IMAGE CODING USING HYBRID VECTOR QUANTIZATION

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ABSTRACT

Statistic approach of vector quantization uses code-books of rounded vectors which allow quasi-optimal coding for a given rate. Therefore these code-books have no structure and require big memory size. Regular lattices of points give the possibility of generating a great number of points from a short number of vectors. This can permit us to solve the problems of the statistic approach. However the lattices presents their own problem, effectively they are only applied for uniform distribution sources. In order to avoid these two kinds of problems, another approach is investigated; It consists of designing a new quantizer by combining the two techniques. In this paper, we present this approach which we call hybrid; first few statistic vectors of the source are performed by using the LBG (Linde, Buzo and Gray) algorithm [2], then each of them leads to a Gosset lattice. The results obtained in image coding are also presented.

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I. NTRODUCTION

Image compression is essential for many applications such as TV transmission, video conferencing, graphic images, etc. Many vector quantization algorithms have been used in order to reduce the transmission bit rate and the memory size required for image storage. Unfortunately every algorithm presents its own inconvenience. Vector quantizers could be divided into two different families :
- Statistic vector quantizers and,
-Algebraic vector quantizers.

Both of them presents advantages and inconvenience. The first one presents the advantage to give a code-book of rounded vectors which allows a quasi-optimal coding for a given bit rate, however it presents the inconvenience of technology limits of storage and calculus complexity. The second one presents the advantage to generate a great number of points from a reduced number of vectors, however this family presents the inconvenience to be applied only for coding uniform distribution sources.

In this paper we present a new approach of vector quantization by the design of a hybrid quantizer (Statistic & Algebraic), this is performed by combining the two techniques in order to avoid the inconvenience found by using each approach alone.

In section II we introduce the vector quantization theory, in section III we present the statistic algorithm LBG, in section IV we define the lattices of regular points and we present an efficient and quick algorithm to generate the Gosset lattices, in section V we present the hybrid technique and in section VI we present the results obtained in image coding.

II. VECTOR QUANTIZATION

A vector quantization (VQ) is an application from a set of input vectors **x**, generally the real k dimension Euclidean space R^k, in a finite sub-set C called code-book of R^k. The *code-book* C is formed by N different elements called *code-words*:

$$C = \{ yi \; ; \; i = 1,2, \ldots , N \} \quad (1)$$

figure(1) illustrates the Principe of vector quantization.

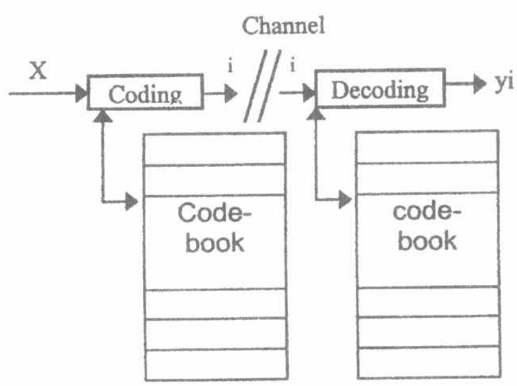
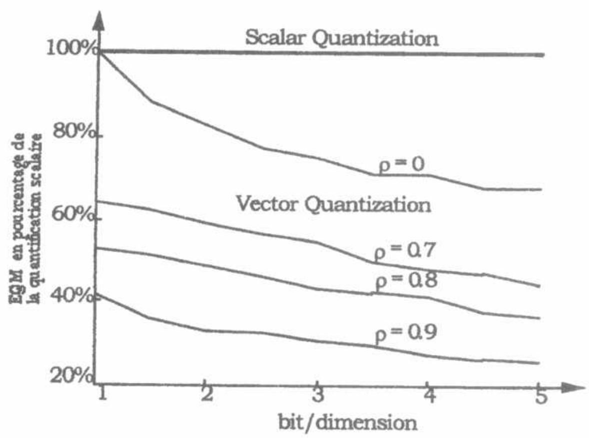


Fig1:Vector Quantization Principle

In the coding part each input vector **x** is associated with a code-word yi of the code-book C according to the nearest neighbour criteria. Only the index i is transmitted to the decoder. There are N indices corresponding to N different code-words of the

code-book C. The same code-book C is used at the decoding part, the code-word yi representant of the vector **x**, is found thanks to the received index **i** [1],[7][10].
Either if the input signal is uncorrolated, or the components of the input vectors are independent, it is always better to use a vector quantization VQ instead of a scalar quantization (SQ); the figure(2) illustrates the performance in terms of mean square error (MSE) of a VQ about two dimensions in percentage of performance of two SQ; the results are given for different values of the correlation and for different rates per dimension [9].



Fig(2):Comparison between performances of two SQ and a VQ applied to a two dimension vector, where each co-ordinate is gaussian with correlation ρ.

III. Statistic VQ design :

This approach is based on an iterative algorithm, so called LBG [2]. This algorithm is used for an unknown distribution training sequence.
Let X = { x_j ; j =1,2, ... , M } a training sequence composed of M vectors about k dimensions,
Y_0 = { y_i ; i =1,2, ... , N } an initial code-book and D0 = ∞ is the initial distortion, the design of a statistic VQ is divided to three steps :
1. For each input vector x_j a reproduction vector yi of the Y_0 code-book is assigned, the partition which minimises the distortion is then found.
2. The average distortion is computed according to this expression :

$$D= \frac{1}{M}\sum_{j=1}^{M} \min_{y\in Y_0} \| \mathbf{x}_j - \mathbf{y} \|^2 \qquad (2)$$

If D did not change, the computation is finished and

$$Y = \{ yi \; ; \; i =1,2, \dots , N \} \qquad (3)$$

represents the optimum reproduction vectors, otherwise continue.
3. We compute the centroïds, then the code-words of Y are replaced by these new centroïds. Notice that if in the partition Ri the number of vectors n_i =0, we let

cent(R_i) = y_i the last code-word then we repeat the steps 1 and 2.
In the above algorithm an initial code-book is assumed in order to start the algorithm. The useful technique to calculate this code-book is the *splitting* one, where the centroid for the training sequence is calculated and splited into two close vectors. The centroids for the two partitions are then computed. Each resulting vector is then splited into two vectors and the above procedure is repeated until an N level initial reproduction vector is created. Splitting is performed by adding a fixed perturbation vector ε to each vector yi producing two vectors $y_i + \varepsilon$ and $y_i - \varepsilon$.

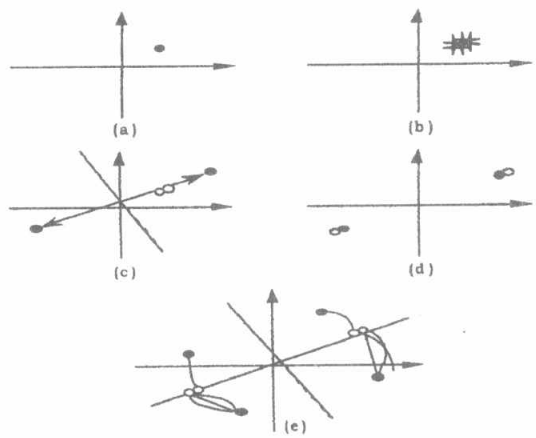


Fig. 3:Design of the initial code-book by splitting.

IV. REGULAR POINTS LATTICES

IV.1. Definition

Let a point of R ^{k}: $\mathbf{x} = (x_1, x_2,., x_k)^T$ $_1$, u_2,., $u_k)^T$ and rayon σ , which represents the set of points **x** :

$$\sum_{i=1}^{k} (x_i - u_i)^2 = \sigma^2 \qquad (4)$$

In a space of k dimension, we consider a regular sphere packing of the same rayon σ, a regular lattice Λ is then formed by the centres of the different spheres, let call O the centre of the lattice. We can easily prove that there is k spheres with centres $\mathbf{v}_i = (v_{i1}, v_{i2},., v_{ik})^T$ and every point of the lattice can be calculated by the expression:

$$\sum_{i=1}^{k} \varepsilon_i \cdot v_i \quad / \quad \varepsilon_i \in Z \qquad (5)$$

The vectors V_i forms the basic cell of the lattice, and they are linearly independent [3],[4],[11],[12]. Figure 4 illustrates a bidimentional lattice.

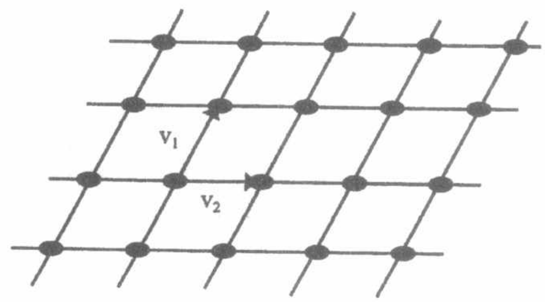


Fig. 4: Bidimentional lattice

The generating matrix of a lattice is given by:

$$M = \begin{pmatrix} v_{11}, v_{12}, \dots, v_{1m} \\ v_{21}, v_{22}, \dots, v_{2m} \\ . \\ . \\ . \\ v_{k1}, v_{k2}, \dots, v_{km} \end{pmatrix} = \begin{pmatrix} v_1^T \\ . \\ . \\ . \\ v_k^T \end{pmatrix}$$

so every point of the matrix can be written as:

$$y = M^T \cdot \varepsilon \text{ with } \varepsilon = (\varepsilon_1, \varepsilon_1, \dots \varepsilon_k)^T / \varepsilon_i \in Z \quad (6)$$

IV.2. The Gosset lattice E_8

The Gosset lattice noted E8 and so called "*diamant*" lattice is defined by the relation [9],[12],[13]:

$$E_8 = D_8 \cup \left[\left[\frac{1}{2} \right] + D_8 \right] \qquad \left[\frac{1}{2} \right] = \left(\frac{1}{2}, \frac{1}{2}, \dots, \frac{1}{2} \right)$$

We can notice that this lattice is obtained by the union of D_8 lattice and the one defined by $\left[\frac{1}{2} \right]$+D8 , we can also define it by the expression:

E_8 = { y'= $(y_1', y_2', .., y_k')^T$, y"= $(y_1'', y_2'', .., y_k'')^T$ /

$y_i' \in Z$ et $\sum_{i=1}^{8} y_i' = 0 \bmod(2)$,

$y_i'' \in (Z + 1/2)$ and $\sum_{i=1}^{8} y_i'' = 0 \bmod(2)$ } (7)

IV.2.1. Quantification with The E8 lattice

Let $E_8(m)$ be the set of points of a sphere and $N_8(m)$ the number of points of this set:

$$E_8(m) = \{y | y \in E_8 \cap S_m\} \qquad (8)$$

Where S_m represents the sphere with the rayon $\sigma = 2\sqrt{2m}$, $E_8 = \bigcup_m E_8(m)$,

if we use $E_8(m)$ like a quantizer, several rates be chosen.

V. Hybrid quantization algorithm

In this approach we consider that the source has globally non-uniform distribution but locally uniform. By using the LBG algorithm we determine some statistic points (4, 8, 16, .); then each point leads us to a regular lattice of points; figure (5) illustrates this approach.

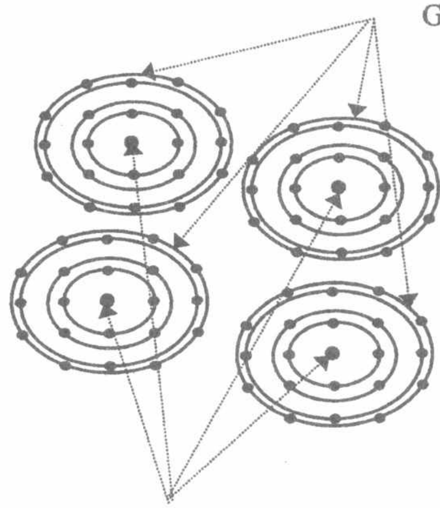


Fig. 5 : Hybrid approach.

The first step consists of computing a statistic point, for this we have to:

- Fix the number N (N little) of points,
- Compute the LBG algorithm,
- Save the statistic points in a file.

The second step, is to generate Gosset lattices centred by each statistic point:
Let **x** be an 8 dimension input vector, and y_i i = 1,2,.,N the statistic code-words

- Compute the euclidean distances $d_i(x, y_i) = \sum_{j=1}^{8} (x_j - y_{ij})^2$ (9)

Choose the code-word index i for which the euclidean distance is minimal

$$d_i(x, y_i) < d_j(x, y_j) \quad \forall j = 1, 2,., N \text{ and } i \neq j$$

- Translate the input vector **x** to y_i ,

$$\widetilde{x} = ((x_1 - y_{i1}), (x_2 - y_{i2}), \ldots, (x_N - y_{iN})) \quad (10)$$

The figure (7) shows that even we have no information about image statistics we can obtain acceptable quality of image with a great compression ratio, thing which confirms the *sphere hardening* theory [9]. Figures (8,9) show that a few statistic vectors could increase sensibly the image quality without affecting considerably the compression ratio.

VII. Conclusion

The hybrid quantization technique, is a new approach which is used in order to avoid the different problems that the statistic and the algebraic approaches usually shows. As illustrated by the obtained results this technique gives satisfaction and more we increase the
number of statistic points (16, 32) the result would be better. However another problem must be resolved. It is important to think about another method that not only resolves the problems treated in this paper, but also to reduce the bit rate which stays important.

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- Compute the euclidean distances between $\tilde{x}$ and the lattice vectors centred by y_i
- Choose the code-word index **j** for which the euclidean distance is minimal
- The couple (i , j) is the index of the input vector x

V.1. Hybrid quantizer decoding algorithm

Let (i , j) be the received code, for decoding it we follow this steps:

- Generation of the lattice vector by the index j , let g_j be this vector
- Research of the statistic vector by it's index i , let y_i be this vector
- Computation of the restated vector $\tilde{x} = g_j + y_i$

VI. RESULTS

Fig.6 : Original image 8 bits / pixel

Fig.7 : Image coded with Gosset lattice E8(1) 1.25 bits/pixel, compression ratio 84.37%

Fig.8 : Image coded with Gosset lattice E8(1)+4 statistic points 1.5 bits / pixel, compression ratio 81.25%

Fig.9 : Image coded with Gosset lattice E8(1)+8 statistic points 1.5 bits / pixel, compression ratio 79.69%