Military Technical College
Kobry El-Kobbah,
Cairo, Egypt

6th International Conference
on Electrical Engineering
ICEENG 2008

Orbit determination using distance measurements

By

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Abstract:

The paper provides methods for precise geostationary satellites orbit determinations using only distance (ranging) measurements. The results for orbit determination using both dual Ranging from two separate earth station locations and classical orbit determination from an earth station are introduced.

A Matlab program is developed to perform orbit determination process. The developed program takes into account the dynamic model of the satellite orbit including orbit perturbations due to non_ spherical earth shape, the gravitational forces of the sun and moon, and the atmospheric drag. Acceptable results where foreseen in comparison to the flight proven software tool.

The limitations of ranging measurements will be demonstrated, and the ways to minimize it will be shown.

Also, spread spectrum orbit determination is provided, and an analysis on the selection of suitable pseudo-random code generator for implementation in spread spectrum orbit determination technique is given using Matlab simulink program.

Keywords:

Orbit, coordinate systems, orbit determination, dual ranging, spread spectrum

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1. Introduction:

Ranging and angular measurements are used together for calculating the trajectory of a satellite with the help of models of orbit evolution, but due to the problems that rise from the use of the angular measurements in the orbit determination process as:

a) The mechanical precession is required which leads to very high operational costs.
b) Azimuth-Elevation accuracy that depends on the mechanical antenna system not on the electrical system as the ranging.
c) The errors on the measurements are often biases, that are slowly evolving, like the alignment of the mechanical axes, or cyclic (due to day-night temperature fluctuations) or variable like the deviation between the targeted radio-frequency direction and the mechanical direction.

Thus, the paper provides a method for orbit determination based only on the ranging measurements (Pseudo-range).

2. Orbital Parameters:

The elements of an orbit [1], [3], [4] are the parameters needed to specify that orbit uniquely. The orbital elements are called the set of Keplerian elements. The Keplerian elements are six: semi-major axis (a), eccentricity of the ellipse (e), inclination angle (i), right ascension of ascending node (Ω) , argument of perigee (ω), true anomaly (ϑ).

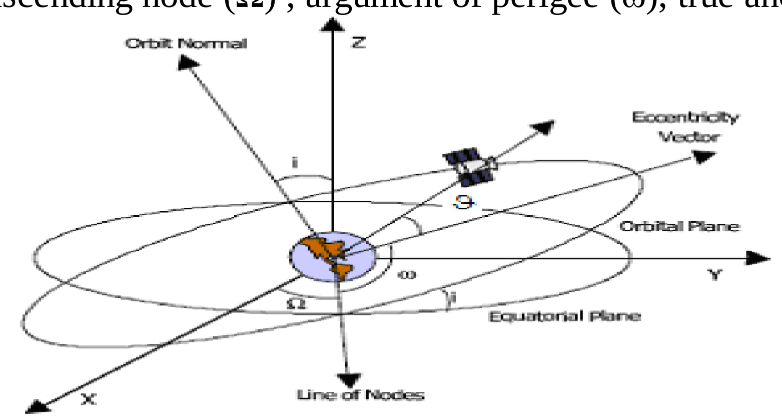


Figure 1*: Orbital angles*

But for geostationary orbit, the inclination angle (i) nearly equal to zero, so the values of ω and Ω can not be given with sufficient accuracy, as the position of the ascending node is not determined accurately. Thus, the parameters in the kepelerian set are slightly modified to include implicitly the parameters (i, ω, Ω) to avoid singularity, as shown:

Semi-major axis:	a	(1)
Eccentricity vector in the x , y directions:	$\bar{e}_x = \bar{e}\cos(\omega+\Omega)$, $\bar{e}_y = \bar{e}\sin(\omega+\Omega)$	(2)
Inclination vector in the x , y directions :	$\bar{i}_x = \sin(i)\cos(\Omega)$, $\bar{i}_y = \sin(i)\sin(\Omega)$	(3)
Longitude:	$l = \omega + \vartheta + \Omega - GAST$	(4)

3. Principle of RF distance measurement

Radio frequency is employed for satellite orbit determination [10], which is accomplished by transmitting a radio frequency signal and receiving - in the unavoidable system noise - the wave reflected or repeated by the target. It is further analyzed at the receiver to determine the target's distance. RF signals are subject to a fixed propagation rate c, the speed of light. A transmitter sends a signal at the instant t_1, which reaches a receiver at the instant t_2. The difference between t_1 and t_2 is directly related to the distance the RF signal has traveled by the speed of light.

$$d = c * (t_2 - t_1) \tag{5}$$

This round-trip time is also known as the time delay in the receiving equipment and permits the distance to the satellite (d/2) to be calculated. The relation between this time delay and the phase shift is:

$$\phi = 2\,\pi\,f\,\tau \tag{6}$$

The distance measurements are performed by means of specific sub-carriers which modulated at the transmitter and then demodulated in the receiver.

Various approaches are possible according to the nature of the sub-carrier, these include:

a) Fixed frequency tone.
b) Variable frequency, modulation by a Pseudorandom (PN) sequence.

As, tone system is used in this paper, in this case the sub-carrier is a sinusoidal wave of fixed frequency.

4. RF distance measurement configurations

The RF ranging techniques can be classified according to their operation methods into two main items [10], [13]:

a) Two-way Distance Ranging: It is customary to use a transponder on the desired target. This device receives the interrogator RF pulse and replies to it with a much stronger pulse, usually on a different frequency, as shown in figure (2).

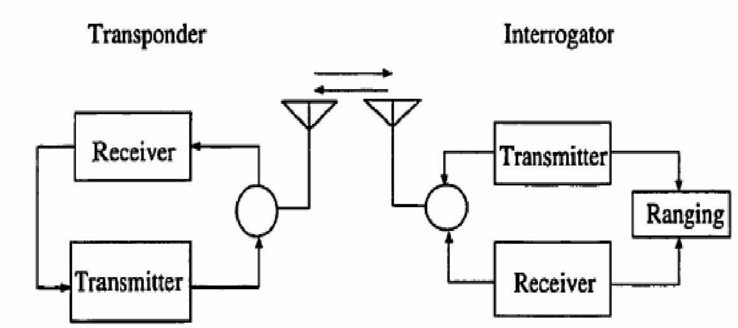


***Figure 2**: Two-way distance ranging*

b) One-way Distance Ranging: If both stations are provided with stable synchronized clocks, the distance between them can be established by a one-way transmission whose elapsed time is measured with reference to the two clocks, as shown in figure (3).

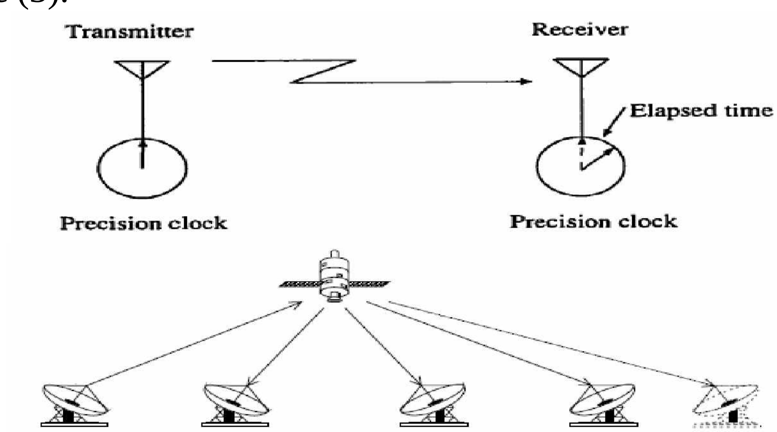


***Figure 3**: One-way distance ranging for single and Multi-station systems*

In this paper two-way distance method is used due to the synchronization problems foreseen in the one-way distance method that makes it difficult to be implemented.

5. Protocols of distance measurement

The distance measurements (ranging) makes use of one major tone for fine measurement and six minor tones for ambiguity resolution. Ranging is performed in two steps, ambiguity resolution and distance measurement.
During the ambiguity resolution phase, all minor tones are sent in sequence, each one being sent together with the major tone.
Ground systems uses different sets of frequencies, as shown in figure (4).

Set A:

f (Hz)	Major tone	Minor tones					
	100,000	20,000	4,000	800	160	32	8
Range R = d/2 (Km)	1.5	7.5	37.5	187.5	937.5	4687.5	18750

Set B:

f (Hz)	Major tone	Minor tones					
	27,777	5556	1111	222	44	9	2
Range R = d/2 (Km)	5.4	26.9	135.01	675.675	3409.09	16666.6	75000

***Figure 4**: Various sets of major and minor tones and their equivalent range ambiguity*

These tables give the distance ambiguity as a function of the frequency of the tone used. The phase shift is measured modulo 2π, the measurement will be the same for all values of R such that $2 \pi f (2 R)/C = K 2\pi$. The distance ambiguity corresponds to the modulus of the distance obtained for K = 1.

For geostationary satellite, the frequency of the tone must be at most 8 Hz for the measurement to be made without ambiguity.
The ranging process behavior could be summarized as shown in figure (5).

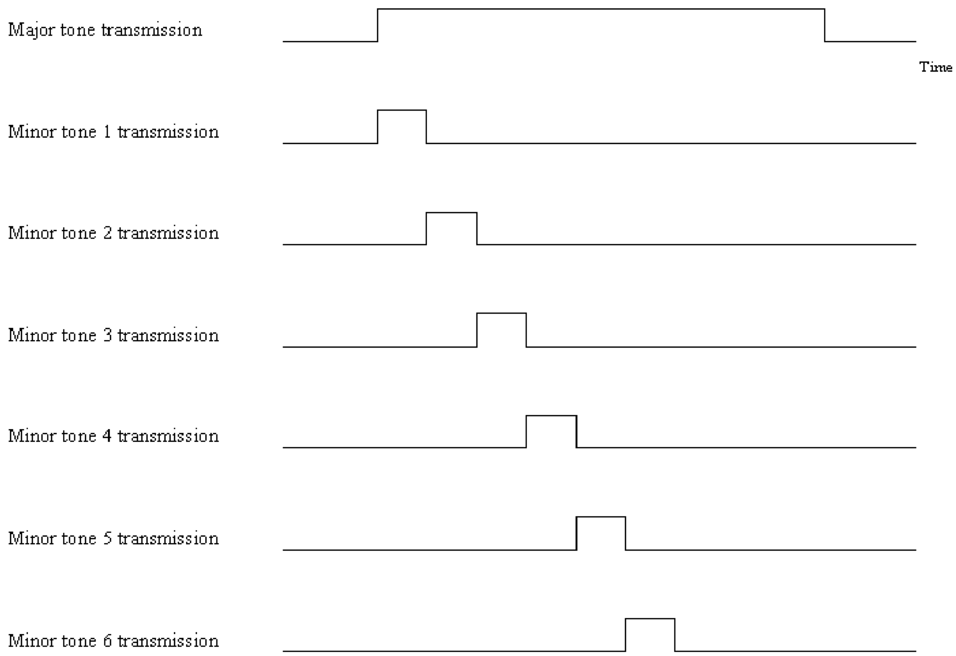


Figure 5*: Ranging sequence performance*

Taking as an example the set A of frequencies specified in figure (4) to demonstrate the procedure for the transmission of the minor tones in order to solve the ambiguity problem is shown in figure (6).

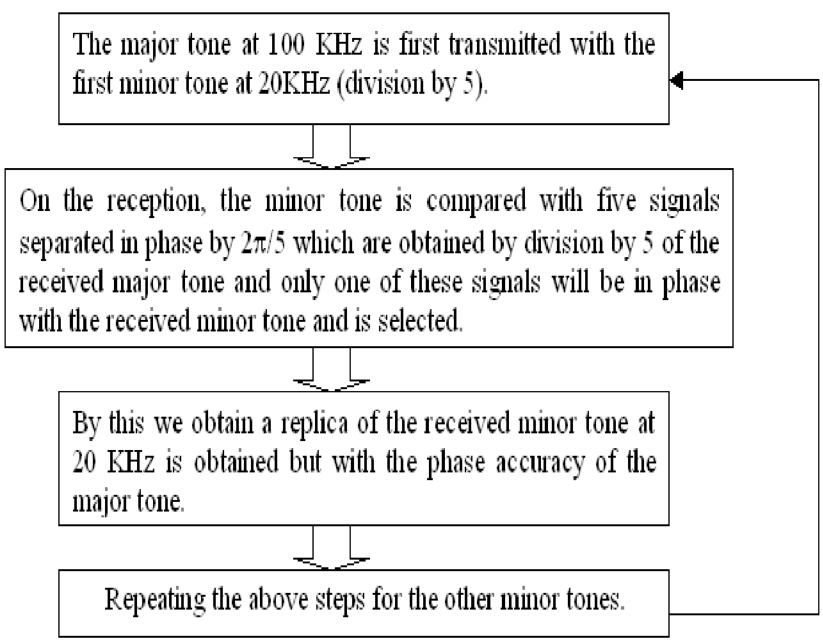


Figure 6*: Tone ranging protocol*

6. Orbit Determination Algorithm

The algorithm presented is based on statistical least squares (LS) method [9].

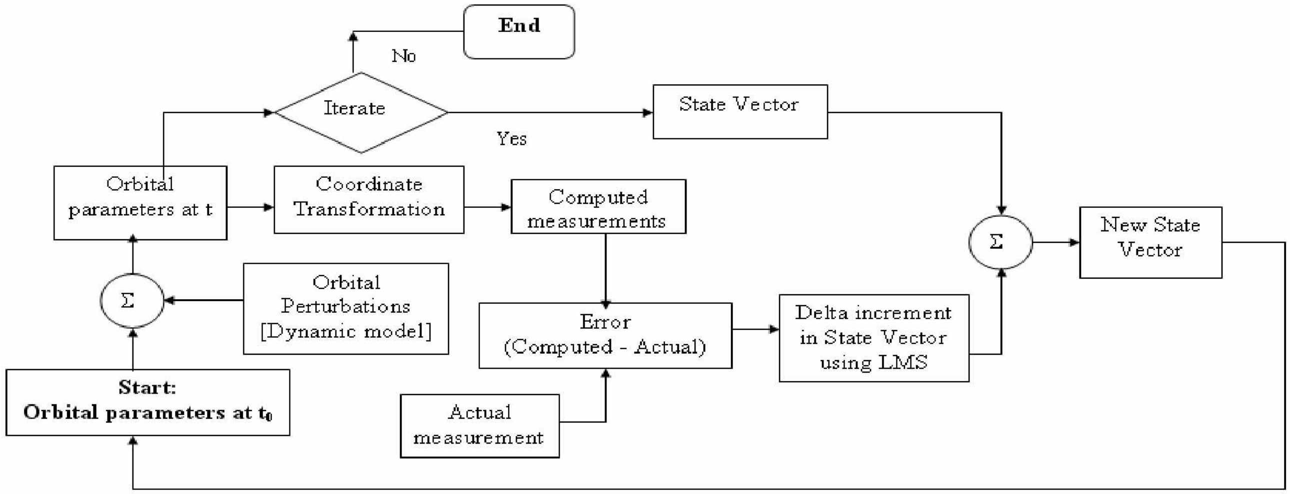


Figure 7*: Simplified Block diagram for Orbit determination*

6.1 Reference Frames

Co-ordinate transformation systems are needed in order to determine the computed range measurement from the given orbital parameters. Thus, four co-ordinate systems [1], [2], [4] are introduced to define the satellite position relative to the ground tracking earth station. These coordinate systems are the following:

1. The Perifocal coordinate (*p*,*q*,*w*).
2. The Geocentric coordinate (*I*,*J*,*K*) ,
3. The Satellite in rotating frame coordinate (*X*,*Y*,*Z*),
4. The Topocentric- Horizon coordinate (*S*,*E*,*Z*).

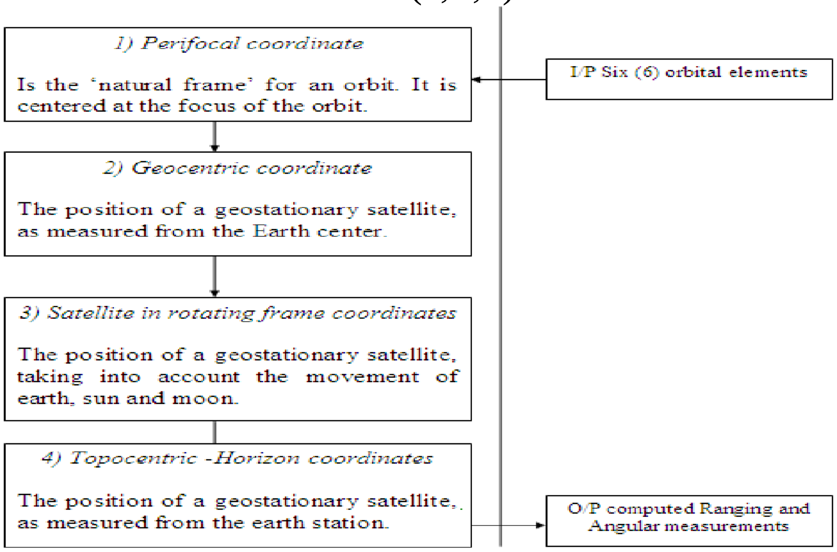


Figure 8*: Coordinate systems transformations*

Each of these reference frames are presented in details;

6.1.1 Perifocal coordinate (p , q ,w)

The perifocal frame is the 'natural frame' for an orbit. It is centered at the focus of the orbit.

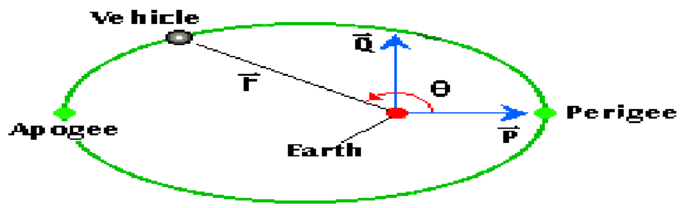


***Figure 9**: Perifocal coordinates*

From this coordinate system the magnitude of the position vector r in the P-Q frame plane is computed;

$r = a *(1-e*\cos(E));$ (7)

$r_p = r*\cos(\vartheta);\ r_q = r*\sin(\vartheta);$ (8)

6.1.2 Geocentric coordinate (I, J, K)

The general Geocentric Equatorial Coordinate System (*IJK*) is also known as the Earth-Centered Inertial (ECI) system. ECI's origin is at Earth's center, and its fundamental plane is the equator.

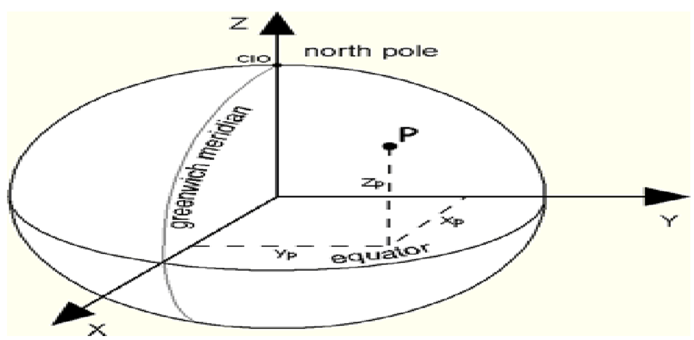


***Figure 10**: Geocentric coordinate*

The *I*-axis (or +*X*-axis) points towards the vernal equinox; the *J*-axis (or +*Y*-axis) is 90^o to the east in the equatorial plane; and the *K*-axis (or +*Z*-axis) points towards the North Pole. Computing the position components X, Y, Z,

$r_i= [\cos(\Omega)*\cos(\omega)-\sin(\Omega)*\cos(i)*\sin(\omega))*r_p + (-\cos(\Omega)*\sin(\omega)- \sin(\Omega)*\cos(i)*\cos(\omega)]*r_q;$ (9)

$r_j=[\sin(\Omega)*\cos(\omega)+ \cos(\Omega)*\cos(i)*\sin(\omega))*r_p +(-\sin(\Omega)*\sin(\omega) + \cos(\Omega)*\cos(i)*\cos(\omega)]*r_q;$ (10)

$r_k= [\sin(i)*\sin(\omega))*r_p + (\sin(i)*\cos(\omega)]*r_q;$ (11)

This coordinate system is considered inertial, but the equinox and plane of the equator move over time. Thus in order to take into account the relative motion of the satellite with respect to the earth, introduce the next coordinate system.

6.1.3 Satellite in rotating frame coordinate (X,Y,Z)

Known also as Satellite Radial coordinate system (*RSW*), moves with the satellite.

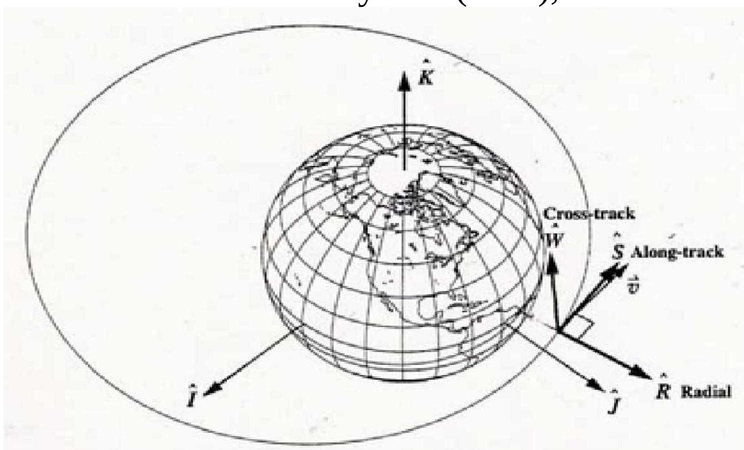


***Figure 11**: Rotating frame coordinate*

Computing the rotating system coordinates:

$$X_r = \cos(GAST)*r_i + \sin(GAST)*r_j \quad (12)$$

$$Y_r = -\sin(GAST)*r_i + \cos(GAST)*r_j \quad (13)$$

$$Z_r = 1*r_k \quad (14)$$

6.1.4 Topocentric-Horizon coordinate (S,E,Z)

The position of a geostationary satellite, as measured from the Earth station, is usually given in terms of the azimuth and elevation angles. The observer's horizon becomes the reference plane and his position, the origin.

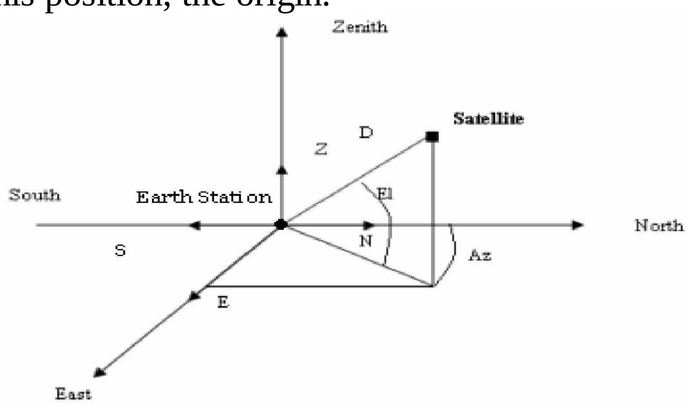


***Figure 12**: Topocentric coordinate*

Now calculating the sub-satellite points (sub-satellite longitude and latitude) as shown in equations (15) and (16) respectively:

$$L_s = \pi/2 - \cos^{-1}[\, Z_r / (X_r^2+Y_r^2+Z_r^2)^{0.5}\,] \quad (15)$$

$$l_s = \begin{cases} -\tan^{-1}(Y_r / X_r) & Y_r > 0, X_r > 0 \\ \pi - \tan^{-1}(Y_r / |X_r|) & Y_r > 0, X_r < 0 \\ \pi/2 + \tan^{-1}(|X_r| / |Y_r|) & Y_r < 0, X_r < 0 \\ -\tan^{-1}(|Y_r| / X_r) & Y_r < 0, X_r > 0 \end{cases} \quad (16)$$

As, distance from center of earth to the satellite (orbital radius) is;

$$r_s = (X_r^2+Y_r^2+Z_r^2)^{0.5} \quad (17)$$

$$\delta = \cos^{-1} [\cos(L_e)*\cos(L_s)*\cos((l_s-l_e)) + \sin(L_e)*\sin(L_s)] \quad (18)$$

Thus, Ranging (R_g) equations is:

$$R_g = r_s*[1+(Re/r_s)^2- 2*(Re/r_s)*\cos(\delta)]^{0.5} \quad (19)$$

6.2 The State Vector

The State Vector is considered another way to determine the orbit rather than the set of orbital parameters, where the orbit is determined through the definition of the position and velocity.

The Matlab program computes the optimum increment in the state vector – equation (26) – and adds it to the initial state vector to produce a new state vector which is more precise. This new state vector is transferred back to the orbital parameters.

Transformation from the orbital parameters to the state vector [2]: position (X, Y, Z) and velocity (dX, dY, dZ), is done as shown in the following equations:

$$X = r*[\cos(\omega+\vartheta)*\cos(\Omega) - \sin(\omega+\vartheta)*\sin(\Omega)*\cos(i)] \quad (20)$$

$$Y = r*[\cos(\omega + \vartheta)*\sin(\Omega) + \sin(\omega + \vartheta)*\cos(\Omega)*\cos(i)] \quad (21)$$

$$Z = r*[\sin(\omega + \vartheta)* \sin(i)] \quad (22)$$

$$dX = -\mu/H * [\cos(\Omega)*(\sin(\omega + \vartheta) + e*\sin(\omega)) + \sin(\Omega)*(\cos(\omega + \vartheta)+ e*\cos(\omega))*\cos(i)] \quad (23)$$

$$dY = -\mu/H * [\sin(\Omega)*(\sin(\omega + \vartheta) + e*\sin(\omega)) - \cos(\Omega)*(\cos(\omega + \vartheta)+ e*\cos(\omega))*\cos(i)] \quad (24)$$

$$dZ = \mu/H * [\cos(\omega + \vartheta) + e*\cos(\omega)]*\sin(i) \quad (25)$$

Based on the Goodyear relations [2], [3], [6], [8] the best estimate increment in the state vector is given by:

$$\Delta x_k = (D^T D)^{-1} * D^T * y \quad (26)$$

Where:

$$D = D'' * \Phi \quad (27)$$

6.3 The Dynamic model

The perturbing forces are taken into account in the dynamic model of the satellite orbit, as shown in the following equations:

Progression of the eccentricity (^{o}/day):

$$\partial e_x/\partial t = -1.5 * (C/(n*a)) * \sin(\Omega_{sun}) , \partial e_y/\partial t = 1.5 * (C/(n*a)) * \cos(\Omega_{sun}) \quad (28)$$

Where:

$C = (1 + \rho) * (S_a/m) * 4.51*10^{-6}$

Progression of the inclination (^{o}/day):

$$\partial i_x/\partial t = -3.6*10^{-4} * \sin(\Omega_M) , \partial i_y/\partial t = (23.4 + 2.7 * \cos(\Omega_M)) *10^{-4} \quad (29)$$

Where:

$\Omega_M = 12.111 – 0.052954 * T$

Progression of the argument of perigee (^{o}/day):

$$\partial\omega/\partial t = (3/4) * n * A* J_2 * [5 * \cos^2 (i) – 1] \quad (30)$$

Where:

$A = Re^2/a^2*(1-e^2)^2$

Progression of the right ascension of ascending node (°/day):

$\partial\Omega/\partial t = (-3/2) * n * A * J_2 * \cos(i)$ (31)

Progression of the mean anomaly:

$\partial M/\partial t = n * [1 + (3/4) * A * (1 - e^2)^{0.5} * J_2 * (3 * \cos^2(i) - 1)]$ (32)

Progression of the longitude (°/day):

$\partial l/\partial t = k_l*(a - a_s) + 2 * e * \cos(M)* \partial M/\partial t + 2*\sin(M)* \partial e/\partial t$ (33)

7. Simulation:

In this section we will provide simulation results for the following:

(a) Orbit determination (OD) example and a generated Matlab results for OD using dual ranging measurements

(b) A three years orbit determination comparison study for two spacecraft using a flight proven orbitoagraphy tool and suggested program generated by Matlab. Each determination was based on two complete days of tracking data using two earth stations located in Cairo and Alex sites.

The flow chart of the developed Matlab program for dual ranging orbit determinations is shown in the next figure.

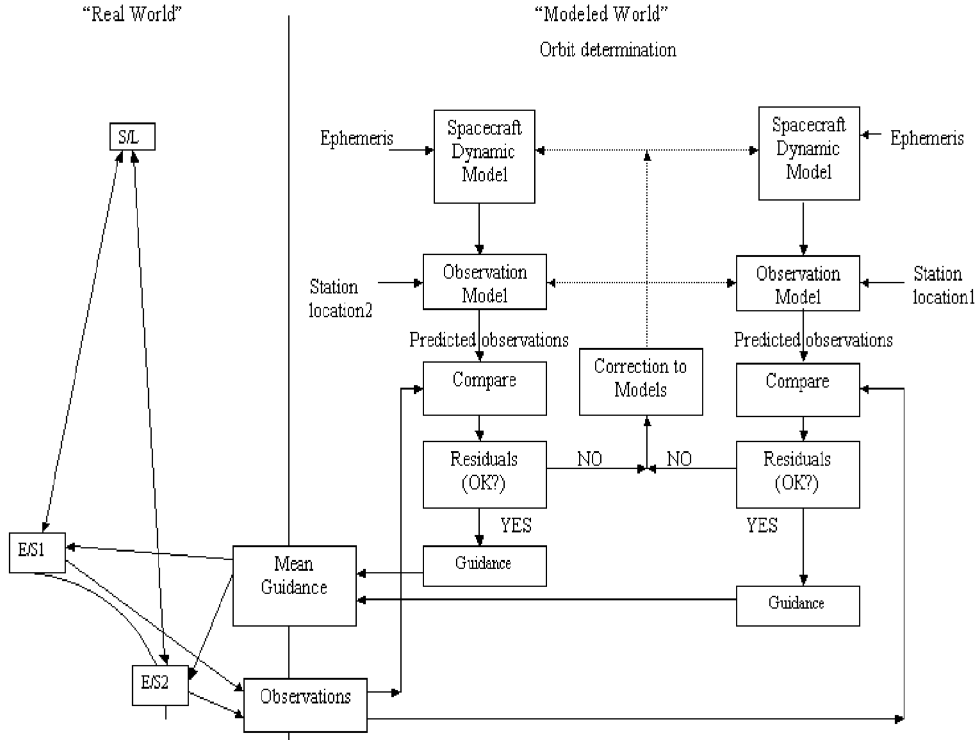


***Figure 13**: Dual ranging orbit determination flow chart*

(a) OD Example

Input: at time : 29/06/06 10:53:17 ,

Initial Orbital parameters : a = 42165.829900000 Km , e_x=-0.000091740 , e_y=0.000511280 , i_x= 0.000419910 rad , i_y= 0.000520490 rad , l= 353.000680 deg

<u>Output</u>: Using Dual ranging OD from two separate earth Stations, and comparing the results from the Matlab program with that provided from a flight proven software tool, the following differences are shown:

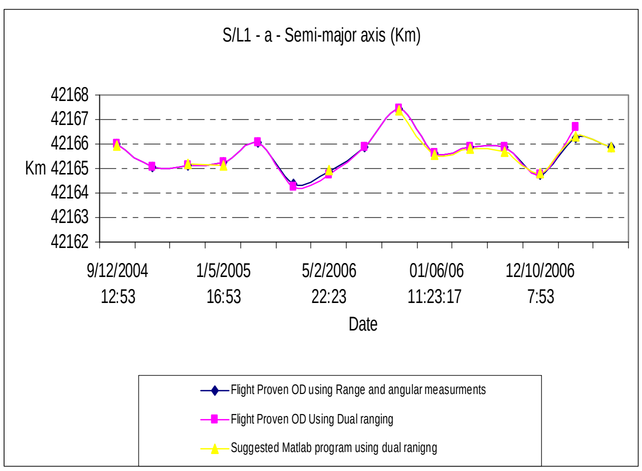
Orbital Parameters	% difference between the Matlab program and the flight proven tool
a	$-6.7343e^{-5}$ %
e_x	3.93216 %
ey	-0.22746 %
i_x	1.623 %
i_y	-0.7869 %
l	$-2.4428e^{-5}$ %

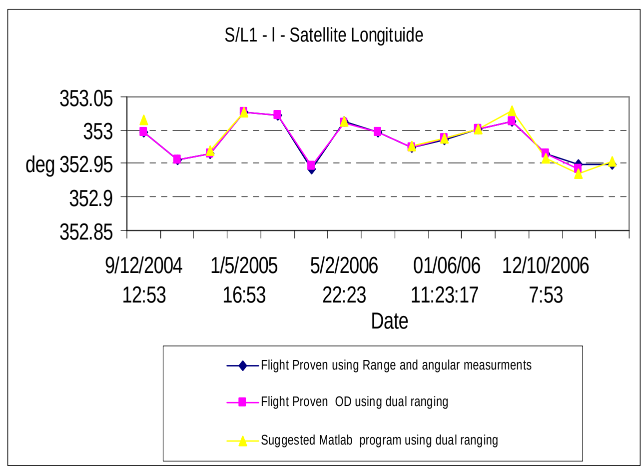
Thus, the Matlab program produce acceptable results compared to the flight proven software tool.

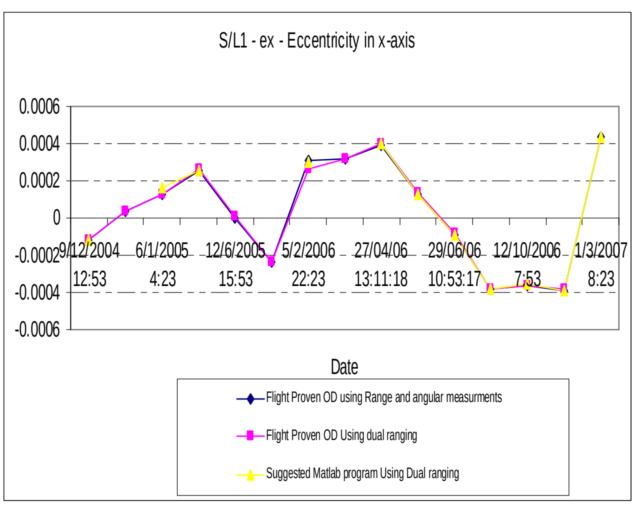
<u>(b) The three (3) years comparison study:</u>

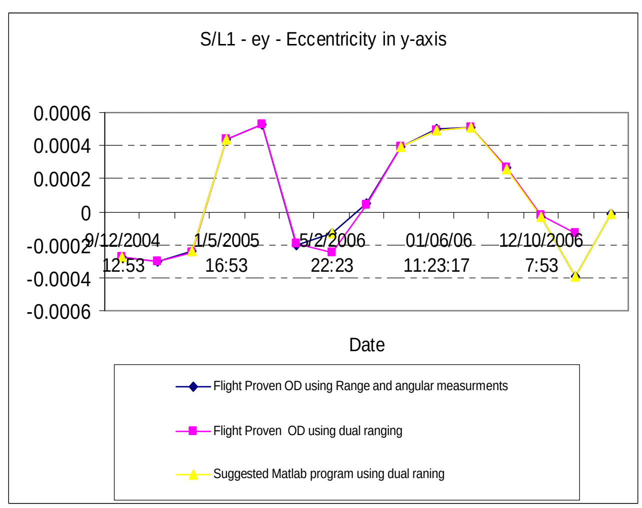
The shown results provide the final modified orbital parameters (semi-major axis, eccentricity vector in the x and y directions, inclination vector in the x and y directions and longitude) for various methods used, and show comparable results with each others.

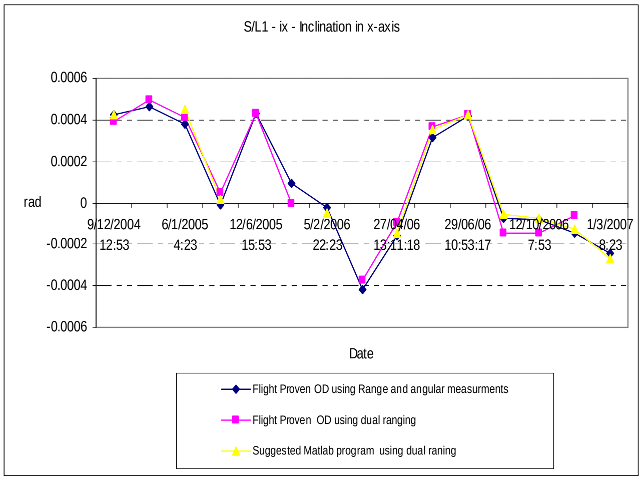
- Spacecraft A final orbital parameters:

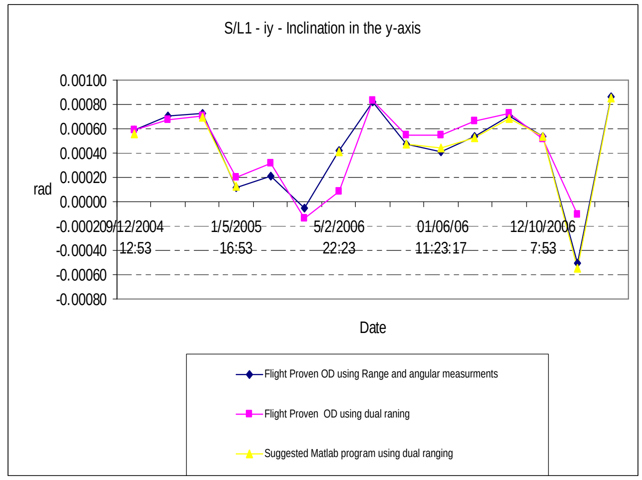




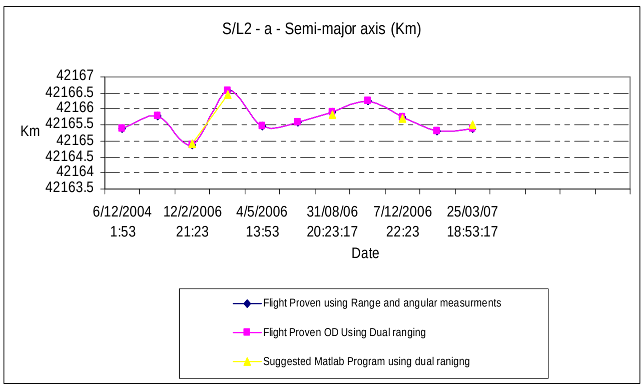


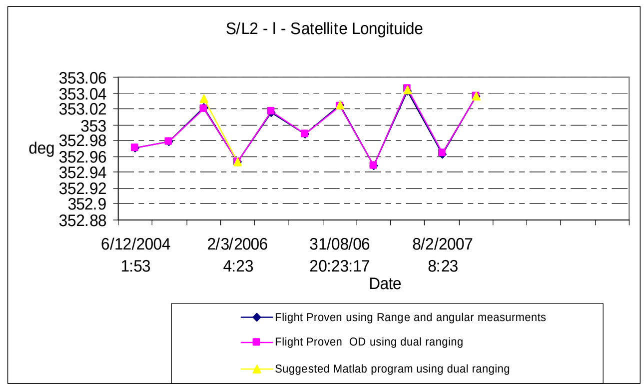


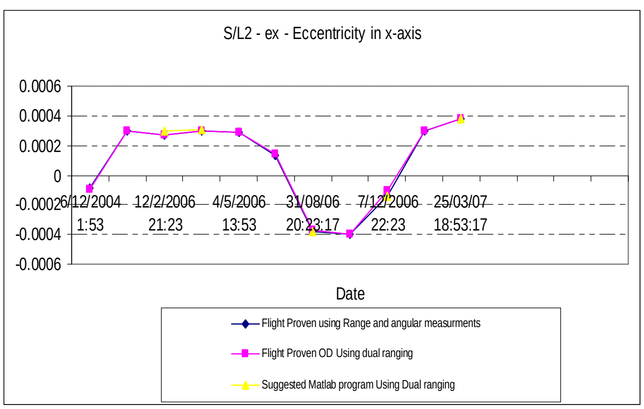


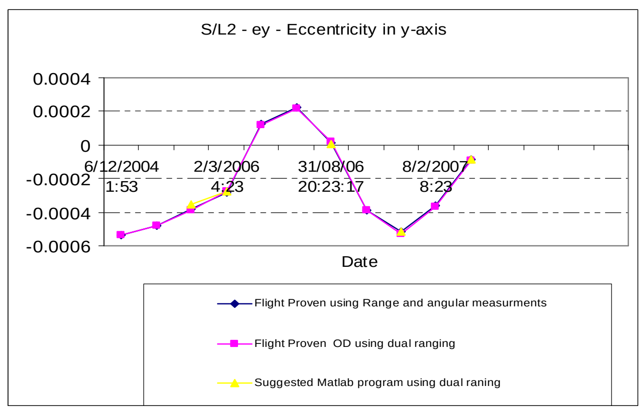


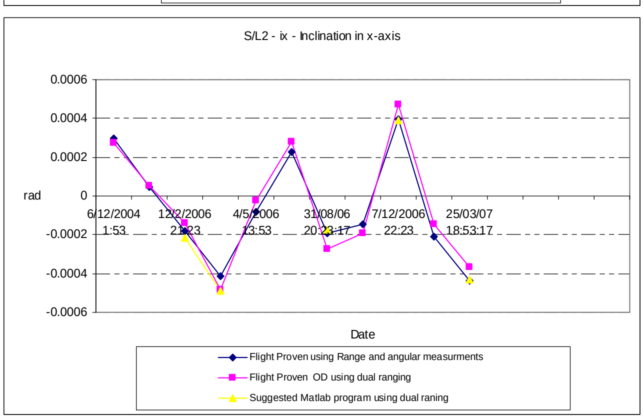
- Spacecraft B final orbital parameters

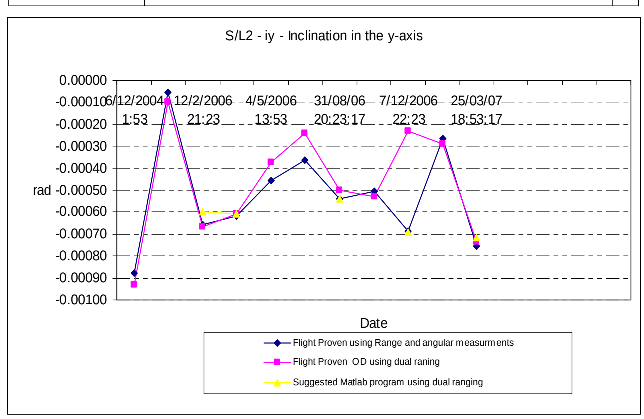












8. RF distance measurement limitations

There are many factors which will degrade the RF signals of distance ranging system along their transmission path, and will limit the range measurement accuracy of a RF ranging system. The most important factors are: Noise and Multipath effects.

8.1 The effect of noise:

Noise enter an RF ranging system from various sources of interferes with range measurement. Noise may mask the measurement signal or cause a spurious signal, as a consequence, the measurement signal may be undetected and even if the RF signal can be recognized with good reliability, the noise may have shifted the original measurement signal in time, changed its amplitude or have had some other detrimental effect on the measurement. Even with good system design, the noise remains as one of the main sources of unpredictable errors, limiting achievable measurement precession.

8.2 The effect of Multipath:

Although only a signal electromagnetic wave is radiated by a transmitting antenna, there are instances in which that wave reaches the receiver by more than one path. Alternate paths involve reflections from the ionosphere, from the ground, or from the buildings and other objects along the propagation path. When more than one wave arrives at the receiving antenna, the net signal at the antenna is phase or sum separated waves.

The effect of ground reflected wave arriving along with the direct wave is important in RF high precision ranging systems. The difficulty arises because the phases of the direct path and the reflected signals are different when they arrive at the receiving antenna and this produce interference effect. Acting as noise, multipath signal will affect the RF signals amplitude and phase and as a result, will interfere with range measurement. The multipath signal is another source of the unpredictable error.

One of the goals of the system design is to depress the multipath signal to the maximum possible extent to improve the measurement precession. Using spread spectrum technique [13] – as shown in the next section - is an effective way to combat the effects of multipath signals.

9. Spread Spectrum Technique

Introducing spread spectrum [13] as a ranging method for orbit determination. It is transmitted through the Satellite payload channels, which is achievable as the transmitted signal is spread in such away that it become noise like and thus would not interfere with the payload traffic and then reproduce the signal again at the receiver level, and from the detection of the time or phase delay between the transmitted and received signal, the operator could compute the distance.

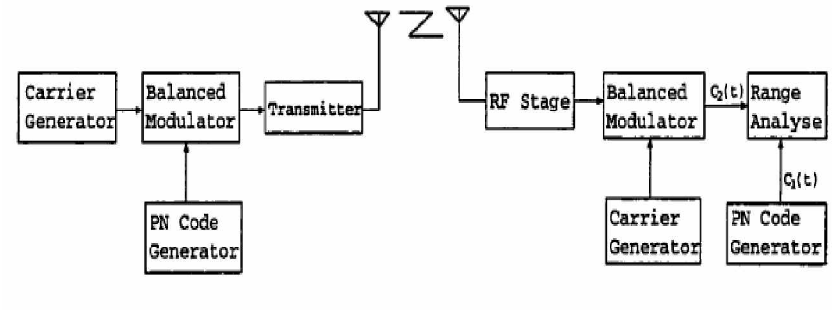


Figure 14*: Direct Sequence spread spectrum system*

Compared to classical Pseudo-range systems, the use of payload signals has various advantages for the satellite operator [13]. The payload based ranging signal is always present, and have no impact on the satellite control activities, while classical Pseudo-range actually has to be activated by satellite commanding. For these reasons, tone ranging campaigns are limited in time, in contrary to payload methods, which provide continuous range data. Also, spreading provide advantages for anti- jamming and anti-interference systems, provide low probability of intercept and could be used for multiple user random access communications with selective addressing capability. Last but not least, the accuracy of payload ranging is in general superior, mainly due to the large bandwidth and high signal-to-noise ratio. The latter two effects also make payload ranging an optimal solution for orbit determination.

Spread Spectrum signals use fast codes that run many times the information bandwidth. These codes are called "Pseudo Random" or "Pseudo Noise" codes. The *Pseudo Noise* (PN) [14] signals are the key factor in Spread spectrum systems as they are the ones responsible for the spreading and de-spreading of the Base band signal. PN sequences are considered to have a noise like properties for an outsider, but they are known to the two devices using it. They are considered *Pseudo Random* because the sequences are deterministic and known to both the transmitter and the receiver.

Due to the usage of the PN code, the spread spectrum technique has the ability to discriminate interference signals and detect the received signal by matching received PN code with the local PN code and measuring the number of chips of the code delay between the signal being transmitted and received, and thus determine uniquely the range from the transmitter to the receiver without ambiguity [10]. Consequently the spread spectrum technique has its advantage in that its phase is easily resolved.

There are three basic properties that can be applied to a periodic binary sequence (PN sequence) as a test of the appearance of randomness, they are:

1. *Balance Property*: Good balance requires that in each period of the sequence, the number of binary *Ones* differs from the number of binary *Zeros* by at most one digit.
2. *Run Property*: A run is defined as: sequence of a single type of binary digits. The appearance of the alternate digit in a sequence starts a new run. It is desirable that about one half the runs of each type are of length 1, about one fourth of length 2, one eighth are of length 3, and so on.

3. *Correlation Property*: If a period of the sequence is compared term by term with any cyclic shift of it self, it is best if the number of agreements differs from the number of disagreements by not more than one count.

Generating a Matlab program as shown in figure (15), to check the autocorrelation property for different codes.

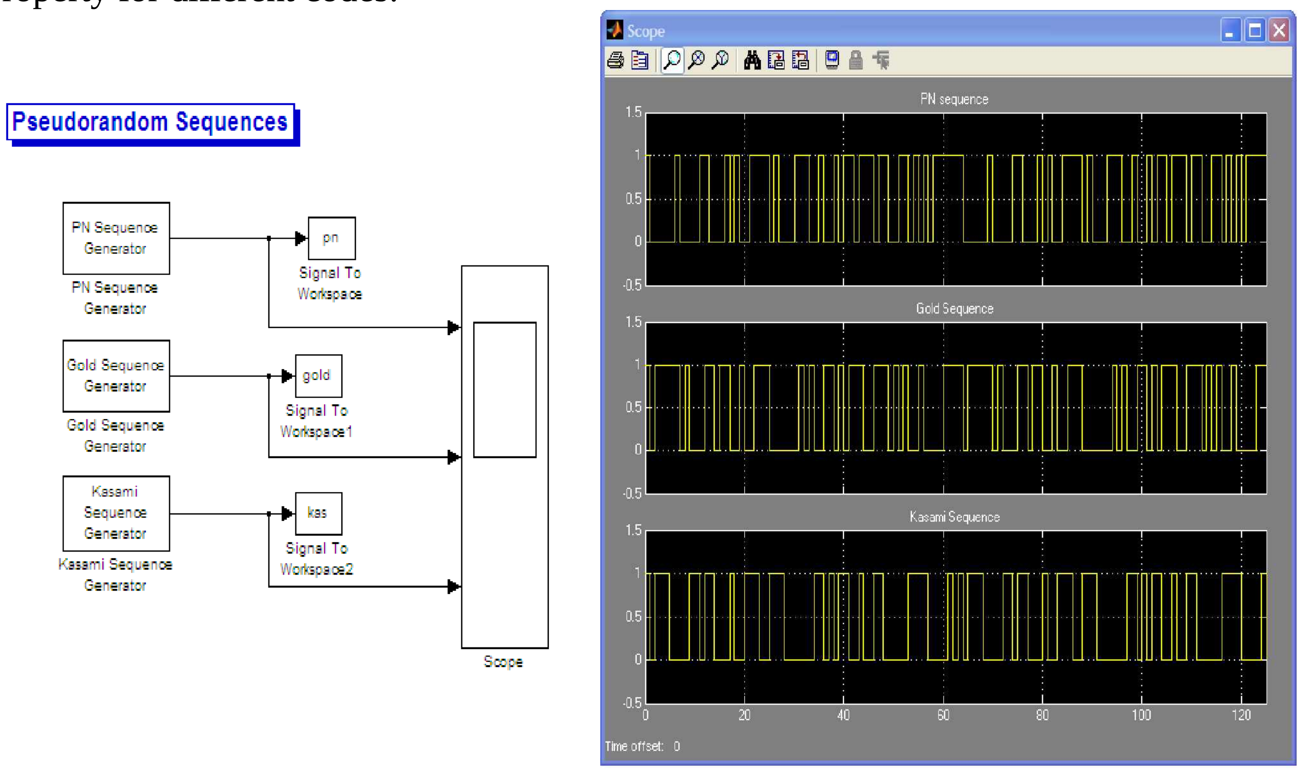


Figure 15: *Pseudorandom Sequences -*

The following autocorrelation results were generated:

PN = -1 63	GOLD = -17 -1 15 63	KASAMI = -9 -1 7 63

From the three types of sequences [13], the PN sequences are considered best suited for synchronization because the autocorrelation takes on just two values. However, the Gold and Kasami sequences provide a larger number of sequences with good cross-correlation properties than do the PN sequences.

The PN sequences is considered best suited for synchronization because the autocorrelation takes on just two values. However, the Gold and Kasami sequences provide a larger number of sequences with good cross-correlation properties than do the PN sequences.

As in the future we will try to use the spread spectrum technique in orbit determination, and then the PN sequence is considered the best suited for this application due to its good synchronization properties.

10. Conclusion

From the simulations shown in section 7, it appears that an Orbit determination using ranging data from two stations apart by around 250 Km and located within different longitudes from the tracking satellite, is feasible and produces acceptable results.
This conclusion lead to accepting the orbit computed from only the ranging data using two stations, thus in case of problems in the limited motion antenna campaign could be performed only using the fixed motion antenna and provide accurate and acceptable results.

11. Future Work

Next we will try to introduce different approach for the orbit determination by using spread spectrum technique instead of the normal Pseudo-range technique in order to reduce the limitations of orbit determination using Pseudo-range technique, discussing the capabilities and potentials provided using spread spectrum in orbit determination.
Also, we will try to implement residue number system to the spread spectrum transmitter/receiver system in order to provide security and efficiency to the system which is not achieved in the case of the spread spectrum transmitter/receiver system that don’t use residue number systems. The signal-to-noise ratio (SNR), and bit error probability (P_e) will be measured and evaluated.

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Nomenclatures:

Ω	Right ascension of ascending node
ω	Argument of perigee
ϑ	True anomaly
GAST	Grinitch apparent sidereal time
ϕ	phase shift
τ	time delay = $(t_2 - t_1)$
H	Magnitude of the angular momentum,
r	Magnitude of the position vector (r) in the P-Q frame plane Earth
μ	Gravitational constant =3.986e^5 km^3/sec^2.
Φ	State transition matrix
y	The deviation in the measurements (actual - predicted measurements)
D	Observation sensitivity matrix
$(D^T D)^{-1}$	Covariance matrix.
Φ	∂(State vector)/ ∂(orbital parameter)|at t_0*[∂(state vector)/∂(orbital parameter)]$^{-1}$|at t
D″	Observation-state mapping matrix = the partials of each observation with respect to the reference state component.
k_l	-0.0128^o /day.Km
a_s	Semi-major axis of the geosynchronous orbit = 42165.8 Km
L_e , l_e	Earth station latitude, and longitude
Re	Earth radius = 6378.13649 Km.
Ω_{sun}	Right ascension of the sun
n	Satellite mean motion of geostationary satellite
S_a/m	Effective satellite area- to – mass ratio
T	Number of days since 1/1/1950
J_2	Zonal coefficient: due to flatness of earth
M	Mean anomaly