

**Military Technical College
Kobry El-Kobbah,
Cairo, Egypt**



**9th International Conference
on Civil and Architecture
Engineering
ICCAE-9-2012**

Passive earth pressure against retaining wall using log-spiral arc

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Abstract

Passive earth pressure against retaining wall depends on a number of factors such as, soil friction angle ϕ , soil wall friction angle δ , backfill angle (ground surface inclination behind wall β , inclination of wall face on horizontal α , and surface of rupture. Several theory have been developed to over come this problem, i. e., determine the coefficient of passive earth pressure using the plane surface of rupture. One of the important parameter which effect on the coefficient of the passive earth pressure surface of rupture. In the present paper , formulation is proposed for calculating coefficient of passive earth pressure on a rigid retaining wall undergoing horizontal translation base on surface of rupture consisting log-spiral and linear segments assisted by computer program (MATLAB program used). The present study is compared with coulomb's results. The comparison between present study and coulomb's values shows the present study predict values of earth pressure much less than those of coulomb's values specially if $\delta \geq 0.3 \phi$. Those results agree with anthers researches

In order facilitate calculation coefficient of passive earth pressure, using the proposed equations, a modified coefficient of passive earth pressure is provided. It is function of ($\phi, \delta, \beta, \alpha$).

Key words Passive earth pressure, retaining wall, surface of rupture, log- spiral

Introduction

Retaining structures are vital geotechnical structures; because the topography of the earth surface is a combination of plain, sloppy and undulating terrain. The retaining wall has traditionally been applied to free-standing walls whose resists thrust of the bank of earth as well as providing soil stability of a change of ground elevation. The design philosophy of the wall deals with the magnitude and distribution of the lateral pressure between a soil mass and wall.

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Estimation of passive earth pressure acting on the rigid retaining wall is very important in the design of many geotechnical engineering structures; particularly retaining wall. Passive earth pressure calculations in geotechnical analysis are usually performed with the aid of Rankine[24] or Coulomb[4] theories of earth pressure based on uniform soil properties. These traditional earth pressure theories are derived from equations of equilibrium along on an assumed planar failure surface passing through the soil mass. Both assume that the distribution of the passive earth pressure exerted against the wall is triangular. However, the distribution of the earth pressure on the face of rough wall depends on the wall movement (rotation about top, rotation about bottom and horizontal translation) and is nonlinear. This is different from the assumption made by both Rankine[24] and Coulomb[4].

The Coulomb theory is more versatile in accommodating complex configurations of backfills and loading conditions as well as frictional effects between walls and backfills. However, both theoretical and experimental studies have shown that the Coulomb assumption of plane surface sliding is not perfectly valid when the wall is rough, especially in the passive case when interface friction is more than 1/3 of internal soil friction. The curvature of the failure surface behind the wall needs to be taken into account. Hence, Coulomb's theory leads to largely overestimation of the passive earth pressure.

The Rankine's theory is applicable to the calculation of the earth pressure on a perfectly smooth and vertical wall, but most retaining walls are far from frictionless soil structure interface.

The passive earth pressure problem has been widely treated in the text books, literature and articles [1-22]. Theoretical procedures for evaluating the earth pressure using different approaches (the limit equilibrium method [11] and [8], the slip line method [5], [15], [22] and [14], the upper- and lower bound theorems of limit analysis [23] and numerical computation.

Rupaand Pise, [19] used a circular arc due to arching effect for determining the passive earth pressure coefficient. Janbu[13] used a method of slices with bearing capacity factors to calculate passive pressure resultants. These different approaches generally confirm the accuracy of the Log Spiral Theory [5] for a wide range of values of the internal soil friction and the soil-structure interface friction angle. Similarly, Martin [10] and Benmebarek et al. [17] who used FLAC^{2D} numerical analysis to evaluate passive earth pressures have found fairly close agreement with Log Spiral Theory. In spite of recent published methods, the tendency today in practice is to use the values given by Caquot and Kérisel[5] and Kérisel and Absi[15].

Many studies have investigated the capacity and load-deflection relationships for walls under passive conditions using finite element and finite difference methods. Duncan and Mokwa[7] review the results of many of those studies, and report that they have generally found the log-spiral surface to accurately reflect the computed failure surface from the models. Moreover, they found that log-spiral solutions for passive

capacity are much more compatible with results of element modeling than the Coulomb model. Smith and Griffiths [21] used the finite element method to estimate the earth pressure uses an elastic-perfectly Mohr-Coulomb constitutive model with stress redistribution achieved iteratively using a reduced integration elasto-viscoplasticity algorithm

In order to appreciate the accuracy of the present analysis, the theoretical approach of Coulomb is used for comparison.

Coefficient of passive earth pressure

Lateral earth pressure is the pressure that soil exerts in the horizontal plane. To describe the pressure a soil will exert a lateral earth pressure coefficient, K, is used. K is the ratio of horizontal pressure to vertical pressure ($K = \sigma_h / \sigma_v$). K used in geotechnical engineering analysis depends on the characteristics of its applications. There are many theories for predictions lateral earth pressure, some are empirically based, and some are analytically derived. In this section we will discuss the theories for the passive earth pressure only.

Coulomb’s theory [4]

Coulomb (1776) first studied the problem of the lateral earth pressure on the retaining structures. He used limit equilibrium theory, which considers the failing soil block as a free body in order to determine the limiting horizontal earth pressure. His theory treats the soil as isotropic and accounts for both internal friction at the wall-soil interface (friction angle δ)

The coefficient of the passive earth pressure based on coulomb’s theory is:

$$K_{pc} = \frac{\sin^2(\alpha - \varphi)}{\sin^2(\alpha) \sin(\alpha + \delta) \left[1 - \sqrt{\frac{\sin(\varphi + \delta) \sin(\varphi + \beta)}{\sin(\alpha + \delta) \sin(\alpha + \beta)}} \right]^2} \tag{1}$$

Where:

K_{pc} = the coefficient of the passive earth pressure based on coulomb’s theory

β = angle between backfill surface lines and a horizontal line

φ = friction angle of the backfill soil

α = angle between a horizontal line and the back face of the wall

δ = angle of wall friction

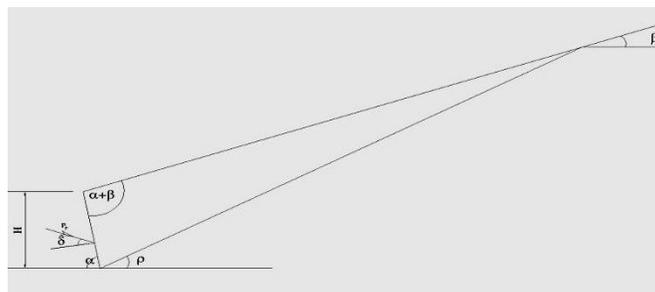


Fig. (1) Schematic forces acting on a retaining wall

Rankine’s theory

The Rankine(1857) method of evaluating passive pressure is a special case of the conditions considered by Coulomb. In particular, Rankine assumes that there is no friction at the wall-soil interface ($\delta = 0$). The coefficient of Rankine’s passive earth pressure can be computed as:

$$K_{pR} = \cos \beta \frac{\cos \beta + \sqrt{\cos^2(\beta) - \cos^2(\varphi)}}{\cos \beta - \sqrt{\cos^2(\beta) - \cos^2(\varphi)}} \tag{2}$$

When the embankment slope angle β equal zero, $K_{pR} = \tan^2(45 + \varphi/2)$.

Properties of logarithmic spiral (after [6])

The equation of the logarithmic spiral generally used in solving problems in soil mechanics is of the form

$$r = r_o e^{\theta \tan \varphi} \tag{3}$$

Where r = radius of the spiral

r_o =starting radius at $\theta=0.0$

ϕ = angle of friction of soil

θ = angle between r and r_o

the basic parameters of a logarithmic spiral are shown in Fig(2)., in which O is the center of the spiral. The area of the sector OAB is given by

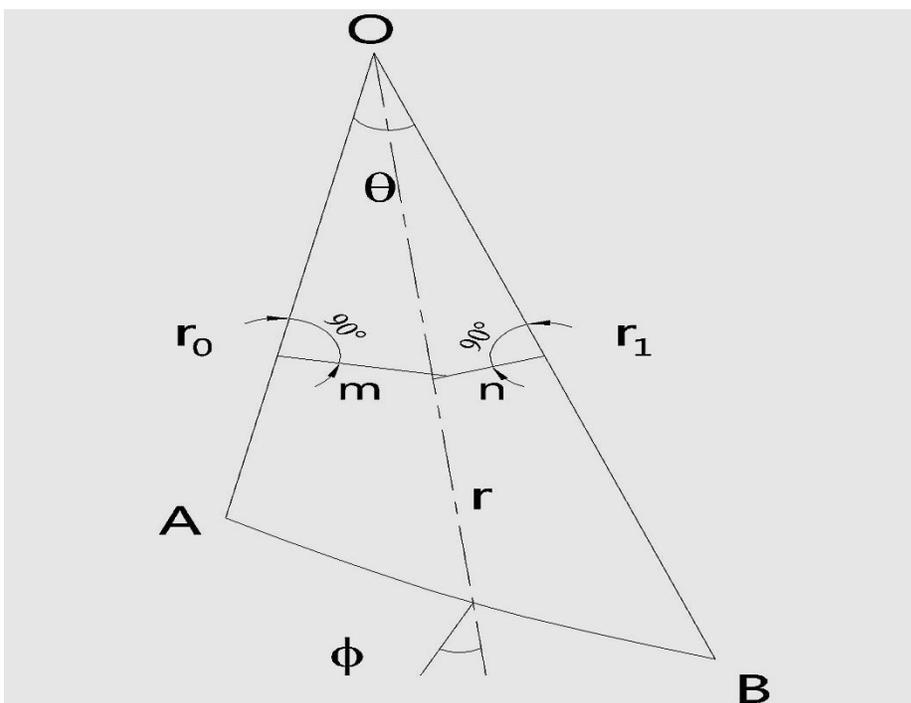


Fig.(2) General parameters of a logarithmic spiral (after Das [6])

$$A = \int_0^\theta \frac{1}{2} r (r d\theta) \quad (4)$$

Substituting the values of r from Eq(3) into Eq(4) , we get

$$A = \int_0^{\theta_1} \frac{1}{2} r_o^2 e^{2\theta \tan \phi} d\theta$$

$$= \frac{r_1^2 - r_o^2}{4 \tan \phi} \quad (5)$$

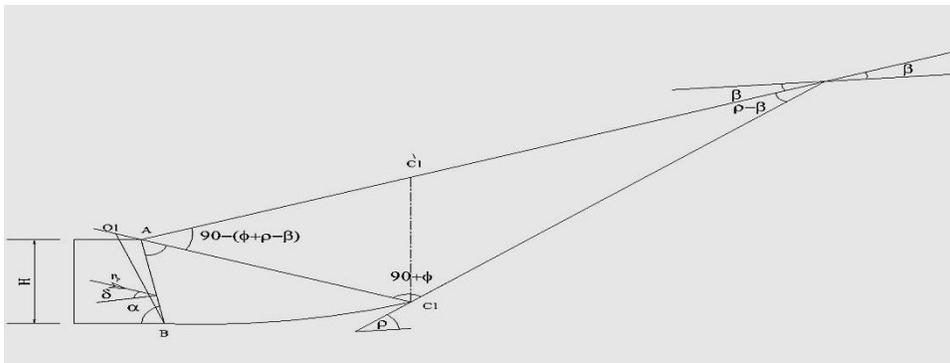
The location of the centroid can be defined by the distances \bar{m} and \bar{n} Fig (2). , measured from OA and OB respectively, and can be given by the following equations (Hijab, 1956):

$$\bar{m} = \frac{4}{3} r_o \frac{\tan \phi}{(9 \tan^2 \phi + 1)} \left[\frac{\left(\frac{r_1}{r_o}\right)^3 (3 \tan \phi \sin \theta - \cos \theta) + 1}{\left(\frac{r_1}{r_o}\right)^2 - 1} \right] \quad (6)$$

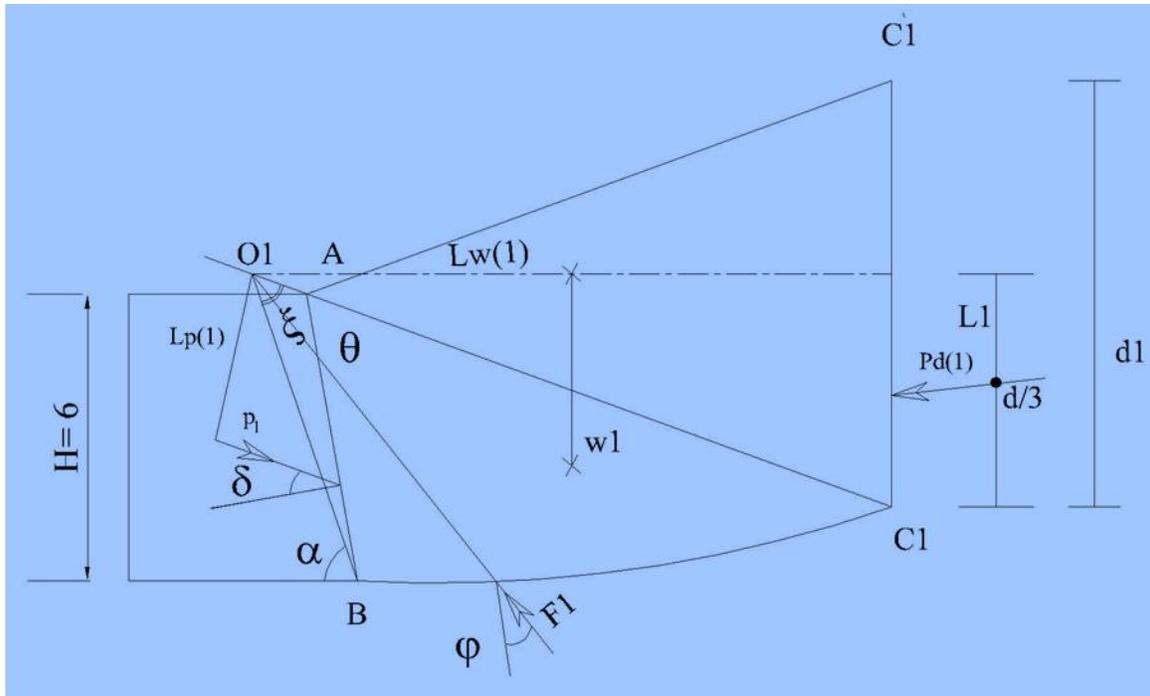
$$\bar{n} = \frac{4}{3} r_o \frac{\tan \theta}{(9 \tan^2 \theta + 1)} \left[\frac{\left(\frac{r_1}{r_o}\right)^3 - 3 \tan \theta \sin \theta - \cos \theta}{\left(\frac{r_1}{r_o}\right)^2 - 1} \right] \quad (7)$$

Another important property of the logarithmic spiral defined by Eq. is that any radial line makes an angle ϕ with the normal to the curve drawn at the point where to radial line and spiral intersect. This basic is particularly useful in solving problem related to lateral earth pressure.

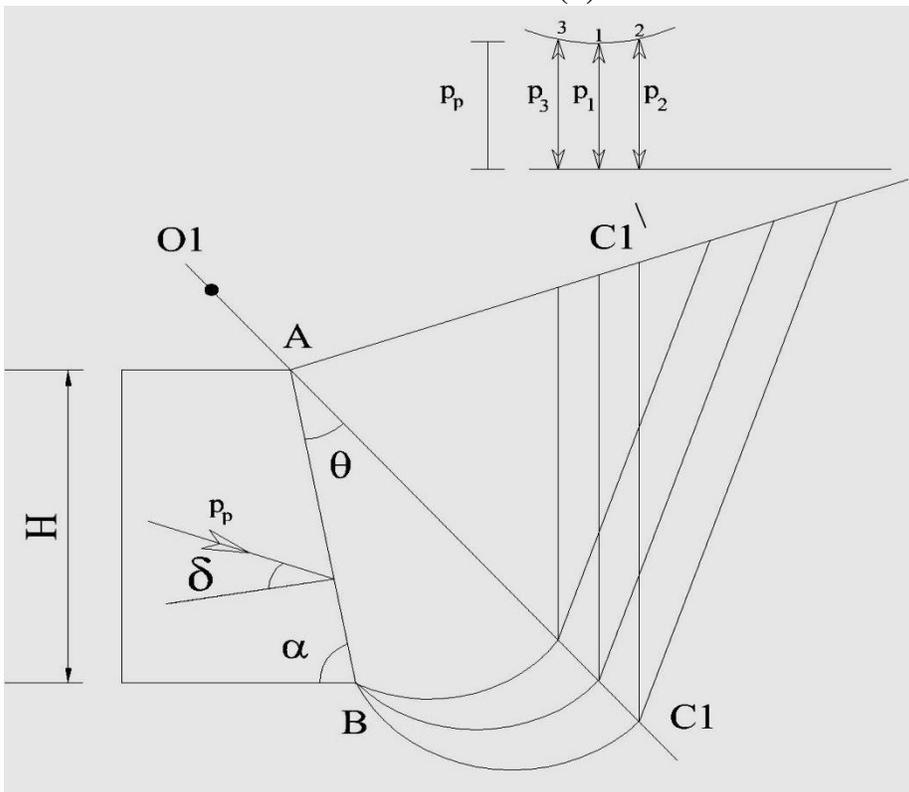
Procedure for determination of passive earth pressure (cohesionless backfill)



(a)



(b)



(c)

Fig. (3) Passive earth pressure against retaining wall with curved failure surface

Fig.(3a) shows the curved failure in the granular backfill of a retaining wall of height H. the shear strength of the granular backfill is expressed as $\tau_f = \sigma \tan \phi$. The curved lower portion BC₁ of the failure wedge is an arc of logarithmic spiral defined by Eq.(3) The center of the log spiral lies on the line C₁A (not necessarily within the limits of the points(C and A). the upper portion C₁D is a straight line that make angle of (ρ)degrees with the horizontal. (ρ) defined by the following eq.

$$\rho = \omega - \phi$$

Where ω as follows:

$$\tan \omega = \frac{\tan(\phi + \beta) + \sqrt{\tan(\phi + \beta)[\tan(\phi + \beta) + \cot(\phi + \alpha)][1 + \tan(\delta - \alpha) \cot(\phi + \alpha)]}}{1 + \tan(\delta - \alpha)[\tan(\phi + \beta) + \cot(\phi + \alpha)]}$$

The soil in the zone AC₁D is in Rankine’s passive state.Fig.(3) shows the procedure for evaluating the passive resistance by trial wedges (Terzaghi and Peck, 1967). The retaining wall is first drawn to scale as shown in Fig.(3a). The line C₁A is drawing in such a way that it makes an angle of ($\rho-\beta$) degrees with the surface of the backfill. BC₁D₁ is trial wedge in which BC₁is the arc of a logarithmic spiral according to the equation Eq. (3). O₁ is the center of the spiral (note: O₁B = r₀and O₁C₁ = r₁ and angle BO₁C₁ = angle between two radial lines of spiral (refer to Fig. (3b). Now let us consider the stability of the soil mass ABC₁C₁’ (Fig. (3b). For equilibrium the following forces per unit length of the wall are to be considered

- 1- Weight of the soil in zone ABC₁C₁’= W₁ = (γ) (area of ABC₁C₁’)(1)
- 2 -The vertical face, C₁C₁’, is the zone of Rankine’s passive state; hence, the force acting on this face is

$$P_{d(1)} = \frac{1}{2} \gamma (d_1)^2 \tan^2 \left(45 + \frac{\phi}{2} \right)$$

Where d₁ = C₁C₁’, P_{d(1)} acts parallel to the ground surface at a distance of d₁/3 measured vertically upward from C₁

- 3- F₁ is the resultant of the shear and normal forces that act along the surface of sliding BC₁. At any point on the curve, according to the property of the logarithmic spiral, a radial line makes an angle ϕ with the normal. Because the resultant, F₁ makes an angle ϕ with the normal to the spiral at its point of application, its line of application will coincide with a radial line and will pass through the point O₁.
- 4- P₁ is the passive force per unit length of the wall. It acts at distance of H/3measured vertically from the bottom of the wall. The direction of the force P₁ is inclined at an angle δ with the normal drawn to the back face of the wall.

Now, taking the moment of W₁,P_{d(1)}, F₁ and P₁ about the point O₁ for equilibrium, we have

$$W_1 [l_{w(1)}] + P_{d(1)} [l_1] + F_1 [0] = P_1 [l_{p(1)}]$$

$$P_1 = \frac{1}{l_{p(1)}} [W_1 l_{w(1)} + P_{d(1)} l_1]$$

where $l_{w(1)}, l_1$ and $l_{p(1)}$ are moment arms for the forces $W_1, P_{d(1)}$ and P_1 , respectively.

The preceding procedure for finding the trial passive force per unit length of the wall is repeated for several trial wedges such as those shown in Fig. (3c). Let $P_1, P_2, P_3, \dots, P_n$ be the forces that corresponding to trial wedges 1, 2, 3, ..., n. The lowest point of the smooth curve defines the actual passive forces, P_p , per unit length of the wall. The coefficient of the passive earth pressure $K_p = 2P_p/\gamma H^2$.

It is worthwhile mentioning here that when we did not get a clear minimum coefficient of passive earth pressure, take $k_p(\min.)$ corresponding the angle BO_1C between $O_1B = r_0$ and $O_1C_1 = r_1$ equal to $(\rho - \beta)$, where ρ inclined angle of tangent on the horizontal and β inclined of the ground surface

Main goal of the present work

The main goal of the present work is the transfer the shown case of passive earth pressure against rigid retaining wall using surface of rupture consisting log- spiral curve and linear segments as depicted in Fig.(3) into group of equations can solved easily by computer with high accuracy.

Parameters used in the program

Wall geometry: height of the wall, H, inclination of the back wall on the horizontal, α , =90, 80 and 70

Ground surface slope of the backfill $\beta = (0, 0.2, 0.4, 0.6 \text{ and } 0.8) \phi$

Soil properties: angle of internal friction, ϕ , =5, 10, 15, 20, 25, 30, 35, 40 and 45

Friction between wall and soil $\delta = (0, 0.2, 0.4, 0.6, 0.8 \text{ and } 1) \phi$

Procedure of calculations

- 1- For a constant $\alpha = 90$; ϕ is changed nine times (5, 10, 15, 20, 25, 30, 35, 40 and 45) and the corresponding minimum coefficient of passive earth pressure are found using proposed as discuss before.
- 2- The value δ is change six times and step No. 1 is repeated.
- 3- The value β is changed five times and steps No. 1 and 2 are repeated.
- 4- For $\alpha = 90, 80$ and 70 degree steps No. 1, 2 and 3 are repeated.
- 5- Results for steps No. 1, 2, 3 and 4 are shown in Table 1, 2 and 3

Table 1 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 90^\circ$

ϕ	$\beta = 0.0$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.218	1.225	1.233	1.233	1.240	1.247
10	1.495	1.510	1.527	1.547	1.575	1.598
15	1.811	1.862	1.918	1.971	2.039	2.109
20	2.224	2.310	2.428	2.556	2.709	2.892
25	2.712	2.893	3.120	3.395	3.740	4.175
30	3.319	3.672	4.100	4.661	5.429	6.425
35	4.120	4.712	5.532	6.703	8.450	10.417
40	5.140	6.168	7.746	10.301	14.089	18.047
45	6.484	8.305	11.427	17.381	25.307	34.026
ϕ	$\beta = 0.2$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.255	1.252	1.259	1.266	1.273	1.273
10	1.567	1.594	1.609	1.628	1.656	1.679
15	1.987	2.022	2.080	2.143	2.213	2.286
20	2.519	2.624	2.748	2.883	3.056	3.260
25	3.208	3.427	3.695	4.012	4.414	4.924
30	4.156	4.564	5.108	5.824	6.771	7.977
35	5.458	6.280	7.369	8.940	11.238	13.722
40	7.379	8.919	11.202	14.970	20.097	25.462
45	10.203	13.274	18.356	27.648	39.359	51.956
ϕ	$\beta = 0.4$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.282	1.288	1.284	1.291	1.297	1.304
10	1.653	1.664	1.691	1.706	1.734	1.755
15	2.132	2.201	2.241	2.306	2.378	2.458
20	2.813	2.922	3.070	3.222	3.403	3.620
25	3.743	4.007	4.295	4.658	5.113	5.690
30	5.098	5.589	6.237	7.091	8.233	9.642
35	7.088	8.126	9.561	11.566	14.446	17.479
40	10.262	12.459	15.636	20.834	27.556	34.460
45	15.681	20.272	28.161	41.812	58.329	75.369
ϕ	$\beta = 0.6$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.306	1.312	1.317	1.313	1.319	1.324
10	1.720	1.745	1.755	1.781	1.796	1.824
15	2.295	2.342	2.397	2.458	2.539	2.616
20	3.114	3.246	3.370	3.541	3.724	3.948
25	4.323	4.592	4.888	5.293	5.782	6.415
30	6.111	6.688	7.433	8.384	9.703	11.289
35	8.978	10.311	11.962	14.419	17.869	21.400
40	14.053	16.804	20.921	27.694	36.103	44.495
45	23.270	29.626	41.012	60.019	82.058	103.672

Table 1 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 90^\circ$ (continous)

ϕ	$\beta = 0.8$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.327	1.332	1.336	1.340	1.335	1.339
10	1.779	1.801	1.822	1.830	1.854	1.868
15	2.448	2.485	2.530	2.583	2.646	2.729
20	3.397	3.517	3.627	3.791	3.981	4.196
25	4.838	5.094	5.414	5.816	6.324	6.973
30	7.134	7.696	8.511	9.528	10.934	12.604
35	11.009	12.400	14.249	17.039	20.899	24.723
40	18.046	21.292	26.249	34.531	44.296	53.707
45	32.072	40.413	55.714	80.184	107.414	132.636

Table 2 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 80^\circ$

ϕ	$\beta = 0.0$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.253	1.253	1.2538	1.2548	1.256	1.2575
10	1.568	1.569	1.5641	1.5763	1.5776	1.5908
15	1.850	1.876	1.8977	1.9319	1.9644	2.0035
20	2.218	2.257	2.3294	2.4063	2.4993	2.6101
25	2.624	2.750	2.8837	3.0558	3.2707	3.5347
30	3.136	3.361	3.6346	3.9872	4.4435	5.0564
35	3.792	4.158	4.6703	5.3675	6.3587	7.7397
40	4.569	5.218	6.1662	7.565	9.8053	12.6693
45	5.561	6.673	8.4238	11.373	16.5068	22.5112
ϕ	$\beta = 0.2$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.286	1.287	1.287	1.288	1.289	1.290
10	1.648	1.661	1.655	1.661	1.668	1.677
15	2.030	2.058	2.079	2.115	2.148	2.188
20	2.525	2.574	2.653	2.735	2.843	2.963
25	3.133	3.278	3.440	3.643	3.894	4.201
30	3.949	4.225	4.582	5.023	5.591	6.348
35	5.040	5.575	6.278	7.226	8.544	10.360
40	6.600	7.591	8.985	11.037	14.284	18.265
45	8.809	10.687	13.584	18.384	26.352	35.386

Table 2 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 80^{\circ}$ (continuous)

ϕ	$\beta = 0.4$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.318	1.318	1.318	1.319	1.319	1.320
10	1.667	1.664	1.678	1.693	1.709	1.718
15	2.042	2.079	2.123	2.183	2.237	2.298
20	2.540	2.635	2.749	2.882	3.031	3.215
25	3.160	3.366	3.625	3.944	4.325	4.814
30	3.969	4.407	4.949	5.668	6.638	7.798
35	5.075	5.915	7.078	8.773	11.128	13.520
40	6.608	8.256	10.824	15.078	20.217	25.471
45	8.876	12.191	18.524	28.704	40.596	53.300
	$\beta = 0.6$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.3472	1.347	1.3469	1.3469	1.3471	1.3475
10	1.7065	1.708	1.7263	1.7445	1.7541	1.7746
15	2.1263	2.1745	2.228	2.287	2.3448	2.4195
20	2.6778	2.8046	2.9358	3.0932	3.2796	3.4969
25	3.4052	3.6671	3.9936	4.3817	4.8789	5.4941
30	4.3918	4.9466	5.6697	6.6465	7.9508	9.3019
35	5.7766	6.9429	8.6207	11.164	14.0912	16.9258
40	7.863	10.3493	14.6983	20.709	27.1968	33.8051
45	11.2451	17.0734	28.4959	42.516	58.8074	75.5015
	$\beta = 0.8$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.3745	1.3734	1.3724	1.3713	1.3704	1.3697
10	1.739	1.7433	1.7635	1.7832	1.7941	1.8156
15	2.1967	2.2526	2.3006	2.3679	2.4341	2.5116
20	2.7967	2.9325	3.091	3.2676	3.4801	3.7326
25	3.606	3.9211	4.2943	4.775	5.3872	6.0744
30	4.739	5.433	6.3547	7.6607	9.1817	10.6155
35	6.4319	7.9737	10.4122	13.6467	16.9049	20.0532
40	9.1701	12.9573	19.4407	26.598	34.3447	41.8913
45	14.1789	25.3723	40.2277	58.3373	78.8112	82.6926

Table 3 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 70^\circ$

ϕ	$\beta = 0.0$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.265	1.265	1.265	1.266	1.268	1.269
10	1.523	1.522	1.525	1.530	1.536	1.544
15	1.862	1.861	1.868	1.881	1.899	1.923
20	2.321	2.267	2.294	2.333	2.407	2.460
25	2.681	2.679	2.771	2.883	2.993	3.160
30	3.133	3.214	3.368	3.586	3.874	4.247
35	3.611	3.850	4.197	4.621	5.218	6.041
40	4.333	4.713	5.269	6.130	7.367	9.255
45	5.161	5.731	6.820	8.481	11.244	15.414
ϕ	$\beta = 0.2$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.300	1.300	1.300	1.301	1.303	1.304
10	1.614	1.615	1.618	1.622	1.629	1.637
15	2.047	2.049	2.057	2.072	2.091	2.115
20	2.469	2.509	2.560	2.605	2.680	2.758
25	2.974	3.033	3.160	3.317	3.495	3.722
30	3.546	3.750	3.999	4.334	4.757	5.317
35	4.259	4.647	5.168	5.873	6.838	8.192
40	5.175	5.876	6.913	8.377	10.653	13.681
45	6.394	7.657	9.610	12.844	18.417	25.022
ϕ	$\beta = 0.4$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.332	1.332	1.333	1.334	1.335	1.337
10	1.703	1.705	1.708	1.713	1.719	1.727
15	2.204	2.188	2.207	2.229	2.256	2.288
20	2.643	2.691	2.777	2.854	2.949	3.062
25	3.229	3.363	3.538	3.753	4.003	4.318
30	3.945	4.265	4.647	5.124	5.743	6.567
35	4.902	5.508	6.293	7.368	8.903	10.838
40	6.143	7.287	8.944	11.442	15.268	19.448
45	7.916	10.086	13.616	20.060	28.988	38.690

Table 3 Coefficient of passive earth pressure using log-spiral curve failure surface at $\alpha = 70^0$ (continuous)

	$\beta = 0.6$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.363	1.364	1.364	1.365	1.366	1.367
10	1.791	1.792	1.795	1.799	1.805	1.811
15	2.293	2.294	2.326	2.360	2.385	2.428
20	2.809	2.890	2.984	3.079	3.207	3.348
25	3.487	3.670	3.901	4.183	4.517	4.933
30	4.376	4.792	5.310	5.963	6.836	7.926
35	5.570	6.403	7.547	9.153	11.417	13.841
40	7.264	8.959	11.546	15.769	21.044	26.431
45	9.785	13.346	19.925	30.654	43.218	56.680
	$\beta = 0.8$					
	δ					
	0	0.2 ϕ	0.4 ϕ	0.6 ϕ	0.8 ϕ	ϕ
5	1.393	1.393	1.393	1.393	1.393	1.393
10	1.876	1.876	1.877	1.879	1.881	1.885
15	2.372	2.386	2.427	2.455	2.502	2.543
20	2.933	3.045	3.150	3.275	3.422	3.595
25	3.705	3.958	4.238	4.572	4.985	5.510
30	4.747	5.276	5.933	6.796	7.964	9.227
35	6.207	7.327	8.881	11.226	14.032	16.772
40	8.372	10.822	14.916	20.838	27.230	33.677
45	11.967	17.830	29.230	43.394	59.907	76.544

Analysis and discussion

The discussion illustrates the effect of the parameters study on the coefficient of passive earth pressure. The main investigated parameters are:-

- Angle of internal friction of soil
- Interface friction angle between soil and wall
- Ground surface slope
- Inclined of back surface

A comparison was made between the results of present work and some researches using different surface failure, to evaluate the coefficient of the passive earth pressure. The deduced formula for calculation K_p corresponding to Coulomb's coefficient.

Relation between ϕ and K_p

The relation between ϕ and K_p is plotted and shown Figs (4-5), it is clear that with increasing ϕ the value of K_p increases, and K_p increasing with increases δ for constant value of β . Figs (4-5) has the same trend for the given values of $\beta = (0.0, 0.8)$ ϕ

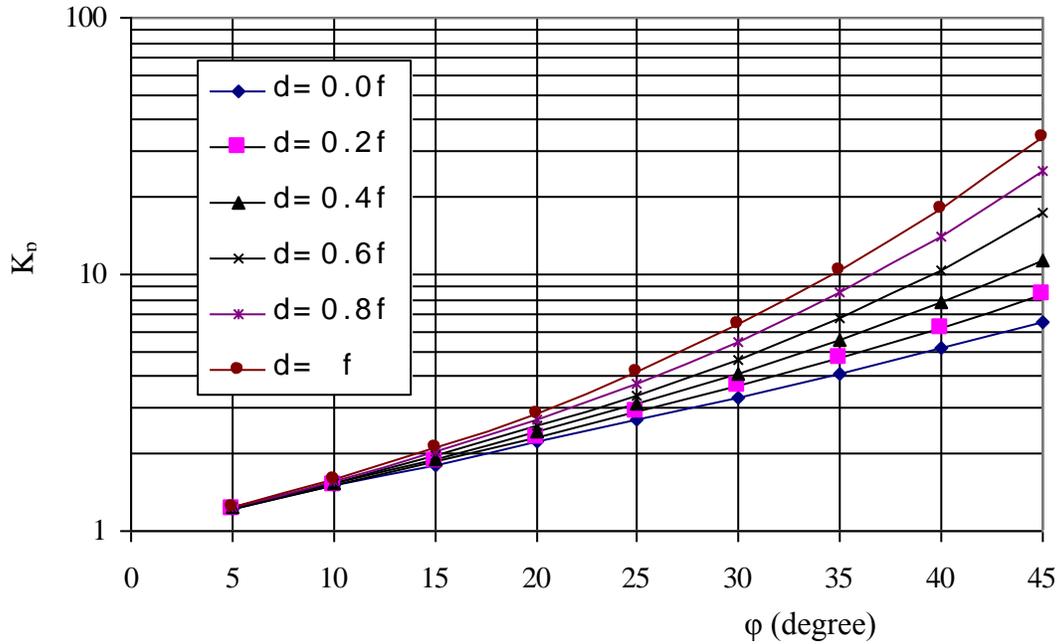


Fig.(4) K_p versus ϕ at $\beta = 0.0$ and $\alpha = 90$

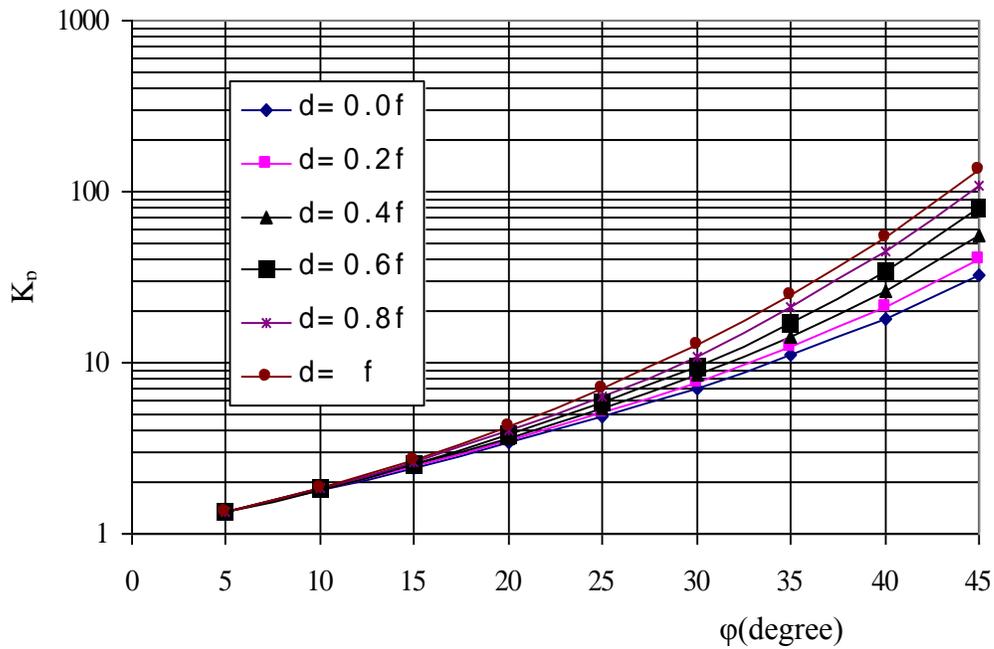


Fig.(5) K_p versus ϕ at $\beta = 0.8$ and $\alpha = 90$

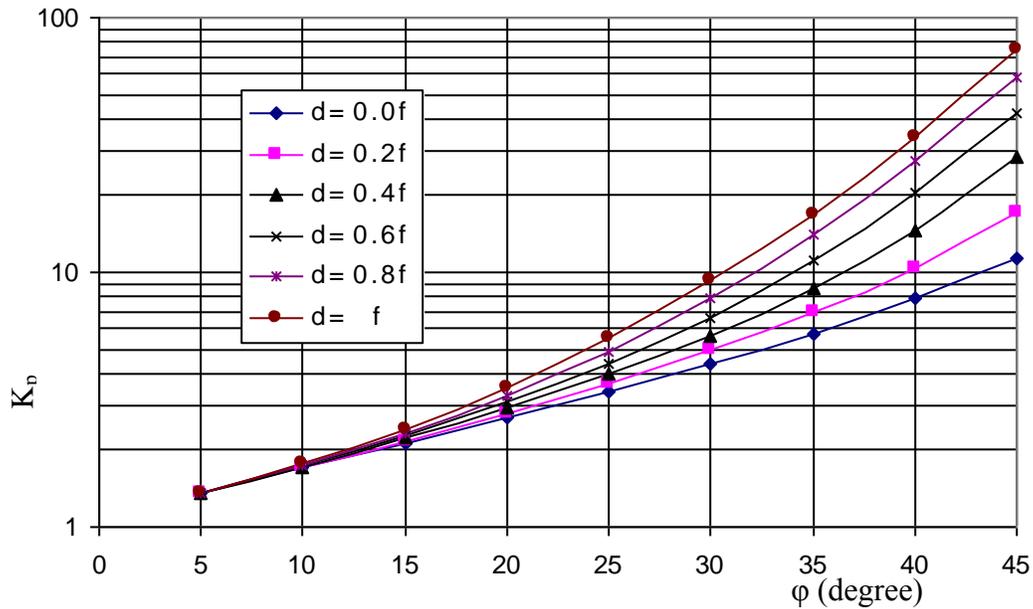


Fig.(6) K_p versus ϕ at $\beta = 0.8 \phi$ and $\alpha = 80$

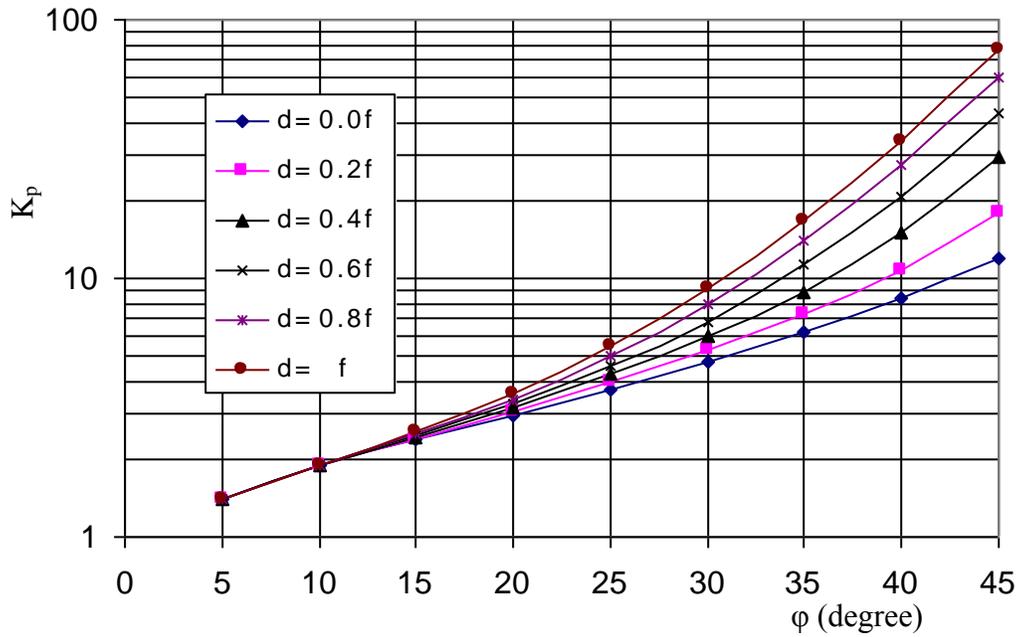


Fig.(7) K_p versus ϕ at $\beta = 0.8 \phi$ and $\alpha = 70^\circ$

Figs.(5-7) show the relation between K_p and ϕ at $\beta=0.8 \phi$ for different values α . It is evident that K_p decreases with decreasing α .

Ground surface slope β

The relation between K_p and β is plotted and shown Fig (8), it is clear that with increasing β the value of K_p increases, and decreases with decreasing α for constant value δ . Figs (8) have the same trend for the given values of $\delta = (0.0, 0.2, 0.6, 0.8 \text{ and } 1) \varphi$.

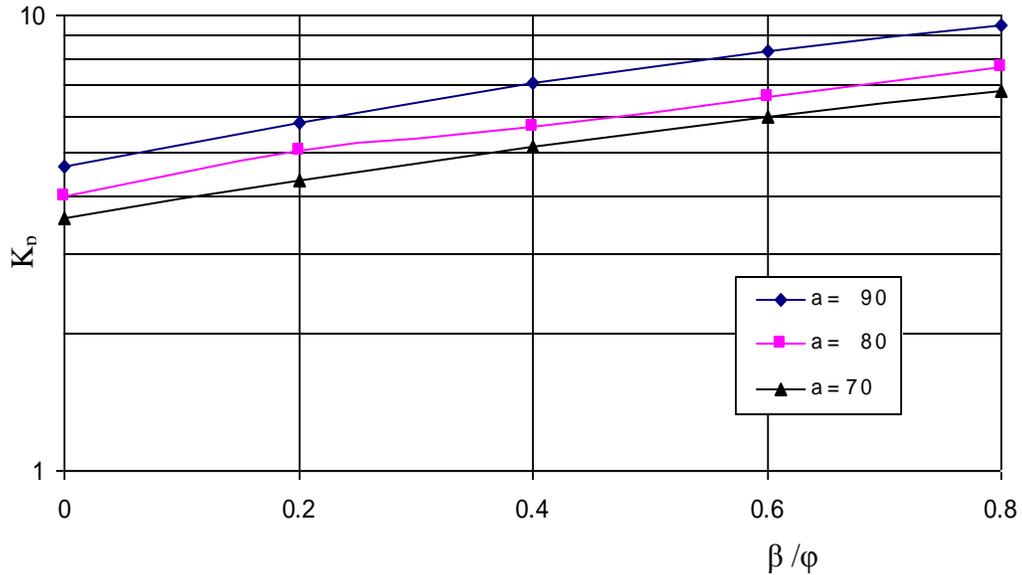


Fig.(8) K_p versus β / φ at $\varphi = 30^\circ, \delta = 0.6\varphi$

Interface angle of internal friction between wall and soil δ

The relation between K_p and δ is plotted and shown Fig (9), it is clear that with increasing δ the value of K_p increases, and decreases with decreasing α for constant value β . Figs (8) have the same trend for the given values of $\beta = (0.0, 0.2, \text{ and } 0.8) \varphi$.

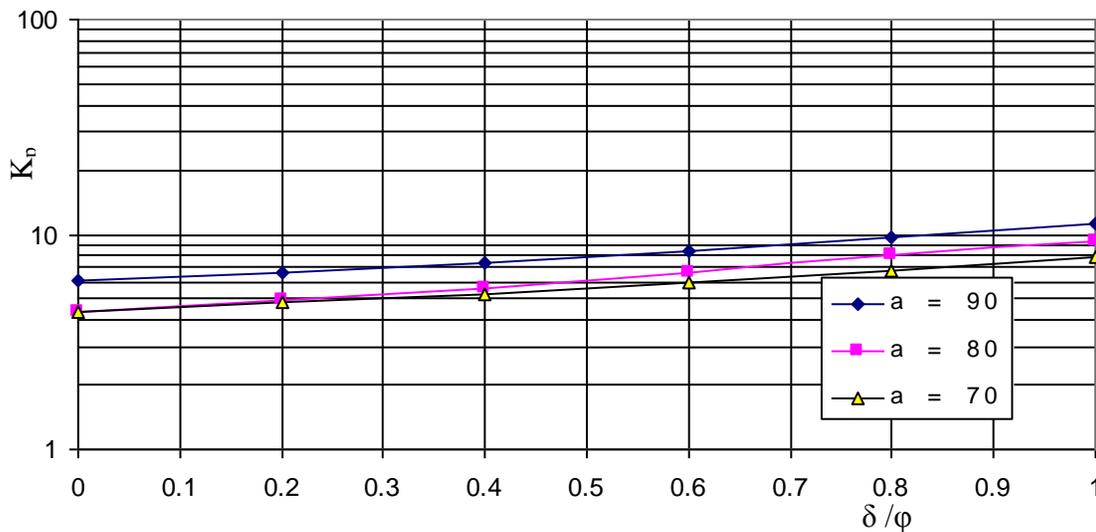


Fig.(9) K_p versus δ / φ at $\varphi = 30^\circ, \beta = 0.6\varphi$

Inclination of the back wall face α

The relation between K_p and α is plotted and shown Fig (10), it is clear that with increasing α the value of K_p increases, and increases with increasing δ for constant value β . Figs (8) have the same trend for the given values of $\beta = (0.0, 0.2, \text{ and } 0.8) \varphi$.

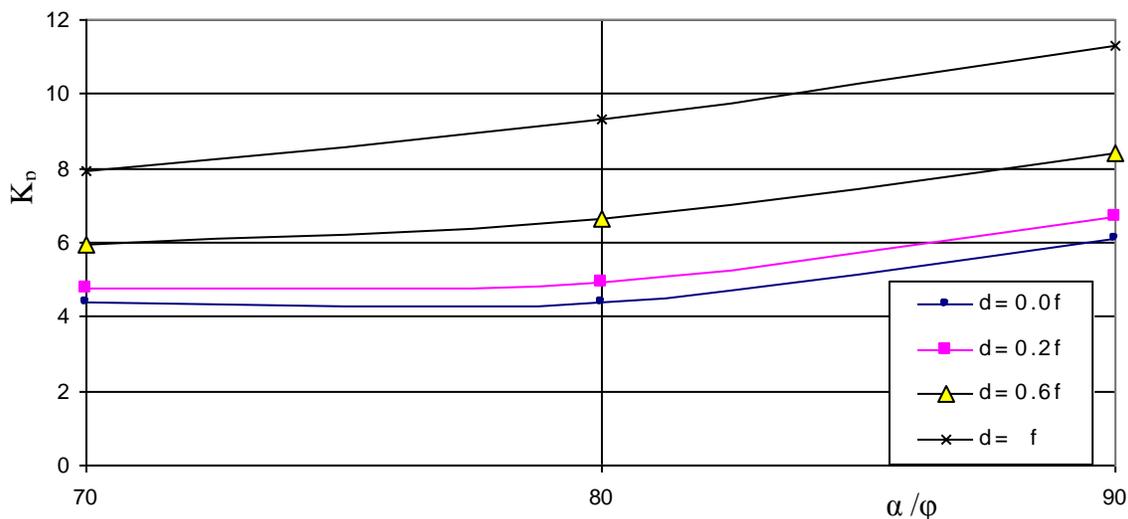


Fig.(10) K_p versus α/φ at $\varphi = 30^\circ, \beta = 0.6\varphi$

The deduced formula for calculation of K_p corresponding K_{pc} (Columb’s coefficient)

Where the magnitude of friction is low so that the angle (δ) is small, the rupture surface is approximately planner. As the angle δ increases, however, the lower zone failure wedge becomes curved for values of, ($\delta > \varphi/3$), up to about one-third of φ . But as δ becomes larger, the error in the computed K_p increasingly greater, whereby the actual passive is less than the computed value (using Eq. (1)). For larger δ analysis of force resulting from passive pressure should be based on a curved surface of rupture. When $\varphi < 20^\circ$ the difference between planner and curve surface failure little and may be neglect. In this section we will try found the relation between k_p and K_{pc} for ($\delta > \varphi/3, \varphi > 20^\circ$) with different anther study parameters.

Based on data recorded in tables 1, 2 and 3 and values of K_{pc} (Columb’s coefficient) which computed using Eq. (1)

The relation between $\frac{K_p}{K_{pc}}$ for different values of φ at certain δ, β and α may be

represented by the following expression:-

$$\frac{K_p}{K_{pc}} = -a \tan (\varphi) + b$$

Where a and b are coefficient obtained by regression formula depend on on δ, α and β are listed in Tables 4 and 5 respectively.

Table 4 Coefficient a

$\alpha = 90^\circ$				
β / φ	δ / φ			
	0.4	0.6	0.8	1.0
0.0	0.37	0.647	1.136	1.456
0.2	0.638	1.024	1.294	1.63
0.4	1.035	1.283	1.61	1.907
0.6	0.766	1.062	1.287	1.594
0.8	1.578	1.826	1.859	2.319
$\alpha = 80^\circ$				
0.0	0.173	0.378	0.639	1.07
0.2	0.419	0.671	1.068	1.402
0.4	0.713	1.08	1.401	1.668
0.6	1.102	1.409	1.659	1.893
0.8	1.422	1.652	1.868	2.044
$\alpha = 70^\circ$				
0.0	0.065	0.219	0.405	0.676
0.2	0.262	0.447	0.697	1.093
0.4	0.491	0.734	1.104	1.441
0.6	0.788	1.127	1.455	1.677
0.8	1.171	1.47	1.676	1.746

Table 5 Coefficient b

$\alpha = 90^\circ$				
β / φ	β / φ			
	0.4	0.6	0.8	1.0
0.0	1.132	1.20	1.386	1.449
0.2	1.187	1.293	1.33	1.395
0.4	1.302	1.323	1.380	1.419
0.6	1.288	1.323	1.325	1.369
0.8	1.354	1.366	1.285	1.392
$\alpha = 80^\circ$				
0.0	1.127	1.163	1.220	1.361
0.2	1.204	1.247	1.364	1.436
0.4	1.285	1.378	1.440	1.469
0.6	1.40	1.447	1.465	1.474
0.8	1.456	1.458	1.454	1.439
$\alpha = 70^\circ$				
0.0	1.177	1.176	1.198	1.263
0.2	1.241	1.247	1.291	1.408
0.4	1.303	1.331	1.428	1.501
0.6	1.385	1.454	1.518	1.523
0.8	1.495	1.53	1.523	1.454

Application of the program and comparison with others

Some examples were solved using program and are compared with the references given in Figs. (11-14). Fig.(11) shows the K_p versus ϕ at $\alpha = 90^\circ$, $\beta/\phi = 0.0$, $\delta/\phi = 0.6$ using different method. It is clear that where the magnitude of friction is low so that the angle (δ) is small the K_p is the same for different methods. After that clear difference between planner surface and log-spiral surface failure

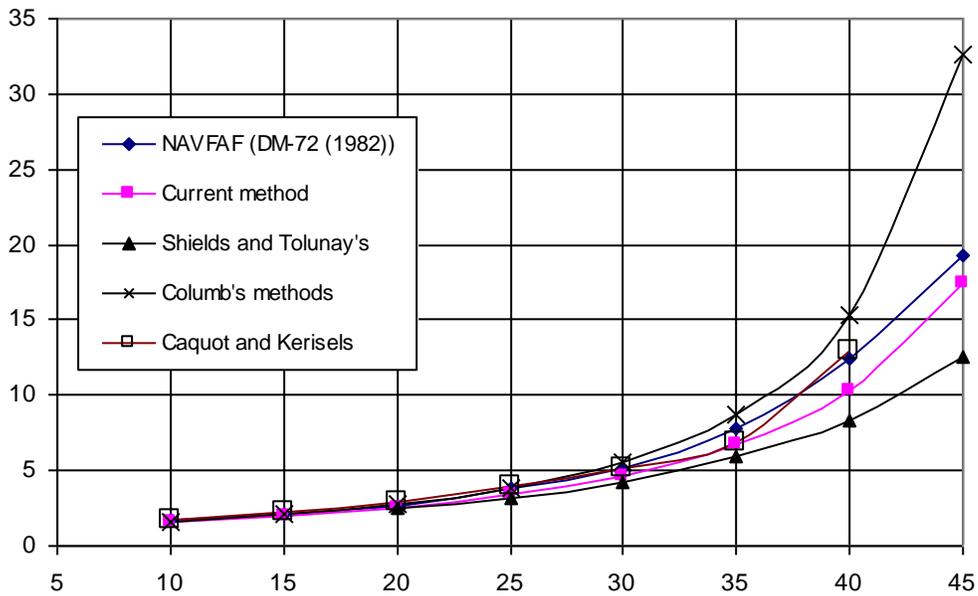


Fig. (11) K_p versus ϕ at $\alpha = 90^\circ$, $\beta/\phi = 0.0$, $\delta/\phi = 0.6$ using different method

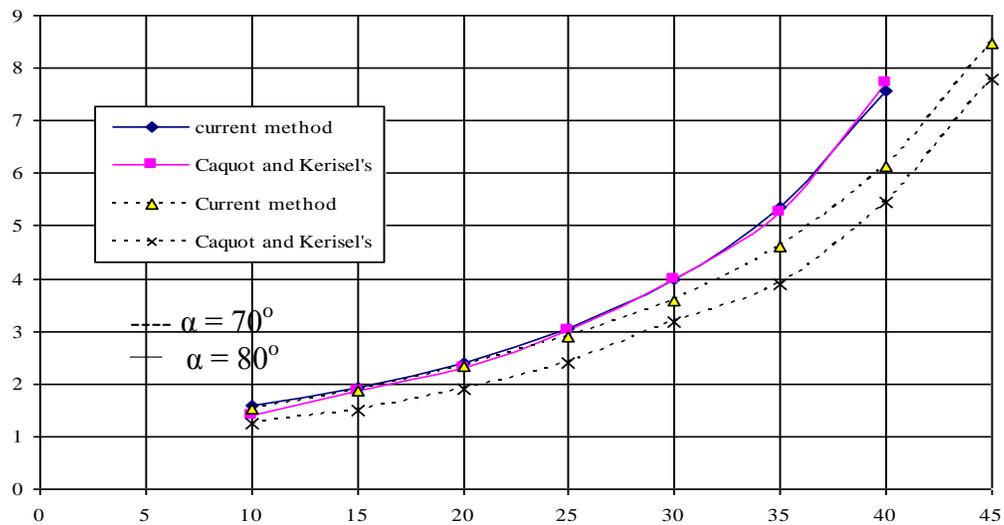


Fig.(12) K_p versus ϕ at $\alpha = 80^\circ, 70^\circ$, $\beta/\phi = 0.0$, $\delta/\phi = 0.6$ using different method

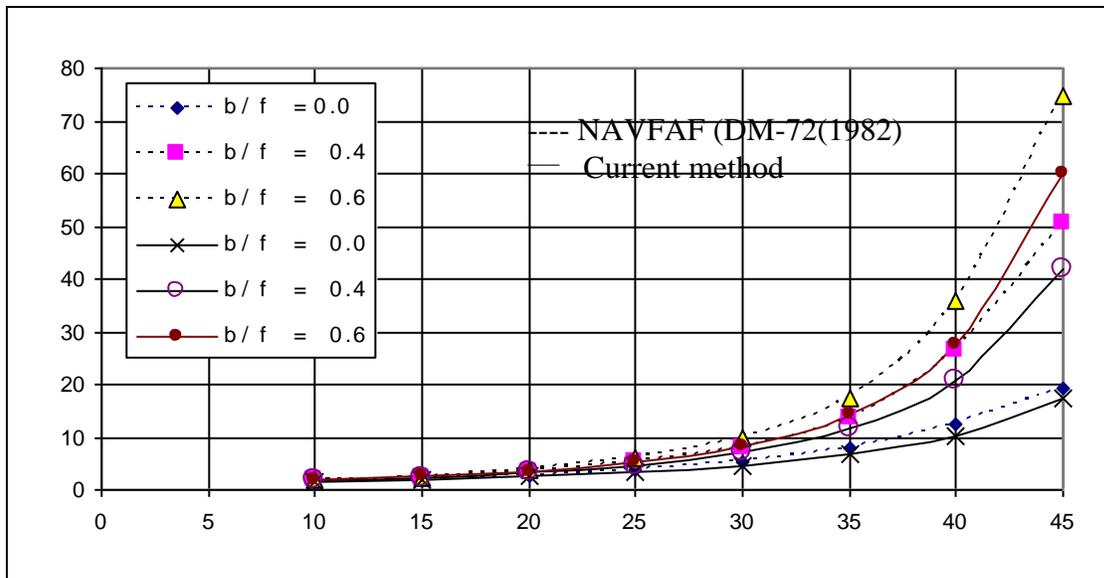


Fig.(13) K_p versus ϕ at $\alpha = 90^\circ$, $\delta / \phi = 1.0$ using different method

Conclusions

The main conclusions of the present study can be drawn as follows:-

- Coefficient of the passive earth increasing with increases angle of internal friction of soil.
- Coefficient of the passive earth increasing with increases δ / ϕ .
- Coefficient of the passive earth increasing with increases β / ϕ .
- Coefficient of the passive earth decreasing with decreases α .
- Where the magnitude of friction is low so that the angle (δ) is small, the rupture surface is approximately planner. As the angle δ increases, however, the lower zone failure wedge becomes curved for values of, ($\delta > \phi/3$). But as δ becomes larger, the error in the computed K_p increasingly greater, whereby the actual passive is less than the computed value (using Columb's theory)). For larger δ analysis of force resulting from passive pressure should be based on a curved surface of rupture. When $\phi < 20^\circ$ the difference between planner and curve surface failure little and may be neglect.

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