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Space-time processing for clutter rejection under jamming condition

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Abstract:

The two-dimensional filtering approach often referred to as space-time processing, (STP) has been the subject of considerable research interest over the past two decades. The space-time processing (STP) is a crucial technique for the new generation airborne radar with high air-to-ground performance. Slowly moving ground targets produce a reflected signals which could not be distinguish from the surrounding clutter reflections, The results in either space processing, (adaptive array) or temporal processing, (MTI) will be miss detection of the desired target which has Doppler frequency and angle of arrival near to that ones of the clutter. Moreover ground based jammers could produce a wide frequency spread jamming signal with relatively high power to confuse airborne radars. However STP could significantly reduces the effects of both the clutter and jammers.

I- Introduction:

In a look down airborne radar, target signals often have to compete with strong ground clutter returns. Owing to the platform motion, the spread in Doppler of the clutter return can be significant, and clutter suppression is far more problematical than in ground-based radar applications. A one dimensional temporal filtering technique, e.g. conventional moving target indicator (MTI), can only successfully achieve sufficient rejection over the full clutter bandwidth at the expense of attenuation of the returns from slow moving targets. Since there is an interdependence of clutter Doppler and clutter angular location, significantly better performance can be achieved with two dimensional filters, which utilise both spatial frequency and temporal frequency characteristics to discriminate targets from clutter. The space-time adaptive processor consists of  $N$  antenna elements, these elements provide spatial sampling of the back scattered wave field, each of the array channels includes amplification, complex demodulation and digitization. Each channel is followed by  $M$  shift registers to store subsequent echo samples; this is the temporal domain of the space-time processing. All the spatial-temporal data are filtered by a space-time matched filter including the coefficients of the signal reference (beam-former and Doppler filter coefficients). These coefficients are adapted by calculating covariance matrix, of the received signal to reduce the effect of the clutter and other interference sources, while it maintains the desired signal. A test function is then calculated based on the actual output signals of the Doppler filter bank and is fed into detection device. The effect of MTI under jamming conditions has been discussed by [1,2], assuming that the jammer bandwidth is narrow compared to usable Doppler bandwidth.

In this paper the effect of additional jamming on the performance of the space time clutter filters will be analyzed. Assuming that the jammers transmit continuously (CW signals) over a broadband, which means that the jamming bandwidth is larger than the usable Doppler bandwidth. The jammer and clutter rejection perform in two steps; the first step includes the estimation of the spatial jammer covariance matrix in a passive radar mode, i.e. before transmitting, to make sure that the covariance matrix estimate is free of clutter. In the second step the jammer and clutter covariance matrix is estimated after transmit and after the spatial anti jamming filter. The resulting space-time clutter filter has to cope with clutter only if the jammer cancellation is perfect.

## II- Signal, clutter, and jammer representation

### II-1 Signal model

Consider a stationary point target, located at certain point  $p$  on the ground, is illuminated by the airborne radar beam. Fig.1 shows the geometry of the airborne array radar. The radar platform is assumed moving in the  $x$ -direction which is considered as a zero azimuth. One will utilize the following notations along the paper to describe the relative positions of the airborne radar and the target.  $\theta$ , is the depression angle,  $R_s$  is the slant range,  $R_g$  the ground range,  $\varphi$  denotes azimuth angle,  $v_p$  the platform velocity.

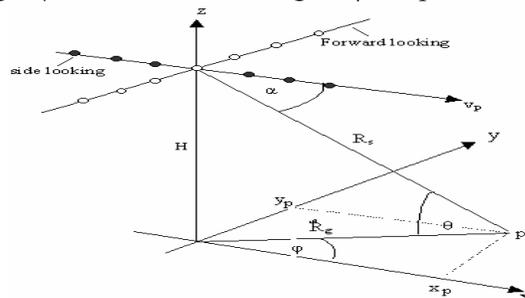


Fig.1 Geometry of airborne antenna array

There are many methods for the array orientation along the radar platform. The most commonly used are the linear sideways looking array (SLA) and the linear forward-looking

array (FLA). These two types of arrays are depicted on Fig.1. Assuming that, the transmitted radar signal is a pulse waveform, which has the following shape:

$$S_t(t) = \text{Re}[A_t E(t) \exp(j\omega_c(t+mT))] \quad , m = 1 \dots M \quad (1)$$

where:  $A_t$ ,  $\omega_c$  are the transmitted signal amplitude and the angular carrier frequency respectively.  $E(t)$  is the envelope of the transmitted waveform,  $T$  is the pulse repetition period and  $M$  is the number of echoes from a target at given range received by the phased array antenna of the STP. The received signal can be analyzed in two dimensions, the spatial dimension and the time dimension. The spatial processor utilizes the relative phase shift of the antenna array output to exploit the directions of arrival of the received signals. Consequently one could de-correlate them and enhance the signal of interest and remove the undesired signal, based on their directions. Meanwhile the temporal processor utilizes the relative delays between successive received pulses to exploit the Doppler spread of the received signal. Consider a sensor with the coordinates  $x_i, y_i, z_i$  relative to the array origin 0. The signal received by this sensor from a stationary scattered point on the ground is phase shifted relative to the origin by:

$$\Delta\Phi = j \frac{2\pi}{\lambda} ((x_i \cos \varphi + y_i \sin \varphi) \cos \theta - z_i \sin \theta) \quad i = 1 \dots N \quad (2)$$

Where  $N$  is the number of array elements.

The received signal after demodulation and matched filtering becomes [3]:

$$S_r(R_s, x_i, y_i, z_i) = A \exp[ j \frac{2\pi}{\lambda} (x_i \cos \varphi + y_i \sin \varphi) \cos \theta - z_i \sin \theta ] \quad (3)$$

In a pulse Doppler radar the Doppler frequency is measured by phase comparison between echo signals from a transmitted coherent pulse train. The received signal from a moving target after demodulation and matched filtering becomes:

$$S_r = A_r \exp[ j \frac{2\pi}{\lambda} 2 v_{rad} (mT) ] \quad (4)$$

Where,  $m = 1 \dots M$ , and  $v_{rad}$ : the radial velocity between the target and the platform of the radar. It is clear from the geometry shown in Fig.1, that the radial velocity is given by  $v_{rad} = v_p \cos \varphi \cos \theta$ . Thus the received signal at the elements of the temporal processor, (4) becomes:

$$S_r = A_r \exp[ j \frac{2\pi}{\lambda} 2 v_p mT \cos \varphi \cos \theta ] \quad (5)$$

From previous discussion it is noted obviously the temporal processor only provides information about the relative motion of the radar and target, while as the spatial processor provides only the directivity of the received signal. The space-time processor combines the effect of the two dimensions on received signal. Thus a received signal by STP is expressed as:

$$S_r = A(\varphi) \exp[ j \frac{2\pi}{\lambda} ((x_i + 2v_p mT) \cos \varphi + y_i \sin \varphi) \cos \theta - z_i \sin \theta ] \quad (6)$$

It is worse to note that (6) expresses the received signal at the  $i^{\text{th}}$  element of the antenna array and the  $m^{\text{th}}$  tap of the tapped delay following that antenna element. Obviously the STP provides an  $NM$  slightly varying replicas of the received signal. This large number of simultaneously existing samples of the received signal, in the space-time processor, enhances the target detection capabilities of the airborne system as will be shown later. In the SLAR, the airborne radar looks for targets at sides of the platform and (6) becomes:

$$S_r^{SLAR} = A(\varphi) \exp[ j \frac{2\pi}{\lambda} (x_i + 2v_p mT) \cos \varphi \cos \theta ] \quad (7)$$

## II-2. Clutter model

A model for airborne clutter has already been presented by Ringel model [4]. In this model the clutter power contributions of individual range-Doppler cells of single channel radar are

computed. Owing to the nonlinear boundaries of range- Doppler cells, this is a complex procedure; on the other hand the Ringel model computes clutter power only. For the purpose of analyzing space-time processing algorithms a simpler model is sufficient for clutter echoes representation, one utilise the following usual assumptions:

The contributions of different scatterers to the clutter echo are statistically independent; since the received clutter echoes are the sum of a large number of scatterers. Hence they are asymptotically Gaussian.

Temporal clutter fluctuations are slow compared with observation time of space-time processing.

The total clutter echo is an integral over the various contributions from all ground scatters in the visible range. The clutter echo could be obtained for a single range increment at the sensor number  $i$  at the  $m^{\text{th}}$  instant of time by integration of (6) [3]:

$$c_{im} = \int_{\varphi=0}^{2\pi} s_r(\varphi) d\varphi \quad (8)$$

$$c_{im} = \int_{\varphi=0}^{2\pi} A \exp[j \frac{2\pi}{\lambda} ((x_i + 2v_p m T) \cos \varphi + y_i \sin \varphi) \cos \theta - z_i \sin \theta] \quad (9)$$

Where A is a circular Gaussian distributed random variable (Gaussian amplitude and uniformly distributed phase) expresses the amplitude of the received clutter echo signal.

### II- 3 Jamming model

The capability of radar for detection of targets is limited not only by the noise and clutter returns, but also by the unlimited number of interference signals. There are many sources of interference such as co-channel radars, microwave and satellite communications. In this paper one will concentrate on the effect of continuous wave (CW) noise jamming. The reason for choosing this type of jamming is that there are many sources of interferences, in the environments, which add together to a Gaussian random variable accompanying the received signal [3,10]. The jamming signal received by the  $i^{\text{th}}$  sensor of the array owing to J jammers is given by

$$c_i^{(j)} = \sum_{l=1}^J A_l(m) \exp[j \frac{2\pi}{\lambda} (x_i \cos \varphi_l + y_i \sin \varphi_l) \cos \theta_l - z_i \sin \theta_l] \quad (10)$$

Where  $A_l$ ,  $\varphi_l$ ,  $\theta_l$  are the jammar amplitude, the azimuth and depression angles respectively which determine the position of jammers. Further more it is assumed that the jamming signals are Doppler broadband. Therefore, the jammers amplitudes,  $A_l$ , are modeled as complex Gaussian variables. We assume that the different jammer signals are temporally uncorrelated, that the output of an array element at the  $m^{\text{th}}$  and  $p^{\text{th}}$  time instants satisfy that.

$$E\{A_l(m)A_l^*(p)\} = \begin{cases} p_l & : m = p \\ 0 & : m \neq p \end{cases} \quad (11)$$

Further more it is assumed that the jammers are mutually uncorrelated, thus

$$E\{A_l A_k^*\} = \begin{cases} p_l & : l = k \\ 0 & : l \neq k \end{cases} \quad (12)$$

However, the sensor output signals owing to the  $l^{\text{th}}$  jammer are mutually fully correlated according to following relation:

$$E\{c_i^{(j)}(l)c_k^{(j)*}(l)\} = p_l \exp[j \frac{2\pi}{\lambda} ((x_i - x_k) \cos \varphi_l + (y_i - y_l) \sin \varphi_l) \cos \theta_l - (z_i - z_l) \sin \theta_l] \quad (13)$$

This expression includes the jammer power and the phase differences between different sensors.

### II-4. Noise model:

The main limiting factor in radar detection is the noise generated in radar receiver, which is dominated by the first amplifier in receiver chain. It is assumed that the noise to be uncorrelated in time and space

$$E\{n_m n_p^*\} = \begin{cases} p_n & : m = p \\ 0 & : m \neq p \end{cases} \quad (14)$$

Fig.2 shows the trajectories of typical jammer-clutter scenario for a SLAR in the  $\varphi$ - $f_D$  plane. Although the space-time clutter spectrum extends along the diagonal of the plot, the jammer is located at certain position and extends over the whole Doppler domain.

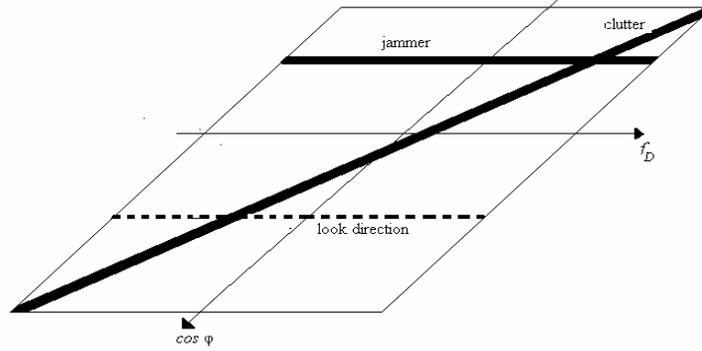


Fig.2 Trajectories of clutter and jammer.

### III- Performance evaluation of the space-time adaptive processor

In order to detect the presence of the signal  $S(t)$ , the STAP must be tuned to the signal vector  $\underline{S}$  in such manner that, the effect of noise and interferences are minimized. This procedure is occurred by maximization signal to interference plus noise ratio at the STAP output. The output of the STAP is weighted sum of the component of the observation vector  $\underline{X}$ , which is given as:

$$X(t) = [X_1^T(t), X_2^T(t), X_3^T(t), \dots, X_M^T(t)]^T \quad (15)$$

$X(t)$  is  $(NM \times 1)$  stacked snapshot vector at all of the data within the processor. The weights are arranged in the  $(NM \times 1)$  vector as:

$$\underline{w} = [w_1 \ w_2 \ \dots w_m \ \dots w_M]^T \quad (16)$$

Where  $w_m$  the  $(N \times 1)$  vector applied at the  $m$ -tap of the processor is given as:

$$\underline{w}_m = [w_{1m} \ w_{2m} \ \dots w_{nm} \ \dots w_{Nm}]^T \quad (17)$$

It is a straight forward to show that the weight vector which maximizes the output signal to noise (clutter plus jamming plus thermal noise) ratio is given by the Winner solution as the following equation [5,6],

$$w = \gamma Q^{-1} s \quad (18)$$

where  $\gamma$  is an arbitrary scalar,  $Q$  is the  $(NM \times NM)$  covariance matrix of the data and  $\underline{S}$  is the  $(NM \times 1)$  signal steering vector which matches the array response to the desired signal.

Then output  $Y$  of the processor is expressed as:  $Y = \underline{w}^T \cdot \underline{X} \quad (19)$

In the analysis of space-time adaptive processing for clutter cancellation one is interested in the improvement factor ( $IF$ ) of the system because the expression  $IF$  is commonly used for characterizing temporal filter for clutter rejection, and the same formula may be used for spatial applications in the context of the interference suppression. The improvement factor is defined as:

$$IF = \frac{\frac{P_s^{out}}{P_n^{out}}}{\frac{P_s^{in}}{P_n^{in}}} = \frac{W^* S(\varphi, f_D) S^*(\varphi, f_D) W}{W^* Q W \cdot S^*(\varphi, f_D) S(\varphi, f_D)} \text{tr}(Q) \quad (20)$$

In practice, a prior data of the interference (clutter or/and jammer) situation cannot be determined. Therefore the space-time adaptive processing (STAP) must specify a learning period to estimate the spatial covariance matrix  $Q_{\sim}$  of the clutter, the processor using “K” observation vector for matrix estimation, this number is given by [7]:

$$K = Nu (2 MN), Nu \geq 2. \tag{21}$$

The space-time covariance matrix  $Q_{\sim}$  is estimated as as :

$$Q_{\sim} \approx \frac{1}{K} \sum_{j=1}^K X_j \cdot X_j^T \tag{22}$$

If the interference field changes, then the weight vector  $W_{opt}$  is no longer optimum to maximize  $SINR_o$  (signal to interference plus noise ratio on the output) at the output of processor. Therefore, the weight vector must be update to adapt the new interference (clutter or / and jammer) conditions. The new optimum weight vector is obtained based on set of new observation vectors to estimate the covariance matrix [8,9].

#### IV- Simulation results

A sideway looking airborne radar (SLAR) geometry was simulated with a twelve elements linear array aligned along the flight direction. The transmitted power was chosen to produce a clutter to thermal noise ratio of approximately 20 dB at a single element. The parameters used in the simulation are summarized below:

Platform velocity	90 m/s
Pulse repetition frequency, PRF	12 k Hz
Number of receive elements, N	12
Number of taps per channel, M	5
Sensor spacing, d	$d = \lambda / 2$
Clutter to noise ratio, CNR	20 dB
Signal to noise ratio, SNR	20 dB
Receiver noise	White
Look direction, $\phi_L$	45o
Range	10 km
Altitude	3 km

#### IV-1 Estimation of clutter power spectra

During the simulation the clutter received signal is sampled properly at the PRF rate. The covariance matrix of the clutter is approximated according to time averaging rather than the mathematical expectation. Good estimation of the space time covariance matrix play a good role in the optimization of the space-time processor to reduce the clutter and other types of interference. The subspace method based on the eigenvalue decomposition, (EVD) of the covariance matrix of the received data or the equivalent singular value decomposition, (SVD) of the received data matrix gives a good estimate for both the clutter plus signal subspace and the noise subspace. The number of non-zero eigenvalues indicates the number of present signals. Further more the number of eigenvalues related to the clutter, are determined as function of the array orientation, number of array elements and number of Doppler processor taps. The eigen spectrum of the clutter, for the considered array and the space-time covariance matrix of the homogeneous ground clutter obtained by that processor is shown in Fig. 3 A second important aspect is clutter power spectra. Fig. 4 shows the clutter power spectra calculated using the minimum variance estimation (MVE) method. It is plotted against normalized Doppler frequency  $f_D$  and the angle  $\phi$ . It can be seen from the figure that the spectrum of sideways looking airborne radar (SLAR) is a knife-edge along the diagonal of an angle- Doppler plane.

#### IV-2 Behavior of optimum space-time processor in presence of clutter

Assume that, there is a desired target at looking direction  $\varphi = 45^\circ$  and depression angle  $\theta = 25^\circ$  and, it has Doppler frequency equals to 5 kHz. Fig. 5 shows the power spectrum of signal plus clutter. The target has a Doppler frequency close to the clutter Doppler frequency at the same direction. In one-dimension processing (temporal or spatial) this target is hard to detect. This is because adaptive array only will steer null in the direction of the clutter, while as the MTI processor, will not pass Doppler components of clutter, and consequently this target. The improvement factor of space-time adaptive processing, given by (20) is shown in Fig.6. The improvement factor is plotted against the normalized Doppler frequency and look direction. It can be seen that along the  $\varphi$ -F clutter trajectories a narrow trench has now been formed by the optimum processor. By comparison between Fig.4 and Fig.6 one notices that the improvement factor is just the inverse of minimum variance clutter spectrum, so that, outside the clutter trench it can be found that an improvement factor plateau detection of moving target is optimum, (flat surface with level 0 dB along the  $\varphi$ - $f_d$  plane except at the clutter trench where the processor IF is about -25 dB). Fig.7 shows the contours of the clutter spectrum, clutter and signal spectrum, and the improvement factor. Figure (7-c) shows that the space-time processing can reject the clutter without delete the target signal.

#### IV-3 Performance evaluation of clutter rejection under jamming Condition:

In practice, the search radar system is almost jammed from its side lobes i.e, by stand off jamming sources. This section is devoted to the effect of jamming on clutter rejection. It will be focused at the clutter plus jammer covariance matrix and its eigenspectra, power spectrum, and the improvement factor of the STAP system. Assume that, there is one continuous wave jammer has the following parameters, looking direction  $\varphi_j$  equals to  $135^\circ$ , depression angle  $\theta_j$  equals to  $\arcsin(3/10)$ , and the jammer to noise ratio (JNR) equals to 20 dB. The jammer signals are assumed of Doppler broadband. Therefore the jammer amplitude is modeled as a complex Gaussian variable. Fig.8 illustrates the clutter plus jammer power spectrum plotted against normalized Doppler and azimuth angle. It is clear that the footprint of clutter is a straight line along the diagonal of  $f_d$ - $\varphi$  plane, and the jammer appears a thin wall along the Doppler axis at the angle  $135^\circ$ . The improvement factor for jammer and clutter rejection is shown in Fig.9, it can be clearly seen the trench stop bands of clutter and jammer.

#### V- Conclusions

This paper is devoted to evaluate the space-time adaptive processing for clutter and jamming rejection in airborne radars. The performance of the processor is evaluated using many aspects such as the covariance matrix, the power spectra and the improvement factor obtained by optimizing the STAP. The contributions of this chapter are summarized at the following notes.

The clutter Doppler frequency of an individual scatterer is proportional to the angle between flight axis and direction of arrival.

For a SLAR the clutter power spectra is concentrated on the diagonal across the  $f_d$ - $\varphi$  plane, while the wide band jammer spread its power spectrum along the doppler frequency band from certain direction on the  $f_d$ - $\varphi$  plane.

The improvement factor for STAP is just the inverse of the clutter plus jammer power spectra. This leads to removing the undesired clutter and jamming while it maintains the desired signal.

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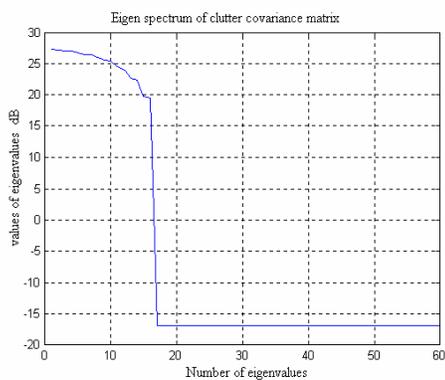


Fig. 3 Eigenspectra of airborne clutter

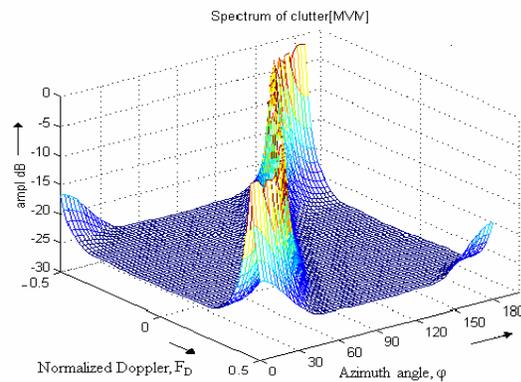


Fig. 4 clutter power spectrum for SLAR

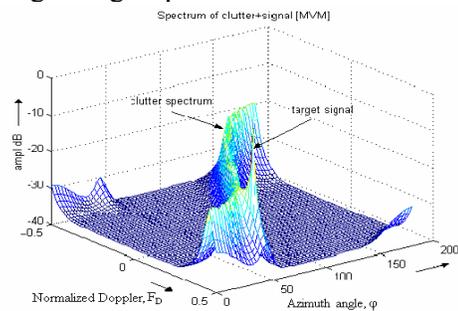


Fig.5 Power spectrum of clutter plus signal for SLAR

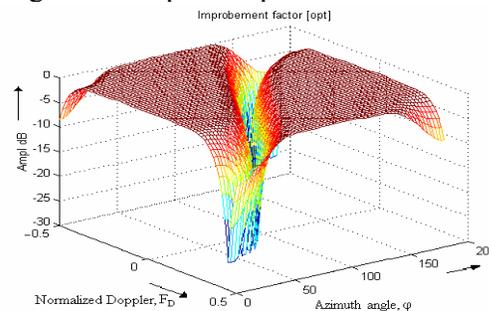


Fig. 6 Improvement factor for SLAR

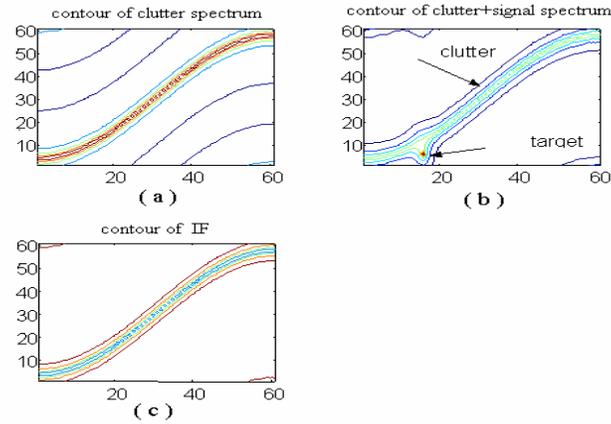


Fig.7 Contours of clutter, clutter plus signal spectrum and IF  
 ( a ) Contours of clutter spectrum.  
 ( b ) Contours of clutter plus signal spectrum.  
 ( c ) Contours of improvement factor.

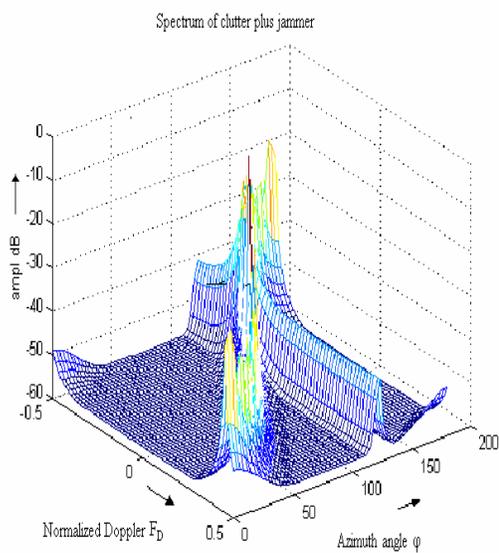


Fig. 8 Clutter plus jammer power spectrum.

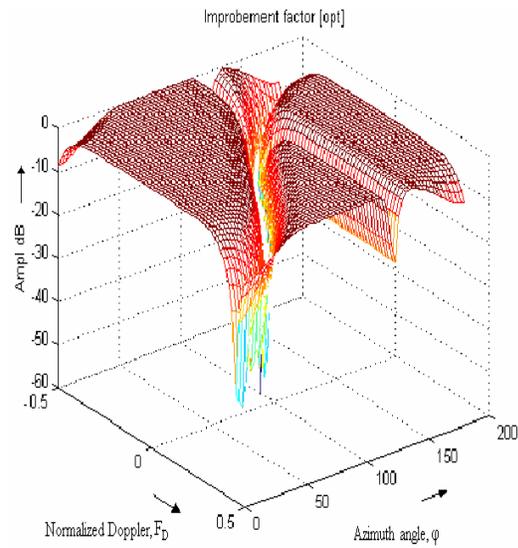


Fig.9 Improvement factor for clutter and jammer