**Military Technical College
Kobry El-Kobbah,
Cairo, Egypt**

**6th International Conference
on Electrical Engineering
ICEENG 2008**

Modeling and Control of Multi Flexible Link Robot: Using Classical and Intelligent controllers

By

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Abstract:

The objective of this paper is concentrated on position and vibration control of multi flexible links robots which take a wide place of research now, especially in aerospace applications (e.g. the space shuttle). A discussion of the kinematics and dynamics of the flexible multi-link robots was established based on Newton-Euler formulation leading to a nonlinear dynamic model was simulated. A linearization of nonlinear model of two flexible links was followed result a linear model which was formed as a state space form, so an optimal control was applied. Fuzzy control was also applied to the linear model obtained from the above linearization. Finally a nonlinear model of two flexible links was simulated and fuzzy control was applied with different suggested trajectories and Simulation results for control demonstrate that the controllers perform very well for the tracking the desired trajectories.

Keywords:

Flexible link, fuzzy control, optimal control, kinematics, dynamics nonlinear model.

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1. Introduction:

Recently, the study the behavior of the multi flexible links robot is very important. This study includes the kinematics, dynamics and control models of the robot, in other words, the kinematics and dynamics of the systems must be represented by several methods like; Hamilton equation or expressed by Euler-Lagrange equation. For control multi flexible links robots, since these robots are light weighted, in order to improve these robots' performance, the vibration resulting from the structure flexibility must be suppressed as quickly as possible while the gross motion is controlled. Although, many control methods have been proposed to solve this kind of control problems such as inverse dynamic control [2], adaptive control, robust control, fuzzy control [4, 8], passivity-based control, but how to control flexible robots, especially two-link flexible manipulator more effectively is still an interesting point to study.

In this paper, the study of kinematics and dynamics of two flexible links with the zero tip constraint, the use of this constraint results in simpler mathematical models, first develops a linear model of two flexible links model; the state space model with optimal control is studied. Then, the fuzzy controller is applied next with the suggested trajectories. A complete non linear model of two flexible links is built and the final control system is design with fuzzy controller. The final results of the non linear model with different suggested trajectories are presented.

2. Modeling of Two Links Flexible Robot:

Modeling of multi flexible links robot has been considered by many researchers. In order to simplify partial differential equations associated with flexible links some of them used assumed modes technique to derive equations of motion [3, 6]. Others used discretization of the links like finite element method [5], or finite segment modeling and other derived equation of motion by using both assumed modes method and finite element method [6].Constrained generalized coordinates is used to derive the equations of motion and the flexible deformation of the links are described in terms of moving coordinate systems. A convenient kinematics description could be used including both the rigid body motion and flexible deformation and recursive procedures can be set up for open chains with flexible links. Then, differential kinematics relationships are needed for computing kinetic and potential energy, within a Lagrangian approach.

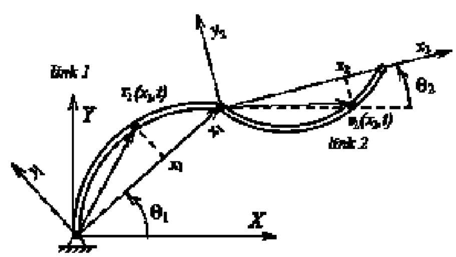
The motion of flexible bodies undergoing combined rigid and elastic motion can be analyzed by viewing the motion from a moving reference frame. The use of such a frame imposes a constraint on the motion viewed from this frame. This constraint is usually taken into consideration by selecting the trial functions that describe the motion viewed from the moving reference frame. One of the methods is the zero tip constraints [6], which is more suitable when dealing with multi flexible links system. The use of this constraint results is a simple mathematical models.

2.1 The Zero Tip Deformation Constraint

Considering the zero tip deformation constraint, which is invoked by drawing a straight line between the link ends x = 0 and x = L. The configuration is represented by the coordinate system xy in Figure 1. As a result, the boundary conditions on the secondary motion become those of a pinned-pinned beam, so that one can use simple sine functions to expand the secondary motion. There are many advantages associated with using the zero tip deformation constraint which are; It is easy to orient the reference frame, The trial functions used to expand the secondary motion have no numerical or sensitivity problems, The force balances at the boundaries are always satisfied, regardless of whether an end is free or connected to another link and The resulting equations are simpler which makes simulating them easier.

2.2 Kinematics of Two Links Flexible Robot

The model consists of two flexible links moving in horizontal plane as shown in Figure 1. The two links are chosen to be identical with the following properties;



***Figure(1)** The two flexible link robot*

The direct kinematics equations establish the functional relationship between the joint variables and the end effector position and orientation, with two flexible links undergoing small bending deformations. For each link *i* the rigid motion is described as clamped angle $\theta_i(t)$; lateral bending $W_i(x_i, t)$. The position vectors for points on the two links can be expressed as,

$$r_1(x_1,t)=x_1 i_1 + W_1(x_1,t)\, j_1 \quad (1)$$

$$r_2(x_2,t) = r_1(L_1,t) + x_2 i_2 + W_2(x_2,t)\, j_2 \quad (2)$$

$$\begin{bmatrix} i_1 \\ j_1 \end{bmatrix} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} i_2 \\ j_2 \end{bmatrix} \quad (3)$$

Where; $c = \cos(\theta_2 - \theta_1)$
$s = \sin(\theta_2 - \theta_1)$

and θ_1 and θ_2 are the rigid motion of the coordinate frames of two links. The secondary motions of the two links are expended as;

$$W_i(x_i,t) = \sum_{k=1}^{ne} \phi_{ik}(x_i)\delta_{ik}(t) \quad (4)$$

where, i = number of links and,

ne= number of selected modes

using (1) and (2) and the zero tip deformation constraint;

$$r_1(L_1,t) = L_1 i_1 \tag{5}$$

and

$$\dot{r}_1(L_1,t) = L_1\dot{\theta}_1 j_1 \tag{6}$$

where, $\dot{\theta}_1$ is the angular velocity of the fist link.

from (5) and (2), $r_2(x_2,t)$ could be written as;

$$r_2(x_2,t) = L_1 i_1 + x_2 i_2 + W_2(x_2,t) j_2 \tag{7}$$

the velocities of points on the links are derived by differentiate (1) and (2) with respect to the time as follow,

$$\dot{r}_1(x_1,t) = -\dot{\theta}_1 W_1 i_1 + (\dot{W}_1 + \dot{\theta}_1 x_1) j_1 \tag{8}$$

$$\dot{r}_2(x_2,t) = L_1\dot{\theta}_1 j_1 - \dot{\theta}_2 W_2 i_2 + (\dot{W}_2 + \dot{\theta}_2 x_2) j_2 \tag{9}$$

2.3 Dynamics of Two Links Flexible Robot

The dynamic analysis is based on Euler-Lagrange equations, starting from calculation of both kinetic and potential energy of each link. The kinetic energy is defined as;

$$Ke_i = \frac{1}{2}\int_0^{L_i} \rho_i \left(\dot{r}_i \bullet \dot{r}_i\right) dx_i \tag{10}$$

from (8) into (10) and integrate, the kinetic energy of link1 is found to be,

$$Ke_1 = \frac{1}{2} J_{o1}\dot{\theta}_1^2 + \frac{1}{2} m_1(\dot{\delta}_{11}^2 + \dot{\delta}_{12}^2) + \frac{1}{2} m_1 \dot{\theta}_1^2 (\delta_{11}^2 + \delta_{12}^2) + a_{11}\dot{\delta}_{11}\dot{\theta}_1 + a_{21}\dot{\delta}_{21}\dot{\theta}_1 \tag{11}$$

and from (9) into (10) and integrate the kinetic energy of link 2 is,

$$Ke_2 = \frac{1}{2} J_{o2}\dot{\theta}_2^2 + \frac{1}{2} m_2 \dot{\theta}_1^2 L_1^2 + \frac{1}{2} m_2 \dot{\theta}_1 \dot{\theta}_2 L_1 L_2 c + \frac{1}{2} m_2(\dot{\delta}_{21}^2 + \dot{\delta}_{22}^2) + \frac{1}{2} m_2 \dot{\theta}_2^2 (\delta_{21}^2 + \delta_{22}^2) \tag{12}$$

$$+ a_{21}\dot{\delta}_{21}\dot{\theta}_2 + a_{22}\dot{\delta}_{22}\dot{\theta}_2 - m_2\dot{\theta}_1\dot{\theta}_2 L_1(p_{21}\delta_{21} + p_{22}\delta_{22})s + m_2\dot{\theta}_1 L_1(p_{21}\dot{\delta}_{21} + p_{22}\dot{\delta}_{22})c$$

where,

$$a_{ik} = \int_0^{Li} \rho_i x_i \phi_{ik}(x_i) dx_i \tag{13}$$

$$p_{ik} = \int_0^{Li} \rho_i \phi_{ik}(x_i) dx_i \tag{14}$$

and, i is the number of links, and k is the number of selected modes.

The potential energy is expressed as;

$$U_i = \frac{1}{2}\int_0^{Li} EI_i \left(\frac{d^2 W_i}{dx_i^2}\right)^2 dx_i \tag{15}$$

As the selected study case is the planer two flexible links acting on a horizontal plane the effect of gravity on the potential energy is negligible and take only the effect of elastic energy of the link, from (4) and (15) the potential energy equation could be expressed in algebraic form as follow;

$$U_i = \frac{1}{2}\sum_{k=1}^{ne} K_{ik}\delta_{ik}^2 \tag{16}$$

where, $K_{ik} = J_{Lk}\omega_{ik}^2$ and $\omega_{ik}^2 = \beta_{ik}^4 \frac{EI_i}{\rho_i}$

the Lagrangian formula is expressed in terms of *N* +*M* generalized coordinates, where $M = \sum_{i=1}^{N} ne_i$ and N the number of links,

$$L = Ke - U = L(\{\theta_i, \delta_{ik}, \dot{\theta}_i, \dot{\delta}_{ik}\}) \tag{17}$$

from (11),(12) and(16) into(17) get that,

$$L = Ke_1 + Ke_2 - (U_1 + U_2) \tag{18}$$

the Lagrange's equations are expressed as,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = T_i \tag{19}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\delta}_{ik}}\right) - \frac{\partial L}{\partial \delta_{ik}} = 0 \tag{20}$$

where, T_i the torque delivered by the actuator at joint *i* of each link.

Then, substituting from (18) into (19) and (20) formed six equations which represent the dynamic model of two flexible links, the model can be arranged in matrix form (mass-damper-spring form) as follows,

$$M\ddot{q} + C\dot{q} + Kq = [T] \tag{21}$$

where,

$$q = [\theta_1 \quad \theta_2 \quad \delta_{11} \quad \delta_{12} \quad \delta_{21} \quad \delta_{22}]^T$$

$$T = [T_1 \quad T_2 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

M is the inertia matrix, C is the damping matrix, and k is the stiffness matrix.

3. Control of Linear Model of Two Links Flexible Robot

In order to design an optimal controller for the dynamic model of two flexible links in equation (21), the model must be linearized. A simple way to get the linear model is considering the equilibrium state of the system. If the angles of rigid motion are assumed to be small; when the links are near to their equilibrium points.

3.1 Linearized Model

Consider that, for small values of θ_1 and θ_2:

$$c = \cos(\theta_2 - \theta_1) \approx 1$$

$$s = \sin(\theta_2 - \theta_1) \approx (\theta_2 - \theta_1) \approx 0,$$

$$\delta_{11}^2 \approx \delta_{12}^2 \approx \delta_{21}^2 \approx \delta_{22}^2 \approx 0$$

and $\dot{\theta}_1^2 \approx \dot{\theta}_2^2 \approx 0$

refer to the nonlinear model (21) of two flexible links the resultant linear model will be,

$$M_L\ddot{q} + Kq = [T] \tag{22}$$

which shows the un-damping dynamic model of two flexible links.

The model in (22) is called the dynamic inverse simulation; (calculate the required torque from trajectory), while the mathematical form for the state space model for linear model can be formed as,

$$\ddot{q} = M_L^{-1}\{[T] - Kq\} \qquad (23)$$

The open loop simulation for the state space model of two flexible links is shown in Figure (2). The open loop performance is; ramp output for both angles and velocities distribution for unit step inputs for input torques.

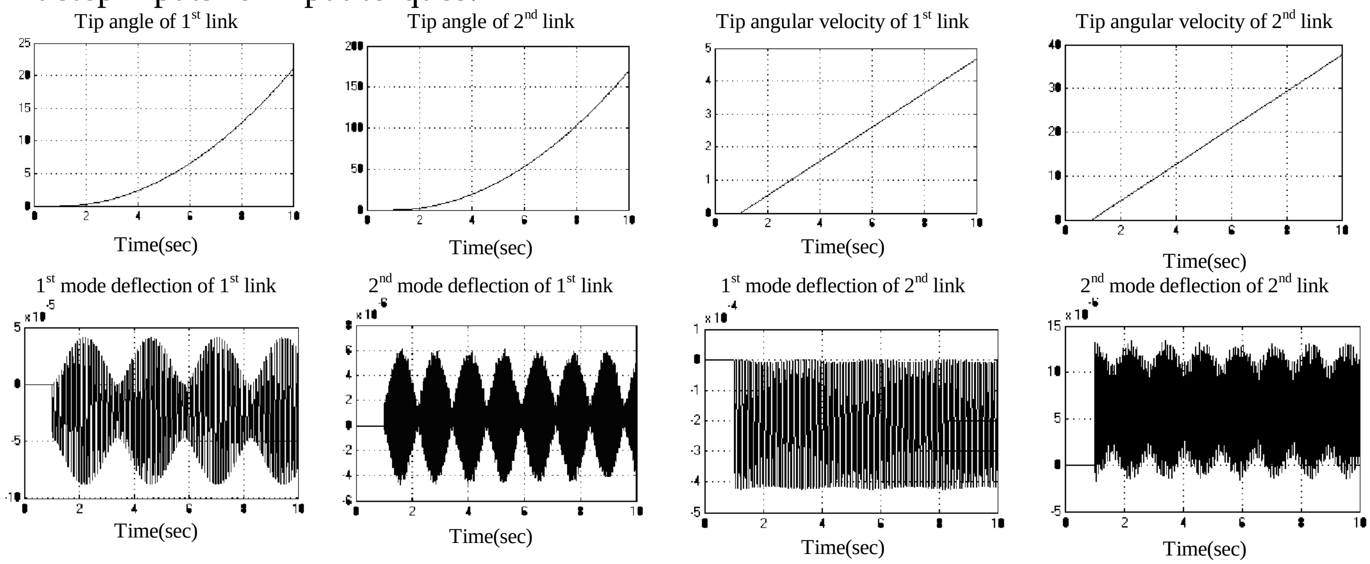
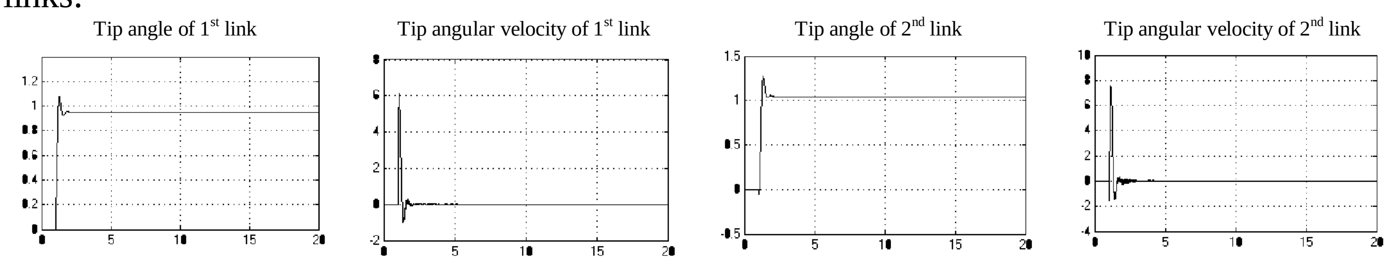


Figure (2) *Simulation of open loop of linear model of two flexible links.*

3.2 Optimal Control

The optimal linear quadratic regulator (LQR) is applied to the linear model of two flexible links to obtain the gain matrix K. The optimal controller $u = -K_o e$ is designed such that a given performance index $J = \int (x^t Q x + u^t R u)dt$ is minimized. The performance index is selected to give the best performance, the choice of the elements of Q and R allows the relative weighting of individual state variables and individual control input. The performance of the two flexible links when applied the optimal controller is shown in Figure (3) for unit step inputs for both links.



1st, 2nd modes deflection of 1st and 2nd link

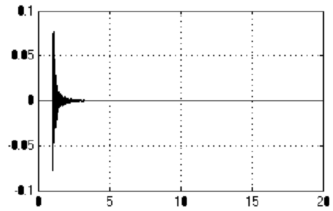


Figure (3) *Performance of two flexible links with optimal controller.*

The performance of the two flexible links with optimal controller shows that; for the 1st link, the overshoot is about 0.1 at 2 sec, the settling time is 3 sec and the steady state error is equal to - 0.05. While for the 2nd link, the overshoot is about .25 at 2 sec, the settling time is 3 sec and the steady state error is 0.01 (less then in 1st link). The modes deflection tends to zero approximately after 3 sec. The velocities of both links show overshoot (about 6 and 7) at 2 sec, which is extremely high and may cause failure in the structure of the model itself.

4. Fuzzy Control

Recently, one of the most important control techniques is the intelligent control; which is defined as a combination of control theory, operation research and artificial intelligence. Fuzzy logic is one the most popular area based on artificial intelligence, which is attached to a large number of research and industrial applications. Fuzzy controllers are expert control system that uses common sense rules and natural language statement. In many practical control systems several tasks are done by human, those tasks must be performed based on the evaluation of the measured data according to a set of rules which the human expert has learned from experience or training. As, there are many complex systems which is highly nonlinear, and/ or ill defined, such systems could be controlled by human without the need to the mathematical models. Fuzzy logic control has many advantages that make it very powerful when applied to those systems; as it is suitable for both linear and nonlinear systems, it is more robust than the classical control, and it can deal with systems which is ill mathematical defined or have no mathematical models at all. Fuzzy systems can be used in as closed-loop controllers. In this case the fuzzy system measures the outputs of the process and takes control actions on the process continuously. The fuzzy controller uses a form of quantification of imprecise information (input fuzzy sets) to generate by an inference scheme, which is based on a knowledge base of control force to be applied on the system [1, 7, and 8]. This establishes a fuzzy logic controller to control nonlinear vibration of a one flexible link. Therefore, a fuzzy logic controller design is adopted and will be applied for such system. In feedback loop of the control system, a fuzzy logic controller is used to provide control signals for the system and used to generate the joint torques and to enhance the performance of the system in vibration process with different suggested trajectories focus on follow the trajectory with minimum possible tip vibration.

4.1 Fuzzy Controller Design

Fuzzy logic consists of three parts: Fuzzification, inference and defuzzification. Fuzzification is an interface that produces a fuzzy subset from the measurement; that is, it is a mapping from the set of measurements. The inference is an interface that produces a new fuzzy subset from the result of the fuzzification using for example a set of fuzzy (If- Then) rules. The antecedent and consequent of the fuzzy rules may be the system states, or the system error, error rate, error integral; in this case it is similar to PID controller. The results of the inference are a fuzzy subset associated with the output. The defuzzification is an interface that produces a crisp output from the results of the interface [1].In the fuzzy logic controller, the rule based from human experts, which is not available for any system and may lead to incomplete and conservative control strategies. Also, the memory requirement for fuzzy logic controllers grows exponentially with the dimension of the system variables used in control rule base. The number of rules is n^k, where k is the number of system variables in a rule and n is the number of fuzzy sets (e.g. for PD fuzzy controller and 7 fuzzy sets defined over each variable, the number of rules equal to $7^2 = 49$ rules.).

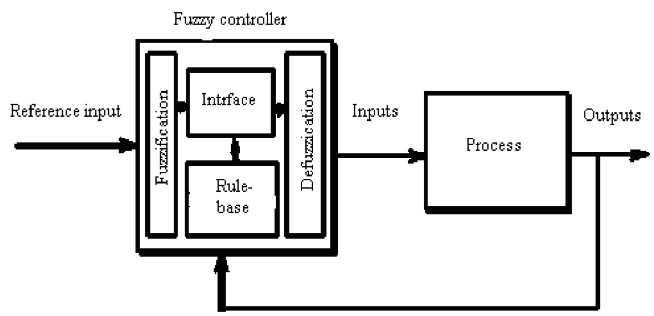
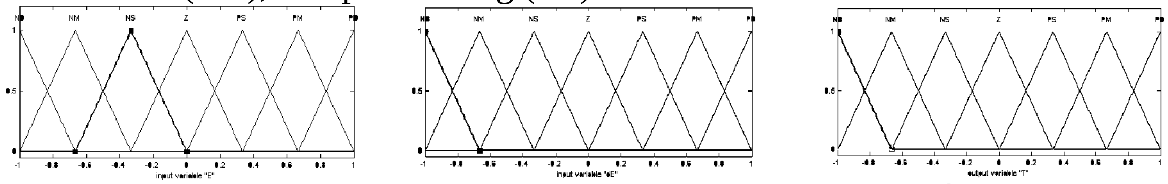


Figure (4) *The basic configuration of the fuzzy system.*

The design of a fuzzy logic controller consists of the selection of membership functions and definition of a rule base. In this study, fuzzy logic controller has two inputs and one output: the position error (e), change in the error (de) and the torque (u). Input and output fuzzy members functions are symmetric and shown in Figure (5). Triangle member functions were used in membership functions. The position error, the change of the error and the output controller are partitioned into seven fuzzy sets: negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM), and positive big (PB).



The position error (e) Change in the error (de) The torque (u)

Figure (5) *Inputs and Output fuzzy membership functions.*

One of the important steps of fuzzy logic controller is rule table. Fuzzy rules can be derived based on some knowledge of the controller process, or by analyzing the behavior of the controlled process in the time domain. The total number of rules is forty seven in this study. The control rules are in the form:

$$\textbf{If } e \textbf{ is } E_i \textbf{ and } de \textbf{ is } De_i \textbf{ then } u \textbf{ is } U_i$$

where, E_i = linguistic term of error.

De_i = linguistic term of change in error.

U_i = linguistic term of torque.

The scaling factors; which scale the real system values into the normalized one are K_p, K_d and K_u such that,

Inputs scaling; $e_n = e * K_p$

$de_n = de * K_d$

Output scaling ; $u = u_n * K_u = K_u *(e * K_p + de * K_d)$

The selection of the scaling factors plays a very important role in the performance of the fuzzy controller; however there is no systematic method to find the optimal scaling factors rather than using the simulation results. Also, there are some guidelines which help in selection the scaling factors like; maximum error, maximum change in error, and maximum control output. The control output from the PD fuzzy controller is similar to the form of the classical PD controller. The PD fuzzy controller rule table for robot is shown in Table (1) [1].

Table (1): *Fuzzy logic rules*

Torque	Derivative of error						
Error	NB	NM	NS	ZO	PS	PM	PB
NB	PB	PB	PB	PM	PM	PS	ZO
NM	PB	PB	PM	PM	PS	ZO	NS
NS	PB	PB	PS	PS	ZO	NS	NM
ZO	PB	PM	PS	ZO	NS	NM	NB
PS	PM	PS	ZO	NS	NS	NB	NB
PM	PS	ZO	NS	NM	NM	NB	NB
PB	ZO	NS	NM	NM	NB	NB	NB

The input for the defuzzification process is an aggregate output fuzzy set and the output is a single number. As much as fuzziness helps the rule evaluation during the intermediate steps, the final desired output for each variable is generally a single number. However, the aggregate of a fuzzy set encompasses a range of output values, and so must be defuzzified in order to resolve a single output value from the set. There are five defuzzification methods: centroid, bisector, middle of maximum, largest of maximum, and smallest of maximum. Perhaps the most popular defuzzification method is the centroid calculation, which returns the centre of area under the curve. Centroid of area defuzzification scheme is used for obtaining a crisp output in this study.

4.2 Fuzzy Control of Linear Model of Two Links Flexible Robot

Two fuzzy controllers applied to the two links, so that the output from the fuzzy controller consid torque for each link. The controller inputs are the error and its derivative. The performance of shown in Figure (6), for the 1^{st} link; the overshoot is about 0.2 at 8 sec, the settling time is appr with zero steady state error. While the 2^{nd} link, the overshoot is 0.25 at 8 sec, the settling time is a performances of the modes of the two links have a settling time of 35 sec too. The velocity of 1 overshoot about 0.3 at 5 sec with settling time about 35 sec, while the 2^{nd} link velocity shows an 2.5 sec with the same settling time as 1^{st} link. The performance of the fuzzy controller is better controller as; the overshoot of the velocities of both links is less than those in the optimal c eliminate the possibility of structure faller. However, the settling time of the system is greater tha controller; the system slowly follows the input.

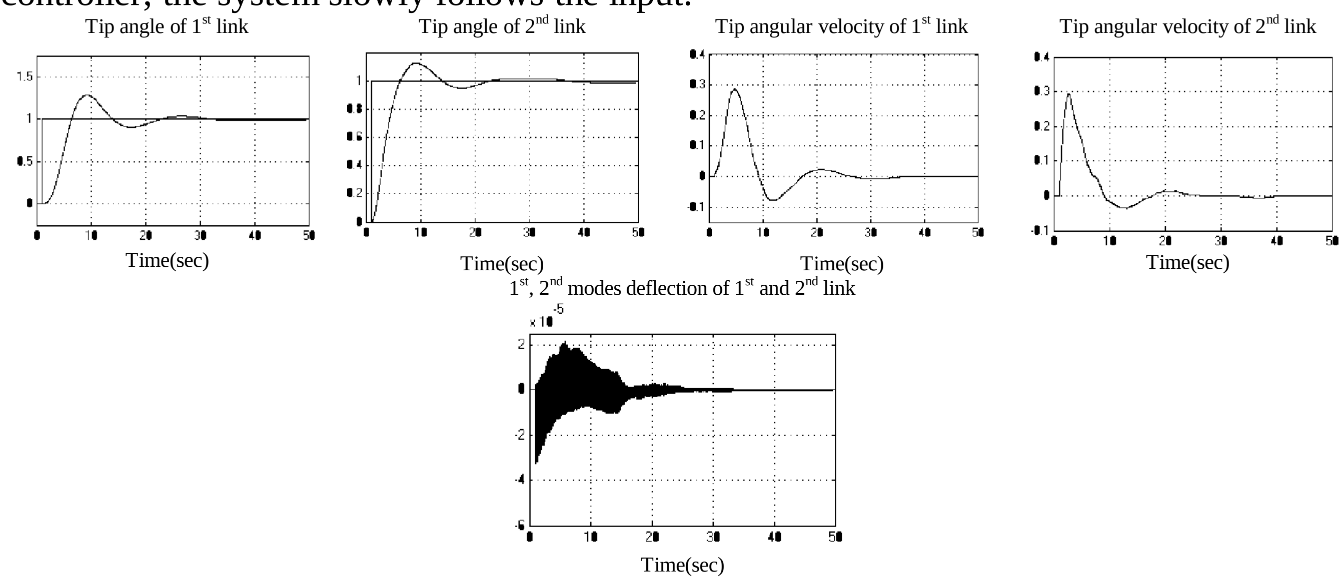


Figure (6) *Performance of two flexible links with PD fuzzy controller*

As shown in figure (6) (c), the modes of the two flexible links are about 2 $*10^{-5}$ and tends to zero reduce the values of those modes; let the control outputs be the derivatives of the torques and inte performance of the modes of the two links is going better as shown in Figure (7); the modes ar equal to zero and high vibrations at the beginning are minimize.

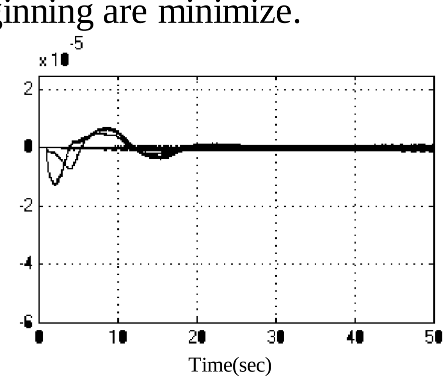


Figure (7) *The effect of PI fuzzy controller on the flexible modes.*

Now, consider the control system with the desired trajectory and the performance of the two f

are shown in figure (8). The figures show that the two flexible links follow the both desired trajector
way.

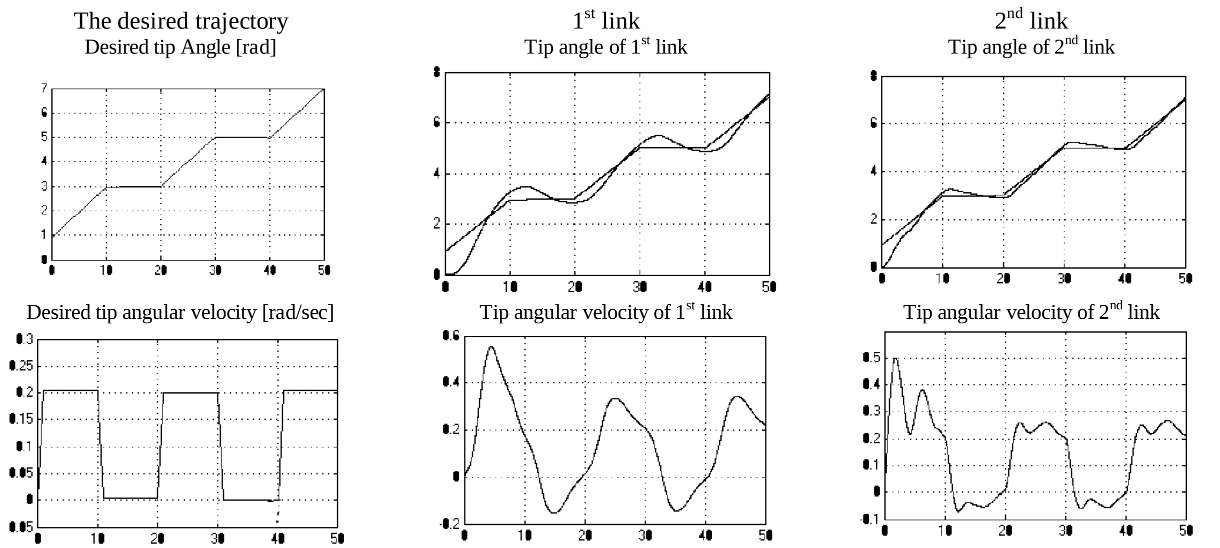


Figure (8) *The simulation results of the desired trajectory of two flexible link*

4.3 Fuzzy Control of Non-linear Model of Two Links Flexible Robot

However, the nonlinear model of two flexible links shows a good performance for both the op
controllers, the increase in rigid angles values will force the linear model to act incorrectly as the
linearity is lost. In this case the study of the non linear model of the two flexible links (the real mo
to take place. The non linear model of two flexible links expressed in equation (21) is simulated;
of open loop system is shown in Figure (9) when applying unit step inputs for both links. The pe
open loop is unstable as it acts as a ramp for unit step inputs. Also, the increase of the rigid angles
rad) the high effect of the vibration occur as shown in Figure (9), which has a very bad effect
performance.

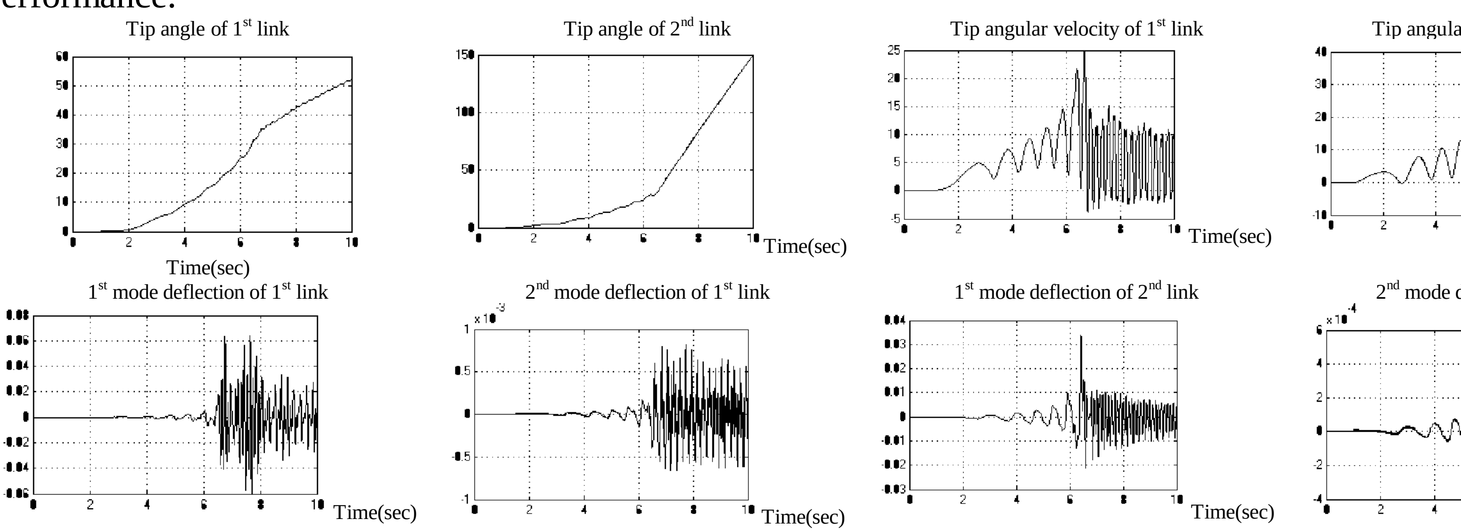


Figure (9) *Simulation of open loop of non linear model of two flexible links.*

The fuzzy controller is applied to the non linear model of two flexible links with unit step inputs fo output performance is shown in figure (10); the 1st link, the angle overshoot is 0.5 at 10 sec overshoot is 0.35 at 5 sec while the setting time is about 38 sec, and the 2nd link; the angle oversho and the velocity overshoot is 0.35 at 2.5 sec while the settling time is about 40 sec. the flexible links tend to zero in about 15 sec.

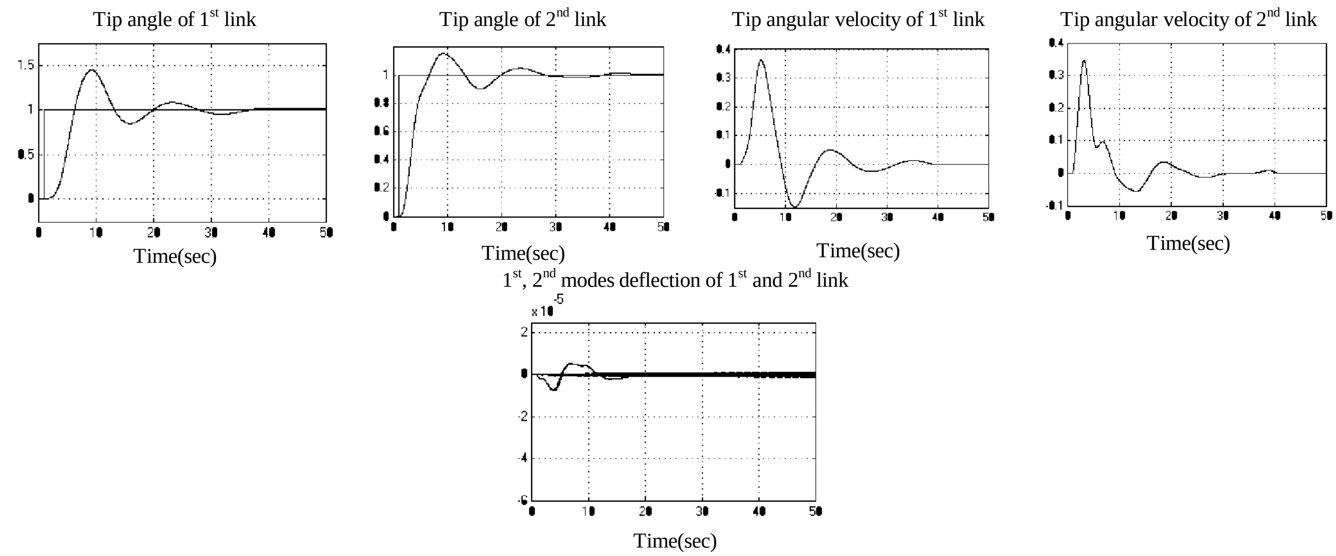


Figure (10) *The simulation results of the fuzzy controller of non linear model of two flexible links.*

Consider the system of fuzzy controller of non linear model with the desired trajectory descr to study the performance of the two flexible links and show how the controller succeeded to fo follow the given desired trajectories as shown in Figure (11).

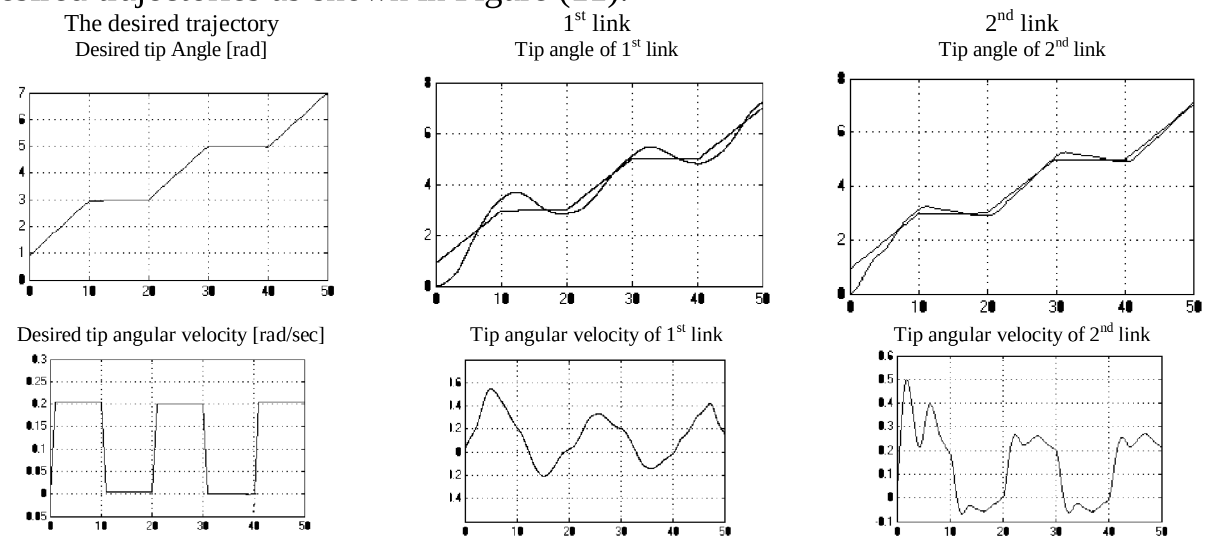


Figure (11) *The simulation results of the desired trajectory of two flexible links.*

5. Conclusions:

In the modeling and control of two flexible links are shown above, some remarked could be summ The kinematics and dynamics model is expressed and the model is highly non linear one. The linear model shows unstable performance with continually vibration caused from the flexibility o the optimal controller shows a stable system, may the high velocity overshoots is extremity danger of the links. The fuzzy controller gives a better performance to solve the problem of the high ve but the response to the inputs is slower than in the optimal control. Also, the steady state error fuzzy controller forced the two flexible links robot linear model to follow the given trajectories model of the two flexible links open loop shows unstable responses to unit step inputs, especially the angles values. The fuzzy controller is applied to the non linear model; shows a good perform as they follow the desired trajectories with approximately zero vibration.

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