DJS Vol. 46(3) (2023) –pp. 99-110 ISSN: 1012-5965

Delta Journal of Science
Available online at
https://djs.journals.ekb.eg/

Research Article

MATHEMATICS

The Effect of Peristaltic Motion on Nano Fluid (CuO/Water) Inside a Vertical Cylindrical Tube

Mohammadein S. A.[*], Atef A. Hemada, Magdy A. El-Tantawy, Muhammad A. Baghagho, M. S. Ali

Department of Mathematics, Faculty of Science, Tanta University, Egypt.

[*]***Corresponding author:*** Selim A. Mohammadein   ***e-mail:*** selimali40_43@yahoo.com

Received: 18/4/2023   **Accepted:** 8 /6/2023

KEY WORDS

Vertical cylindrical tube, Copper Oxid (CuO), Newtonian liquid, Grashof number, Pressure gradient, Long wavelengths, Navier-stokes equations.

ABSTRACT

In this paper, the incompressible viscous Newtonian nano fluid (CuO/Water) flow through a vertical cylindrical tube under the effect of peristaltic motion is studied. The mathematical model is formulated by mass, Navier-stokes, heat and volume rate equations. The nonlinear and linear partial differential equations are solved analytically in terms of temperature, velocity, stream function and pressure gradient distributions of the nano fluid for long wavelengths. The obtained results are affected by amplitude ratio, heat transfer coefficient, void fraction of nanoparticles and Grashof number. The nano fluid temperature is proportional inversely with amplitude ratio and void fraction of nanoparticles. The nano fluid velocity is proportional directly with heat transfer coefficient, amplitude ratio and Grashof number values. The void fraction of nanoparticles values affects velocity and pressure gradient distributions of nano fluid inversely. The nano fluid pressure gradient is increasing with Grashof number values. Moreover, the concluded remarks proved the validity of the proposed model. The physical model can be extended for different nanoparticles under the effect of some dominant physical parameters of fluid and flow.

Introduction

Peristaltic force is an organic siphon which employs episode wave-like squeezing movement which voyages down a vessel and forces the stuffing of the vessel **(Abu-Nab et al., 2022; Mohammadein and Abu-Nab, 2019; Darzi et al., 2012).** This mechanism also occurs in many applications involving bio-mechanical systems such as finger and roller pumps, heart lung machine, blood pump machine, dialysis machine and transport of noxious fluid in nuclear industries **(Mohammadein and Gad El-Rab, 2001)**. In recent years, considerable efforts have been usefully devoted to the study of peristaltic flow of Newtonian nanofluids because a practical and fundamental constitutive relation that can be used for all fluids and flow is not available **(Mohammadein and Abu-Nab, 2019; Mohammadein and Gouda, 2006; Nasiri et al., 2011)**. The production of particles with sizes on the order of nanometers (nanoparticles) can be achieved with relative ease with the recent improvements in nanotechnology **(Nasiri et al., 2011; Mohammadein and Gouda, 2006)**. As a consequence, the idea of suspending these nanoparticles in a base liquid for improving thermal conductivity has been proposed recently **(Suresh et al., 2012; Sundar and Sharma, 2010; Prasher et al., 2005)**. Such suspension of nanoparticles in a base fluid is called a nano fluid. Due to their small size, nanoparticles fluidize easily inside the base fluid, and as a consequence, clogging of channels and erosion in channel walls are no longer a problem.

It is even possible to use nano fluids in microchannels **(Darzi et al., 2012)**. When it comes to the stability of the suspension, it was shown that sedimentation of particles can be prevented by utilizing proper dispersants **(Suresh et al., 2012; Sundar and Sharma , 2010)**. Recently, the analytical solutions of fluid velocity field and stream function are be prevented by Mohammadein, 2017.

In this paper, the incompressible Newtonian nano fluid (CuO/Water) through a vertical cylindrical tube under the effect of peristaltic motion is studied. The mass, Navier-stokes, heat and volume rate equations represent the mathematical model of present problem. The analytical solution is obtained for temperature, velocity, pressure gradient distribution of the nano fluid under the effect of Grashof number G_r , heat source parameter β, amplitude ratio e, heat transfer coefficient α and void fraction of nanoparticles φ_p . Results are valid for long wavelength only.

Analysis

Consider the axisymmetric of a Newtonian fluid in circular cylindrical tube with a sinusoidal wave of small amplitude traveling down its wall. The wall of the tube is given by the equation:

$$H(z,t) = a + b\sin(2\pi z) \tag{1}$$

where a is the average radius of the original undisturbed tube, b is the amplitude of the wave, λ is the wavelength and c is the wave speed, r and z are the cylindrical polar coordinates with z measured along the axis of the tube and r is in the radial direction. Let u and w be the velocity components in the radial and axial directions, respectively. The peristaltic bubbly flow of a viscous incompressible Newtonian fluid through a vertical tube. The relations between physical parameters (density, thermal conductivity and viscosity of (CuO/Water)) of nano state in terms of nanoparticles and water are in the following form:

$$(\rho)_{nf} = \varphi_p(\rho)_p + (1-\varphi_p)(\rho)_l \tag{2}$$

$$(\rho c_p)_{nf} = \varphi_p(\rho c_p)_p + (1-\varphi_p)(\rho c_p)_l \tag{3}$$

$$(\mu)_{nf} = (\mu)_f(1-\varphi_p)^{-2.5} \tag{4}$$

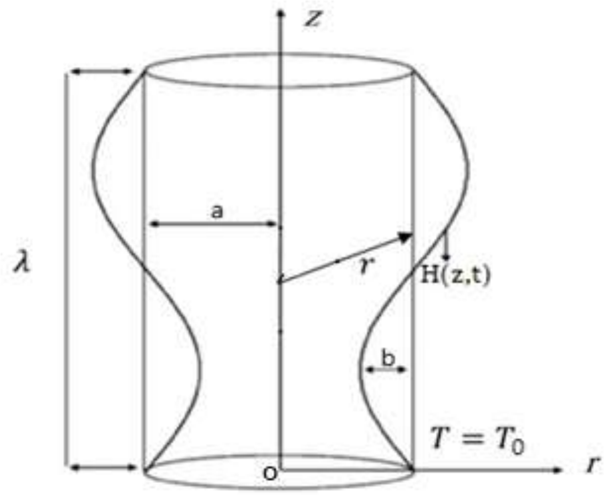

Fig. (1): The Problem Sketch

The mathematical model of the physical problem is described by the conservative nano mass, Navier - Stokes heat transfer and volume flow rate equations as follows:

Mass equation:

$$\frac{1}{\bar r}\frac{\partial}{\partial \bar r}(\bar r\,\bar u) + \frac{\partial \bar w}{\partial \bar z} = 0 \tag{5}$$

Navier-Stokes equations

$$r:\ (\rho)_{nf}\left(\bar u\frac{\partial \bar u}{\partial \bar r} + \bar w\frac{\partial \bar u}{\partial \bar z}\right) = -\frac{\partial \bar P}{\partial \bar r} + (\mu)_{nf}\left\{\frac{1}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial \bar u}{\partial \bar r}\right) - \frac{\bar u}{\bar r^2} + \frac{\partial^2 \bar u}{\partial \bar z^2}\right\} \tag{6}$$

$$z:\ (\rho)_{nf}\left(\bar u\frac{\partial \bar w}{\partial \bar r} + \bar w\frac{\partial \bar w}{\partial \bar z}\right) = -\frac{\partial \bar P}{\partial \bar z} + (\mu)_{nf}\left\{\frac{1}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial \bar w}{\partial \bar r}\right) + \frac{\partial^2 \bar w}{\partial \bar z^2}\right\} + g(\rho\alpha)_{nf}(\bar T - \bar T_0) \tag{7}$$

Heat equation:

$$(\rho c_p)_{nf}\left(\bar u\frac{\partial \bar T}{\partial \bar r} + \bar w\frac{\partial \bar T}{\partial \bar z}\right) = (k)_{nf}\left(\frac{\partial^2 \bar T}{\partial \bar r^2} + \frac{1}{\bar r}\frac{\partial \bar T}{\partial \bar r} + \frac{\partial^2 \bar T}{\partial \bar z^2}\right) \tag{8}$$

Volume flow rate equation:

$$\bar q = 2\int_0^h \bar r\,\bar w(\bar r,\bar z)\,d\bar r \tag{9}$$

The above system (5-9) represents a current problem in a dimensional form. The dimensionless variables as follows:

$$\bar{r} = ar, \quad \bar{z} = \lambda z, \quad \bar{w} = cw, \quad \bar{u} = \frac{c\delta}{\lambda} u,$$

$$\bar{P} = \frac{c\lambda\mu_l}{a^2} P, \quad \bar{T} = T_0\theta_{nf} + T_0,$$

$$\delta = \frac{a}{\lambda}, \quad e = \frac{b}{a}, \quad G_r = \frac{a^2\rho_l g\alpha_l T_0}{c\mu_l},$$

$$P_r = \frac{\mu_l (c_p)_l}{k_l}, R_e = \frac{\rho_l c\lambda}{\mu_l}$$

$$h = 1 + e\sin(2\pi z)\ , \bar{q} = c\ aq \tag{10}$$

Substituting by the equation (10) into the equations (5-9), we obtain the non-dimensional equations in the form:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u) + \frac{\partial w}{\partial z} = 0 \tag{11}$$

$$\frac{(\rho)_{nf} R_e \delta^4}{\rho_l \lambda}\left(\frac{\delta}{a} u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -(\frac{\partial P}{\partial r})_{nf} + \frac{(\mu)_{nf}\delta^2}{\mu_l \lambda}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{u}{r^2} + \delta^2\frac{\partial^2 u}{\partial z^2}\right\} \tag{12}$$

$$\frac{R_e \delta^2}{\rho_l}(\rho)_{nf}\left(\frac{\delta}{a} u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -(\frac{\partial P}{\partial z})_{nf} + \frac{(\mu)_{nf}}{\mu_l}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right\} + \frac{\delta^2}{\rho_l{}^2}\frac{\partial^2 w}{\partial z^2} + \frac{(\rho\alpha)_{nf}}{(\rho\alpha)_l} G_r\theta_{nf} \tag{13}$$

$$\frac{R_e P_r \delta^2 (\rho c_p)_{nf}}{(\rho c_p)_l}\left(\frac{\delta}{a} u\frac{\partial \theta_{nf}}{\partial r} + w\frac{\partial \theta_{nf}}{\partial z}\right) = \frac{(k)_{nf}}{k_l}\left(\frac{\partial^2\theta_{nf}}{\partial r^2} + \frac{1}{r}\frac{\partial\theta_{nf}}{\partial r} + \delta^2\frac{\partial^2\theta_{nf}}{\partial z^2}\right) \tag{14}$$

Volume flow rate equation of the nano fluid:

$$q_{nf} = 2\int_0^h r\, w_{nf}(r,z)dr \tag{15}$$

The above system (11-15) is difficult to solve in an analytical way. In contrary, when we take in our consideration that, the wavelength is long $(\delta \ll 1)$ and the Reynolds number is quite small $(R_e \to 0)$ then, equations (12-14) are simplified as follows:

$$(\frac{\partial P}{\partial r})_{nf} = 0 \tag{16}$$

$$(\frac{\partial P}{\partial z})_{nf} = \frac{(\mu)_{nf}}{\mu_l}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right\} + \frac{(\rho\alpha)_{nf}}{(\rho\alpha)_l} G_r\theta_{nf} \tag{17}$$

$$\left(\frac{\partial^2\theta_{nf}}{\partial r^2} + \frac{1}{r}\frac{\partial\theta_{nf}}{\partial r}\right) = 0 \tag{18}$$

The boundary conditions of heat transfer are given by:

$$\frac{\partial\theta_{nf}}{\partial r} = \frac{(\alpha)_{nf}}{(k)_{nf}} \quad at \quad r = -h \tag{19}$$

$$\theta_{nf} = (\theta_{nf})_0 \quad at \quad r = h \tag{20}$$

Solving Eqn. (18) using the boundary conditions (19-20), then the nano fluid temperature distribution θ_{nf} is given by:

$$\theta_{nf}(r,z) = (\theta_{nf})_0 + c_1 \ln\frac{r}{h} \tag{21}$$

where,

$$c_1 = \frac{-h(\alpha)_{nf}}{(k)_{nf}} \tag{22}$$

Substituting by Eq. (21) into the Eqn. (17) and solving Eqn. (17) with the following boundary conditions:

$$\frac{\partial w}{\partial r} = 0 \quad at \quad r = 0 \tag{23}$$

$$w = A_0 \quad at \quad r = h \tag{24}$$

Then putting $(w = w_{nf})$, we get the nano fluid velocity w_{nf} in the direction of cylinder axis z in the form:

$$w_{nf}(r,z) = A_0 + \frac{1}{4}(r^2 - h^2)\left(\frac{\partial P}{\partial z}\right)_{nf} - G_r\{\frac{(\theta_{nf})_0}{4}(h^2 - r^2) + \frac{c_1}{8}\left\{(r^2 - h^2) - 2r^2\ln\frac{r}{h}\right\}\} \tag{25}$$

Substituting from Eqn. (25) into the Eqn. (15) and solving Eqn. (15), then the nano fluid pressure gradient $(\frac{\partial P}{\partial z})_{nf}$ becomes:

$$(\frac{\partial P}{\partial z})_{nf} = \frac{8}{h^2}\left(A_0 - \frac{q}{h^2}\right) + G_r \left((\theta_{nf})_0 - \frac{c_1}{4}\right) \quad (26)$$

On the basis of equations (11) and (25), the nano fluid stream function ψ_{nf} is given by:

$$\psi_{nf}(r,z) = \int_0^r r w_{nf}(r,z)dr$$

$$\psi_{nf}(r,z) = A_0 r^2 + \frac{1}{16} r^2(r^2 - 2h^2)$$

$$(\frac{\partial P}{\partial z})_{nf} - G_r\{\frac{r^2(\theta_{nf})_0}{16}(2h^2 - r^2) + \frac{c_1}{64}\{r^2(r^2 - 2h^2) - 4r^4\left(\ln\frac{r}{h} - \frac{1}{4}\right)\}\} \quad (27)$$

where

$$u_{nf}(r,z) = -\frac{1}{r}\frac{d\psi_{nf}}{dz},$$

$$\frac{dh}{dz} = 2\pi e \mathrm{Cos}(2\pi z) \quad (28)$$

The nano fluid velocity in the radial direction of cylinder u_{nf} is given by:

$$u_{nf}(r,z) = -2\pi e \mathrm{Cos}(2\pi z)\{\frac{r}{16}\{-4h\left(\frac{dp}{dz}\right)_{nf}\} - \frac{G_r r}{8}\{2h(\theta_{nf})_0 - \frac{(\alpha)_{nf}}{4(k)_{nf}}\{G_r\left((\theta_{nf})_0 + \frac{(\alpha)_{nf}}{(k)_{nf}}\right) + \frac{4q}{h^5} - \frac{16A_0}{h^3}\}(r^2 - 2h^2) - 4h\left(\frac{dP}{dz}\right)_{nf}\} - \frac{G_r r}{8}\{2h(\theta_{nf})_0 - \frac{(\alpha)_{nf}}{4(k)_{nf}}(r^2 - 6h^2) - 2r^2\left(\ln\frac{r}{h} - \frac{5}{4}\right)\}\} \quad (29)$$

Discussion and Results

The peristaltic flow of incompressible nanoparticles CuO/Water problem is studied. The physical problem is described by mass, Navier-Stokes, heat and volume flow rate equations (5-9) respectively for Newtonian nano fluid flow throughout a vertical cylindrical tube. The dimensional system (5-9) is transformed to the non-dimensional one (11-15). The system (11-15) is reduced to the system (16-18) when $(\delta \ll 1)$. This system is solved analytically in a simple way.

The mathematical model is solved analytically in case of long wavelength in the peristaltic vertical cylindrical tube. The nano fluid temperature distribution is obtained by equation (21).The nano fluid velocity components are given by equations (25) and (29). The nano fluid pressure gradient is defined by equation (26). Moreover, the stream function of the nano fluid is derived by relation (27). The nano fluid flow is affected by some physical parameters such as, Grashof number G_r , amplitude ratio e , heat transfer coefficient α and void fraction of nanoparticles φ_p . The peristaltic flow of nano fluid (CuO/Water) problem is studied in a steady state. The numerical values of physical parameters **(Suresh et al., 2012)** used in the present problem is tabulated in the following Table.

Table (1): The physical properties of pure water and CuO.

Materials	**Density** $\rho(kg/m^3)$	**Specific heat** $Cp(J/kg\ K)$	**Thermal conductivity** $k(W/m\ K)$
Pure Water	958.3	4240	0.6857
(CuO)	6320	531.8	18

Moreover,

$A_0 = 0,\ \ e = 0.2,$

$(\theta_{nf})_0 = 10^{-2},\ \ \alpha = 0.2,$

$\varphi_p = 0.01,\quad G_r = 0.4\ and\ q = 0.01$

The nano fluid temperature distribution is clearly increasing with the increasing of heat transfer coefficient α. On the other side, it is proportional inversely with the increasing of the amplitude ratio e and the void fraction of nanoparticles φ_p as shown in Figs. 2(a-c).

The gradient pressure distribution of the nano fluid is with the increasing of heat transfer coefficient α, amplitude ratio e and the Grashof number G_r values. On contrary, it is proportional inversely with the void fraction of nanoparticles φ_p as shown in Figs. 3(a-d).

The nano fluid velocity increases with heat transfer coefficient α, amplitude ratio e and the Grashof number G_r values. On contrary, it is proportional inversely with the void fraction of nanoparticles φ_p as shown in Figs. 4(a-d).

The nano fluid stream function is plotted for three different amplitude ratio values e .It is observed that the density of streamlines is proportional directly with amplitude ratio values e as shown in Figs. 5(a-c).

In Figs. 6(a-c), the nano fluid stream function is plotted for three different Grashof number values G_r .It is observed that the density of the nano fluid stream lines is proportional inversely with Grashof number values G_r.

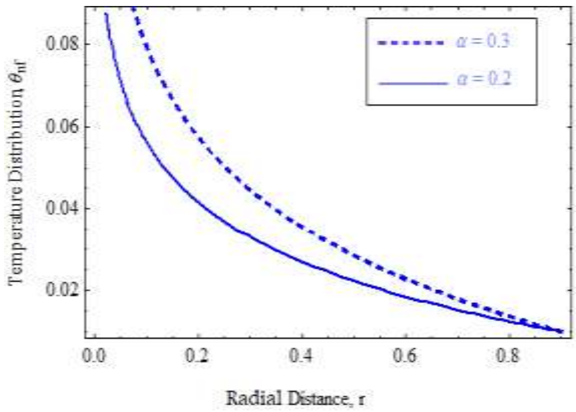

Fig. 2(a): Variation of nano fluid temperature with the heat transfer coefficient α

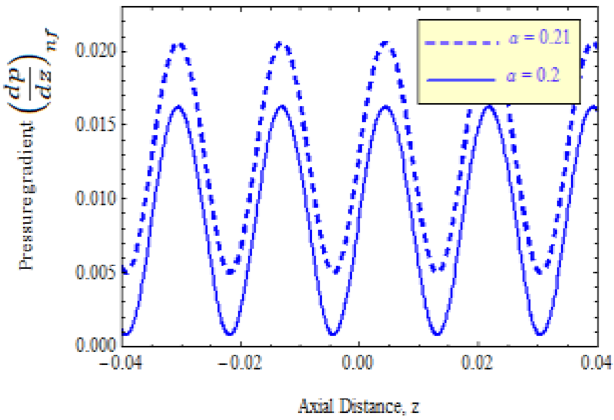

Fig. 3(a): Variation of nano fluid pressure gradient with the heat transfer coefficient α

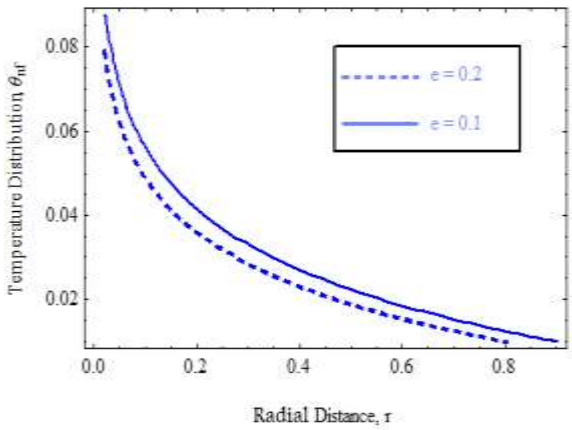

Fig. 2(b): Variation of nano fluid temperature with the amplitude ratio e

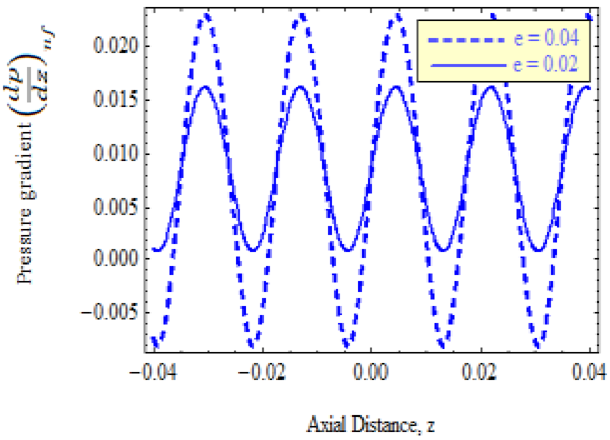

Fig.3(b): Variation of nano fluid pressure gradient with the amplitude ratio e

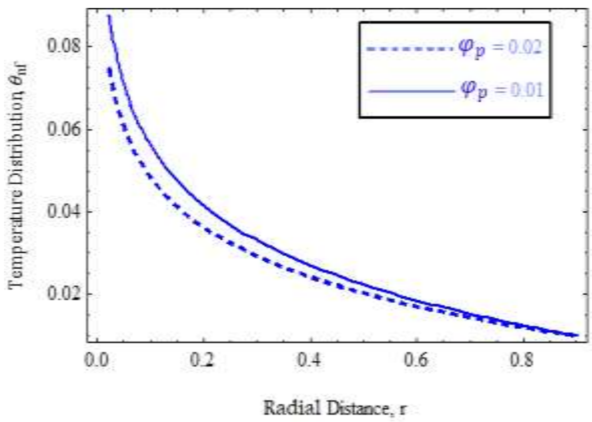

Fig. 2(c): Variation of nano fluid temperature with the void fraction of nanoparticles φ_p

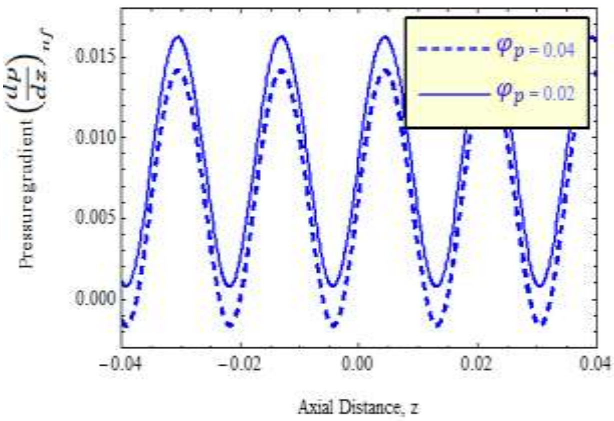

Fig. 3(c): Variation of nano fluid pressure gradient with the void fraction of nanoparticles φ_p

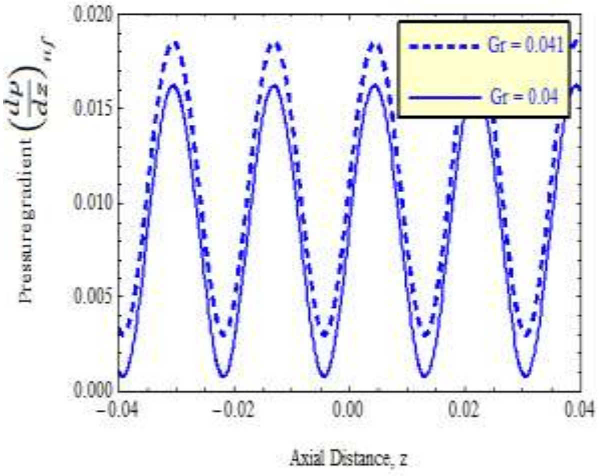

Fig. 3(d): Variation of nano fluid pressure gradient with Grashof number G_r

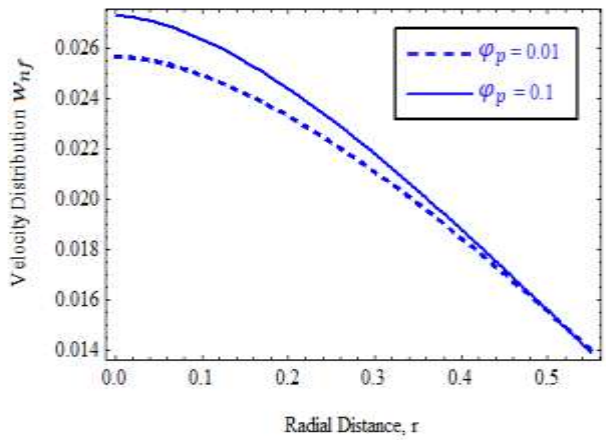

Fig.4(c): Variation of nano fluid velocity with the void fraction of nanoparticles φ_p

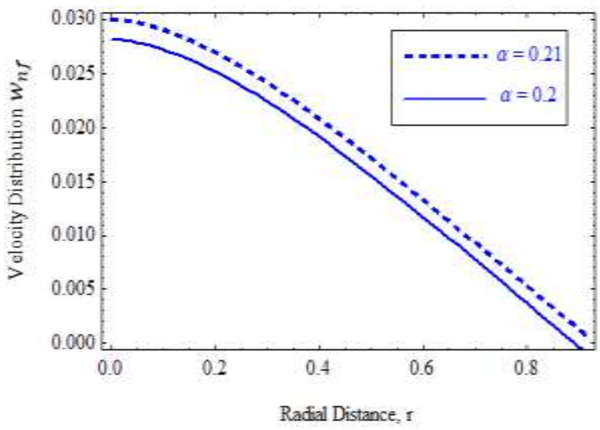

Fig. 4(a): Variation of nano fluid velocity the heat transfer coefficient α

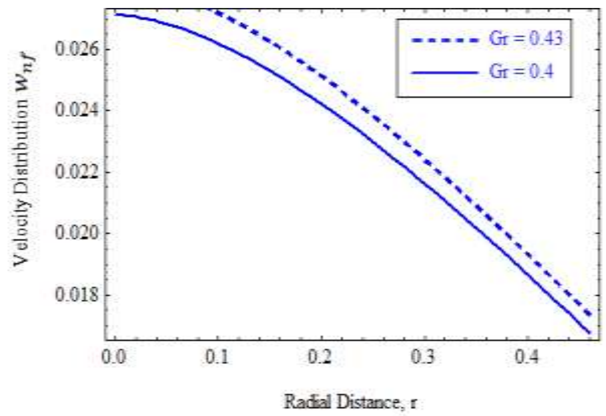

Fig.4(d): Variation of nano fluid velocity with Grashof number G_r

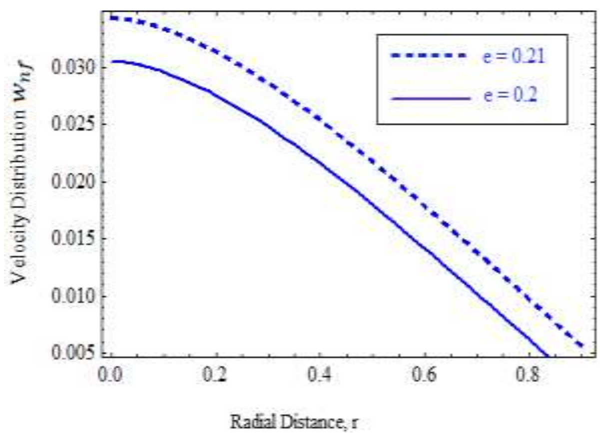

Fig.4(b): Variation of nano fluid velocity with the amplitude ratio e

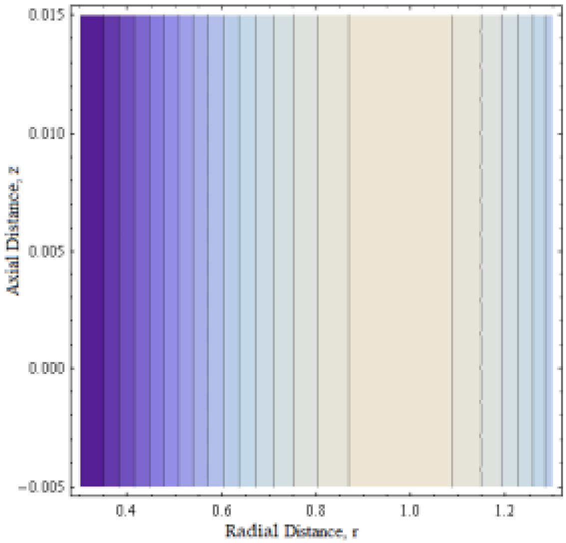

Fig.5(a): Nano fluid stream function at the amplitude ratio $e = 0.2$

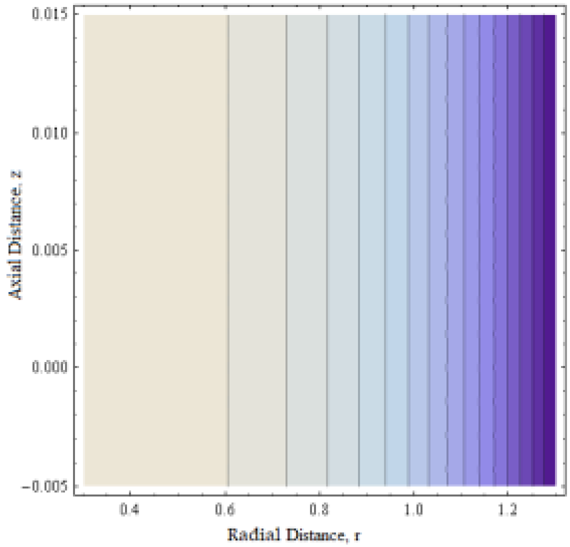

Fig.5(b): Nano fluid stream function at the amplitude ratio $e = 0.3$

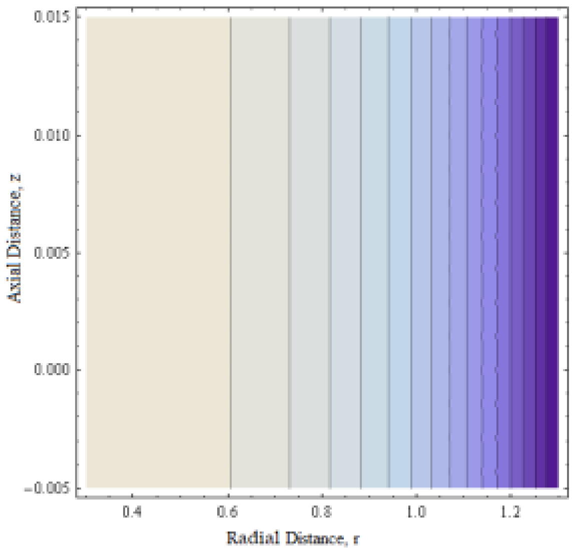

Fig.5(c): Nano fluid stream function at the amplitude ratio $e = 0.5$

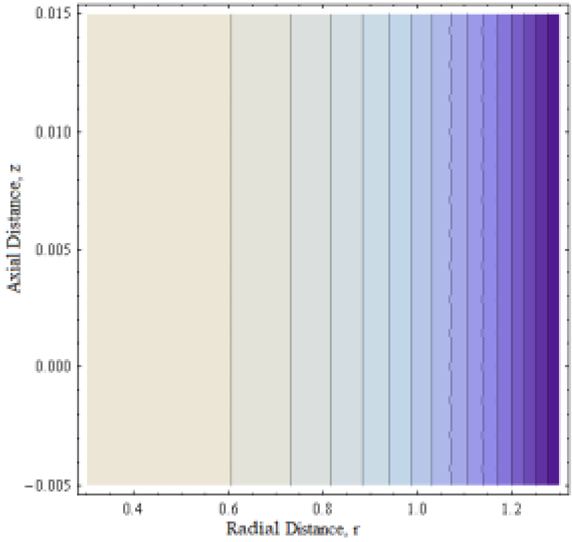

Fig.6(a): Nano fluid stream function at Grashof number $G_r = 0.4$

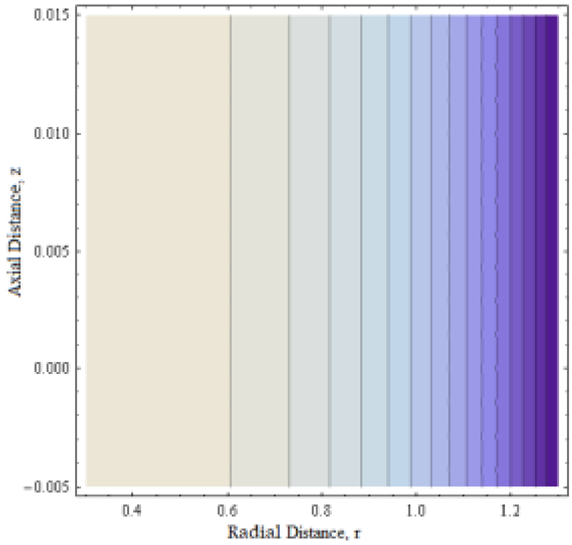

Fig.6(b): Nano fluid stream function at Grashof number $G_r = 0.6$

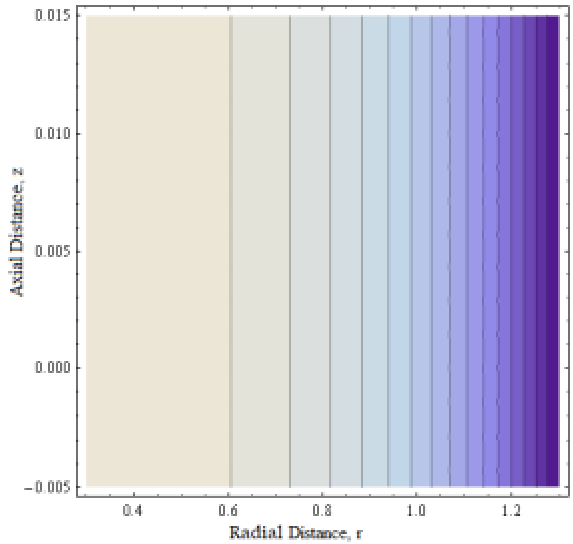

Fig.6(c): Nano fluid stream function at Grashof number G_r =0.9

Conclusions

The current problem is solved analytically under the effect of peristaltic motion for long wavelength and low Reynolds number. The nano temperature, axial velocity, pressure gradient, stream function, and radial velocity of the nano fluid (CuO/Water) are obtained by equations (21), (25), (26), (27), and (28) respectively. The effect of amplitude ratio, heat transfer coefficient α and void fraction of nanoparticles φ_p are observed. The discussion of results and figures concluded the following remarks:

1-The nano temperature distribution of the nano fluid is proportional directly with heat transfer coefficient α and inversely with amplitude ratio e and void fraction of nanoparticles φ_p .

2-The gradient pressure distribution of the nano fluid is proportional directly with heat transfer coefficient, amplitude ratio e and the Grashof number G_r values and inversely with the void fraction of nanoparticles φ_p .

3-The nano fluid velocity distribution is proportional directly with heat transfer coefficient, amplitude ratio e and the Grashof number G_r values. On contrary, it is proportional inversely with void fraction of nanoparticles φ_p .

4-The density of the nano fluid stream lines is proportional directly with amplitude ratio values e.

5-The density of the nano fluid stream lines is proportional inversely with Grashof number values G_r.

6-The nano fluid (CuO/Water) flow represents a dominant factor in vertical peristaltic motion. The proposed model can be extended for more properties of fluid and flow.

Nomenclature

T Temperature of liquid (K^0).
P Pressure (N/m^2).
T_0 Initial temperature of liquid (K^0) .
$(\theta_{nf})_0$ Initial nano fluid temperature.
e Amplitude ratio.
G_r Grashof 's number.
q Dimensionless volume flow rate.
P_r Prandtl number.
δ Wave number.
a Radius of the tube.
b Amplitude of the wave.
λ Wavelength.
c_p Specific heat of liquid at constant pressure $(\mathrm{Kg}^{-1}\mathrm{J/k})$.
ρ_l Density of liquid $(\mathrm{Kg\ m}^{-3})$.
$(k)_{nf}$ Thermal conductivity of nano fluid.
k_l The thermal conductivity of liquid.
$(\rho)_{nf}$ Density of nano fluid $(\mathrm{Kg\ m}^{-3})$.
h peristaltic boundary

References

Abu-Nab A.K., Omran M. H. and Abu-Bakr A. F. (2022). Theoretical Analysis of Pressure Relaxation Time in *N*-Dimensional Thermally-Limited Bubble Dynamics in Fe_3O_4/Water Nanofluids. *JON*, 11(3): 410 – 417.

Mohammadein S. A. and Abu-Nab A. K. (2019). Peristaltic Flow of a Newtonian Fluid Through a Porous Medium Surrounded Vapour Bubble in a Curved Channel. *JON*, 8(3): 651–656.

Mohammadein S. A. (2017). The Peristaltic Motion Inside a Vertical Cylindrical Tube Surrounded Vapour Bubble with Two-Phase Density Flow. *ABB*, 5(4):71.

Darzi A. A. R., Farhadi M., Sedighi K., Shafaghat R. and Zabihi K. (2012). Experimental Investigation of Turbulent Heat Transfer and Flow Characteristics of SiO_2/Water Nanofluid within Helically Corrugated Tubes. *ICHMT*, 39: 1425–1434.

Suresh S., Venkitaraj K. P., Selvakumar P. and Chandrasekar M. (2012). A Comparison of Thermal Characteristics of Al2O3/Water and CuO/Water Nanofluids in Transition Flow Through a Straight Circular Duct Fitted with Helical Screw Tape Inserts. *Therm. Fluid Sci.*, 39:37-44.

Nasiri M., Etemad S. Gh. And Bagheri R. (2011). Experimental Heat Transfer of Nanofluid Through an Annular Duct. *ICHMT*, 38(7): 958-963.

Sundar L. S. and Sharma K. V. (2010). Heat Transfer Enhancements of Low Volume Concentration Al_2O_3 Nanofluid and with Longitudinal Strip Inserts in a Circular Tube. *Int. J. Heat and Mass Transfer*, 53(19-20): 4280–4286.

Sundar L. S. and Sharma K. V. (2010). Turbulent Heat Transfer and Friction Factor of Al_2O_3 Nanofluid in Circular Tube with Twisted Tape Inserts. *Int J. Heat and Mass Transfer*, 53(7-8): 1409–1416.

Evans, W., Fish, J., and Keblinski P. (2006). Role of Brownian Motion Hydrodynamics on Nanofluid Thermal Conductivity. *Appl. Phys. Lett.*, 88(9): 093116-3.

Mohammadein S. A. and Gouda Sh. A. (2006). Temperature Distribution in a Mixture Surrounding a Growing Vapour Bubble. *J. Heat and Mass Transfer*, 42: 359-363.

Prasher R., Bhattacharya P. and Phelan P. E. (2005). Thermal Conductivity of Nanoscale Colloidal Solutions (Nanofluids). *Phys. Rev. Lett.*, 94(2): 025901.

Divinis N., Karapantsios T. D., Kostoglou M., Panoutsos C. S., Bontozoglou V., Michels A. C., Sneep M. C., De Bruijn R. and Lotz H. (2004). Bubbles Growing in Supersaturated Solutions at Reduced Gravity. *AIChE J.*, 50 (10): 2369- 2382.

Mohammadein S. A. and Gad El-Rab R.A. (2001). The Growth of Vapour Bubbles in Superheated Water Between Two Finite Boundaries. *Can. J. Phys.*, 79(7): 1021–1029.

تأثير الحركة التمعجية على السائل النانُوي (CuO / الماء) داخل أنبوب إسطواني عمودي

أ.د/ سليم علي محمدين ـ د/ عاطف عبد الجليل حميدة ـ د/ مجدي الطنطاوي ـ

د/ مها سليم علي – أ/ محمد عبدالله عبد الغنى بغاغو

قسم الرياضيات ـ كلية العلوم ـ جامعة طنطا

ـ في هذا البحث تمت دراسة تدفق السائل النانُوي النيوتوني اللزج غير القابل للإنضغاط (CuO / الماء) عبر أنبوب إسطواني عمودي تحت تأثير الحركة التمعجية. تمت صياغة النموذج الرياضي بواسطة معادلات الكتلة و Navier-stokes والحرارة ومعدل الحجم. تم حل المعادلات التفاضلية الجزئية الخطية وغير الخطية بشكل تحليلي للحصول على درجة الحرارة والسرعة ودالة التدفق وتوزيعات التدرج في الضغط للسائل النانوي فى حالة الأطوال الموجية الطويلة. النتائج المحصول عليها تتأثر بنسبة السعة ومعامل الإنتقال الحرارى والكسر الفراغي للجسيمات النانوية وقيم رقم Grashof. حيث تكون درجة حرارة السائل النانُوي متناسبة عكسيًا مع نسبة السعة والكسر الفراغى للجسيمات النانُوية. وسرعة السائل النانُوي تتناسب بشكل مباشر مع معامل إنتقال الحرارة ونسبة السعة وقيم رقم Grashof. كذلك يتأثر الكسر الفراغى لقيم الجسيمات النانوية بسرعة وتوزيع تدرج الضغط للسائل النانُوي بصورة عكسية،بينما يتزايد تدرج الضغط للسائل النانُوي مع قيم أرقام Grashof. علاوة على ذلك ، أثبتت الملاحظات الختامية صحة النموذج المقترح. لذلك يمكن تمديد النموذج الفيزيائي للجسيمات النانوية المختلفة تحت تأثير بعض الخواص الفيزيائية السائدة للمائع والإنسياب.