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Notes on Morphological Accuracy Reduction

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1. ABSTRACT.

Attribute reduction is an important pre-processing step for data mining and has become a hot research topic in machine learning which involves high-dimensional descriptions of input features (attributes). It is therefore not surprising that a lot of research has been done on dimension reduction. Information systems are the most known forms of knowledge representation. In this paper, we propose a novel attributes' reduction technique for information systems. The proposed technique [namely, morphological accuracy reduction (for short, MAR)] is based on computing a morphological accuracy using the morphological operators (neighborhood-erosion, neighborhood-dilation). Which the ratio between the neighborhood-erosion, neighborhood-dilation of the set-in order to gouge its stability; comparing with existing reduction techniques including "a nano topology" and "dimensionality reduction", experimental results show that the proposed MAR method is an efficient algorithm for attributes reduction and calculating the core attributes. Also, the main advantage of the new method is that it helps to reduce data without losing useful information, as well as it saves time and reaches the best core set in fewer steps.

Key words:

rough set theory, attributes reduction, information system, mathematical morphology, morphological accuracy, neighborhood structure element, nano topology, and approximation spaces.

2. INTRODUCTION.

Rough set theory was developed by Pawlak [20, 21, 22] in the early 1980's the rough set of any set is defined by a pair of the lower and upper approximation of the set. As demonstrated by Pawlak applying the lower and upper approximation operators to the rough set is the same as applying the interior and closure operators to the set, respectively. They can also be seen as a closure operator and an interior operator of the topology induced by an equivalence relation on a universe. The classical rough set theory based on equivalence relations has been extended to tolerance relations, similarity relations, and general binary relations [12, 8]. Rough approximation operators and values of accuracy are the most significant characteristic of rough set theory. In literature there exist serval techniques to improve and increase the stability of the set under the approximation operators. For instance, those techniques introduced by Abu-Donia [5], Hosny [10], Abo-Tabl [3] and Al-shami et.al. [15, 16, 17].

In data mining and machine learning attributes' reduction become an important tool for the purpose of improving the efficiency and quality, as well as using the processing time. In real-life applications, we deal with information systems that hold too many data attributes, which consume time and cost in processing- yet some attributes might be unnecessary or redundant. Therefore, a data attributes' reduction is needed to maintain the core attributes. In literature, there is a large body of research working on handing a way to reduce the data attributes; among those are the techniques based on the concepts of topological spaces. For information systems, the idea of attributes' reduction was firstly introduced by Pawlak [20, 21]; mainly, a simple mechanism for determent a minimum reduct was used, by locating all the possible reducts, hence, to select the one with the lowest cardinality and the highest dependency. The work for reducing the data attributes information systems was carried out in serval ways, (see for instance, [2]).

In this paper, we propose a new technique for attributes' reduction, based on the topological neighborhood along the basic morphological operators. In section 2, we give a brief revision for some concepts of information systems, the most used forms for sorting data. Sections 3 and 4 are devoted to introducing the basic definitions for the rough set theory, as well as a quick review for some concepts of mathematical morphology from a topological neighborhood point of view. In section 5, we demonstrate the methods for attributes' reduction produced by EL-Sayed in [11]. In section 6, we introduce a new technique for attributes' reduction based on computing the accuracy of the set using morphological operators. Finally, in section 7, we summarize the work done through this paper.

3. DECISION SYSTEMS.

In this section, we recall the definitions of the decision system, which is an information system accompanied by distinguished attribute called, the decision attribute.
A decision system is represented as a table, where each row is devoted to the case (an event, a patient, or simply an object). whereas each column represented an attribute (a variable, an observation, a property, characteristic condition, etc.), as well as a distinguished column for some decision [1].

Definition 2.1 [4].
Information systems can be formulated as a pair $(U, X \cup \{C\})$, where U is the universe, X is a nonempty finite set of attributes is called conditional attributes and C is the decision attribute; where $C \notin X$. Each attribute of X is defined as a function: $p: U \to V_p$, where V_p is the set of all the membership values of each object of values of U in the attribute column p.

Definition 2.2 [4].
Consider a finite non-empty set $U \neq \emptyset$, with a binary relation $R \subseteq U \times U$. For any information system (U, X) and a set of attributes $P \subseteq X$, The indiscernibility relation of $(IND(p))$, is the set of all pairs of objects having some membership values in each attribute in P. $IND(P) = \{(x, y) \in R: v_{p_k}(x) = v_{p_k}(y), \forall\, k \in 1: |P|\}$, Where v_P is the set of membership values defined in Definition (2.1).

Remark 2.1.
For any object $x \in U$, the indiscernibility equivalence class of x denoted by $[x]_p$, is the set contains all the objects related to x with respect to the indiscernibility relation. $[x]_P = \{y \in U | (x, y) \in IND(P)\}$. For the set U, the family of all the equivalence class of each object in U with respect to the set of attributes P is denoted by $U/IND(P)$ and is defined as follows: $U/IND(P) = \{[x]_P: \forall x \in U\}$.

Definition 2.3 [18].
Consider an information system (U, X) for a set of objects U and a set of attributes X. The core attributes of X is these attributes with high influence over the information system. For any information system a decision can, be made considering only the core attributes. To construct a core set of the

attributes' set X, we commence with excluding the attributes that maintain the lowest classification ability, that is for $P \in X$, if $U/IND(X) = U/IND(X - \{P\})$. Then P is not a core attribute, the set of all core attribute of X is denoted by $CORE\ (X)$.

4. ROUGH SET THEORY [16].

The theory of rough sets has been under continuous development for the last decades. The rough set is a pair of precise concepts called the lower and the upper approximations, The lower approximation is the objects that belong to the subset of interest for sure, whereas the upper approximation is a description of the objects which possibly belong to the subset.

Definition 3.1 (Types of Relations) [8].

The Cartesian product of two sets A and B, is denoted by $A \times B$; the set of all ordered pairs (a, b), where a is in A and b is in B. That is, $A \times B = \{(a, b): a \in A, b \in B\}$. A relation is a subset of the Cartesian product and it's a form of connection between the elements of one set and the elements of another set: $R \subseteq A \times B$. Some of the important types of relations are as follows.

i. **Reflexive Relation:** A Relation R defined on Set A is said to be reflexive if each element of the set is mapped to itself. $R = \{(a, a), \forall\ a \in A\}$.

ii. Symmetric Relation: A relation R is said to be a symmetric relation if $, R = \{(a_1, a_2), (a_2, a_1); \forall\ a_1, a_2 \in A\}$.

iii. Transitive Relation: A relation R is said to be a transitive relation if $, R = \{(a_1, a_2), (a_2, a_3), (a_1, a_3); \forall\ a_1, a_2, a_3 \in A\}$.

iv. Equivalence Relation: A relation R on a set A, if it is a reflexive, symmetric, and transitive relation, then it is called an equivalence relation.

v. **inverse Relation:** The inverse relation of a relation R is denoted by R^{-1} and is obtained by interchanging the elements of each ordered pair of R.

Definition 3.2 (Approximation Space).

The notion of the approximation space of some set is a fundamental concept in the rough set theory. In [20, 21, 22], Pawlak constructed the approximation space from a relation defined for the elements of the set.

The lower approximation of a set A, that is $\underline{P}(A) = \{x: [x]_p \subseteq A\}$,

The upper approximation of a set A, that is $\overline{P}(A) = \{x: [x]_p \cap A \neq \emptyset\}$.

Definition 3.3 [21].

The boundary region is provided a simple approach to find the boundary of any set by combining the lower and upper approximations. So, the boundary is defined as: $\mathbb{B}(A) = \overline{P}(A) - \underline{P}(A)$.

5. NEIGHBORHOOD MORPHOLOGICAL OPERATORS.

In mathematical morphology a subset of the universe set called the structure element played an important role in defining the morphological operators. In image processing, the structure element is a fixed small subset of the image [6, 7].

Also, we suggest constructing a family of structure elements instead of using a fixed structure element; namely, a family of neighborhood structure elements. The constructed neighborhood structure elements are based on the topological concepts of the neighborhood of each element of the universe set.

Definition 4.1.

The neighborhood structure element (SE_x) is a subset of the neighborhood of the element $x, \forall x \in U$, so that U is the universe set. defined; $SE_x = \{\ SE_x \subseteq U; A \in U, x \subseteq A \subseteq\ SE_x\}$.

Definition 4.2.
For any set U and two subsets $A, SE_x \subseteq U$, we define the two operators, namely, neighborhood-erosion $n\_\varepsilon(A)$ and neighborhood-dilation $n\_\delta(A)$ as follows:

i. $n_{\varepsilon(A)} = \{x: x \in U | SE_x \subseteq A\}$,

ii. $n\_\delta(A) = \{x: x \in U | SE_x \cap A \neq \emptyset\}$.

5. ATTRIBUTES' REDUCTION BASED ON NANO-TOPOLOGY.

In [11] the author used the concept of nano-topology founded by Lellis Thivagar et. al [9] to construct an attributes' reduction method based on the nano-topology deduced from the decision system under study. Also, they are deduced the approximation space for subsets of the decision system by using the indiscernibility equivalence classes which were introduced by Pawlak [22]

Definition 5.1.

Consider some information system(U, X), subsets $A \subseteq U$ and $P \subseteq X$. A nano-topology (denoted by τ_P) with respect to the set of attributes P is a collection of the approximation space of the set A with boundary of A, as well as both the universe set and the empty set, that is $\tau_P = \{U, \emptyset, \underline{P}(A), \overline{P}(A), \mathbb{B}(A)\}$; where

$\underline{P}(A), \overline{P}(A), \mathbb{B}(A)$ are respectively the lower approximation, the upper approximation and the boundary region of the set A; as defined in section 3.

5.1 The nano-topology reduction algorithm [11].

1. Read the decision system $(U, X \cup \{C\})$,
2. Cluster the system according to decision attribute (C),
3. Define the approximation space and the boundary region for each cluster,
4. Construct the nano-topology for each cluster using the set of attributes X,
5. Construct a new set of attributes X' by removing one attribute from X.
6. Read the modified decision system $(U, X' \cup \{C\})$,
7. Repeat steps (2-4), to find a base nano-topology for each cluster using X',
8. Repeat steps (5-7), for the remain attributes of X,
9. Compare the base nano-topology deduced from the original decision system, with the base nano-topologies deduced using the modified decision systems produced through the process to find the core of the decision system,
10. Stop.

Empirical Example 5.1.

In [11], a simple decision system is given in (Table 1) including eight objects $U = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and four attributes X = (Trance. Line a, Trance. Line b, Trance. Line e, Trance. Line f).

Table 1: *decision system*

X \ U	Trance. Line (a)	Trance. Line (b)	Trance. Line (e)	Trance. Line (f)	$(\boldsymbol{C})$
v_1	Medium	Medium	Medium	Medium	Normal
v_2	Medium	Low	Low	Medium	Rest
v_3	Medium	Low	Low	Medium	Normal
v_4	Low	Low	Low	High	Rest
v_5	Low	Medium	Medium	Medium	Rest
v_6	Medium	Medium	Low	High	Normal
v_7	Medium	Medium	Low	Low	Rest
v_8	Medium	Medium	Low	High	Normal

To commence, the author in [11], cluster the system according to the decision column into two clusters C_1 and C_2, where $C_1 = \{\upsilon_1, \upsilon_3, \upsilon_6, \upsilon_8\}$, $C_2 = \{\upsilon_2, \upsilon_4, \upsilon_5, \upsilon_7\}$.

Let $C_1 = \{\upsilon_1, \upsilon_3, \upsilon_6, \upsilon_8\}$,

The family of equivalence classes is: $U/IND(X) = \{\{\upsilon_1\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \{\upsilon_5\}, \{\upsilon_6, \upsilon_8\}, \{\upsilon_7\}\}$.

The approximations space of C_1 with its boundary is,

$$\underline{P}(C_1) = \{\upsilon_1, \upsilon_6, \upsilon_8\}, \overline{P}(C_1) = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_8\}, \text{ and } \mathbb{B}(C_1) = \{\upsilon_2, \upsilon_3\}.$$

Hence, we deduce nano-topology with respect to X is,
$\tau_X = \{U, \emptyset, \{\upsilon_1, \upsilon_6, \upsilon_8\}, \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_8\}, \{\upsilon_2, \upsilon_3\}\}$. Finally, a base of this nano-topology can be constructed by using the following: $\beta_X = \{U, \{\upsilon_1, \upsilon_6, \upsilon_8\}, \{\upsilon_2, \upsilon_3\}\}$.

In a similar way, we can deduce the following after removing the attributes, one at a time.

Table 2: *nano-topology and base of nano-topology for decision system*

equivalence classes	nano-topology	base of nano-topology
$U/IND(X-\{a\}) = \left\{\begin{matrix}\{\upsilon_2, \upsilon_3\}, \{\upsilon_1, \upsilon_5\}, \\ \{\upsilon_4\}, \{\upsilon_6, \upsilon_8\}, \{\upsilon_7\}\end{matrix}\right\}$	$\tau_{X-\{a\}} = \left\{\begin{matrix}U, \emptyset, \{\upsilon_6, \upsilon_8\}, \{\upsilon_1 \upsilon_2, \upsilon_3, \upsilon_5\}, \\ \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_5, \upsilon_6, \upsilon_8\}\end{matrix}\right\}$	$\beta_{X-\{a\}} = \left\{\begin{matrix}U, \{\upsilon_6, \upsilon_8\}, \\ \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_5\}\end{matrix}\right\}$
$U/IND(X-\{b\}) = \left\{\begin{matrix}\{\upsilon_1\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \\ \{\upsilon_5\}, \{\upsilon_6, \upsilon_8\}, \{\upsilon_7\}\end{matrix}\right\}$	$\tau_{X-\{b\}} = \left\{\begin{matrix}U, \emptyset, \{\upsilon_1, \upsilon_6, \upsilon_8\}, \{\upsilon_2, \upsilon_3\}, \\ \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_8\}\end{matrix}\right\}$	$\beta_{X-\{b\}} = \left\{\begin{matrix}U, \{\upsilon_2, \upsilon_3\}, \\ \{\upsilon_1, \upsilon_6, \upsilon_8\}\end{matrix}\right\}$
$U/IND(X-\{e\}) = \left\{\begin{matrix}\{\upsilon_1\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \\ \{\upsilon_5\}, \{\upsilon_6, \upsilon_8\}, \{\upsilon_7\}\end{matrix}\right\}$	$\tau_{X-\{e\}} = \left\{\begin{matrix}U, \emptyset, \{\upsilon_1, \upsilon_6, \upsilon_8\}, \{\upsilon_2, \upsilon_3\}, \\ \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_8\}\end{matrix}\right\}$	$\beta_{X-\{e\}} = \left\{\begin{matrix}U, \{\upsilon_2, \upsilon_3\}, \\ \{\upsilon_1, \upsilon_6, \upsilon_8\}\end{matrix}\right\}$
$U/IND(X-\{f\}) = \left\{\begin{matrix}\{\upsilon_1\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \\ \{\upsilon_5\}, \{\upsilon_6, \upsilon_7, \upsilon_8\}\end{matrix}\right\}$	$\tau_{X-\{f\}} = \left\{\begin{matrix}U, \emptyset, \{\upsilon_2, \upsilon_3, \upsilon_6, \upsilon_7, \upsilon_8\}, \\ \{\upsilon_1\}, \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_7, \upsilon_8\}\end{matrix}\right\}$	$\beta_{X-\{f\}} = \left\{\begin{matrix}U, \{\upsilon_1\}, \\ \{\upsilon_2, \upsilon_3, \upsilon_6, \upsilon_7, \upsilon_8\}\end{matrix}\right\}$

When comparing the base nano-topology deduced from the original decision system, with the base nano-topology deduced using the modified decision systems produced through the process, a CORE for the decision system can be found as: CORE = {Trance. Line *(a)*, Trance. Line (*f*)}.

6. MORPHOLOGICAL ACCURACY REDUCTION.

This section is devoted to proposing a strategy for attributes' reduction based on computing a morphological accuracy. Measure using the mathematical morphological operators. In literature, the reduction techniques are classified into two groups, one is for objects' reduction and other is for attributes' reduction. Attribute reduction can be defined as the process for determining a minimal subset of attributes from an original set of attributes; the reduced subset is called the core.

The view is divided into the following steps. Firstly, we define the classes of neighborhood structure element for each object, Secondly, we cluster the system according to the decision attribute. Thirdly, we apply morphological operators to get morphological accuracy. Finally, order the values of morphological accuracy to define the core of attribute.

Definition 6.1.
For any set $A \subseteq X$, the morphological accuracy of A is defined to be the ratio between the size of the neighborhood-eroded set of A and the size of the neighborhood-dilated set of A namely, morphological accuracy; $\mathcal{A}_{\mathcal{C}}(A) = \frac{|n\_\varepsilon(A)|}{|n\_\delta(A)|}$, where $|n\_\delta(A)| \neq 0$.

6.1. The proposed methodology.

Consider decision system $(U, X \cup \{C\})$, where U is a universe set of objects, X is a non-empty set of attributes and C is a decision attribute; we propose the following procedure to define a set of core attributes for the system under consideration:

1. Read data decision system $(U, X \cup \{C\})$,
2. Determine the membership value for decision system,
3. Construct attribute coding,
4. Cluster U according to decision attribute (C),
5. Construct a new set of attributes X' by removing one attribute from X, for the cluster $U_i \subset U$.
6. Read the modified decision system $(U, X' \cup \{C\})$,
7. Determine the neighborhood structure element for each cluster using X',
8. Apply the morphological operators to evaluate the morphological accuracy of U_i,
9. Repeat the steps (5-8), for all the attributes by removing one attribute at a time.
10. Find the core of the decision system, that is, the minimum value of morphological accuracy,
11. Stop.

Empirical Example 6.1.

To demonstrate, the proposed methodology, we use the system from Example (5.1), to commence, cluster U according to the decision attribute into two clusters U_1 and U_2; that is; $U_1 = \{\upsilon_1, \upsilon_3, \upsilon_6, \upsilon_8\}$, $U_2 = \{\upsilon_2, \upsilon_4, \upsilon_5, \upsilon_7\}$. Hence, we construct the following new sets of attributes, namely, $X_k = X - \{k\}$, $k = \text{Line } a, \text{Line } b, \text{Line } e, \text{Line } f$. Now, for the cluster U_1, we have the following new decision systems, $(U_1, X_k \cup \{C\})$, $k = \text{Line } a, \text{Line } b, \text{Line } e, \text{Line } f$.

The next step is to find the structure element for each deduced decision system $(U_1, X_k \cup \{C\})$, we chosen the equivalence classes, to be the structure elements that is:

$SE_a = U/X_a = \{\{\upsilon_1, \upsilon_5\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \{\upsilon_6, \upsilon_8\}, \{\upsilon_7\}\}$,

$SE_b = U/X_b = \{\{\upsilon_1\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \{\upsilon_5\}, \{\upsilon_6, \upsilon_8\}, \{\upsilon_7\}\}$,

$SE_e = U/X_e = \{\{\upsilon_1\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \{\upsilon_5\}, \{\upsilon_6, \upsilon_8\}, \{\upsilon_7\}\}$,

$SE_f = U/X_f = \{\{\upsilon_1\}, \{\upsilon_2, \upsilon_3\}, \{\upsilon_4\}, \{\upsilon_5\}, \{\upsilon_6, \upsilon_7, \upsilon_8\}\}$.

Then**,** we apply the morphological operators (neighborhood-erosion and neighborhood-dilation) using the previous structure elements. Hence, we compute the morphological accuracy as shown is in the following table:

Table 3: *morphological operators and morphological accuracy for decision system*

equivalence classes	neighborhood-erosion	neighborhood-dilation	Morphological accuracy
U/X_a	$\{\upsilon_6, \upsilon_8\}$	$\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_5, \upsilon_6, \upsilon_8\}$	0.33
U/X_b	$\{\upsilon_1, \upsilon_6, \upsilon_8\}$	$\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_8\}$	0.6
U/X_e	$\{\upsilon_1, \upsilon_6, \upsilon_8\}$	$\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_8\}$	0.6
U/X_f	$\{\upsilon_1\}$	$\{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_6, \upsilon_8\}$	0.2

Finally, we choose the attributes corresponding to the equivalence classes with the smallest accuracy to be the core attributes for the original system. CORE = {Trance. Line-a, Trance. Line-f}.

Empirical Example 6.2.

For more verification for the proposed technique in (Table 4), we give an example for a simple decision system from the work done by the authors in [13].

The given decision system includes six students. $U = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and four subjects $X = (subject\ a, subject\ b, subject\ e, subject\ f)$, accompanied by a decision attribute $C = \{A, B, C\}$. The following the steps of the proposed technique, we get the following results:

Table 4: *decision system*

U \ X	subject (a)	subject (b)	subject (e)	subject (f)	$(\boldsymbol{C})$
v_1	81	77	84	83	B
v_2	100	81	93	85	A
v_3	71	78	89	60	C
v_4	93	82	88	60	B
v_5	97	87	91	85	A
v_6	93	68	87	90	C

we construct morphological accuracy reduction as following:

Step 1: Determine the membership values for the decision system denoted by, $\mu_{v_{p_k}(x)}$; where,

$$\mu_{v_{p_k}(x)} = \frac{v_{p_k}(x)}{\max_{k\in 1:|P|} v_{p_k}}.$$

Table 5: membership value for decision system

U \ X	subject (a)	subject (b)	subject (e)	subject (f)	$(\boldsymbol{C})$
v_1	0.81	0.88	0.90	0.92	B
v_2	0.100	0.93	0.10	0.94	A
v_3	0.71	0.89	0.95	0.66	C
v_4	0.93	0.94	0.94	0.66	B
v_5	0.97	0.10	0.97	0.94	A
v_6	0.93	0.78	0.93	0.100	B

Step 2: Construct attributes' coding:

$$\varsigma = \begin{cases} 1 & \propto < \mu_{ij} \leq 2\propto \\ 2 & 2\propto < \mu_{ij} \leq 3\propto \\ 3 & 3\propto < \mu_{ij} \leq 4\propto \\ 4 & 4\propto < \mu_{ij} \leq 5\propto \end{cases}, \text{ where } \propto = \min_k \left(min\ \mu_{v_{p_k}} \right) = 0.2098.$$

Table 6: *attributes coding for decision system*

U \ X	subject (a)	subject (b)	subject (e)	subject (f)	$(\boldsymbol{C})$
v_1	4	5	5	5	B
v_2	5	5	5	5	A
v_3	4	5	5	4	C
v_4	5	5	5	4	B
v_5	5	5	5	5	A
v_6	5	4	5	5	C

Step 3: To commence, cluster the system according to the decision column into three clusters U_1, U_2 and U_3, where $U_1 = \{v_1, v_4\}, U_2 = \{v_2, v_5\}$, $U_3 = \{v_3, v_6\}$.

Step 4: For the cluster $U1$, construct a new decision system excluding one attribute at a time $(U_1, X_k \cup \{C\})$, $k = subject\ a, subject\ b, subject\ e, subject\ f$.

Step 5: Find the structure element for each deduced decision system:

$$SE_a = U/X_a = \{\{v_1, v2, v5, v6\}, \{v3, v4\}\},$$
$$SE_b = U/X_b = \{\{v1\}, \{v2, v5, v6\}, \{v3\}, \{v4\}\},$$

$$SE_e = U/X_e = \{\{v1\},\{v2,v5\},\{v3\},\{v4\},\{v5\},\{v6\}\},$$
$$SE_f = U/X_f = \{\{v1,v3\},\{v2,v4,v5\},\{v6\}\}.$$

Step 6: Apply the morphological operators to evaluate the morphological accuracy for each decision system. The results are obtained in (Table 7).

Table 7: morphological operators and morphological accuracy for decision system

equivalence classes	neighborhood-erosion	neighborhood-dilation	Morphological accuracy
U/X_a	0	$\{v1,v4\}$	0
U/X_b	$\{v1,v4\}$	A	$1/6$
U/X_e	$\{v1,v4\}$	$\{v1,v4\}$	1
U/X_f	0	$\{v1,v3,v2,v4,v5\}$	0

The final step: To choose the core attributes corresponding to the equivalent classes with the smallest accuracy. CORE = {*subject* a, *subject* f}.

Remark 6.1:
Comparing the results deduced using the proposed technique to the work done in [17] and [3], we obtain the same core attributes, each attribute is accompanied with an accuracy value, which, enables us to rearrange the core attributes according to its priority.

7. CONCLUSION.

In this paper, a new technique to serve the purpose of attributes' reduction, was presented the proposed technique, which we call "the morphological accuracy reductio", is based on using an accuracy measure which was introduced in section [6], this accuracy measure uses the basic morphological operators to compute the ratio between the neighborhood-erosion and neighborhood-dilation of the set-in order to gauge its stability. The obtained morphological accuracy provided the ability to exclude the objects with the lowest morphological accuracy, hence, a core set of attributes is deduced. Furthermore, the proposed technique is tested and compared with existing attributes' reduction methods; The obtained results showed that the proposed Morphological Accuracy Reduction technique (MAR) is effective in reducing a core attribute, as well as, saving time and effort.

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