كلية الهندسة بشبرا
FACULTY OF ENGINEERING AT SHOUBRA
ERJ
PRINT ISSN 3009-6049
ONLINE ISSN 3009-6022

ENGINEERING RESEARCH JOURNAL (ERJ)
Volume (53),Issue (2)
April 2024, pp:60-67
https://erjsh.journals.ekb.eg

Numerical Analysis of GFRP-Reinforced Concrete Continuous Deep Beams

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Abstract: This paper presents a numerical investigation of Glass Fiber-Reinforced Polymer (GFRP)-reinforced concrete continuous deep beams using the nonlinear finite element program ANSYS. A 3-D numerical model is proposed for the geometrical and material modeling of the studied deep beams. A validation study has been conducted to affirm the efficacy of the ANSYS program as a reliable analysis tool. The comparison between the experimental results from the literature and the numerical results revealed good harmony between the two, indicating the effectiveness of the numerical simulation in capturing the key aspects of the structural behavior of continuous deep beams reinforced with GFRP. A parametric study was carried out to study the effectiveness of various structural parameters on the shear capacity of continuous concrete deep beams reinforced by GFRP bars and steel stirrups. This investigation shows that the ultimate shear capacity of the studied beans increased by 19.70%, 8.30%, 15.50%, 24.90%, 6.90%, and 8.00%, respectively, due to the increase in concrete ultimate strength, GFRP ultimate strength, beam with, beam depth, bottom reinforcement ratio, and top-to-bottom GFRP reinforcement ratio by 40%, 25%, 20%, 20%, 40%, and 12%, respectively. The results show that the ultimate load capacity increased by 71% due to the decrease of the shear span-to-span ratio by 50%. Also, the results show that the vertical web reinforcement has a considerable effect on the ultimate shear capacity of these beams compared to the horizontal web reinforcement, where the ultimate shear capacity increased by 7.1% and 1.8% due to increases in both the vertical and horizontal web reinforcement by 33%.

Keywords: Reinforced concrete; continuous deep beams; glass fiber-reinforced polymer (GFRP); numerical; parametric study.

RESEARCH SIGNIFICANCE

1. Proposing a 3-D numerical model for the geometrical and material modeling of continuous concrete deep beams reinforced with GFRP bars using ANSYS as a nonlinear finite element software.
2. Conducting a validation study through the ANSYS program to verify and validate the numerical predictions with previous experimental findings.
3. Performing a parametric study using ANSYS to explore the influence of specific parameters on the behavior of continuous concrete deep beams reinforced with GFRP.

1. Introduction

In reinforced concrete (RC) buildings, deep beams find extensive applications. It can be used as a load distribution element in various structures, including tank walls, pile caps, foundation walls, transfer girders, folded plates, diaphragms, and shear walls [1-6]. In continuous RC-deep beams, shear deformations play a significant role, rendering conventional beam theory inadequate for predicting ultimate loads. Deviations from normal beam behavior in continuous deep beams contribute to an incomplete understanding of their structural performance. There has been extensive research on simple and continuous RC deep beams reinforced with traditional steel reinforcement bars. Some literature studies are reported for simple deep beams with fiber reinforcement polymer (FRP) bars, but research specifically on continuous deep beams with glass fiber reinforcement polymer (GFRP) bars is limited. In certain regions, steel reinforcement bars may be both expensive and unavailable in the required diameters and quantities. Moreover, steel reinforcement bars are prone to rapid corrosion when exposed to harsh conditions, especially in sustainable infrastructure and marine structures.

To address these challenges, there is a growing trend toward using GFRP bars as an alternative to steel reinforcement. GFRP bars offer high ultimate strength, cost-effectiveness, and corrosion resistance, making them a viable and sustainable choice for reinforcing continuous deep beams in various structural applications. [1] conducted a study involving seven simply supported and seventeen two-span-deep beams. Their investigation encompassed beams both with and without vertical web reinforcement, including those with a high ratio of vertical stirrups. The analysis of the results revealed that beams lacking stirrups experienced sudden failure with minimal or no plastic deformation. In contrast, beams with a substantial quantity of stirrups exhibited ductile failure. [2] presented test results for eight RC continuous deep beams. A slight discrepancy was observed when comparing the test outcomes with the prevailing codes of practice. [2] specifically highlighted the influence of web reinforcement types on shear capacity, noting that vertical web reinforcement had a more significant impact on shear than horizontal web reinforcement. The research findings emphasized the crucial role of web reinforcement in determining the shear behavior of RC continuous deep beams.

[6] conducted an experimental investigation to examine the behavior and shear strength of concrete-deep beams reinforced with FRP bars. The findings indicated a substantial increase in the ultimate shear strength of the tested beams with a decrease in the shear-span-to-depth ratio. Furthermore, the study compared the predicted shear strength of the tested beams using shear design provisions recommended by the new version of the Canadian code with the experimental results. The study demonstrates good agreement. In a related study, [7] presented a parametric analysis focusing on support settlement and different support conditions for two-span continuous deep beams. The aim was to understand the extent of the effects occurring in continuous reinforced concrete (RC) deep beams under various conditions. Notably, [7] observed that the numerical finite element model could effectively predict failure loads and deflections, yielding results that were nearly identical to those obtained experimentally. This underscores the reliability of numerical simulations in capturing the behavior of continuous RC deep beams under different support conditions.

[8] investigated the shear behavior of deep concrete beams reinforced with GFRP for flexure and without shear reinforcement. The findings suggested that the stiffness of concrete deep beams reinforced by steel bars is higher when compared to beams reinforced with GFRP bars. These highlight a distinction in the shear behavior between concrete beams reinforced with traditional steel and those incorporating GFRP reinforcement. Similar findings have been recorded by [9-10].

[11] presented a numerical study involving twelve concrete-deep beams reinforced with GFRP bars, specifically without web reinforcement. The finite-element modeling results exhibited excellent agreement with experimental observations, demonstrating the accuracy and reliability of the numerical approach in capturing the behavior of GFRP-reinforced concrete deep beams. [12] developed a numerical model to investigate the behavior of concrete-deep beams reinforced with GFRP. Their study concluded that the developed numerical models proved to be efficient tools for conducting extensive parametric investigations related to the behavior of this specific structural member. This emphasizes the utility of numerical simulations in gaining insights into the performance of GFRP-reinforced concrete deep beams. In a study by [13], the structural response of continuous concrete deep T-beams reinforced with Carbon Fiber Reinforced Polymer (CFRP) under shear failure conditions was explored. The findings highlighted the substantial influence of the shear-span-to-depth ratio on shear strength and collapse loads. [13] concluded that CFRP-reinforced T-beams could achieve higher shear strength values compared to similar steel-reinforced T-beams. [14] conducted tests on three continuous deep beams and three simply supported GFRP-reinforced deep beams. The results revealed that a decrease in the shear span-to-span ratio led to a significant increase in the load-carrying capacity of the beams. Additionally, simply supported beams exhibited lower load capacity compared to their continuous deep beams. This underscores the importance of considering the shear span-to-span ratio in the design and analysis of GFRP-reinforced deep beams.

Given the limited prior investigations on the behavior of continuous deep beams reinforced with glass fiber-reinforced polymer (GFRP) bars, this research specifically focuses on addressing this gap in the existing knowledge. The study aims to provide valuable insights into the structural performance, shear behavior, and overall characteristics of continuous deep beams reinforced with GFRP bars, contributing to a better understanding of their behavior.

[15] investigated the shear behavior of reinforced concrete (RC) continuous deep beams with glass fiber reinforced polymer (GFRP) bars. Four large-scale beams with a constant shear span-to-overall depth ratio of 1.2 and different GFRP shear reinforcement ratios were tested. [15] concluded that the moment redistribution in the deep beams was very limited.

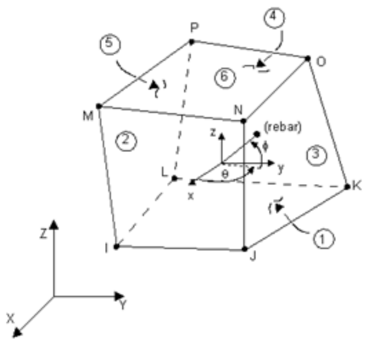
[16] studied experimentally the effect of some parameters on the behavior of concrete beams reinforced by GFRP bars and GFRP stirrups. [16] compared the shear capacity from the experimental results with that from Egyptian, American, and Canadian codes. [16] concluded that the analytical results are conservative in some cases and unconservative in others, while the analytical results in general are conservative. The Canadian code is unconservative compared to the experimental results for the range of the studied parameters and specimens.

2. NUMERICAL MODEL and VALIDATION STUDY

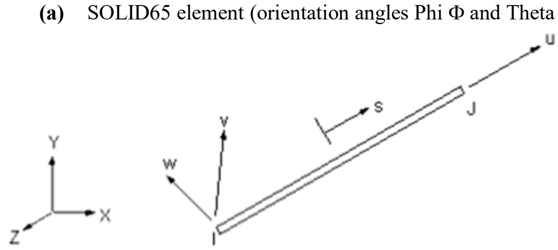
Finite element analysis stands out as the most extensive, powerful, and mature technology in both research and engineering applications. This study employs the structural analysis program ANSYS V.15 [17] to investigate the nonlinear shear behavior of continuous concrete deep beams reinforced with Glass Fiber-Reinforced Polymer (GFRP) bars and steel stirrups.

2.1 Modeling of geometry, loads, and boundary conditions

For the concrete modeling, an eight-node solid element (SOLID65) [17] was employed. This element features three translational and additional rotational degrees of freedom at each node. Special considerations were given to the unique features of SOLID65, including plasticity, cracking, small and large deflection capabilities, and the ability to accommodate plastic deformation. The mesh size utilized for beam modeling was set at 50x50x50mm. GFRP bars and steel stirrups were modeled using the Link180 element. This element is characterized by two nodes with three degrees of freedom of translation at each node in the X, Y, and Z directions. The assumption of a perfect bond was made for the connection between GFRP bars, steel stirrups, and concrete. To simulate loading and bearing plates, SOLID185 elements were used to model steel plates, as depicted [17] in Figure 1.



(a) SOLID65 element (orientation angles Phi Φ and Theta θ)

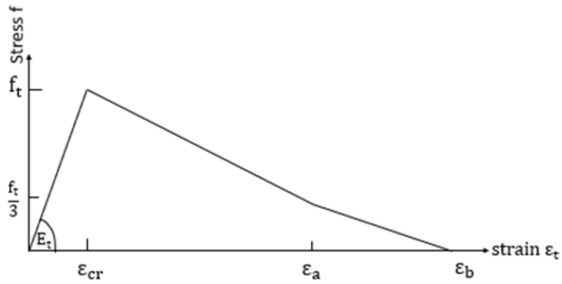


(b) Link180-element

FIGURE 1. Idealization for beams by ANSYS [17]

2.2 Material Model

In tension, the representation of concrete utilized a bilinear-softening (trilinear) model. Meanwhile, concrete in compression was modeled using the unconfined concrete model. Fiber-Reinforced Polymer (FRP) bars and steel stirrups were idealized through linear modeling, as illustrated [17] in Figure 2.



(a) Concrete stress-strain curve in tension

$$f = E_t * \varepsilon_t \quad \text{for } 0 \le \varepsilon_t \le \varepsilon_{cr} \tag{1}$$

$$f = \frac{2}{3} f_t \frac{(\varepsilon_a - \varepsilon_t)}{(\varepsilon_a - \varepsilon_{cr})} + \frac{1}{3} f_t \quad \text{for } \varepsilon_{cr} \le \varepsilon_t \le \varepsilon_a \tag{2}$$

$$f = \frac{1}{3} f_t \frac{(\varepsilon_b - \varepsilon_t)}{(\varepsilon_b - \varepsilon_a)} \quad \text{for } \varepsilon_a \le \varepsilon_t \le \varepsilon_b \tag{3}$$

$$\varepsilon_{cr} = 0.1\varepsilon_0 \tag{4}$$

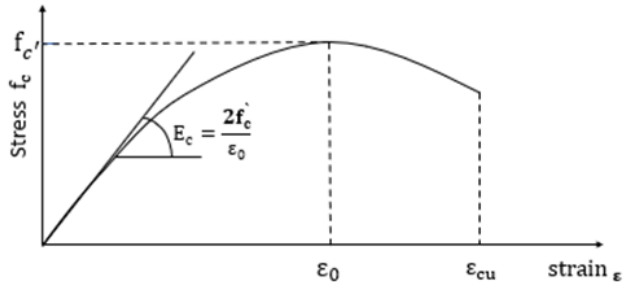
$$\varepsilon_a = 3\varepsilon_{cr} = 0.3\ \varepsilon_0 \tag{5}$$

$$\varepsilon_b = 10\varepsilon_{cr} = \varepsilon_0 \tag{6}$$

$$\varepsilon_0 = 0.003 \tag{7}$$

$$E_t = \frac{f_t}{\varepsilon_{cr}} \tag{8}$$

f_t is the concrete's tensile strength = 0.56 $\sqrt{f_c`}$ , where $f`_c$ is the concrete cylindrical compressive strength.



(b) Concrete stress-strain curve in compression

$$f_c = f_c` \left[2\left(\frac{\varepsilon}{\varepsilon_0}\right) - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] \quad 0 \le \varepsilon \le \varepsilon_{cu} \tag{9}$$

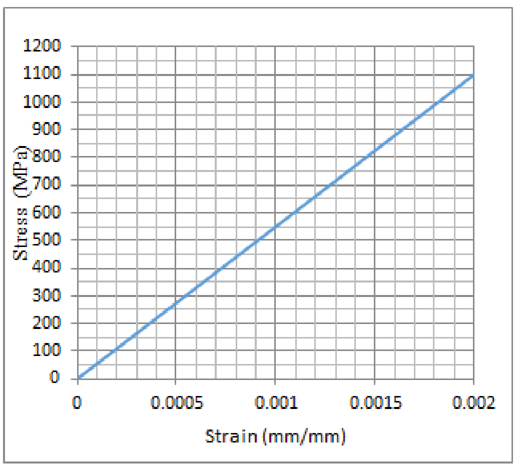
$$f_c = 0 \quad \varepsilon > \varepsilon_{cu} \tag{10}$$

$$\varepsilon_0 = 0.003 \tag{11}$$

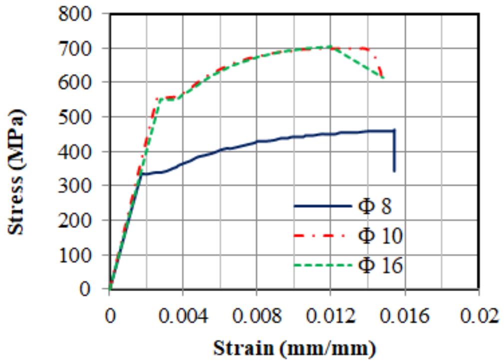
$$\varepsilon_{cu} = 0.004 \tag{12}$$

Where : $f_c`$ and E is the concrete cylindrical compressive strength (MPa) and its elastic modulus.

f_c and ε_c are the concrete compressive stress and strain.



(c) Stress-strain curve for GFRP bars



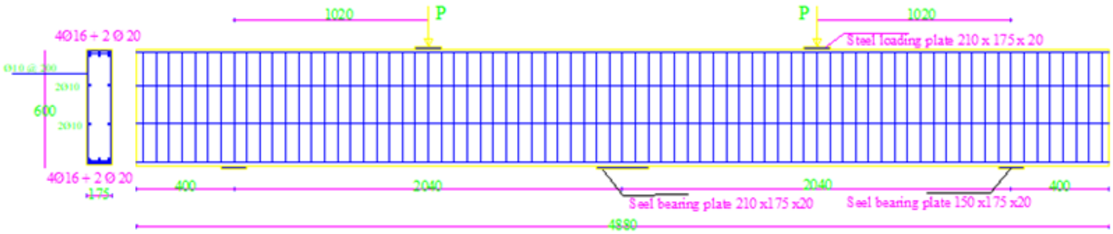
(d) Stress-strain curve for steel stirrups

FIGURE 2. Idealized stress-strain curves for concrete in tension, compression, GFRP bars, and steel stirrups

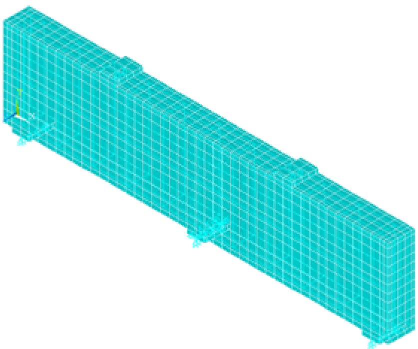
2.3 Verification of the Numerical Model

The numerical simulation of GFRP-reinforced concrete deep beams using the aforementioned models was employed to validate the numerical results. Specifically, Beam G1.7-600-W, as tested by Zinkaah et al. [18], served as a verification example, as depicted in Figure 3. The predicted load-deflection curve obtained from ANSYS [17] was compared with experimental data from the literature [19], as illustrated in Figure 4. Remarkably, Figure 4 demonstrates a strong alignment between the simulation results and experimental observations across all stages of behavior, from the initial loading up to failure.

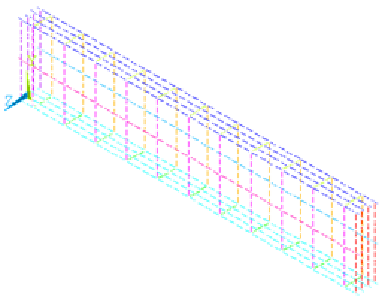
Furthermore, Figure 5 presents the deformed shape of beam G1.7-600-W, showcasing the agreement between the numerical simulation and the experimental setup. Additionally, Figure 6 displays the numerical crack propagation at 50%, 75%, and 100% of the failure load. The crack pattern predicted by the numerical model, as depicted in Figure 6, closely resembles the experimental results (P_{ANSYS} / P_{Exp} = 1.13).



(a) Concrete dimension and cross-section



(b) Typical idealization of the concrete elements



(c) Typical idealization of the reinforcement

FIGURE 3. Beam G1.7-600-W (Zinkaah et al. [18]) details, concrete, and reinforcement idealization

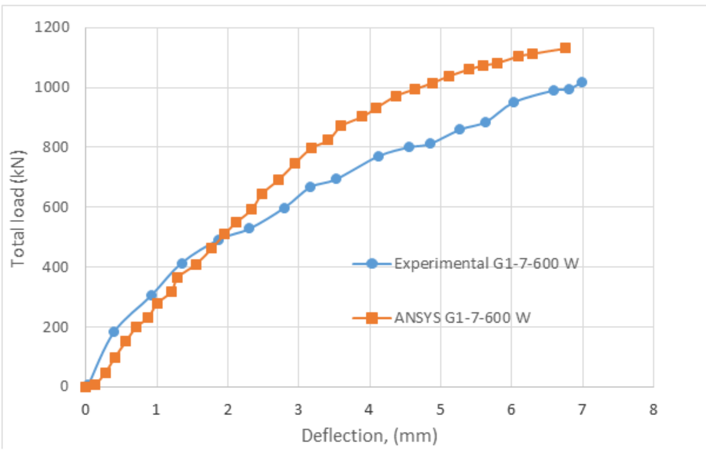


FIGURE 4. Numerical vs. experimental load-deflection curves for beam G1.7-600-W Zinkaah et al. [18]

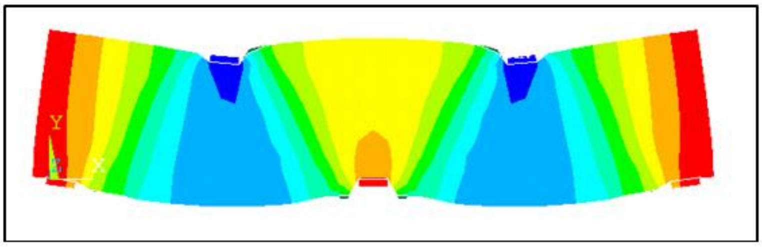
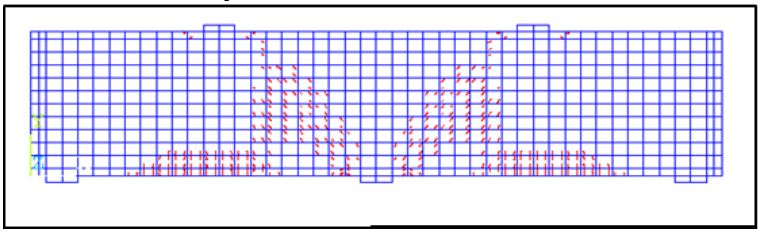
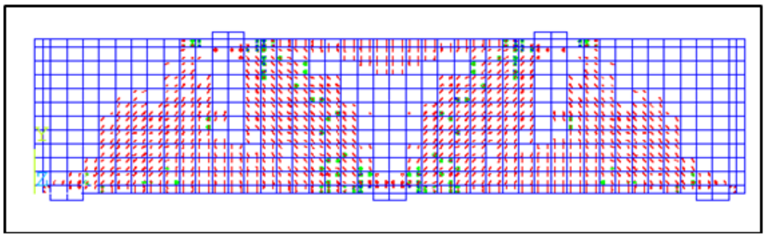


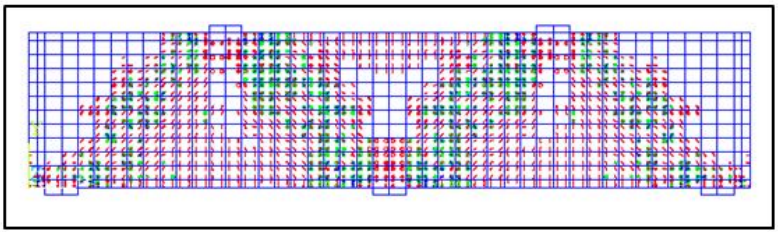
FIGURE 5. Deformed shape of beam G1.7-600-W Zinkaah et al. [18] at 100% failure load



(a) At 50% P failure



(b) At 75% P failure



(c) At 100% P failure

(d) Experimental crack pattern

FIGURE 6. Numerical and experimental crack propagation for beam G1.7-600-WZinkaah et al. [18]

In [19], several validation studies are also reported where a good agreement with the measured experimental results was achieved. [20], [21], and [18], continuous deep beams were employed as verification examples for the ANSYS [17] modeling. Notably, a strong agreement was observed between the results obtained from ANSYS [17] and those derived from the experimental work, particularly in terms of support reactions and the total applied load for all beams. This alignment further underscores the reliability and accuracy of the ANSYS [17] modeling in capturing the behavior of continuous deep beams in accordance with experimental findings.

3. PARAMETRIC STUDY

3.1 Design of a Parametric Study

A parametric study was conducted [19] to comprehensively understand the actual behavior of continuous reinforced concrete deep beams reinforced with glass fiber-reinforced polymer (GFRP) bars. This investigation was carried out numerically using the ANSYS [17] program. The objective was to examine the impact of various influencing factors on the strength of continuous deep beams reinforced with GFRP bars. For the parametric study, two spans of continuous concrete deep beams reinforced with GFRP bars were employed, resulting in a total of 19 numerical specimens categorized into 9 groups. The detailed material properties for each beam and the effects of the selected parameters are outlined in Table 1 and illustrated in Figure 7.

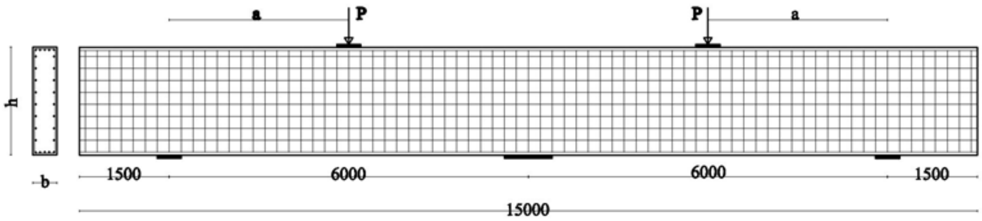


FIGURE 7. Details of the typical beam used for parametric study (mm.)

TABLE 1. Parameters utilized in the parametric study and their effect on shear capacity compared to the reference specimen

Studied parameters	Symbol	Studied parameters									P/P Reference%
		f_{cu} (MPa)	f_{fu} (MPa)	(b/L)	(t/L)	$\mu_{f\ Bottom}/\mu_b$	μ f top /μ f bottom	(a/L) Shear span-to-span ratio	Vl. web RFT%	Hz. web RFT%	
Reference Specimen	B1	30	1000	0.06	0.30	1.0	1.25	0.50	0.30	0.30	100.00
fcu	B2	25	1000								89.74
	B1	30									100.00
	B3	35		0.06							109.05
f fu	B4		800								91.70
	B1		1000		0.30						100.00
	B5		1200			1.0					106.90
Beam width	B6			0.05							84.50
	B1			0.06			1.25				100.00
	B7			0.07				0.50			114.70
Beam depth	B8				0.25						75.10
	B1				0.30				0.30		100.00
	B9				0.35					0.30	126.00
$\mu_f/\mu b$	B10					1.2					103.60
	B1					1.0					100.00
	B11	30				1.4					106.90
A_f top to A_f bottom	B12						1.15				97.00
	B1		1000				1.25				100.00
	B13			0.06			1.40				108.00
Shear span-to-span ratio	B14							0.33			157.00
	B1				0.30			0.50			100.00
	B15					1.0		0.25			171.00
vl. web RFT%	B16								0.00		97.79
	B1						1.25		0.30		100.00
	B17							0.50	0.4		107.1
Hz. web RFT%	B18								0.30	0.0	99.50
	B1									0.30	100.00
	B19									0.40	101.80

Where:

- L is the length of each span at 6000 mm.
- The balanced reinforcement ratio of the GFRP. $\mu_{fb} = 0.8*0.67*(f_{cu}/f^{*}_{fu}) * \varepsilon_{cu} / (\varepsilon_{cu} + \varepsilon^{*}_{fu})$
- For tension failure, $\mu_{fb} < \mu_f < 1.4\ \mu_{fb}$
- The GFRP ratio $\mu_f = A_f / (b_w.\ d)$
- A_f is the area of the GFRP bars.
- $b_w = b$ is the beam width.
- d is the beam effective depth, which equals the total depth (h) without the concrete cover.
- Take $\varepsilon_{cu} = 0.003$
- $f^*_{fu} = CE.\ f_{fu}$
- f_{fu} is the ultimate strength of the GFRP bars.
- CE is the environmental reduction factor = 0.5 to 1.0; assume CE = 1 for analysis, then $f^*_{fu} = f_{fu}$.
- $\varepsilon^{*}_{fu} = CE.\ \varepsilon_{fu}$, where $\varepsilon_{fu} = f_{un}/E_f$
- Take $\varepsilon^{*}_{fu} = \varepsilon_{fu} = f_{lu} / E_f$ (for CE = 1).
- E_f is the elastic modulus of GFRP bars at 40 GPa to 110 GPa; assume E_f = 70 GPa.
- f_{fu} = 400 MPa to 2000 MPa; take f_{fu} = 800, 1000, and 1200 MPa.
- Vertical and horizontal stirrups are made of steel reinforcement with f_y = 350 MPa and spacing s of 200 mm with two branches; the diameter of stirrups ϕ is variable.
- $\mu_{stirrups} = A_{stirrup} / (b_w.\ s)$
- $A_{stirrup}$ is the area of the stirrups.
- f_{cu} is the concrete cubic compressive strength (MPa) (f_{cu} = 25, 30, and 35 MPa).
- $f_c` = 0.8\ f_{cu}$
- $f`_c$ is the concrete cylindrical compressive strength (MPa).
- $E_c = 4400\sqrt{f`c}\ (MPa)$; $f_{ct} = 0.56\sqrt{f`c}$ (MPa).
- The dimensions of the loading and exterior bearing plates are $400*b_w$ with a thickness of 30 mm.
- The dimensions of the interior bearing plate are $800*b_w$, with a thickness of 30 mm.

3.2 Findings of the parametric study

The numerical results for all specimens are compared to the results of reference specimen B1, as depicted in Figure 8. The following conclusions can be drawn for each studied parameter in general:

1. The ultimate shear strength of continuous deep GFRP-reinforced concrete beams is affected by the ultimate strengths of the used concrete and GFRP bars. As shown in Fig. (8-a), increasing the compressive strength of concrete from 25 to 30 and 35 MPa enhances the shear capacity by 11.26% and 19.76%, respectively. Figure (8-b) indicates that the increase in the ultimate strength of GFRP bars by 25% increases the shear capacity by 8.3%.
2. Beam width and depth have significant effects on the ultimate shear capacity. Shear strength demonstrated an increase with the increase of the beam with-to-span ratio and total-depth-to-span ratio of 15.5% and 24.90%, respectively, due to an increase of 20% in beam width and beam depth, as shown in Fig. (8-c) and Fig. (8-d).
3. The ultimate shear strength increases with an increase in the flexural main-bottom reinforcement ratio and the top-to-bottom reinforcement ratio. As shown in Fig. (8-e), increasing the ratio of the GFRP reinforcement area from μ_b (balanced reinforcement ratio) to 1.4 μ_b enhances the load capacity by 6.90%. Figure (8-f) shows that increasing the ratio of the top-to-bottom GFRP area by 12% increases the load capacity by 8.0%.
4. The horizontal web reinforcement has an insignificant effect on the ultimate shear strength, while the vertical web reinforcement has a considerable effect. As shown in Fig. (8-g) and Fig. (8-h), the escalation in the vertical and horizontal reinforcement percentage by 33% resulted in an ultimate load increase of 7.1% and 1.8%, respectively.
5. The ultimate shear capacity considerably increases with a decrease in the shear-span-to-span ratio. The shear strength capacity increased by 71% due to the decrease in the shear-span-to-span ratio by 50%, as indicated in Fig. (8-i)

4. CONCLUSIONS

Nonlinear finite element analysis was conducted using the ANSYS V 15 [17] program to simulate the behavior of continuous concrete beams reinforced by Glass Fiber-Reinforced Polymer (GFRP) concerning failure load, crack patterns, and load-deflection behavior. The concluded points provide valuable insights into the behavior of continuous reinforced concrete deep beams reinforced with GFRP bars, offering a comprehensive understanding of the influence of various parameters on their shear strength and overall performance. For continuous reinforced concrete deep beams reinforced with GFRP bars and within the range of the studied parameters, the following conclusions can be drawn:

1. The proposed spatial idealization and constitutive material models in conjunction with the ANSYS computer program, are excellent tools for nonlinear finite element analysis of continuous concrete deep beams reinforced with GFRP. The comparison between the experimental results from the literature and the numerical results of the 3-D model revealed good harmony between the two, indicating the effectiveness of the numerical simulation in capturing the key aspects of the structural behavior of such deep beams.
2. The shear capacity of continuous GFRP-reinforced concrete deep beams increases moderately with the increase in strength of the used materials. Increasing the compressive strength of concrete from 25 to 30 and 35 MPa enhances the shear capacity by 11.26% and 19.76%, respectively. The increase in the ultimate strength of GFRP bars by 25% increases the shear capacity by 8.3%.
3. The predicted shear capacity of continuous GFRP-reinforced concrete deep beams is significantly sensitive

to the variation of geometrical parameters. Shear strength demonstrated an increase with the increase of the beam with-to-span ratio and total-depth-to-span ratio of 15.5% and 24.90%, respectively, due to an increase of 20% in beam width and beam depth. The shear strength capacity increased by 71% due to the decrease in the shear-span-to-span ratio by 50%.

4. For continuous GFRP-reinforced concrete deep beams, the predicted shear capacity is moderately sensitive to the variation of longitudinal reinforcement parameters. Increasing the ratio of the GFRP reinforcement area from μ_b (balanced reinforcement ratio) to 1.4 μ_b enhances the load capacity by 6.90%. Increasing the ratio of the top-to-bottom GFRP area by 12% increases the load capacity by 8.0%.
5. For the continuous GFRP-reinforced concrete deep beams, the horizontal web reinforcement has an insignificant effect on the ultimate shear strength compared to the effect of the vertical web reinforcement. The escalation in the vertical and horizontal reinforcement percentage by 33% resulted in an ultimate load increase of 7.1% and 1.8%, respectively.

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