of growth of the overall produtivity of labour so that parameter α' amounted to 0,68 on the average. Its magnitude (less than one) demonstrates from the points of view of machinery and equipment capital absorbing technical progress. This technical progress was labour-saving but made considerable demands on the active component of the fixed assets.

If we further assume the other variables of the model on the level of the average for the same period, then twenty years later, i.e. in 1985 the share of gross investments in the national income would attain approximately 42% and 23% into machinery and equipment. The machinery and equipment capital-output ratio would also rise substantially, from the calculated 0.985 in 1955 to 1.327 in 1975 and to 1.788 in 1985.

An unfavourable development of the capital as well as investment intensity of the growth of the national income would have been accompanied by a substantial decrease of the share of consumption in the national income as well as by a decrease of its rate of growth. The consumption would decrease from 80% in 1965 to 63% in 1985. Even if we were to consider a correction (according to the previous explanation) to the usual conception of consumption (with a difference between the depreciation fund and the value of the retired fixed assets) only the percentages in the above mentioned years would change, but the decreasing trend of the percentage and of the rate of growth, which interested us more in this projection than its absolute level, would not change very much.

It appears therefore that a pronounced increase of the effectiveness of capital accumulation, which will lead to a growth of parameter α is a necessary condition for maintaining the past rates of growth of the national income. To maintain the rates of growth of the fixed assets under an otherwise unchanged situation and thus to increase the share of the gross productive investments in the national income above the level of 1960 and 1961 when it already exceeded 18% (11, 8% in 1955) is evidently beyond the material, labour and organisational possibilities of the Czechoslovak capital construction activities, apart from the effectiveness of such a procedure. The new system of planning and management of the national economy should certainly be able to expand our possibilities in this respect, but increasing the volume of investments without a corresponding increase of their effectiveness is in our conditions definitely not an acceptable way of long-term development.

The only possible way to achieve a solution lies in decreasing the rate of growth of the fixed assets, which should permit to spend a greater part of the gross productive investments on renovation and reconstrution, particularly of machinery and equipment. At the same time it will of course require the strict fulfillment of the condition that the new machinery and equipment will have a much higher technical standard both with regard to labour requirements as well as to production efficiency. I have already shown that the objectively possible and effective rates of growth of the fixed assets in the future decades depend after all on the technical standard of the fixed assets.

The most concentrated and at the same time the most synthetic criterion of the effectiveness of technical progress and in general of the effectiveness of the internal reproduction process is in this case the magnitude of parameter α . When the rates of growth of the fixed assets will be reduced then the rates of growth of the national income can be maintained only by a substantial increase of the effectiveness of the technical and organisational progress in comparison with both the average of 1955 - 1961 and particularly with the standard of recent years.

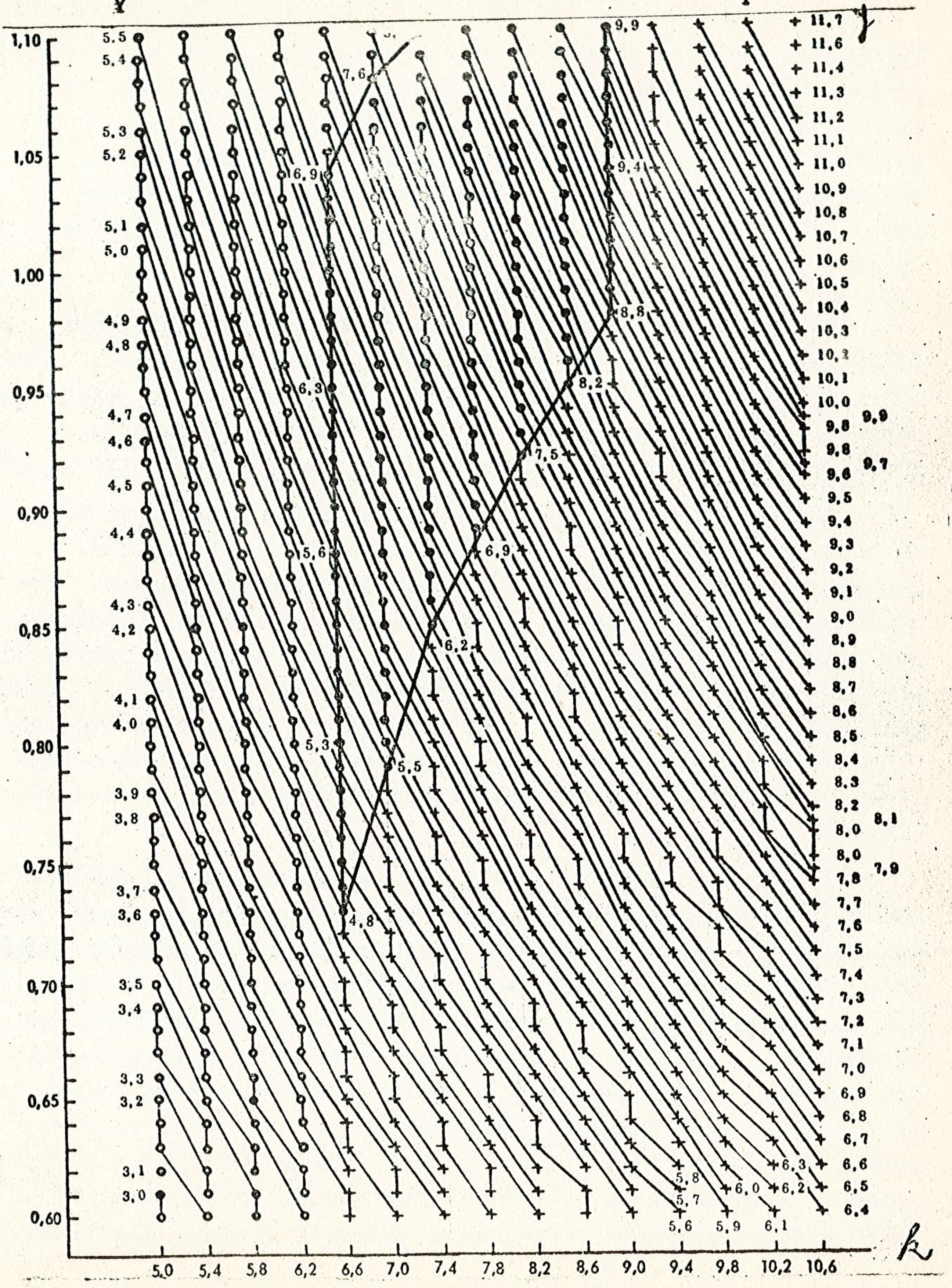
I would like to show it on a graph.

The graph of isotemps of the national income is a very clear illustration of the conditions required to reach a certain rate of growth of the national income, which would not, within a period of twenty years exceed the ratio of gross productive invesments to the national income within the limits of 14% - 21%.

The average annual rate of growth of the fixed assets (machinery and equipment) k is plotted along the horizontal x-axis. Parameter α is plotted along the vertical y-axis. The oblique lines in the graph - the individual isotemps - indicate the same average annual rate of growth of the national income for the individual combinations of parameter α and the rate of growth of machinery and equipment fixed assets k during the twenty years under investigation. For better orientation the more important rates of growth of the national income (4%, 5%, 6%, 7% etc) are drawn in heavy lines.

The area bounded by heavy lines within the graph defines the area of those long-term rates of growth of the national income, which for the given combination parameter α and an average annual rate of growth of machinery and equipment, at a ratio of the overall investments to the volume of investments into machinery and equipment q = 1.83 and at a ratio of gross and net investments into machinery and equipment h = 1.3 (i.e. q = 3.33) based on initial values of the year 1965 (for Y_o, K_o) will not require during the entire twenty years period a lower ratio of the gross productive investments to the national income than 14% nor a ratio higher than 21%. The points marked by a small circle in-

ISOTENPS OF THE NATIONAL INCOME USING 1965 AS A BASIS



dicate that either during the first years or during the entire twenty years period the given rate of growth of the national income does not reach the 14% share; the points marked by a cross indicate that the 21% limit has been exceeded towards the end or throughout the entire period. Those variants, which are marked by a full circle remain throughout the entire period within the 14% - 21% limits for the ratio of the gross productive investments to the national income. For the sake of simplicity the interpolation between the various points has been dune linearly.

For example it can be seen from the graph that for the chosen range of parameter α and the rate of growth of the fixed assets <u>k</u> the quantities q and <u>h</u> defined above and on the basis of the initial year 1965 the variants of the rates of growth of the national income lying within the area of admissible solutions are those from approximately 5% and higher, at minimum values of parameter $\alpha = 0{,}73$ and a minimum rate of growth of machinery and equipment of 6.6% annually on the average. The "area of admissible solutions" widens gradually with the growing rate of growth of fixed assets <u>k</u>, at a minimum value of parameter $\alpha = 0{,}73$, all this up to k = 9%. From the magnitude of $\alpha = 1.04$, which represents the type of capital saving technical progress, the area of admissible solutions begins to narrow from the left because even at relatively higher rates of growth of machinery and equipment these variants do not reach the chosen 14% limit on the ratio of gross productive investments. With a growing parameter α and therefore with the intensification of the type of technical progress with economical requirements on fixed assets the "area of admissible solutions" would close completely from the left. All variants would lie below the chosen ratio of 21%.

It is quite evident of course that all the variants to the left of the "area of admissible solutions", which lie below the lower limit of 14% of the ratio of the gross productive investments to the national income are acceptable. Those variants, which evidence comparatively the same long-term rate of growth of the national income (the same isotemps) are even relatively more effective. The same rate of growth of the national income is attained with a lower capital accumulation.

On the other hand these variants do not exhaust the entire volume of investment funds, which could be devoted to the development and under otherwise identical conditions thus ensure an acceleration of the rate of growth.

The "area of admissible solutions" should therefore be understood as including those variants, which fulfill the limiting conditions of a ratio of the gross productive investments to the national income of not less than 14% and not more than 21%.

The graph clearly shows that rates of growth of the national income of approximately 7% at values of parameter α= 0,65 - 0.70 and growth rates k in excess of 10% are all very far from the so-called area of admissible growth because they highly exceed the 21% for the ratio of gross productive investments on the national income. And that is a situation which was characteristic for the already mentioned extrapolation variant based on the conditions of the period 1955 - 1961.

The projection of various variants has also brought interesting results concerning the analysis of the fixed assets and investment a capital-output ration capital-labour ratio, labour productivity, the development of the consumption, of other indicators.

The results provided by this simple model contribute certain useful considerations about the possibilities of our further economic growth. Of course it is not a quite comprehensive survey, not even on a macroeconomic level. The model itself as I have shown was improved further by new relations. In spite of that, even in the original form, which reflects the consequences of the pressing need to know the answers as soon as possible because "yesterday was already late", it has provided data for important economic conclusions for the draft long-term outlook of Czechoslovak Economy. It represents for us also a certain point of departure for the development of the methodology of long-term projections of an aggregated nature, which is followed by a disaggregation by means of structural growth models. And it is subject of my next lecture