



Cnoidal and solitary dust ion-acoustic waves in Jupiter ionosphere

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ABSTRACT

The nonlinear wave structure of small-amplitude dust ion-acoustic cnoidal and solitary waves are investigated in a five-component plasma consisting of positive proton beam from solar wind, positive ion fluid, two electrons population, one of them from the solar wind and stationary positive dust grains. The physical parameters in the system such as, the temperature ratio of proton beam and positive ion, magnetic field and streaming velocity of the beam plays the dominant role in the profile of the amplitude and the width of the wave. Using the reductive perturbation theory, the basic set of equations is reduced to a Zakharov–Kuznetsov equation. The latter has been solved using the Painlevé analysis to obtain a set of analytical solutions, which reflects the possibility of the propagation of various nonlinear structures. The solutions of Zakharov–Kuznetsov equation present a positive potential, which corresponds to a compressive wave profile. The findings of this investigation are used to interpret the electrostatic cnoidal and solitary waves that may be observed in the Jupiter ionosphere. dust-ion-acoustic waves; Jupiter ionosphere; cnoidal and solitary waves

Key Words:

dust-ion-acoustic waves; Jupiter ionosphere; cnoidal and solitary waves.

1. INTRODUCTION

Understanding the nonlinear phenomena in the planet Jupiter magnetosphere is important to the entire field of space physics. Such nonlinearity depends on the properties of all the physical plasma parameters as well as on the nature of the planetary obstacle. Studies the plasma phenomena in the Jupiter ionosphere using fluid equations and magnetohydrodynamic equations were performed by many authors. Exploring a

plasma containing dust grains is of important to understand different astrophysical, space phenomena, and industrial physical applications [1, 2]. The dust grains could modify the plasma modes. For example, the ion-acoustic waves can be modified to dust-ion-acoustic waves (DIAWs). Shukla and Silin [3] expected the existence of DIAWs. Later, Barkan et al. [4] noticed the DIAWs in laboratory. Barkan et al. [5] detected that the negatively charged dust grains can enhance the phase velocity of the propagating waves. Also, it reduces the Landau damping. Indeed, the study of DIAWs are limited by complexity of the problem. The plasma cannot be described in this complex form [6]. Therefore, different studies were limited to a simple plasma cases. Neglecting some effects could modify the wave propagation properties as shown for example in Ref. [7, 8, 9, 10, 11]. Jaiswa et al. [12] studied the effect of magnetic field on the phase transition in a dusty plasma. At strong magnetic field, there is a variation in the angular velocity of the moving particles. The presence of charging process can give rise to new anomalous dissipation. It makes possible existence of a kind of shocks. In the absence of dissipation, then the balance becomes between dispersion and nonlinearity, which creates cnoidal and/or solitary waves [13, 14, 15, 16, 17]. These studies considered unmagnetized dusty plasma. Thus, our aim in this work is to explore the characteristic profile of the DIAWs in a magnetized dusty plasma. The skeleton of the paper is as follows: in Section 2, the basic set of fluid equations describing the system is presented. Then the Zakharov–Kuznetsov equation is deduced. Section 3 has the mathematical solution and discussion. Section 4 contains conclusions of the paper.

2. Plasma model

The proposed dusty plasma is considered as collisionless and magnetized, where the dust grains are stationary positive charge with fluid ions, isothermal distribution background electrons as well as solar wind protons and electrons. The external magnetic field is $B = B_0 \hat{z}$. The dynamics of the DIAWs are given by the following equations [11, 13]. The proton beam equations:

$$\frac{\partial n_p}{\partial t} + \nabla(n_p u_p) = 0, \quad (1)$$

$$m_p n_p \left(\frac{\partial u_p}{\partial t} + u_p \nabla u_p \right) - n_p \left(-\nabla \varphi + \frac{1}{c_0} u_p \times B \right) + \nabla P_p = 0. \quad (2)$$

The basic fluids equations for the positive ion are

$$\frac{\partial n_h}{\partial t} + \nabla(n_h u_h) = 0, \quad (3)$$

$$m_h n_h \left(\frac{\partial u_h}{\partial t} + u_h \nabla u_h \right) - n_h \left(-\nabla \varphi + \frac{1}{c_0} u_h \times B \right) + \nabla P_h = 0. \quad (4)$$

The electrons have Maxwellian distributions

$$n_e = n_{e0} \exp\left(\frac{e\varphi}{K_B T_e}\right), \quad (5)$$

$$n_{es} = n_{es0} \exp\left(\frac{e\varphi}{K_B T_{es}}\right), \quad (6)$$

The system of Eqs. (1)–(6) is closed by the Poisson equation

$$\nabla^2 \varphi = 4\pi e(n_e + n_{es} - n_h - n_p - Z_{d0} n_{d0}). \quad (7)$$

Here, n_p is the solar wind positive proton beam number density, n_h is the positive ion number density, n_e is electron number density, n_{es} is the solar wind electron number density, φ is the electrostatic potential, K_B is the Boltzmann constant, $P_{p,h}$ is the thermal pressure where $P_{p,h} = \frac{K_B T_{p,h} n_{p,h}^\gamma}{(n_{p0,h0})^{\gamma-1}}$, $T_{p,h}$ are the temperature of the two positive ions and γ is the ratio of specific heat at constant pressure to constant volume where $\gamma = 5/3$, T_e is the temperature of the electrons, the Debye length $\lambda_{De} = \left(\frac{K_B T_e}{4\pi e^2 n_p}\right)^{1/2}$, the inverse of the ion plasma frequency $\omega_{pp}^{-1} = \left(\frac{m_p}{4\pi e^2 n_p}\right)^{1/2}$, ω_{co} is the cyclotron frequency for positive ion, ω_{ch} is the cyclotron frequency for positive ion, where c_0 is the speed of light at vacuum, $\mu_h = m_p/m_h$, $C_{so} = \left(\frac{K_B T_e}{m_p}\right)^{1/2}$ is the ion-acoustic speed.

Equations (1)–(7) can be rewritten in the following form

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p u_{px}) + \frac{\partial}{\partial y}(n_p u_{py}) + \frac{\partial}{\partial z}(n_p u_{pz}) = 0, \quad (8)$$

$$\frac{\partial u_{px}}{\partial t} + u_{px} \frac{\partial u_{px}}{\partial x} + u_{py} \frac{\partial u_{px}}{\partial y} + u_{pz} \frac{\partial u_{px}}{\partial z} + \frac{\partial \varphi}{\partial x} - w_{sp} u_{py} + \frac{5}{3} \sigma_p n_p^{-1/3} \frac{\partial n_p}{\partial x} = 0, \quad (9)$$

$$\frac{\partial u_{py}}{\partial t} + u_{px} \frac{\partial u_{py}}{\partial x} + u_{py} \frac{\partial u_{py}}{\partial y} + u_{pz} \frac{\partial u_{py}}{\partial z} + \frac{\partial \varphi}{\partial y} + w_{sp} u_{px} + \frac{5}{3} \sigma_p n_p^{-1/3} \frac{\partial n_p}{\partial y} = 0, \quad (10)$$

$$\frac{\partial u_{pz}}{\partial t} + u_{px} \frac{\partial u_{pz}}{\partial x} + u_{py} \frac{\partial u_{pz}}{\partial y} + u_{pz} \frac{\partial u_{pz}}{\partial z} + \frac{\partial \varphi}{\partial z} + \frac{5}{3} \sigma_p n_p^{-1/3} \frac{\partial n_p}{\partial z} = 0, \quad (11)$$

$$\frac{\partial n_h}{\partial t} + \frac{\partial}{\partial x}(n_h u_{hx}) + \frac{\partial}{\partial y}(n_h u_{hy}) + \frac{\partial}{\partial z}(n_h u_{hz}) = 0, \quad (12)$$

$$\frac{\partial u_{hx}}{\partial t} + u_{hx} \frac{\partial u_{hx}}{\partial x} + u_{hy} \frac{\partial u_{hx}}{\partial y} + u_{hz} \frac{\partial u_{hx}}{\partial z} + \mu_h \frac{\partial \varphi}{\partial x} - w_{sh} u_{hy} + \frac{5}{3} \mu_h \sigma_h n_h^{-1/3} \frac{\partial n_h}{\partial x} = 0, \quad (13)$$

$$\frac{\partial u_{hy}}{\partial t} + u_{hx} \frac{\partial u_{hy}}{\partial x} + u_{hy} \frac{\partial u_{hy}}{\partial y} + u_{hz} \frac{\partial u_{hy}}{\partial z} + \mu_h \frac{\partial \varphi}{\partial y} + w_{sh} u_{hx} + \frac{5}{3} \mu_h \sigma_h n_h^{-1/3} \frac{\partial n_h}{\partial y} = 0, \quad (14)$$

$$\frac{\partial u_{hz}}{\partial t} + u_{hx} \frac{\partial u_{hz}}{\partial x} + u_{hy} \frac{\partial u_{hz}}{\partial y} + u_{hz} \frac{\partial u_{hz}}{\partial z} + \mu_h \frac{\partial \varphi}{\partial z} + \frac{5}{3} \mu_h \sigma_h n_h^{-1/3} \frac{\partial n_h}{\partial z} = 0, \quad (15)$$

$$n_e = \exp(\varphi), \quad (16)$$

$$n_{es} = \exp(\sigma_{es} \varphi), \quad (17)$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \delta_e n_e - \delta_{es} n_{es} + n_p + \delta_h n_h + \delta_d = 0, \tag{18}$$

where $w_{sp} = \omega_{co}/\omega_{p0}$, $w_{sh} = \omega_{ch}/\omega_{p0}$, $\sigma_p = T_p/T_e$, $\sigma_h = T_h/T_e$, $\delta_e = n_{e0}/n_{p0}$, $\delta_{es} = n_{es0}/n_{p0}$, $\delta_d = n_{d0}Z_{d0}/n_{p0}Z_d$, and $\delta_h = n_{h0}/n_{p0}$.

To study the DIAWs with two positive ion fluids in Jupiter ionosphere with electrons have Maxwellian distributions, the reductive perturbation method is used [14]. Thus, the following space–time variables are introduced

$$\zeta = \varepsilon^{1/2}(z - Vt), \eta = \varepsilon^{1/2}y, \xi = \varepsilon^{1/2}z \quad \text{and} \quad \tau = \varepsilon^{3/2}t, \tag{19}$$

where ε is small parameter less than unity and V the phase speed to be determined later. The physical quantities appearing in Eqs. (8)–(18)

$$\begin{bmatrix} n_p \\ u_{pz} \\ n_h \\ u_{hz} \\ n_e \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 \\ u_{p0} \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_{p1} \\ u_{pz1} \\ n_{h1} \\ u_{hz1} \\ n_{e1} \\ \varphi_1 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_{p2} \\ u_{pz2} \\ n_{h2} \\ u_{hz2} \\ n_{e2} \\ \varphi_2 \end{bmatrix} + \varepsilon^3 \begin{bmatrix} n_{p3} \\ u_{pz3} \\ n_{h3} \\ u_{hz3} \\ n_{e3} \\ \varphi_3 \end{bmatrix} + \dots, \tag{20}$$

and for u_{px} , u_{py} , u_{hx} and u_{hy}

$$\begin{pmatrix} u_{px} \\ u_{py} \\ u_{hx} \\ u_{hy} \end{pmatrix} = \varepsilon^{3/2} \begin{pmatrix} u_{px1} \\ u_{py1} \\ u_{hx1} \\ u_{hy1} \end{pmatrix} + \varepsilon^2 \begin{pmatrix} u_{px2} \\ u_{py2} \\ u_{hx2} \\ u_{hy2} \end{pmatrix} + \varepsilon^{5/2} \begin{pmatrix} u_{px3} \\ u_{py3} \\ u_{hx3} \\ u_{hy3} \end{pmatrix} + \dots \tag{21}$$

The charge-neutrality condition is maintained through the relation

$$1 + \delta_d + \delta_h - \delta_e - \delta_{es} = 0. \tag{22}$$

Using Eqs. (20) and (21) into Eqs. (8)–(18) and collecting the lowest-order in ε , we get

$$n_{p1} = \frac{3}{3(u_{p0}-V)^2-5\sigma_p} \varphi_1, u_{pz1} = \frac{3(V-u_{p0})}{3(u_{p0}-V)^2-5\sigma_p} \varphi_1, \tag{23}$$

$$n_{h1} = \frac{3\mu_h}{3V^2-5\mu_h\sigma_h} \varphi_1, u_{zh1} = \frac{3V\mu_h}{3V^2-5\mu_h\sigma_h} \varphi_1, \tag{24}$$

$$n_e = \varphi_1, n_{es} = \sigma_{es}\varphi_1, \tag{25}$$

$$u_{px1} = \frac{-3(u_{p0}-V)^2}{3w_{so}(u_{p0}-V)^2-5w_{so}\sigma_p} \frac{\partial \varphi_1}{\partial \eta}, u_{py1} = \frac{3(u_{p0}-V)^2}{3w_{sp}(u_{p0}-V)^2-5w_{sp}\sigma_p} \frac{\partial \varphi_1}{\partial \xi}, \tag{26}$$

$$u_{hx1} = \frac{-3V^2\mu_h}{3w_{sh}V^2-5w_{sh}\mu_h\sigma_h} \frac{\partial \varphi_1}{\partial \eta}, u_{hy1} = \frac{3V^2\mu_h}{3w_{sh}V^2-5w_{sh}\mu_h\sigma_h} \frac{\partial \varphi_1}{\partial \xi}, \tag{27}$$

and the Poisson equation gives the linear dispersion relation

$$\frac{3\mu_h\delta_h}{3V^2-5\mu_h\sigma_h} + \frac{3}{3(u_{p0}-V)^2-5\sigma_p} - \delta_e - \delta_{es}\sigma_{es} = 0. \tag{28}$$

The next-order of the perturbation gives the ZK equation

$$\frac{\partial\varphi_1}{\partial\tau} + A_1\varphi_1\frac{\partial\varphi_1}{\partial\zeta} + B\frac{\partial^3\varphi_1}{\partial\zeta^3} + A_2\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\varphi_1}{\partial\xi^2} + \frac{\partial^2\varphi_1}{\partial\eta^2}\right) = 0, \tag{29}$$

where

$$A_1 = B\left(\frac{81(u_{p0}-V)^2-15\sigma_p}{(3(u_{p0}-V)^2-5\sigma_p)^3} + \frac{3\delta_h\mu_h^2(27V^2-5\mu_h\sigma_h)}{(3V^2-5\mu_h\sigma_h)^3} - \delta_e - \delta_{es}\sigma_{es}^2\right), \tag{30}$$

$$B = \left(\frac{18(V-u_{p0})}{(3(V-u_{p0})^2-5\sigma_p)^2} + \frac{18\delta_h\mu_hV}{(3V^2-5\mu_h\sigma_h)^2}\right)^{-1}, \tag{31}$$

$$A_2 = \frac{9(V-u_{p0})^4}{w_{s0}^2(3(u_{p0}-V)^2-5\sigma_p)^2} + \frac{9\delta_h\mu_hV^4}{(3w_{sh}V^2-5w_{sh}\mu_h\sigma_h)^2}. \tag{32}$$

Indeed, ZK equation is not stable against bending instability, see details in Ref. [18]

3. Mathematical solution and discussion

To get the possible analytic solutions of Eq. (29), we will use Painlevé method [19]. To use this method Eq. (29) have to be integrable equation, which is the case at hand. It is implication of this method is to introduce many different solutions that may appear in nonlinear systems like plasma physics. According to this method, we assume the following expansion

$$\varphi_1 = \sum_{i=0}^{\alpha} f_i \Phi^{i-\alpha}. \tag{33}$$

The last expansion could be resected for α is positive integer and Φ is a singular manifold. Using Eq. (33) into Eq. (29), then balance the dispersion and nonlinear terms, we obtain $\alpha = 2$ in a generalized Laurent series as [20, 21, 22]

$$\varphi_1 = \frac{1}{\Phi^2}(f_0 + \Phi f_1 + \Phi^2 f_2), \tag{34}$$

where Φ , f_0 , f_1 , and f_2 are analytic functions of ζ , ξ , η , and τ . Substituting Eq. (34) into Eq. (29) and equating the coefficients of Φ^{-5} ; Φ^{-4} ; and Φ^{-3} to zero, we obtain

$$f_0 = \frac{-12(BL_1^2+C_1(L_2^2+L_3^2))}{A}(\Phi_Y)^2, \tag{35}$$

$$f_1 = \frac{-12(BL_1^2+C_1(L_2^2+L_3^2))}{A}\Phi_{Y,Y}, \tag{36}$$

$$f_2 = \frac{U}{AL_1} + \frac{3(BL_1^2+C_1(L_2^2+L_3^2))\Phi_{Y,Y}}{AL_1(\Phi_Y)^2} - \frac{4(BL_1^2+C_1(L_2^2+L_3^2))\Phi_{Y,Y,Y}}{A(\Phi_Y)^2}. \tag{37}$$

where $Y = L_1\zeta + L_2\xi + L_3\eta - U\tau$, here $L_1 = (1 - L_2^2 - L_3^2)^{\frac{1}{2}}$, L_1 , L_2 , and L_3 are the direction cosine. Φ_Y , $\Phi_{Y,Y}$ and $\Phi_{Y,Y,Y}$ are the first, second, and third derivatives of Φ .

Inserting Eqs. (35)–(37) in Eq. (34), we obtain the analytical solution of the ZK equation (29). In order to find the solitary/cnoidal wave solutions, we assume that ω has the form [22]

$$\Phi = \frac{2}{\tanh(\omega)-1}, \tag{38}$$

where

$$\omega = \frac{Y}{w_1} + c_1 \operatorname{sn}\left[F\left(\frac{Y}{w_2}, m\right), m\right]. \tag{39}$$

Here, F represents an incomplete elliptic integral of the first kind with moduli m , c_1 , w_1 , and w_2 are arbitrary constants.

Equation (39) can be used to obtain a Zakharov–Kuznetsov equation solution as

$$\begin{aligned} \varphi_1 = & \frac{-12(BL_1^2 + C_1(L_2^2 + L_3^2))(\Phi_Y)^2}{A} + \frac{-12(BL_1^2 + C_1(L_2^2 + L_3^2))\Phi_{Y,Y}}{A} + \\ & \frac{U}{AL_1} + \frac{3(BL_1^2 + C_1(L_2^2 + L_3^2))\Phi_{Y,Y}}{AL_1(\Phi_Y)^2} - \frac{4(BL_1^2 + C_1(L_2^2 + L_3^2))\Phi_{Y,Y,Y}}{A(\Phi_Y)^2}. \end{aligned} \tag{40}$$

We can divide the discussion into two parts. The first part when the arbitrary constant is greater than zero, this gives the cnoidal wave solution. We remark that, when the temperature ratio of proton beam increases the amplitude of the cnoidal wave decreases as shown as figure 1a. The same behavior is happen for the density ratio of positive ion, the amplitude of the cnoidal wave decreases with δ_n so we did not include this figure here. We also find that when the temperature of the positive ions decreases, this leads to a decrease in the amplitude of the cnoidal wave, as shown in Figure 1b. In figure 1c, we find the effect of the magnetic field. When the magnetic field ratio decreases, this leads to a decrease in the amplitude of the cnoidal wave. Finally, we find that when the ratio of the initial velocity of the positive proton beam increases, this leads to a decrease in the amplitude of the cnoidal wave, as shown in the figure 1d.

The second part when the arbitrary constant is equal to zero, this gives the solitary wave solution. When the temperature of the positive proton beam increases, this leads to a decrease in the nonlinearity of the system, which in turn leads to a decrease in the amplitude of the solitary wave, as shown in Figure 2a. Also, when the number density ratio of the positive ion decreases, this leads to an increase in the amplitude of the solitary wave, which is similar to Figure 1a. We find in figure 2b that when the temperature of the positive ions decreases, this leads to a reduction in the nonlinearity of the system, which in turn leads to a decrease in the solitary wave amplitude. Figure 2c depicts the effect of the magnetic field on the pulse profile, which is clear that the amplitude of the solitary wave is slightly increases with the magnetic field. However, the width has not significant change. In Figure 2d, the effect of the initial velocity is depicted. The amplitude and width of the solitary wave are decreases with the increase of streaming speed.

4. Summary

A study of nonlinear DIAWs in a magnetized plasma is considered. The electrons are described by the Maxwellian distributions, while the proton beam and positive ions are described by the hydrodynamic fluid equations. The development of the plasma system is described by Zakharov–Kuznetsov equation. The ZK equation was solved using Painlevé analysis. Two kind of solutions were obtained, that permit

the existence of cnoidal and solitary waves. The present model is employed to recognize a possible nonlinear wave at Jupiter ionosphere.

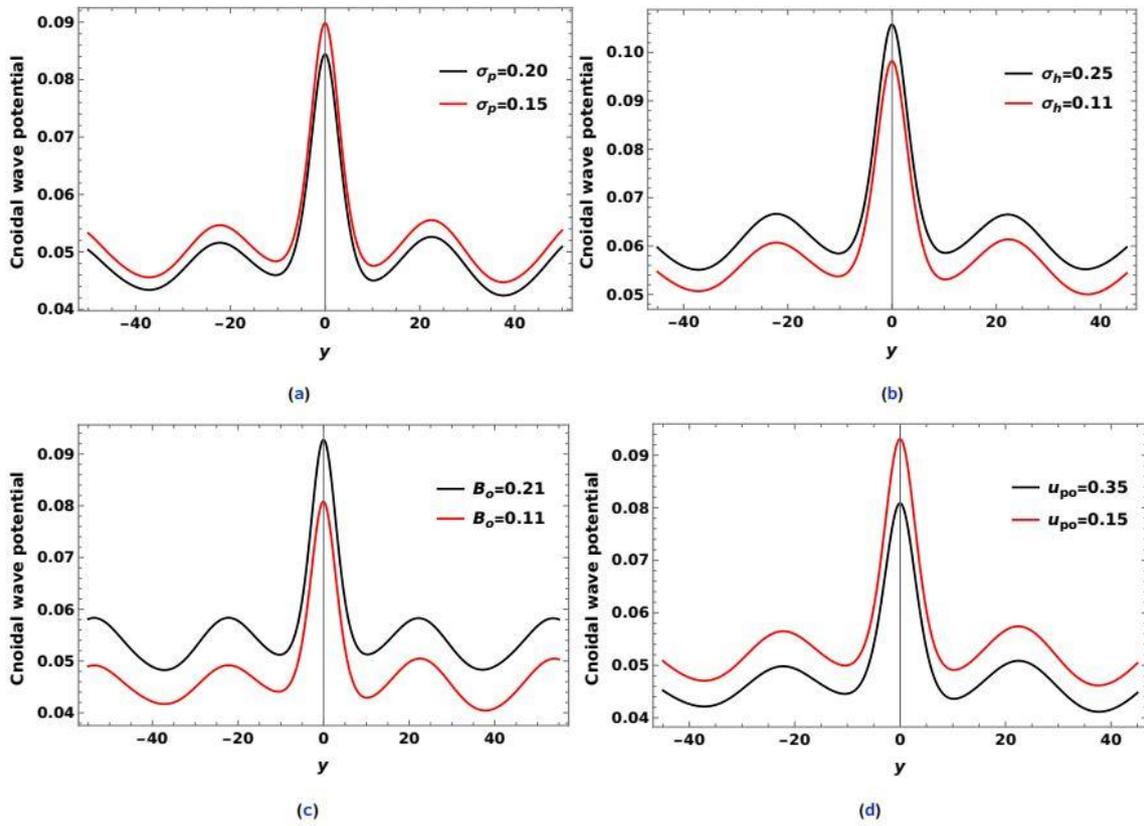


Figure 1: (Color online) The normalized cnoidal wave potential for different values of σ_p , σ_h , B_0 , and u_{po} where the plasma parameters are $\delta_h = 0.45$, $\delta_d = 0.14$, $\delta_e = 0.14$, $T_e = 10$ eV, $\sigma_{es} = 1.5$, $\sigma_e = 2$.

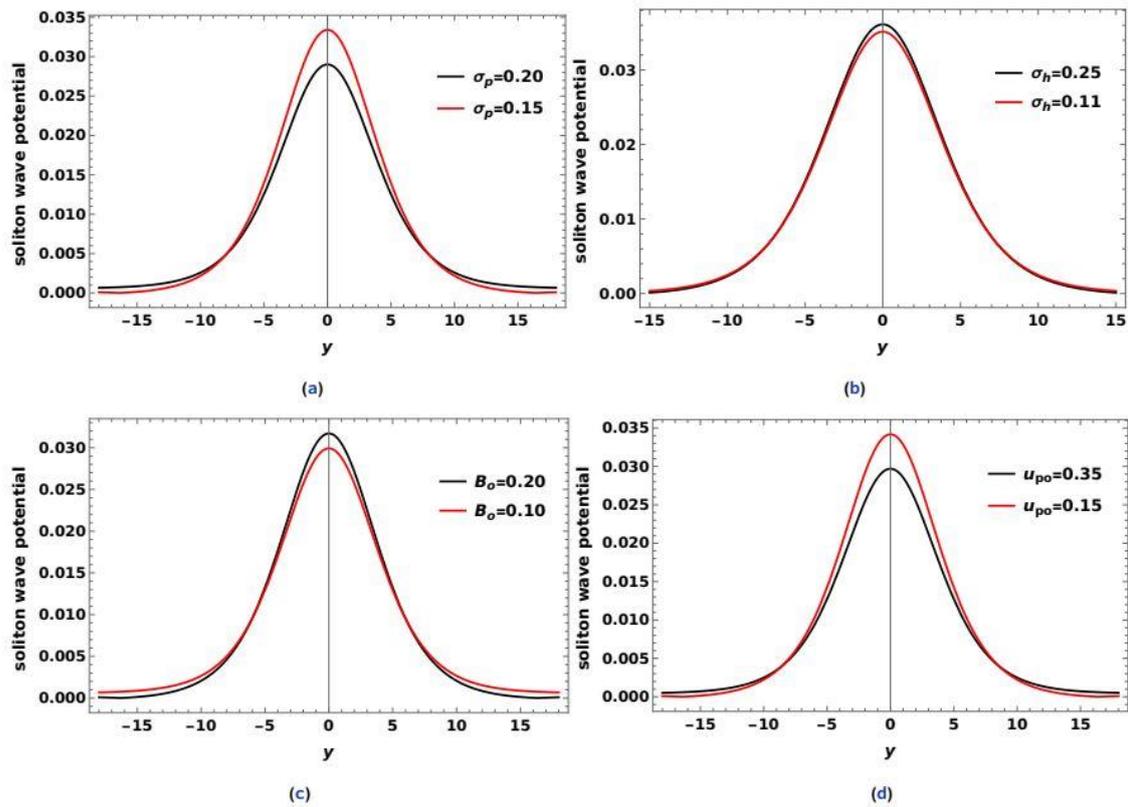


Figure 2: (Color online) The normalized solitary wave potential for different values of σ_p , σ_h , B_0 , and u_{p0} where the plasma parameters are $\delta_h = 0.5$, $\delta_d = 0.14$, $\delta_e = 0.24$, $T_e = 10$ eV, $\sigma_{es} = 1.5$, $\sigma_e = 2$.

5. Reference

- [1] P. K. Shukla and A. A. Mamun, "Introduction to Dusty Plasma Physics," 2002 (IOP Publishing, UK).
- [2] A. Bouchule, "Dusty Plasmas," Journal of Plasma Physics, vol. 64, no. 02, p.201-203, August 2000, doi:10.1017/S002237780023852X.
- [3] P.K. Shukla, and V.P. Silin, "Dust Ion-Acoustic Waves," Physica Scripta, vol. 45, no.5, p.508, 1992, doi: 10.1088/0031-8949/45/5/015.
- [4] A. Barkan, R. L. Merlino, and N. D'Angelo, "Laboratory observation of the dust-acoustic wave mode," Physics of Plasmas, vol. 2, no.10, p.3563, May 1995, doi: 10.1063/1.871121.
- [5] A. Barkan, N.D'Angelo, R.L.Merlino, "Experiments on ion-acoustic waves in dusty plasmas," Planetary and Space Science, vol.44, no.3, P.239-242, 1996, doi:10.1016/0032-0633(95)00109-3.
- [6] S. S. Duha, B. Shikha, and A.A. Mamun, "Nonlinear dust-ion-acoustic waves in a multi-ion plasma with trapped electrons," Pramana- journal of physics, vol.77, no.2, p.357-368, 2011, doi:10.1007/s12043-011-0102-7.
- [7] W. F. El-Taibany, S. K. EL-Labany, A. S. El-Helbawy, and A. Atteya, "Dust-acoustic solitary and periodic waves in magnetized self-gravito-electrostatic opposite polarity dusty plasmas," The European Physical Journal Plus, vol. 137, no.2, p.261, 2022, doi:10.1140/epjp/s13360-022-02461-9.

- [8] R. E. Tolba, W. M. Moslem, A. A. Elsadany, N. A. El-Bedwehy, S. K. El-Labany, "Development of cnoidal waves in positively charged dusty plasmas," *IEEE Transactions on Plasma Science*, vol.45, no.9, p.2552-2560, 2017, doi: 10.1109/TPS.2017.2733085.
- [9] L. B. De Toni and R. Gaelzer, "Effects of dust particles charged by inelastic collisions and by photoionization on Alfvén waves in a stellar wind," *Monthly Notices of the Royal Astronomical Society*, vol. 508, no.1, p.340–351, 2021, doi:10.1093/mnras/stab2603.
- [10] A. Atteya, S. Sultana, R. Schlickeiser, "Dust-ion-acoustic solitary waves in magnetized plasmas with positive and negative ions: The role of electrons superthermality," *Chinese Journal of Physics*, vol. 56, no. 5, p.1931-1939, 2018, doi:10.1016/j.cjph.2018.09.002.
- [11] R. E. Tolba, M. E. Yahia, and W. M. Moslem, "Nonlinear dynamics in the Jupiter magnetosphere: implications of dust-acoustic cnoidal mode," *Physica Scripta*, vol. 96, no. 12, 125637, p. 7, 2021, doi:10.1088/1402-4896/ac42ee.
- [12] S. Jaiswal, T. Hall, S. Leblanc, R. Mukherjee, and E. Thomas, "Effect of magnetic field on the phase transition in a dusty plasma," *Physics of Plasmas*, vol. 24, no.11, p. 113704, 2017, doi:10.1063/1.5003972.
- [13] A. H. Al-Yousef, B. M. Alotaibi, R. E. Tolba, W. M. Moslem, "Arbitrary amplitude dust-acoustic waves in Jupiter atmosphere," *Results in Physics*, vol. 21, p. 103792, 2021, doi:10.1016/j.rinp.2020.103792.
- [14] R. E. Tolba, "Propagation of dust-acoustic nonlinear waves in a superthermal collisional magnetized dusty plasma," *The European Physical Journal Plus*, vol.136, no.1, p.1-15, 2021, doi:10.1140/epjp/s13360-020-01028-w.
- [15] H. Saleem, W. M. Moslem, and P. K. Shukla, "Solar wind interactions with the dusty magnetosphere of Jupiter produce shocks and solitons associated with nonlinear drift waves," *Journal of Geophysical Research: Space Physics*, vol. 117, A08220, 2012, doi:10.1029/2011JA017306.
- [16] F. M. Vogt, S. Gyalay, A. E. Kronberg, J. E. Bunce, S. W. Kurth, B. Zieger, C. Tao, "Solar wind interaction with Jupiter's magnetosphere: A statistical study of Galileo in situ data and modeled upstream solar wind conditions," *Journal of Geophysical Research: Space Physics*, vol.124, no.12, p.10170-10199, 2019, doi:10.1029/2019JA026950.
- [17] M. E. Yahia, S. K. El-Labany, R. Sabry, W. M. Moslem, and E. A. Elghmaz, "Head-On Collision of Electron-Acoustic Solitons in a Magnetized Plasma," *IEEE Transactions on Plasma Science*, vol. 47, no.1, p. 762-769, 2019, doi:10.1109/TPS.2018.2874170.
- [18] W. M. Moslem, S. Ali, P. K. Shukla, X. Y. Tang, and G. Rowlands, "Solitary, explosive, and periodic solutions of the quantum Zakharov-Kuznetsov equation and its transverse instability," *Physics of Plasmas*, vol.14, p. 082308, 2007, doi:10.1063/1.2757612.
- [19] R. E. Tolba, N. A. El-Bedwehy, W. M. Moslem, S. K. El-Labany, and M. E. Yahia, "Nonlinear structures: Cnoidal, soliton, and periodical waves in quantum semiconductor plasma," *Physics of Plasmas*, vol.23, p. 012111, 2016, doi:10.1063/1.4940346.
- [20] S. Mahmood, H. Ur-Rehman, "Electrostatic solitons in unmagnetized hot electron-positron-ion plasmas," *Physics Letters A*, vol.373, no.26, p. 2255-2259, 2009, doi:10.1016/j.physleta.2009.04.050.
- [21] S. Mahmood, F. Haas, "Ion-acoustic cnoidal waves in a quantum plasma," *Physics of Plasmas*, vol.21, no.10, p.102308, 2014, doi:10.1063/1.4899041.
- [22] M. S. Afify, W. M. Moslem, R. E. Tolba, and M. A. Hassouba, "Generation of soliton, cnoidal, and periodic waves during pumping GaAs by an electron beam," *Chaos, Solitons and Fractals*, vol. 124, p.18–25, 2019, doi:10.1016/j.chaos.2019.04.032.