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Shape Optimization of Concrete Slab Subjected to Blast Load

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Abstract

This paper presents design optimization of concrete slab subjected to blast loading of nuclear explosion. It focuses on sizing optimization problem of a concrete slab having elastic foundation and fixed ends. In the formulation of the optimization problem, the objective function is to minimize the maximum existing displacement. The design variables of the optimization problem are the thicknesses of the concrete slab elements. The constraints are the concrete slab are the mass, derivative of the thicknesses and plastic strain. A MATLAB code has been developed to obtain the pressure time history. This code is linked to the finite element software ANSYS to determine the displacement time history , strains, stresses and best cross section shape of the concrete slab.

Keywords: Shape optimization, Blast loading, Nuclear explosion and concrete slab.

Nomenclature

$\mathbf{X}$: Design variables vector
X_L : Lower limit on design variables
X_U : Upper limit on design variables
N_{dv} : Number of design variables
$\mathbf{C}$: Nodal coordinate vector in the FE model
$\mathbf{V}$: Velocity field vector
ε_{max} : Max plastic strain for the material
$\boldsymbol{\varepsilon}$: Plastic strain vector
M : Total mass of the structure
M_{max} : Upper limit for the mass of the structure
t : Thickness of the structure (plate) at any point
t_{min} : Minimum thickness allowed
R : The distance from the center of the charge.

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1. Introduction

Few papers, available in literature, have focused on blast mitigation for concrete and steel sections. Dharaneepathy and Sudhesh [1] investigated the stiffener patterns on a square plate subject to blast loads modeled using Friedlander's exponential function. Xue and Hutchinson [2] and Fleck and Deshpande [3] compared blast resistance of solid versus sandwich panels which were assumed to be ductile to withstand deformation caused by the impulsive blast loads. Yen, *et al* [4] presented an experimentally validated dynamic analysis procedure utilizing LS-DYNA and the ConWep (blast function provided in LSDYNA) air blast function with shock mitigation materials such as honeycomb or foam. The numerical results indicate that significant reduction in the maximum stress amplitude propagating within the protected components can be achieved by suitable selection of honeycomb material with proper crush strength. Liang Yang and Wu [5] investigated the optimum design of panels under blast loading by using a combined algorithm of the feasible direction method (FDM) and the Backtrack Program Method (BPM). This work ignores the effect of nonlinearity and deals with the blast loads as static loads.

Main and Gazonas [6] studied the effect of an air blast on uniaxial crushing of cellular sandwich plates. Initial numerical results show that the capacity of the sandwich plates of shock mitigation can be improved by varying mass fractions of the front and back face sheets. Further, the authors investigated physics of variation in geometry and shock mitigation capacity leading to an interesting optimization formulation to maximize mass distribution of front sheet while minimizing back-face acceleration. Sriram *et al* [7] studied the feasibility of using some lightweight composites (i.e. sandwich composite plates, S2-glass/epoxy laminated face sheets and aluminum foam core) in designing blast-resistant protective structures.

From the literature, it is clear that no application in optimizing shape of concrete slab is studied to resist blast loads. Therefore, an approach and methodology to optimize the shape of concrete slab to withstand blast loading due to nuclear explosion is presented in the current paper. Here, difficulties are also treated in the optimization problem. These difficulties are: i) the nature of blast loading ii) transient dynamic response iii) non-differentiable, nonconvex and computationally expensive functions.

The nonlinearities are pressure time load, deformations and plasticity. The transient analysis generally requires more computer resources. Some preliminary work should be done to understand how nonlinearities affect the structure's response by doing a static analysis first. Here, a MATLAB computer program has been developed to implement the nuclear explosion getting the load time history and linked to ANSYS program. In the shape design optimization process, two different optimization methods (i.e zero order method and first order method) are used to find the optimum shape of a concrete slab.

2. Pressure loads of nuclear explosion

The effects of an explosion can be distinguished in three ranges: *i)* contact detonation *ii)* near zone and *iii)* far zone of the detonation. The size of all these zones depends on many parameters, the most important one is the quantity of the explosive charge. Additional effective parameters are given in curves for the description of the different air blast using a rich body of experimental data, see Kinney [8]. The parameters are presented in double logarithmic diagrams with the scaled distance *(Z)*, and are also available as polynomial equations. The schematic of the concrete slab used for shape optimization is shown in Fig. 1. The standoff distance of the charge is 210 m. It should be noted that the slab is just a part of a fortified structure used to model the experimental condition. With reference to Fig. 1, the basic problem addressed in this paper can be stated as follows: Given a set of basis shapes that controls the shape, a mass limit for the slab, plastic strain limits representing fracture strength, and a minimum thickness for the slab; determine the best possible combination of these restrictions that minimizes the deflection.

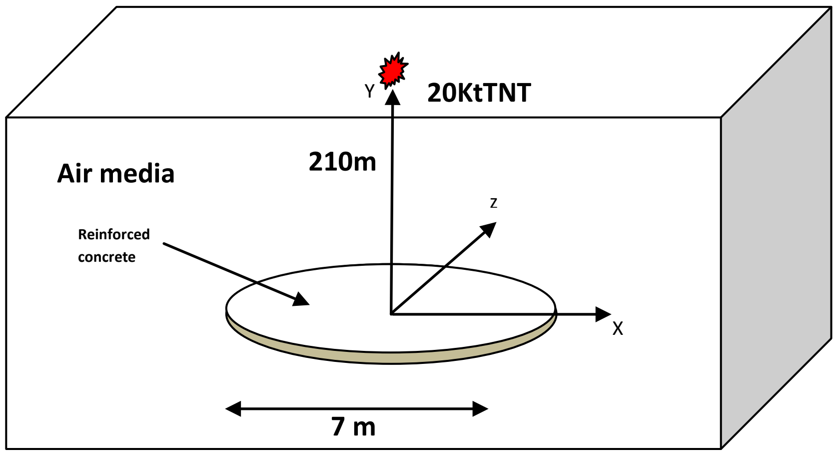


Figure.1 Base line structure model details

Important notes regarding the above problem are:

(i) The y-displacement of nodes in the concrete slab is taken as the objective function

(ii) The displacement is a function of time.

(iii) Plastic strain, also a function of time, stabilizes after a certain simulation time duration.

(iv) M refers to the mass of the concrete slab.

(v) Thickness is computed from nodal coordinates of the plane-25 element in the FE model.

Determine the best possible shape that minimizes the deflection (at the first peak in time).

The pressure at a known point can be described by the modified Friedlander equation from Baker [9] and depends on the time of the arrival of the pressure wave *($t = t_0$-ta)*. The other effects of nuclear explosions as thermal, electromagnetic pulse and radiation are disregarded. The peak over pressure can be calculated using equation (1). This equation is valid for weapon yield (*1kt or 1 Mt*). Using scaling laws,the pressure can be determined for other yields. All parameters of the pressure time curve are normally written in terms of scaled laws utilizing equation (2).

$$p_{os}(r,z) = \frac{10.47}{r^{a(z)}} + \frac{b(z)}{r^{c(z)}} + \frac{d(z).e(z)}{1+f(z)r^{g(z)}} + h(x,r,y) \quad (1)$$

$$\frac{R}{R1} = \left(\frac{W}{W1}\right)^{1/3} \quad , \quad \frac{t}{t1} = \left(\frac{W}{W1}\right)^{1/3} \quad , \quad \frac{I}{I1} = \left(\frac{W}{W1}\right)^{1/3} \quad (2)$$

Where (*W*) is the weapon yield and (*R*)is the distance between the center of the charge and the object.The pressure attains its maximum very fast (extremely in short time called arrival time (t_a) of the shock wave to the point under consideration). Then, the pressure starts decreasing until it reaches the reference pressure (P_0) as shown in figure 1.

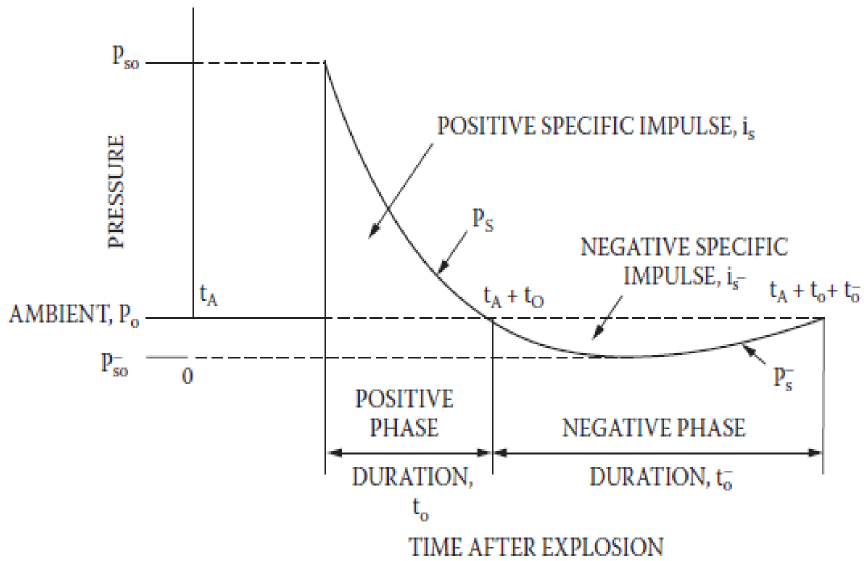


Figure.2 Field pressure- time history (TM 5-855-1, 1986[9])

The time of arrival (t_a) depends on the weapon yield and the distance from the point of burst. The time of arrival can be calculated using:

$$t_a = \begin{cases} u & x < x_m \\ u\dfrac{x_m}{x} + w\left(1 - \dfrac{x_m}{x}\right) & x \geq x_m \end{cases} \tag{3}$$

The positive phase duration(t_d) is the time for reaching the reference pressure. After that the pressure drops below the reference pressure until the maximum negative pressure (P_{min}) is reached.The negative phase duration is denoted as (t_n). The overpressure impulse is the integral of the overpressure curve over the positive phase (t_d) and this can be computed using:

$$t_d = \left(\frac{1640700 + 24629t_a + 416.15t_a^2}{10880 + 619.76t_a + t_a^2}\right)\left[\left(0.4 + \frac{0.001204t_a^{1.5}}{1 + 0.001559t_a^{1.5}}\right) + \left(0.0426 + \frac{0.548t_a^{0.25}}{1 + 0.00357t_a^{1.5}}\right).s\right] \tag{4}$$

The peak value of the dynamic pressure (P_{ds})as a function of peak pressure is:

$$p_{ds} = \begin{cases} \dfrac{5}{2}\dfrac{p_{os}^2}{7p_a + p_{os}} p_{os} < 300\ psi \\ \dfrac{5}{2}\dfrac{p_{os}^2(1 + 0.0604\pi + 0.0345\pi^2)}{7p_a + p_{os} - 72.2\pi^2 + 23.4\pi^3} p_{os} > 300psi \end{cases} \tag{5}$$

In the current study, A MATLAB code has been developed to simulate the nuclear explosion by calculating the pressure time history as shown in figure 3.This program contains yield of the weapon in kt TNT, type of blast (surface or air)and charge location.

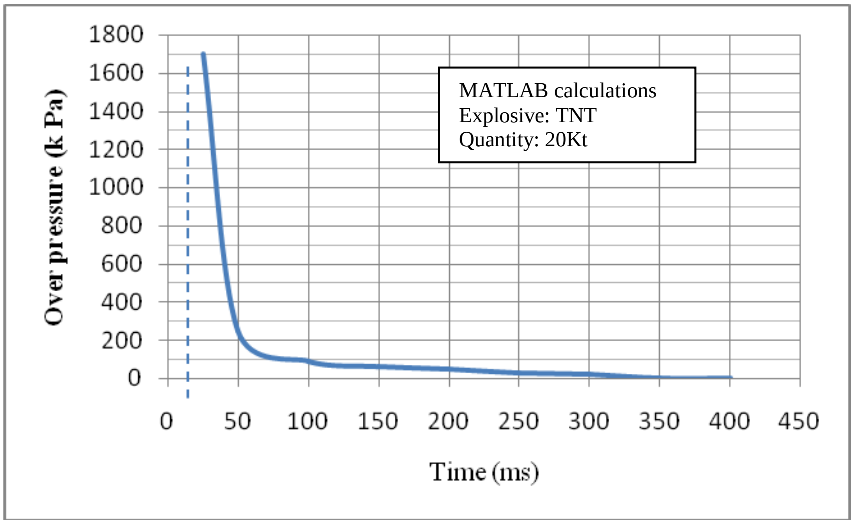


Figure 3: Pressure time history for 20 kt TNT at 210 m from MATLAB program

3. Finite element model consideration

The concrete slab is modeled as cylindrical reinforced concrete slab with thickness *t* =0.2m. The slab is modeled with plane-25 elements and is subjected to blast loading. The blast load information is shown in Table 1.

Table 1: Blast load information

Property	Value
Equivalent mass of TNT	20 kt TNT
Blast Location	(0.0, 0.0,-210) m
Type of Burst	Air Blast

In this section a general procedure of optimization using ANSYS is introduced. The optimization problem can be mathematically expressed as follows:

$$minimize \quad \|d(x)\|_z$$

Subject to

$$\varepsilon_j \leq \varepsilon_{max} \qquad for\ each\ element\ j$$
$$M \leq M_{max}$$
$$x^L \leq x \leq x^U$$

Virtually any aspect of a design can be optimized: dimensions (such as thickness), shape (such as fillet radii), cost of fabrication, material property, and so on. The optimal design has to meet all specified requirements and minimize *d(x)* which represents key items such as weight, size, displacement, stress, cost, and other factors.

The basic problem, in this paper, is a set of constraints to control the shape of concrete slab as follows :i) Plastic strain limits representing fracture strength ii) A minimum thickness for the concrete slab. Concrete slab and the surrounding support are made of concrete with the isotropic material failure criterion model. The model is used to define the failure of the concrete. The concrete slab is modeled using 51902 elements to establish design optimization model elements. The material properties of the concrete slab are given in table2.

Table 2: Material Properties and concrete constants

Property	value
Mass density	2700kg/m^3
Young's modulus	3E10Pa
Poisson's ratio	0.2
shear transfer coefficients for an open crack (βr)	0.4
shear transfer coefficients for a closed crack (βc)	0.8
Uniaxial tensile cracking stress *(fr)*	2E6Pa
Uniaxial crushing stress *(fc)*	3.33E7Pa

4. Computer code development

In the following the design optimization terminology, basic optimization terms and procedures are presented from the Release 11.0 Documentation for ANSYS [10]. An ANSYS input file contains a complete analysis sequence (preprocessing, solution, and post-processing). The file must contain a parametrically defined model, using parameters to represent all inputs and outputs to be used as design variables(DVs), state variables(SVs), and the objective function. The feasible design is a design that satisfies all specified constraints (those on the state variables as well as on the design variables).If any of the constraints is not satisfied, the design is considered 'infeasible'. The best design is the one, which satisfies all constraints and produces the minimum objective function value. The optimization methods are traditional techniques that strive for either minimization or maximization of an objective function subject to constraints. The ANSYS program always tries to minimize the objective function as it efficiently handles the minimization problems.

4.1 Sub-problem approximation method

The ANSYS optimization procedure offers several methods and tools that in various ways attempt to address the mathematical problem stated above. The optimization methods transform the constrained problem into an unconstrained one that is eventually minimized. The design tools, on the other hand, do not directly perform minimization. Use of tools offer alternate means for understanding design space and the behavior of the dependent variables. The optimization method employed in the present stud is sub problem approximation method. The problem was

seen to be clearly non-differentiable, attributable to the dynamic nature of the response. Hence, the use of gradient based optimizers is not appropriate.
This is an advanced zero-order method, which requires only the values of the dependent variables (state variables and the objective function), and not their derivatives. It is a general method, which can be applied efficiently to a wide range of engineering problems. There are two concepts that play a key role in the sub problem approximation method: the use of 'approximations' for the objective function and the 'conversion' of the constrained optimization problem to an unconstrained problem. To convert the optimization problem to an unconstrained one as they can efficiently handle the minimization techniques and this is achieved by introducing a penalty function.

Sub problem approximation method using sequential unconstrained minimization technique (SUMT) at each iteration has proven to be successful. In this method and by using curve fitting, relationships between the objective function and the DVs are established. This is done by calculating the objective function for several sets of DV values (that is, for several designs) and performing a least squares fit between the data points. The resulting curve (or surface) is called an approximation. Each optimization loop generates a new data point, and the objective function approximation is updated. It is this approximation that is minimized instead of the actual objective function. Decision parameters of the optimization methods are *i)* constrain of the derivative defines in the set of ($U_{ad,h}$ admissible displacement), *ii)* constrain for admissible thickness and *iii)* design variable, thickness of the concrete slab (t). State variables are volume of concrete slab, derivative of thickness and plastic strain limit, as shown in Table .3. An approximation is generated for each state variable and updated at the end of each loop.

4.2 Convergence checking

At the end of each loop, a check for convergence is made. The problem is said to be converged if the current, previous, or best design is feasible and any of the following conditions are satisfied:

- The change in objective function from the best feasible design to the current design is less than the objective function tolerance.
- The change in objective function between the last two designs is less than the objective function tolerance.
- The changes in all design variables from the current design to the best feasible design are less than their respective tolerances.
- The changes in all design variables between the last two designs are less than their respective tolerances.

5. Analysis file and solution process

The process involved in design optimization consists of the following general steps. The steps may vary according to optimization interactively performing in batch mode shown in figure 5.
First create an analysis file**.** This file should represent a complete analysis sequence by building the model parametrically (command PREP7), obtain the solution(s) (command SOLUTION) retrieve and assign to parameters which the response quantities will be used as state variables

and objective functions (command POST1/POST26). Finally review the resulting design sets data (command OPT). The procedure is organized into two files. Analysis file typically consists of commands to proceed a finite element analysis of an engineering problem. All the optimization variables must be parameterized. ANSYS-OPT input file contains optimization method, number of iterations, state variables, maximum mass limit, plastic strain limit and geometric limits, Upper and lower limit of each design variable and objective function.
General procedure of ANSYS optimization has been briefly described, see Figure.4. For more details, the ANSYS manual [11], Hutton [12] Haslinger, and Makinen [13] and Sokolowski and Zolesio [14] may consult.

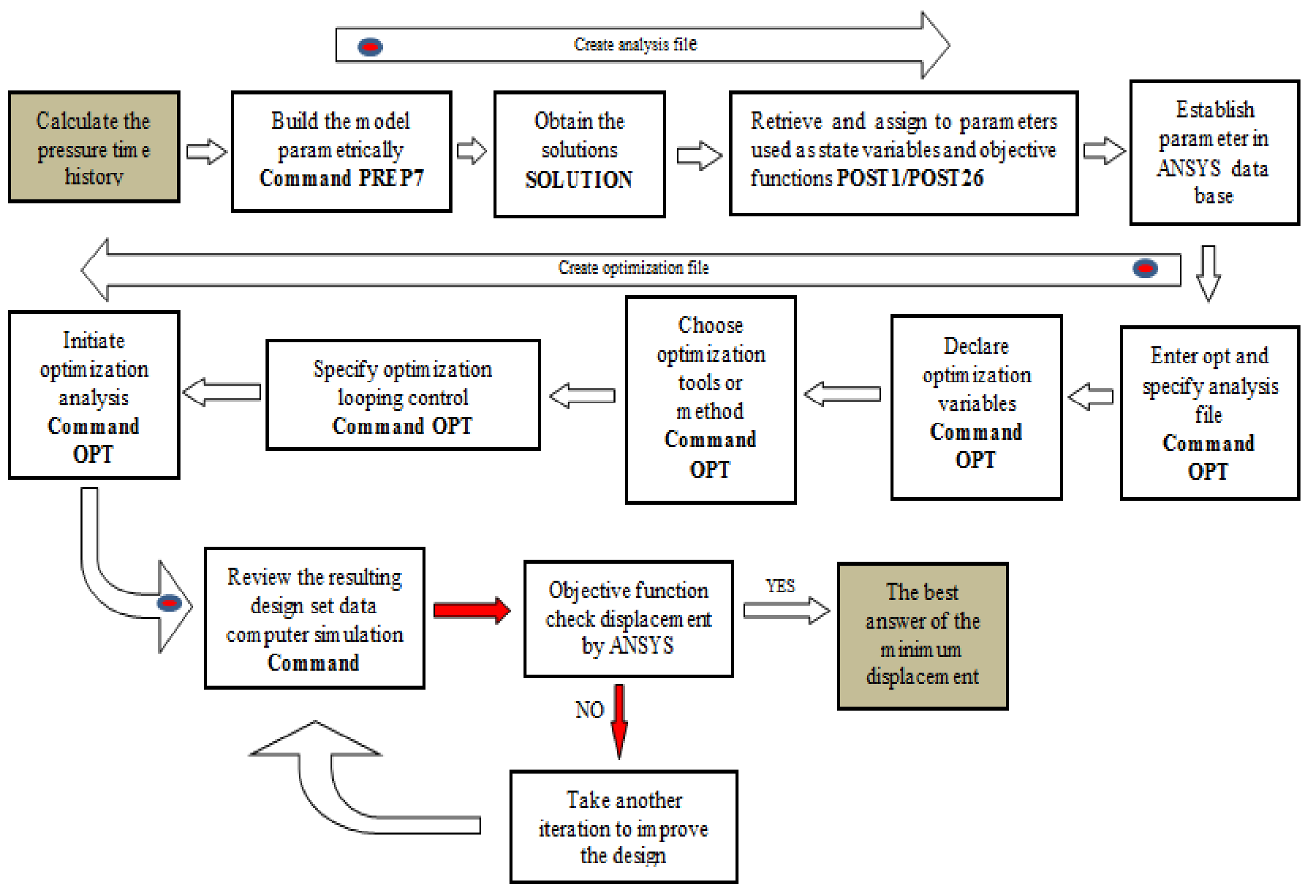


Figure.4 Process flowchart for using optimization routine by ANSYS

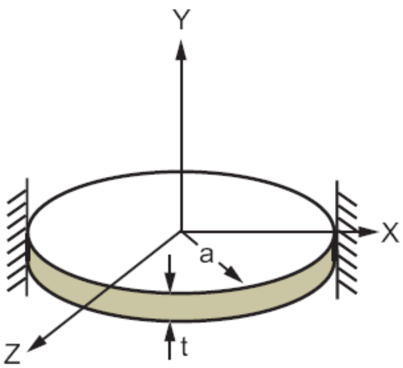
In the finite element problem, the concrete material properties and geometry properties are given in Table .3. Key points, element type and the Young's modulus of elasticity are defined in the analysis file. One set of real constants as AREA1, IZ1, AREA2, IZ2 for each element is defined.

Table 3: Input data of the state problem

Material properties	Geometric properties
E= $3*10^{10}$ Pa	radius =3.6 m

ν = 0.3	Thickness = 0.2m

Uniform cylinder concrete slab is considered as a base shape or base line design. Initial studies were carried out with the base line design to better understand the finite element analysis procedure. All parts are meshed using PLANE25 elements.The area of the element must be nonzero. The element must lie in the global X-Y plane as shown in figures 5 and 6 .The Y-axis must be the axis of symmetry for axisymmetric analyses.



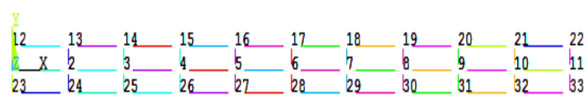


Figure.5. Problem schematic **Figure .6 Representative finite element model**

The following optimization parameter values are used (n) number of equal distance divisions n(h)= 10 and Positive constant restriction for U_{ad}as shown in table 4.

Table.4. Typical values of input parameters used in the input file Parameter Value

Design set	Parameter	Min	Max	Tolerance
Design variable	Thickness of slab	1.96849 in	3.93699in	0.01
State variable	Volume of concrete	689566 in^3	689569 in^3	0.1
	Derivative of the thickness	-1	1	0.01
	Plastic strain limit	0.15		
Objective	Displacement	Minimum		0.00001
Optimization method	Sub-problem approximation			

6. Numerical results

The shape optimization results are presented below for our selected model using thickness design variables (10). Initial base line (flat concrete slab) model and the resulting optimum shape of the slab obtained using sub problem approximation are shown in Figure 7. The result shown in the figure is for the front face. Rear faces is the same due to the high yield of TNT charge.
Due to element distortion, the initial run stops at t = 0.12sec. Rezoning is applied at this time to achieve complete loading. Solution control automatically adjusts solution parameters and attempts to obtain a robust, accurate solution. Element distortion is prevented by computing the determinant of Jacobian ratio from element nodal coordinates in every nodal point during optimization.

Figure.7 Optimized design for the top and bottom surfaces
of concrete slab with thickness (40cm)

Table 4 summarizes the results of concrete model. The 'optimum shape results are slightly better compared to 'base shape results', as is expected. In Table 5, max displacement (the objective function to be minimized) and the total mass are quoted.

Table 5.Results for baseline design and optimized designs

Property	Baseline design	Optimized design	Change
Max Displacement(inch)	0.825inch	0.42 inch	50.9%
Total mass (kg)	5832	6415.2	(1.1%)

Displacement plots for baseline design and optimized design is given in Figure 8. The overall thickness is reduced from 40 cm with the introduction of protrusions on both sides as shown in Figure 9. It can also be shown that the optimum shape reduces the displacement of the concrete slab. Displacement reaches its peak at *t* = 0.003s. The displacement values of optimized concrete slab designs are 50.9% lesser compared to baseline design.

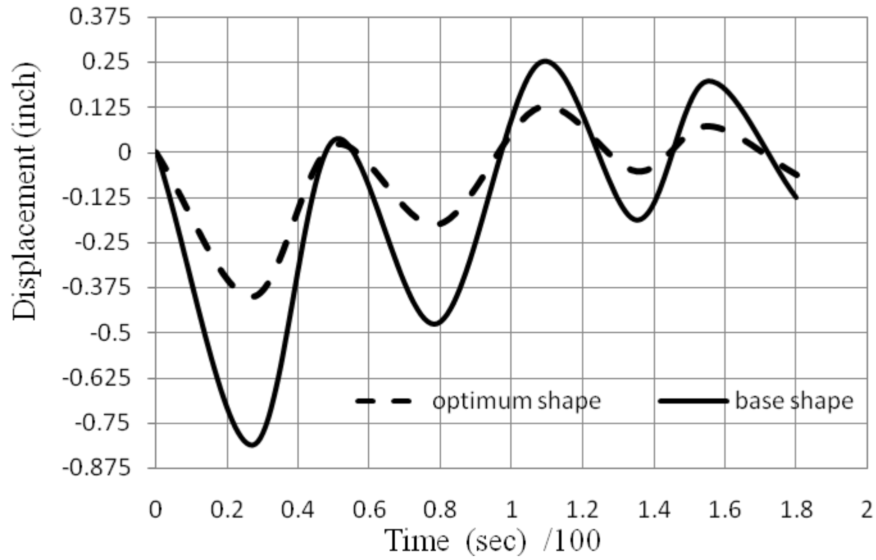


Figure.8 Comparison of displacement time history between basic and optimum shape

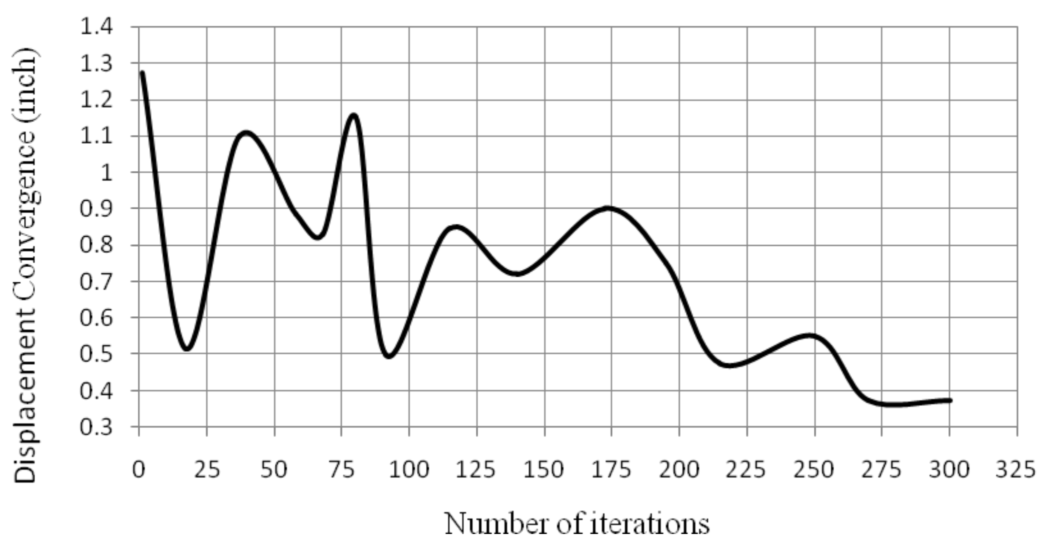


Figure .9 Displacement functional values for sub problem approximation method

7. Conclusions

Shape optimization of a concrete slab under blast load is carried out. This research is timely dependent as very little work has been done in this area. Direct application to design of fortified structures and protection layer. MATLAB is linked to an ANSYS optimizer. Design and state

variables on the base concrete slab shape are chosen, and the shape is optimized within these constraints.
Elastic strain criteria, mass and element distortion are all resulting during optimization. The objective is to minimize the dynamic displacement of the concrete slab. In the chosen constraints of base shapes, and for the given blast load data, the optimized shape came out to be a double protrusion; equal protrusion on both the sides of the concrete slab. Displacement decreased by 50.9% as an average compared to the baseline (flat concrete slab) design. Total mass increased as well and was well within the limit. This work lays down a methodology of shape optimization against blast loading using ANSYS.

8. References

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