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Calculation of the aggregate claim amount distribution using a recursive method with R and Actuar

Apply to one of the comprehensive motor insurance companies

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Abstract

This study aims to compute the aggregate claim amount distribution in one of the comprehensive motor insurance companies. One major issue that actuaries face is computing the distribution of aggregate losses, where computing the aggregate claim amount distribution requires a discrete arithmetic claim amount distribution, in some numerical techniques. This paper concerning with the discretization of continuous claims amount distribution by applying four discretization methods. Then calculate the aggregate claim amount distribution by Panjer recursive technique. Also, this paper presents how to achieve this by actuar which is a software package that provides statistical science functionality to the R statistical system. This package provides also functions to discretize & compute the continuous distributions of the overall loss distribution using a variety of techniques, including methods as recursive and simulation. Results indicate the increasing importance of comprehensive motor insurance recently according to the annual statistics book of insurance last issues. Also presents that for the four discretization methods, both unbiased and rounding seem identical.

Keywords: aggregate claim, claim frequency, claim severity, Collective risk model, Discretization, recursive method.

1. Introduction

Insurance provides protection for the insured from different risks, which happens by replacing the expected large loss with a small certain loss which is called insurance (premiums). Recently, there is increased interest in comprehensive motor insurance cause of requiring the treatment of a large number of risk events. These risks include cases of theft and property damage due to accidents or other reasons like the extent of damage and parties concerned.

In Egypt the published data from Financial Regulatory Authority (FRA), in the annual statistical book for insurance[1], shows the increase in premiums for comprehensive motor insurance, and the next table shows premiums development over five years (2017 – 2021) in thousands of pounds:

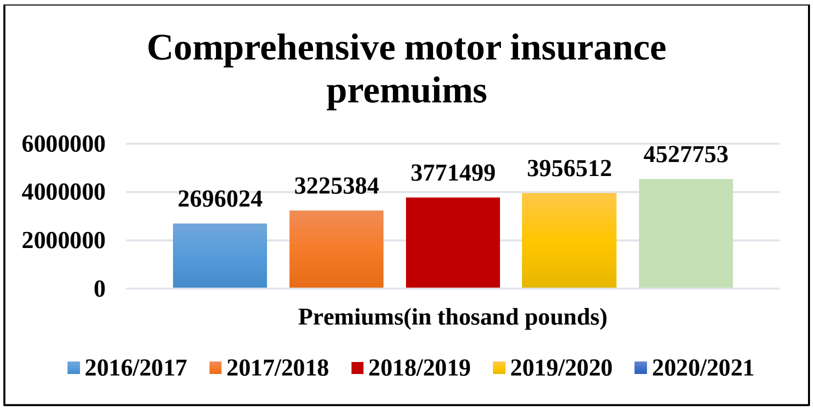
Table (1)
Premiums develop over five years
(In thousands of pounds)

year	Premiums	Premiums development rate
2016/2017	2696024	32%
2017/2018	3225384	19.60%
2018/2019	3771499	16.90%
2019/2020	3956512	4.90%
2020/2021	4527753	14.40%

Source: annual statistical book for Insurance (2021), FRA.

This table shows increasing in premiums value during the period (2017-2021), and also shows the development rates increasing, where figure (1) represents this increase graphically

[1] annual statistical book for insurance 2021,

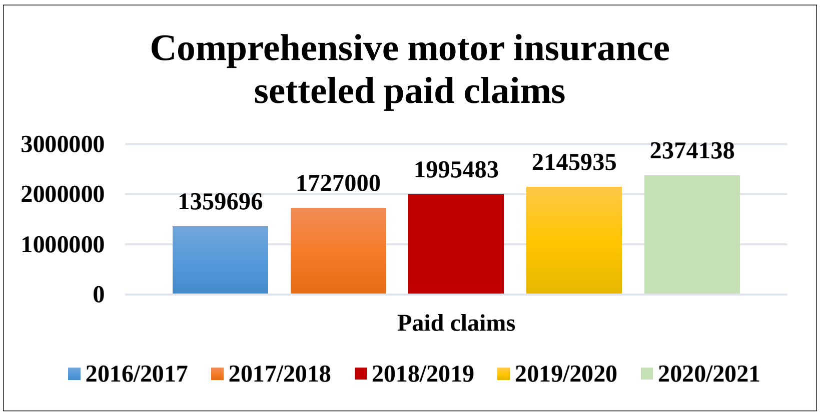


Source: prepared by the researchers based on the annual statistical book for insurance (2021)

Figure (1)

Premiums development rate over five years

This figure shows increases in the values of premiums, which reflects an increasing trend in buying comprehensive motor insurance. also, this development is motivated by changes in society for protection against various types of risks of adverse random events with significant economic consequences. Figure (2) shows also the increase in settled paid claims for this sector which covering by paied premiums in the same duration (2017-2021).



Source: prepared by the researchers based on the annual statistical book for insurance (2021).

Figure (2)

Settled paid claims over five years

Both figures (1) and (2) respectively, shows the increase in buying this type of insurance and also claims resulting from occurring of risks

According to actuarial science, the aggregate loss distribution is derived by modelling separately the two components of claim losses: *claim frequency* & *claim severity*. The majority of general insurance firms use their own historical claim data to develop estimates of claim frequency and severity. This serves as the foundation for administrative choices and is occasionally supplemented with information from outside sources.

According to (Goulet, 2010) actuar is a R statistical system package that adds Actuarial Science functionality. He presented the characteristics of the package focusing on the theory of risk, which deals with modeling and measuring the hazards connected with a portfolio of insurance policies. Also shows that the actuary's main concern is the distribution of total claims over a specified time period, is the distribution of total claims over a specified time period, as modeled by the traditional collective model of risk theory. Finally, this paper also explained that this package offers features to distinguish continuous distributions and to calculate the overall distribution of the number of claims using a variety of techniques, such as recursive method.

Since (Omari, Nyambura, & Mwangi, 2018) mentioned that Property/Casualty insurance claims experience depends on chance events that led to claim frequency and claim severity. Also showed that P/C insurers face two difficulties when modeling claims data; selecting the most suitable distribution of claim data and evaluating how well the selected distribution fits the claims data. According to the study's

findings, a skewed and heavy-tailed distribution is a better model for claims severity distribution than a fitted lognormal distribution, which is a continuous distribution. As opposed to other conventional discrete distributions, negative binomial and geometric distributions are chosen as the most suitable distributions for the claims frequency.

(Ng'elechei, Chelule, Orango, & Anapapa, 2020) offered a methodology for selecting an appropriate probability distribution and fitting it to historical data on motor vehicle claims. And this is because the importance of the frequency of insurance claims and severity when the premiums are priced in general insurance, and also future claims from the current portfolio of policies will be settled. The study found for the severity claim data, the Pareto model provided the greatest fit. Comparing the tested models, it was discovered that the negative binomial distribution model provided the greatest fit for frequency claims.

Based on the above, the problem of this research is the modelling of claim losses, since it is the base of loss distribution which is the first step of all decisions made in the insurance company regarding premium loading, expected profits, and reserves, and these models can be used to determine how *aggregate claim amounts* are distributed. Also, this paper concentrates on the computation of the *aggregate claim amounts* distribution using the functions found in the package actuar for the R statistical system, since actuar provides a powerful and consistent interface to this calculation by means of the function aggregateDist.

And It is noteworthy that the feature set of this package can be split into four main categories: loss distributions modeling, risk theory

(including ruin theory), simulation of compound hierarchical models and credibility theory.

2. Research objectives

The objective of this paper is summarized as follows:

1. **Discretize continuous distributions applying to one of the comprehensive motor insurance company data.**
2. **Compute the distribution of aggregate claim amount via recursive coputation.**

This is all done by the functions (discretize and aggregateDist) found in the package (actuar) in R statistical system.

3. Importance of research

It is crucial for determining insurance pricing and predicting future claims to compute the aggregate claim amounts of an insurance business, which include both claim frequency and claim severity. The majority of insurance companies rely on already-established models from developed markets that are typically tweaked to estimate the distributions of claim frequency as well as severity.

Because diverse markets have distinct characteristics, particular models from different markets cannot be universally applied to accurately estimate the claims data of all insurance firms.

So, the importance of this paper is for both the insurance company and the insured. When insurance company estimate claim frequency and claim severity in accurate value that makes pricing the policy equitable and sufficient. And this affects the confidence of the insureds in the ability of the company to meet future obligations.

4. Research Methodology

This paper gives a general methodology in compute aggregate claim amounts for one of the comprehensive motor insurance companies in Egypt using the functions found in the (actuar) package in the R statistical system. in this paper, for the computation of the aggregate-loss distribution, both recursive and approximate approaches will be taken into consideration. The methodology is organized as follows:

4.1: The Collective risk model.

4.2: Discretization of continuous claim amounts distributions.

4.3: Computation of the aggregate claim severity distribution.

4.1 The Collective risk model

Claim frequency, or the number of claims in a group of insurance policies over time, is a crucial indicator of claim losses. Although claim frequency does not directly reflect financial losses primarily concerned with insurance claims, it is an influential factor when calculating losses. and claim severity (or claim size) refers to the financial value of each claim. The aggregate claim for losses of all claims make up the total claim for losses under the block of policies.

Individual risk modeling and collective risk modeling are the two main methods used to model aggregate loss. In this paper we interest in collective risk model. The primary distribution of the aggregate loss according to the collective risk model is the frequency of claims, and the secondary distribution is the severity of claims. If the claim-frequency distribution falls under the (a, b, 0) class, the Panjer recursion can applied to calculate the distribution of the total loss. A

discrete distribution is used to discretize or approximate the claim severity distribution. And this is discussed in the next section.

When compute the aggregate loss the overall loss assumed that follow a compound distribution, when employing collective risk model. The amount paid on all claims that occurred within a given time frame on a specified set of insurance contracts can be modeled in a variety of ways.

The recording of payments as they are made and subsequent addition is one of them. The total sum of all claims, or (the total amount of claims,) can be represented as a sum S, random variable, of a collection of distinct risks over a predetermined time period.

where N, reflect the portfolio's frequency (or claims count) during that time period using a random variable. in addition to randomness X_j indicate the claims amount j(or severity). The random sum then appears as follows:

$$S = X_1 + X_2 + \cdots + X_N, \qquad N = 0, 1, 2. \ldots \qquad (1)$$

Where:

$$S = 0$$

$$N = 0$$

This equation represents the *collective risk model* with the assumptions:

1. **Unless otherwise stated, X_j will be independent and identically distributed $(i.i.d.)$ random variables.**
2. **The random variables $(X_1 . X_2 . \ldots X_n)$ are $(i.i.d.)$ random variables if $N = n$**
3. **The common distribution of the random variables$(X_1, X_2, \ldots, X_n)$ does not rely on n when N=n.**

4. **There is no relationship between the values of $(X_1, X_2, \dots, X_n)$ and the distribution of N (Klugman, et.al, 2012).**

The current aim is to numerically evaluate the cdf of S, given by

$$F_S(x) = \Pr[S \leq x]$$
$$= \sum_{n=0}^{\infty} \Pr[S \leq x | N = n] p_n$$
$$= \sum_{n=0}^{\infty} F_X^{*n}(x) p_n \qquad (2)$$

Where

- $F_X(x) = P_r[X \leq x]$ **is the typical** cdf **of** $X_1, X_2, \dots, X_n$, $p_n = P_r[N = n]$
- $F_X^{*n}(x) = P_r[X_1 + X_2 + \cdots + X_n \leq x]$: **n-fold convolution of** $F_X(\cdot)$.
- **If X is discrete on $0, 1, 2, \dots$, one has**

$$F_X^{*k}(x) = \begin{cases} I\{x \geq 0\}, & k = 0 \\ F_X(x), & k = 1 \\ \sum_{y=0}^{x} F_X^{*(k-1)}(x-y) fX(y), & k = 2, 3, .. \end{cases} \qquad (3)$$

If A is true, then $I\{A\} = 1$;else, $I\{A\} = 0$ (Dutang, Goulet, & Pigeon, 2009)

4.2 Discretization of continuous claim amount distributions

Calculating the distribution of total losses is one of the biggest issues facing actuaries. The distribution of the Xj 's is often both discrete and continuous. The discrete component is caused by the accumulation of losses at certain quantities, but the continuous part derives from losses of any size (nonnegative).

When an insurance claim is submitted but no payment has been paid by the insurer, these sums are often zero, certain round records

(when claims are resolved for a certain amount, such as 1 million pounds), or the maximum amount payable under the policy (when the insured's loss exceeds the maximum payable).

Specific numerical techniques call for a discrete mathematical claim amount distribution in order to compute the aggregate claim amount distribution. A distribution that is defined on the interval $(0, h, 2h, \dots)$ for some step (or span, or lag) h. Four discretization methods are available. (Goulet, 2010):

a. **Upper discretization, often known as the forward difference of $F(x)$;**
b. **Backward difference, or lower discretization of $F(x)$;**
c. **Midpoint technique or rounding the random variable;**
d. **Unbiased or local matching using the first-moment method, respectively.**

a. Forward difference:

$$f_x = F(x+h) - F(x) \qquad (4)$$

For $x = a, a+h, \dots, b-h$ Always above the true cdf is the discretized cdf.

b. Backward difference:

$$f(x) = \begin{cases} F(a), & x = a \\ F(x) - F(x-h), & x = a+h, \dots, b \end{cases} \qquad (5)$$

The true cdf is always below the discretized cdf.

c. Midpoint technique:

$$f(x) = \begin{cases} F\left(a + \frac{h}{2}\right), & x = 0 \\ F\left(x + \frac{h}{2}\right) - F(x - \frac{h}{2}), & x = a+h, \dots, b-h \end{cases} \qquad (6)$$

The discretized cdf's steps directly correlate to where the true cdf exits.

d. Unbiased:

$$\begin{cases} \frac{E[X \wedge a] - E[X \wedge a + h]}{h} + 1 - F(a), & x = a \\ \frac{2E[X \wedge x] - E[X \wedge x - h] - E[X \wedge x + h]}{h}, & a < x < b \\ \frac{E[X \wedge b] - E[X \wedge b - h]}{h} - 1 + F(b) & x = b \end{cases} \quad (7)$$

The total probability and expected value for both the true and discretized distributions are the same (a, b) and discretize function in actuar package supports these four discretization methods.

4.3 Computation of the aggregate claim severity distribution

As a special front end for various approaches to obtaining or estimating the cdf of the aggregate claim amount random variable S, the function aggregateDist is being used. There are five supported techniques summarize as follow:

4.3.1 Recursive calculation:

Recursive calculation is the method used the most frequently to calculate the cdf of S. which introduced evaluation of a family of compound distributions by recursive computation using power series operations that are familiar in other areas of mathematics, using the algorithm of Panjer (1981) the Panjer recursive requirement:

a. For some monetary units, the distribution of severity is discrete math on$0, 1, 2, \ldots, m$.

b. The frequency distribution must belong to the $(a, b, 0)$ or $(a, b, 1)$ family of distributions (Goulet, 2010)[2].

And the recursive general formula:

$$f_S(x) = \frac{(p_1 - (a + b)p_0)f_C(x) + \sum_{y=1}^{\min(x,m)} \left(a + \frac{by}{x}\right) f_C(y) f_S(x - y)}{1 - a f_C(0)} \qquad (8)$$

Using a starting point $f_S(0) = P_N\big(f_C(0)\big)$

Hence $P_N(\cdot)$: the N\`s probability generating function.

The computation of probabilities continues until their total is determined near to 1. When the predicted number of claims is so high that $f_S(0)$ is equivalent to zero numerically, the recursive technique fails.

In this situation, the solution offered by Klugman et al. (2008) involves dividing the suitable frequency distribution parameter by 2^n where n is selected such that numerically $f_S(0) > 0$

The final distribution is then calculated by convolving the resulting distribution n times with itself using the recursive method used to compute the distribution of total claim amount.

4.3.2 Simulation

By simulating a large enough random sample from S, this method can approximate $F_S(x)$ using the empirical cdf formula

$$F_n(x) = \frac{1}{n} \sum_{j=1}^{n} I\{x_j \le x\} \qquad (9)$$

The frequency and severity distributions can be simulated, which is an easy technique to examine a compound distribution. A simulation

[2] For an arbitrary mass at $x = 0$, the Poisson, binomial, negative binomial, and logarithmic distributions, as well as their expansions, is one of these families.

algorithm, on the other hand, is regularly ineffective & needs a considerable amount of processing power.

4.3.3 Direct calculation by numerical convolutions

This calculation is done by using (2) and (3), and a separate severity distribution is also required, but there is no restriction on the frequency distribution shape. Due to a large number of products and sums performed, this technique is limited to small difficulty in practice.

4.3.4 Approximation distribution

Probably the easiest way to compute $F_S(x)$ is to fit the distribution. The drawback is unknown precision and usually crude tails. This technique supports two approximating methods:

a. The normal approximation method of the cdf, that is

$$F_S(x) \approx \Phi\left(\frac{x-\mu_S}{\sigma_S}\right) \qquad (10)$$

Where $\mu_S = E[S]$ and $\sigma_S^2 = Var[S]$

b. The normal Power II approximation method

$$F_S(x) \approx \Phi\left(-\frac{3}{\gamma s} + \sqrt{\frac{9}{\gamma_s^2} + 1 + \frac{6}{\gamma s}\frac{x-\mu_S}{\sigma_S}}\right) \qquad (11)$$

Where $\gamma_S = \frac{E[(S-\mu_S)^3]}{\sigma_S^{\frac{3}{2}}}$

This approximation applies to $x > \mu_S$ and assuming that it works accordingly $\gamma_S < 1$. The approximation is not appropriate for very strongly skewed distributions.

From above, because the recursive method, is the most popular technique to calculate aggregate claim severity, the following section

provides the applied study to compute aggregate claim severity distribution by the recursive method.

5. Applied study

The data of this research is time series for the duration (2012-2021) from one of the insurance companies for the comprehensive motor insurance sector. The applied study is organized as follows:

5.1: Discretize claim severity distributions with discretization methods.

5. 2: Calculate aggregate claim severity distribution.

And all with R program specifically with actuar package.

5.1 Discretize claims severity distribution.

Fitting the severity data with (Easy fit 5.5 professional) find that data follow a Gamma distribution with parameters $(2,1)$. Next is the application of four discretization methods to discretize Gamma(2,1) distribution on (0,10) with a step of 0.5, using function discretize[3] the results as follow:

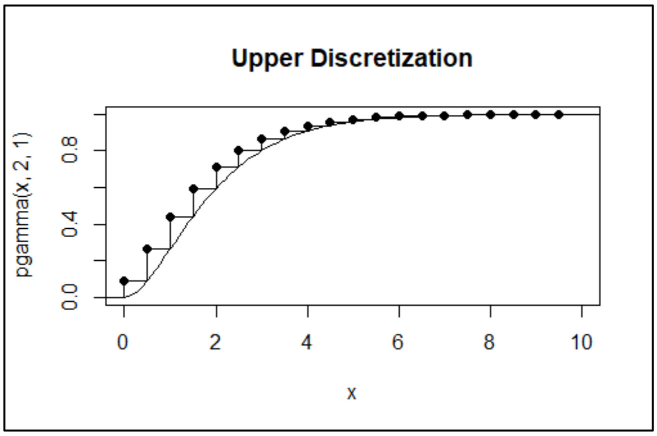
a. Upper discretization

The upper discretization of continuous gamma$(2,1)$ severity data, on$(0,10)$ with a step of 0.5. achieved by using following R codes:

```
fCUP <- discretize(pgamma(x,2, 1), method = "upper",
                          from = 0, to = 10, step = 0.5)
```

And figure (3) display the results of this discretization method

[3] The names of methods here matching with the programe codes.



Source: R programe output.

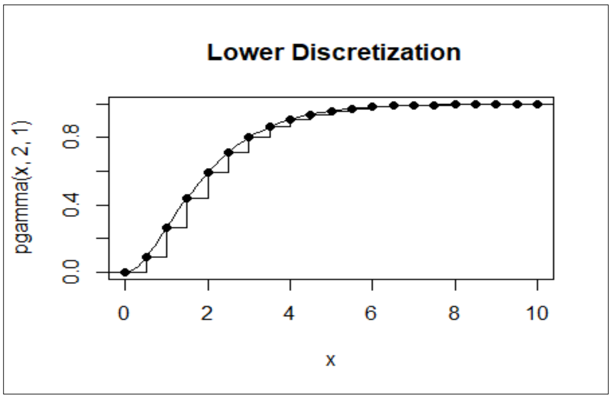
Figure (3)
Upper discretization to continuous severity data

b. Lower discretization

The following R codes achieved different discretization method for continuous gamma $(2, 1)$ severity data, on $(0, 10)$ with a step of 0.5

```
fcL <- discretize(pgamma(x,2, 1), method = "lower",
                   from = 0, to = 10, step = 0.5)
```

And the results represent graphically in figure (4)



Source: R programe output.

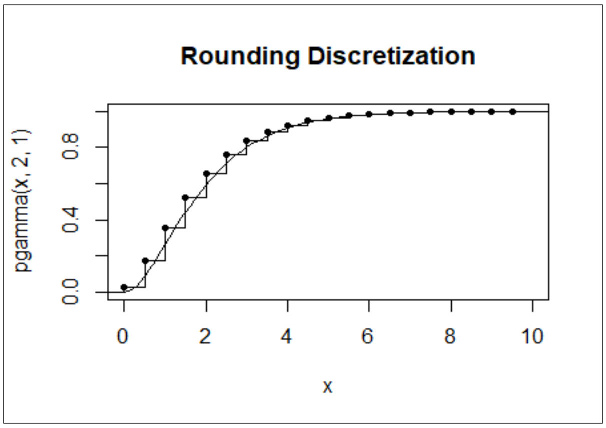
Figure (4)
Lower discretization to continuous severity data

c. Rounding the random variable

To apply rounding of the random variable on the same severity data set, the following codes achieved this

```
fCR <- discretize(pgamma(x,2, 1), method = "rounding",
                                    from = 0, to = 10, step = 0.5)
```

Figure (5) presents the data after discretization



Source: R programe output.

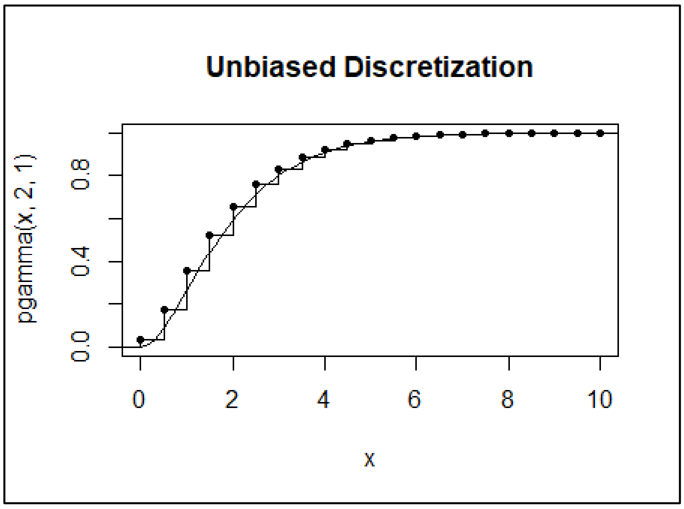
Figure (5)
Rounding discretization to continuous severity data

d. Unbiased discretization

The last discretization method is unbiased, could apply with this codes

```
fCU <- discretize(pgamma(x,2, 1), method = "unbiased",lev=levgamma(x,2,1),
                                    from = 0, to = 10, step = 0.5)
```

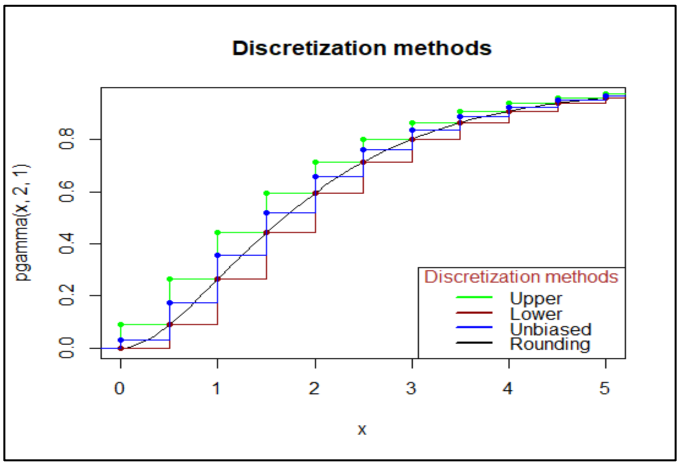
Consider that levgamma is used only with this method. Figure (6) display the results of this method.



Source: R programe output.

Figure (6)
Unbiased discretization to continuous severity data

Figure (7) compares the four discretization methods graphically. The upper and unbiased methods are close from the figure for this data set. where the unbiased and rounding methods seem identical, but the numerical data in tables (2) and (3) respectively show that two methods not identical.



Source: R programe output.

Figure (7)
Comparison of four discretization methods

Table (2)

Unbiased discretization method results

```
> fcU
 [1] 0.0326532986 0.1419700499 0.1800111255 0.1661366708 0.1353115769
 [6] 0.1030229743 0.0751948379 0.0533159151 0.0370128393 0.0252850147
[11] 0.0170560106 0.0113881497 0.0075399682 0.0049569776 0.0032393186
[16] 0.0021059219 0.0013629337 0.0008785967 0.0005643965 0.0003614298
[21] 0.0001325945
```

Table (3)

Rounding discretization method results

```
> fcR
 [1] 0.0264990212 0.1468595115 0.1820056744 0.1667574484 0.1353308647
 [6] 0.1028180003 0.0749390962 0.0530810905 0.0368220648 0.0251399806
[11] 0.0169502574 0.0113132196 0.0074879779 0.0049214754 0.0032153784
[16] 0.0020899415 0.0013523556 0.0008716437 0.0005598535 0.0003584768
```

After discretizing the severity data, compute the cdf of the random variable S, with function (aggregateDist). The distribution of S fitting Poisson distribution, the result of apply (Easy fit 5.5 professional). Parameter $\lambda = 10$, severity distribution gamma(2,1), so summary of aggregate claim amount distribution after applying discretization methods are In tables (4), (5),(6) and (7) as follow

Table (4)

Aggregate claim amount empirical CDF for upper discretization

```
Aggregate Claim Amount Empirical CDF:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
  0.00000  12.50000  17.00000  17.36057  22.00000 250.00000
```

Table (5)

Aggregate claim amount empirical CDF for lower discretization

```
Aggregate Claim Amount Empirical CDF:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
  0.00000  16.50000  22.00000  22.33317  28.00000 250.00000
```

Table (6)

Aggregate claim amount empirical CDF for rounding discretization

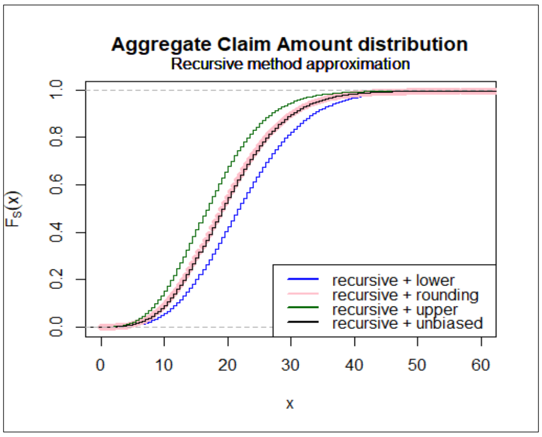
```
Aggregate Claim Amount Empirical CDF:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
  0.00000  14.50000  19.50000  19.80593  25.00000 250.00000
```

Table (7)

Aggregate claim amount empirical CDF for unbiased discretization

```
Aggregate Claim Amount Empirical CDF:
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
  0.00000  14.50000  19.50000  19.84526  25.00000 250.00000
```

Tables (6) and (7) show that the aggregate claim amount empirical CDF for both rounding and unbiased is almost identical, where the results of applying recursive method shown in figure (8) which represent comparison of four discretization methods with recursive method approximation.



Source: R programe output.

Figure (8)

Compare the empirical cdf of S obtained by different methods

6. Conclusion

This study shows that comprehensive motor insurance has an increasing development rate for both (premiums and settled paid claims). Computation of the aggregate claim amount distribution requires a discrete claim amount distribution. applying four discretization methods shows that the results of rounding and unbiased methods seem to be identical. The recursive method is the most popular technique to compute the accumulated distribution function of the overall number of claims.

7. **Recommendation**

Recommendations of this study summarized in using scientific actuarial techniques for calculating aggregate claim amounts. It is also recommend that experts in actuarial science field provides their efforts and contribute code to actuar project, which consider modern actuarial science functionality to the R statistical system.

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