UNITED ARAB REPUBLIC

THE INSTITUTE OF NATIONAL PLANNING

التخطيط القومي

Memo. No. 829

On the Planning Method "PERT" and its Application in the GDR

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Dresden

March 1968

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1. On time planning with "PERT"

Stochastic models of network technics are also based on a network. It shows the flow of the overall process and its single partial processes. In principle this network will be prepared in the same way as it was laready discussed when treating the critical path method. Thus the same aspects are essentially valid for the flow planning. But there some differences exist in time planning.

The method PERT (PERT (Program Evaluation and Review Technique) is based on a stochastic model. The various quantities used are not unambigiuosly determined - as, e.g. in C.P.M.- but the activity time, the dates of events - except the beginning of the total project -, the duration time of the whole process reflected in the network and with that the finish date of the total plan are interpreted stochastically. They are based on the assumption that the activity time cannot only be represented as a time value as in the determinated model, but as a distribution of probabilities. That results in the impossibility of determining the dates unambiguously. There cannot be determined, e.g. : the finish date of that activity will be on July 15th. There can only be stated: the finish date of that activity will be expected with a certain percentage of probability in the time from ... until ...

If the network was prepared for a project to be treated according to PERT, it is possible to being with time planning on this basis. Primarily its task is the investigation of the total time of realizing the total project. It is based on time estimates for each activity. The estimates have to be done by a specialist being able to evaluate the corresponding partial process as well as possible. There must be estimated:

a(i, j) = optimistic time of A(i, j)
m(i, j) = most likely time of A(i, j)
b(i, j) = pessimistic time of A(i, j)

The time being necessary, if this partial process flows better than normally to be expected and if no disturbances or difficulties occur, is meant as an optimistic time of an activity. The most likely time corresponds to that estimate which comes nearest to the actual duration of this activity. The pessimistic time is that time which can be expected to occur, if considerable disturbances and difficulties appear in carrying out the activity referred to. Later the three values estimated for each activity should be changed only, if the expenditure of work, or the number of employed manpower, or the quantity and (or) quality of the means of work was changed.

A fourth value of time is derived from these three, and that is the so-called expected duration time; i.e. that value of time which probably would be obtained as an average duration time of this activity, if often repeated. The duration time is calculated as a weighted mean value of the three estimated time values, and that is

$$te(i, j) = \text{expected duration time of } A(i,j) = \frac{a + 4m + b}{6}$$

The following example characterizes the values mentioned:

Table 1

Activity E(i)	E(j)	a(i, j)	m(i, j)	b(i, j)	te(i, j)
0	1	5	10	15	10
1	2	8	10	12	10
2	3	5	10	12	9,5
3	4	8	10	15	10,5

The distributions corresponding to the values in table 1 are represented in Fig. 1 to 4.

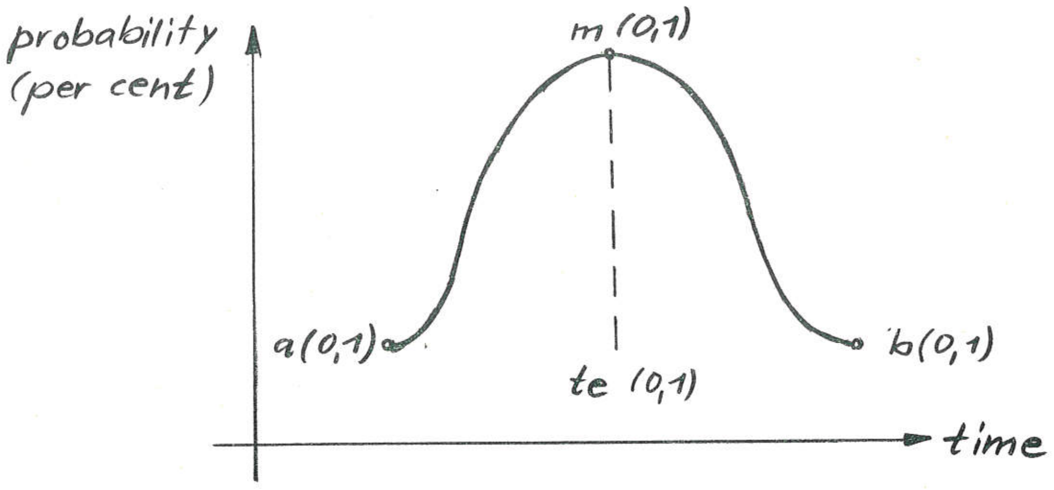


Fig. 1. Distribution of activity (0,1)

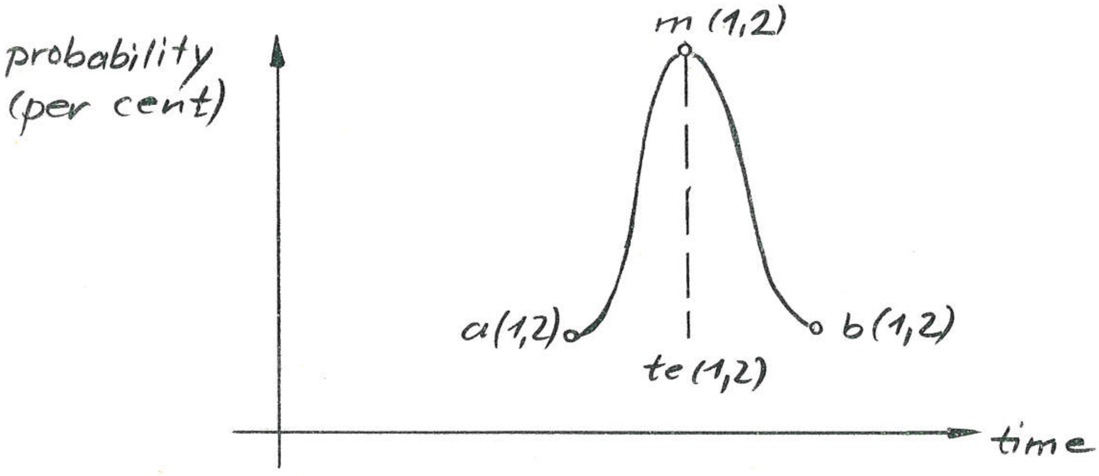


Fig. 2. Distribution of activity (1,2)

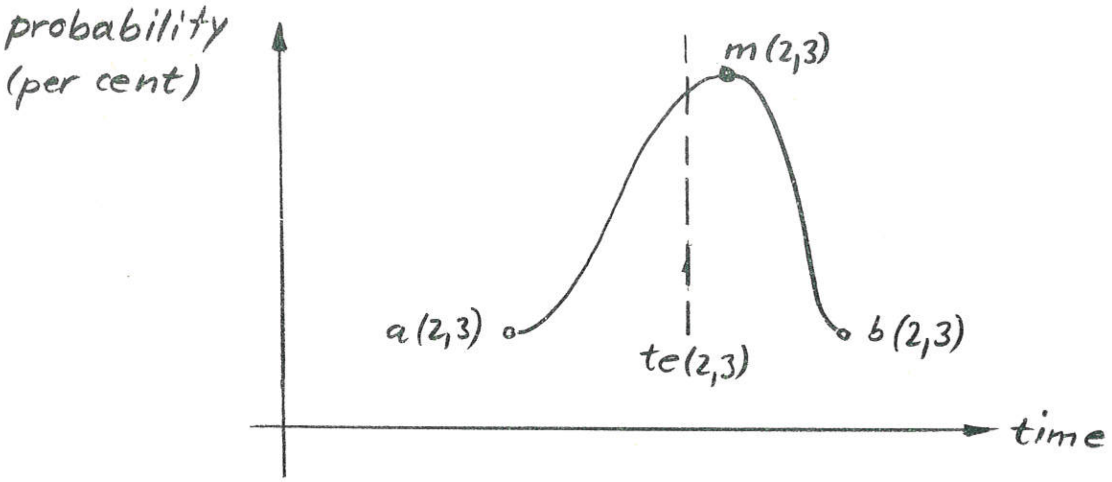


Fig. 3. Distribution of activity (2,3)

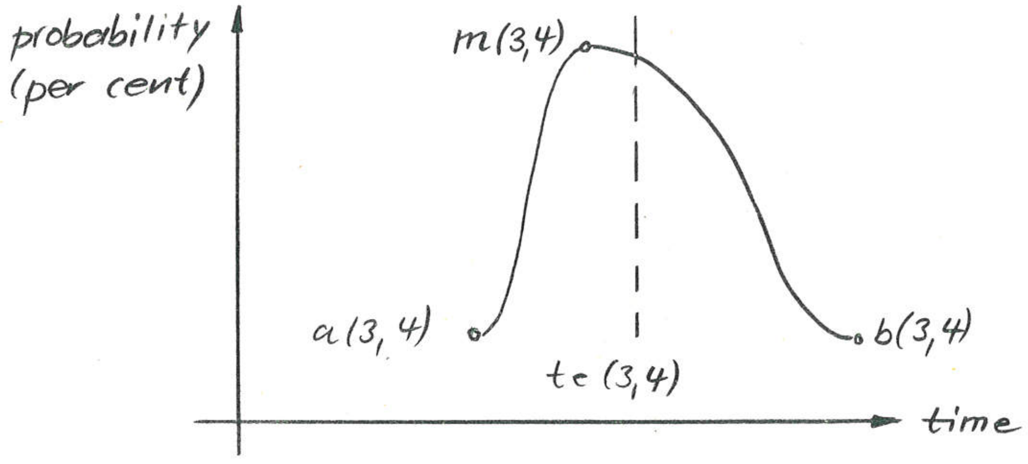


Fig. 4. Distribution of activity (3,4)

The estimates of time are somewhat doubtful. The variance or the mean square deviation is a measure of this uncertainty. As a rule the uncertainty connected with an activity and the distribution corresponding to it is the greater, the farther the values for optimistic and for the pessimistic time lie apart, that is to say, the greater the range of a distribution is. The variance of the expected duration time corresponds to that range of distribution. For that is vlid:

$\sigma^2_{t(i,j)}$ = variance of the expected duration time for the activity time of A(i,j)

$$= \left(\frac{b(i,j) - a(i,j)}{6} \right)^2$$

To illustrate the variance it is assumed that two time estimates were made for one and the same activity.

Table 2

estimator	a(i, j)	m(i, j)	b(i, j)
A	4	8	12
B	9	10	11

The variance is for the estimation of

$$A: \sigma^2_{t(i,j)} = \left(\frac{12-4}{6}\right)^2 = 1{,}77$$

$$B: \sigma^2_{t(i,j)} = \left(\frac{11-9}{6}\right)^2 = 0{,}12$$

The substantially smaller range of distribution of B corresponds to the lower variance. A, however, has a greater range of distribution and consequently a greater variance, too. Estimator B felt surer about the matter than A did. On principle is valid: the greater the variance of the expected duration time of an activity, the more uncertain the time estimate, and vice versa.

Now the earliest date of the start of each event has to be calculated by adding the te(i,j) for the proceding activities. Subsequently the latest possible date of the tail of the events has to be calculated. The corresponding mean values, again resulting from the mean values te (i,j), are taken as basis of these dates. Then there must be determined, which events and which activities are on the critical path. On principle the same method is used as it was explained with the critical path method. The following example informs on small deviations.

2. An example of time planning with PERT

To illustrate the example which shall represent a section taken from the scheduling of a research work consisting of different partial processes, there are still introduced at first:

$f\bar{T}a(i,j)$ = mean earliest start date of the activity (i,j), stochastic variable

$s\bar{T}e(i,j)$ = mean latest finish date of the activity (i,j), stochastic variable

$\sigma^2_{fTa(i,j)}$ = variance - square deviation - of fTa(i,j)

$\sigma^2_{sTe(i,j)}$ = variance - square deviation - of sTe(i,j)

$\bar{T}F(i)$ = mean total floats of the event (i) - stochastic variable -, where $f\bar{T}a(i,j) - s\bar{T}e(i,j) = \bar{T}F(i)$ - end event index j -; This float may assume three shapes due to the fact that quantities distributed according to their probabilities exist:
positive float - resources of manpower and (or) means of work are available, an anticipated finishing of the activity closed by this event may be expected; negative float - manpower and (or) means of work are not adequate. Delated finishing of the activity closed by this event may be expected, float = 0 manpower and (or) means of work are just adequate. A normal finishing in time may be expected.

$\overline{TF}(i,j)$	=	mean total float of activity A(i,j) - stochastic variable -, where $s\overline{T}e(i,j) - f\overline{T}a(i,j) - te(i,j) = \overline{TF}(i,j)$
Tv(j)	=	given temporal fixed value of the tail event (j) - also called milestone
P	=	probability in %.
$\Phi(x)$	=	Table value of the normalized normal distribution

The following activity oriented quantities have been determined for the example[1]:

Table 3

E(i)	E(j)	a(i,j)	m(i,j)	b(i,j)	te(i,j)	$\sigma^2_{t(i,j)}$
0	1	2	3	6	3,33	0,44
1	2	1	2	4	2,16	0,25
1	4	2	3	4	3,00	0,11
2	3	2	5	7	4,83	0,69
3	5	7	8	9	8,00	0,11
4	5	3	5	10	5,50	1,32

1) cp. Gerold Rädisch: On the application of PERT-network medels. The telecommunication expert, No, 1/1967, p. 6. (Gerold Radish: Zur Anwendung von PERT-Netzwerkmodellen. Der Frenmelderpraktiker Heft 1/1967, S. 6)

The network shows the dependences represented in Fig. 5:

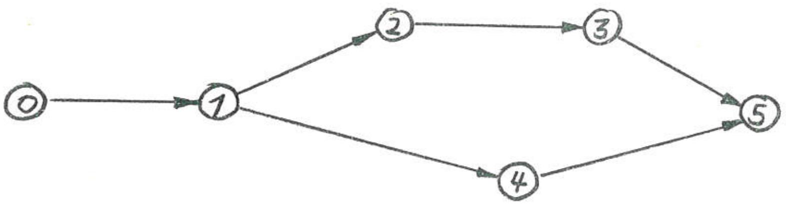


Fig. 5. Network of a PERT - example

On the basis of these initial values, the network matrix can be compiled and calculated - as on principle had already been explained when discussing the C.P.M. Here, however, it must be noted that the variance has to be considered when estimating the stochastic variables. In the cells of the matrix, the lower one of the two values means the mean activity duration te(i,j) and the upper value the variance of activity duration $\sigma^2_{t(i,j)}$.

Table 4

E(i) \ E(j)	0	1	2	3	4	5	$f\bar{T}a(i,j)$	$\sigma^2_{fTa(i,j)}$
0		3,33 0,44					0,00	0,00
1			2,16 0,25		3,00 0,11		3,33	0,44
2				4,83 0,69			5,49	0,69
3						8,00 0,11	10,32	1,38
4						5,50 1,32	6,33	0,55
5							18,32	1,49
$s\bar{T}e(i,j)$	0,00	3,33	5,49	10,32	12,82	18,32		
$\sigma^2_{sTe(i,j)}$	1,49	1,05	0,80	0,11	1,32	0,00		
$s\bar{T}e(i,j) - f\bar{T}a(i,j)$	0	0	0	0	6,49	0		
P	50%	50%	50%	50%	0,01%	50%		

From table 4, the critical events may be read. Thus in this case the determination of the critical activities is possible, too. As, however, $s\overline{T}e(i,j)$ and $f\overline{T}a(i,j)$ are quantities distributed according to probability and lying within an interval, the difference between the two values may also become ≤ 0.

It is necessary additionally to indicate with which probability the latest finish date is smaller than the earliest finish date.

Tables of normal distributions may be used to determine these probabilities. These tables show the values of the distribution function $\Phi(x)$.

If two or more critical paths run through the network in a larger case than this example, the oritical activities must be determined by calculation. That is done after the aspects already explained when discussing the critical path method. An activity will then be critical, if

$$f\overline{T}a(i,j) + te(i,j) \geq s\overline{T}e(i,j).$$

The method PERT offers the possibility to determine the probability with which the total process or certain partial processes will be finished at a fixed given date. But only very important events within the network are fixed. A finish date is assigned to them. This date is marked by the introduced symbol $Tv(j)$.

It is an important help for each person who has to take decisions, when she has the possibility of evaluating the probability of observing given dates. The example explained in the following shows a method.

The probability shall be tested with which the time for the total process is only 18 units of time. In this case Tv(j)=18. Here the value of x is determinded by the following formula:

$$x = \frac{Tv(j) - f\overline{T}a(i,j)}{\sqrt{\sigma^2_{fTa(i,j)}}} = \frac{18 - 18,32}{\sqrt{1,49}}$$

$$= \frac{0,32}{1,22} = - 0,27 .$$

With this calculated value, the table of the distribution function $\Phi(x)$ - normalized normal distribution - is consulted. It can be expected with a probability of P≈40% corresponding to the table value that the total process is already finished after 18 units of time.

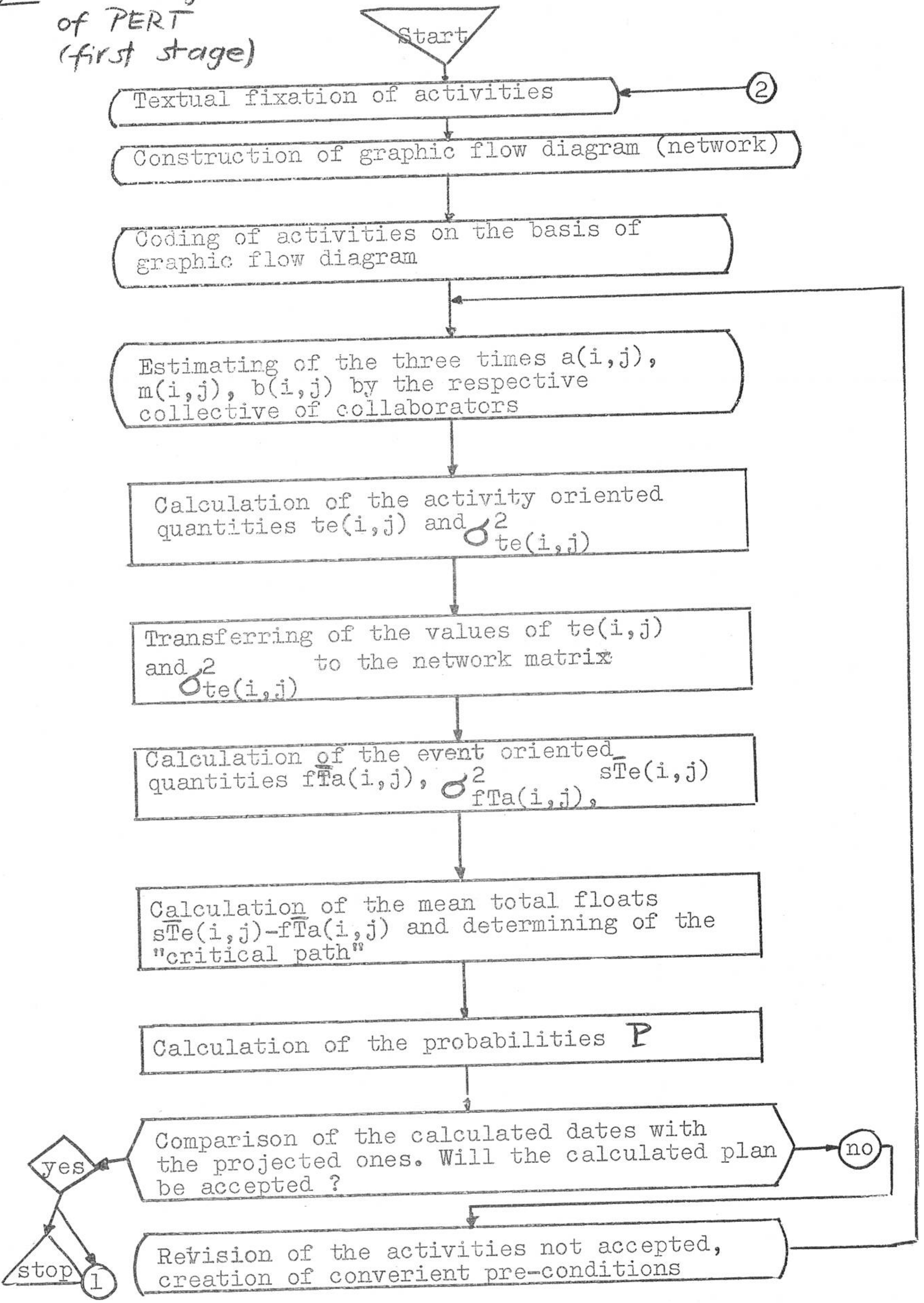
The example shows that time planning after PERT makes possible:

- The events and the activities on the critical path become evident.
- Total floats of events may be determined and, if the occasion arises, subdivided floats of activities may also be found as it is possible with the critical path method.
- It becomes possible to ascertain the probability of observing the given dates by determining x. That is especially relevant to very important intermediate dates and the finish date fixed in advance.

Fig. 6 shows the flow diagram of the stage of the first planning with the help of PERT.[1)]

1) cp. Schreiter/Stempell/Frotscher:Critical Path and PERT. Methods of Planning of processes dependent on time. Verlag Die Wirtschaft, Berlin 1965,p. 61. (Kritischer Weg und PERT. Methoden zur Planning zeitabhängiger Prozesse)

Fig. 6. Flow diagram of PERT (first stage)



3. On correction of plan and on regular check of plan

As a rule the first plan will seldom meet all requirements in regard to the intentions on date of the responsible management. The first stage of planning will have to be completed by further similar planning stages, if it is necessary to change the number of man power or means of work used resp. of activities on the critical path, in order to be able to estimate a shorter time for them. Thus a similar revision must be done as already explained when discussing the critical path method.

If the time period of 18.32 units of time in the example mentioned above was not acceptable by the management, the activities on the critical path would have to be proved with regard to their duration time. For that, however, the corresponding material pre-conditions for the reduction of one or another activity must be created. After that the values of time must be re-estimated and then the proceeding is as represented in Fig. 6.

The activity time must be shortened or re-estimated resp. only corresponding to the manpower or means of work additionally used. There should be considered that a shortening of activity time is generally combined with an increase in cost. Moreover a new critical path may develop with a very radical shortening, eventually showing that part of the additional expenditure is not at all necessary for the activity concerned. Therefore it would be better to carry through one more stage of planning (calculating for network) than very much to increase cost by a considerable additional use of powers and means. A possibility of compensation is also in reducing powers and means for slack activities with floats.

When checking the plan it is further important to increase the probability that a given date falls within a certain interval or that a range of distribution is decreased and thus the reliability and probability resp. of a given date is increased. When checking in order to reduce the variances, the effects must be thought over, in order to eliminate uneconomical measures from the very beginning. Computers are widely used, in order to decrease routine calculating work.

Moreover the electronic computation offers the possibility theoretically to play through the flow of the project before the beginning of realization. On that occasion various possible disturbances may be assumed and the flows resulting from this can be simulated. After a certain number of simulated flows, already substantiated conclusions can be obtained in order further to prepare and carry through the project.

E.g. the calculation could result when planning the time of development of a new technological proceeding:

a) 2.5 years with normal use of manpower and normal disturbances
b) 2.2 years with normal use of manpower and almost undisturbed flow

c) 1.3 years with increased use of manpower and normal disturbances

d) 1 year with increased use of manpower and almost undisturbed flow

In practice it will often be necessary to choose between variants a) to c), because an increased use of manpower and an almost undisturbed flow cannot be obtained in each case.

If an acceptable plan has been obtained after several checks, its realization can be started. Then the keeping of dates must be checked at certain time intervals (e.g. of one or two weeks). These checks are the basic requirement of adjusting the effects of inexact time estimates as far as possible. It is checked which activities were finished and which is the situation of the current partial tasks, i.e. whether there still exists conformity with the dates planned. In case of delays of dates, measures are taken and the program of the total project with the changed dates is recalculated by the computer. There can be seen by the solution, whether eventually the critical path was lengthened, which start and finish dates of the still uncompleted activities and which floats of following partial sections are changing. Activities and events not being on the critical path till then may have become critical. That, too becomes obvious during the recalculation. Thus useful data are received, in order to be able newly to dispose according to the changed situation and nevertheless to keep the planned finish date.

In the case of the check of plan it must be considered for the nominal-actual-comparisons that they are stochastic quantities and consequently the dates can only be kept within certain intervals. Generally only such activities must be adjusted which negatively affect the realization of the total process.

The essential steps of the check of plan and of the sliding planning become obvious from Fig. 7[1]) It is a supplementation of Fig. 6.

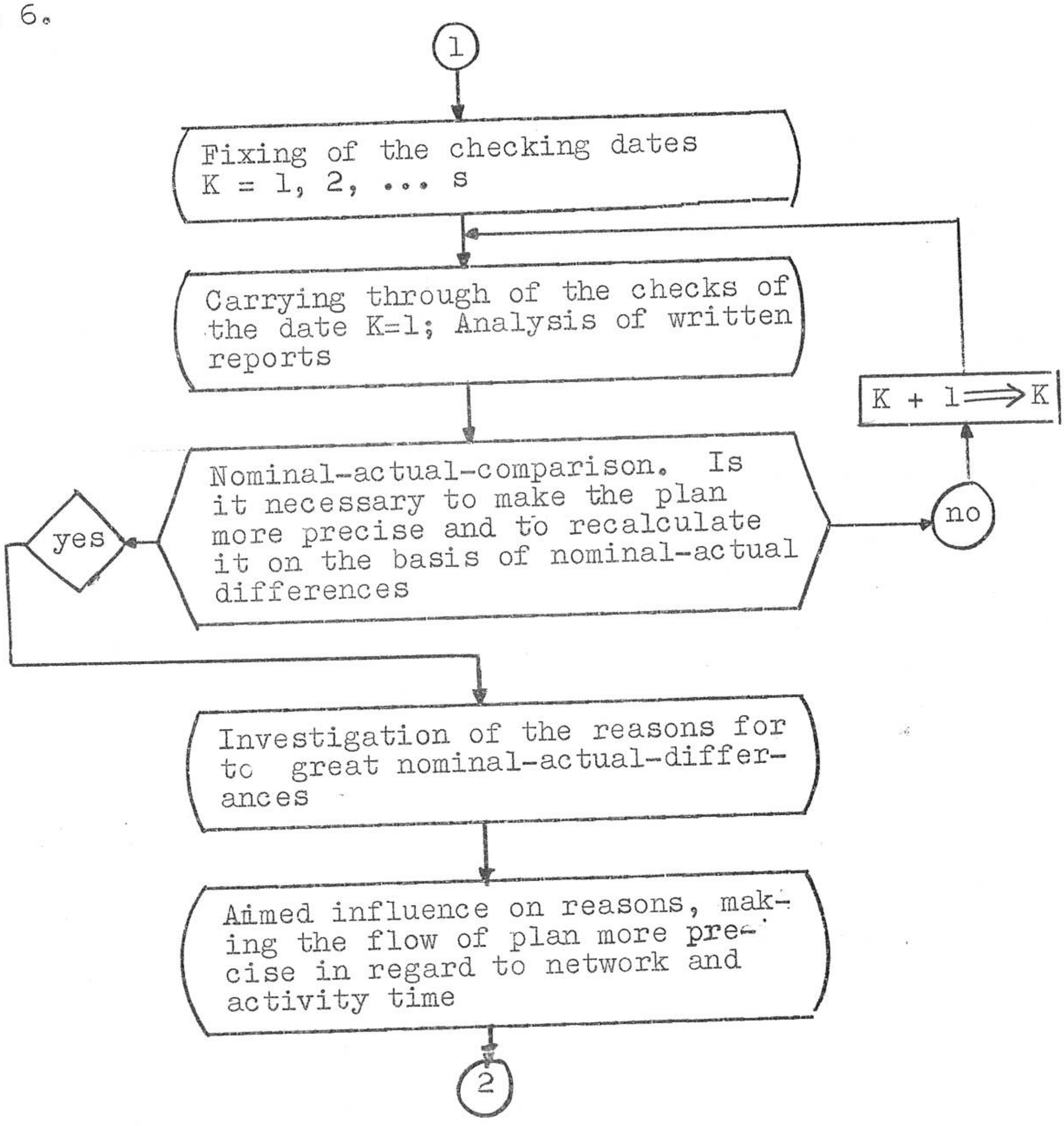


Fig. 7. Flow diagram of PERT (Stage of plan checking and sliding planning)

1) cp. ibid., p. 63

4. On application of methods of network technics

The critical path method having been primarily developed for the realization of production tasks in due time, the method PERT explained here in first line aimed at a good co-operation of many enterprises, or institutions, or research workers, etc. especially in connection with the simultaneous solution of development and production tasks for a project.

In the G. D. R. in first line the stochastic model of network technics was applied to

- planning of research and construction works
- planning of projecting works, which did not have determination time values
- planning of new technologies, flow and time of which did not have exact criteria, too.
- co-operation of numerous enterprises to fulfil one important task of plan
- preparations for the use of new technical equipment with new working operations, inter alia also when using electronic data processing machines.
- organization of works of management, planning, and administration, i.a. complicated projects of investment.

Though here are given some generally valid indications, it has always to be considered when applying methods of network technics, that every project involves specific problems. These problems generally require an individual examination.

The following conditions have to be met, in order to be able to apply the methods of network technics :

- the total process has to be subdivided into exactly to be defined partial processes
- time of each partial process has to be known or suitable to be estimated on the basis of data of experience
- it must be established for every partial process, which other partial processes have to be finished at its beginning and which must follow it.

Mostly - on principle e. g. in projects of investment - as a rule no more initial data are wanted than had been required for the planning of such projects till now. The single partial tasks, the activities, have already been registered in lists. Thus it is easily possible to arrange them from the first as it is requir-ed by the forms of the computing centre.

On the basis of the data and with the help of a checking pro-gram the collaborators of the computing centre are able to change critical activities already during calculation. That requires beside the input data also the network and the activity cards in the computing centre. The computer calculates the critical path and all the other essential data. Computers with printing of tables deliver these data as a block of numbers. Check of net-works and of results can also be undertaken by the computing cen-tre, if desired by the customer.

Generally it can be said that the processing of such plannings of time by the methods of network technics using electronic data processing machines will be economical if the project involves about a 100 or more activities. But the opinions of the various authors differ very much in this point. The data range from 50 to 200 activities.

A further condition not resulting from the critical path model or modern computation is the co-ordination of flow and time planning as well as of material planning with flow and time planning - if necessary with a broad participation of the workers of the industrial plants concerned. Otherwise the results of time planning could be made dubious.

The conventional planning methods of the flow of work and time for that kind of projects dealt with here, e.g. the block diagram, in some cases require less expenditure than the methods of network technics. These, however, offer the essential advantage of actually finding the critical path and of being able quickly to show arbitrarily many variants by variable dates of time for the normal and the minimal time of realization (if necessary the latter subdivided in stages). If difficulties in keeping the dates occur during the flow, in short time a new founded solution can be determined on the basis of the critical path. That is impossible by the conventional planning methods of the flow of work and time. Therefore the methods of network technics have to be preferred.

Thus these methods and the use of modern computation contribute to improve the quality of management and moreover to arrive at a scientific plant management. Therefore the endeavours must be aimed at a multi-purpose appliance of these methods.

5. On further development of the methods of network technics

In recent years the further development of methods of network technics mainly proceeded in three directions:

(1) Involving of calculations of optimization, especially on the basis of cost
(2) Establishing of decisive networks by including alternative events
(3) Simultaneous planning of the use of manpower and means of work.

Here only some statements on the possibility noted in 3. shall follow.

While one project in each case is planned by the methods explained here, the so-called RAMPS-method (Resource Allocation and Multi-Project-Scheduling - distribution of capacities of expenditure and planning of multi-projects) serves simultaneously to calculate several parallel projects, where the same manpower and means of work are used.

In case an advantageous use of the available manpower is aimed at, the flow of time is planned in "mendays" under the following aspects :

- number of manpower to carry out a certain task during a normal flow
- number of manpower in an accelerated flow
- minimal number of manpower to carry out the task concerned. The separation into normal, maximal, and minimal quantities is approximately given by it.

The use of manpower can be planned 20 days or a month in advance. There has to be considered among other points:[1)]

- works, where disturbances are not permissible (planning of of a so-called fine for interruptions)
- total and freely available float (high estimation of those partial tasks, the duration of which cannot be prolonged without lengthening the critical path and thus endangering the finish date)
- shifting of work (at first those works are carried out which are followed by numerous others)
- continuity of work (works already taken up are preferred)
- number of works ready for táking them up (they are estimated in such a manner that the highest possible number of such works can be taken up)
- break in work (all means of work available are preferred)

The program of the computer is set up in such a way (following the authority mentioned in literature) that the answer to the following question can be determined on the basis of the fed data : which work has to be done by which working team, on which day of a production period, with how many workers, machines, and material? The computer delivers a list for each category of work showing which manpower has to do which work on which project at which time. A summarizing survey is printed at the end of the list. For every working day it contains statements on the used manpower, the still available, or the labour additionally to be provided.

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1) cp. Budil, W.: "RAMPS". Industrial Organization, 32 (1963) No. 9 p. 311 ("RAMPS". Industrielle Organisation)

The methods mentioned leads to approximately optimal solutions under the point of view of the use of manpower and means of works as well as cost. Such a program, however, requires the use of a large date processing machine.

I want to conclude my explanations by these final remarks which were intended to give you a little insight into the future application of methods of network technics. I thank you for your attention and for your kindly reception.