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Robust Flight Control System Design Using H_∞ Loop-Shaping and Recessive Trait Crossover Genetic Algorithm

By

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Abstract:

A proposed approach to robust controller design is introduced. This approach combines the Recessive Trait Crossover Genetic Algorithm with the loop shaping design procedure using H_∞ synthesis. The requirements, design and simulation of a flight control system for precision tracking task are considered. The proposed method is applied to design a control system for the F-16 fighter aircraft model. The flight simulations reveal that the desired performance objectives are achieved and that the controller provides acceptable performance in spite of modeling errors and plant parameter variations.

Keywords:

Recessive trait crossover genetic algorithm, H_∞ Loop-Shaping, Flight simulations, Control augmentation system

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1. Introduction:

Over the last decade, genetic algorithms (GAs) have been extensively used as search and optimization tools in various problem domains, including the science, commerce, and engineering. As a general purpose optimization tool, GAs are moving out of academia and finding significant application in many other venues. Their popularity can be attributed to their freedom from dependence on functional derivatives and their incorporation of these characteristics [1]:

1- GAs are parallel search procedures and can be implemented on parallel processing machines.
2- GAs are applicable to any optimization problems.
3- GAs are stochastic and less likely to get trapped in local minima.
4- GAs are flexible for both structure and parameter identification.

GAs are different from more normal optimization and search procedures in four ways [2]:

1- GAs work with a coding of the parameter set, not the parameters themselves.
2- GAs search from a population of points, not a single point.
3- GAs use payoff (objective function) information, not derivatives or other auxiliary knowledge.
4- GAs use probabilistic transition rules, not deterministic rules.

Specifications for the performance of feedback control systems are often expressed in terms of inequalities which need to be satisfied. A separate development has been the use of H_∞ optimization in a variety of approaches to design robust control systems. One such approach is the Loop Shaping Design Procedure (LSDP) [3], [4]. This approach involves the robust stabilization to additive perturbation in the sense of H_∞ norm of normalized coprime factors of a weighted plant. The weighted-plant singular values are shaped by adjusting the weighting functions to give a desired open-loop shape which gives good closed-loop performance with stability robustness. Certain aspects of the LSDP make it suitable to combine this approach with the Recessive trait Crossover Genetic Algorithm (RCGA) to design directly for both closed-loop performance and stability robustness. This brief paper describes this new approach and applies the proposed method to the design of robust controller for a model of the F-16 aircraft.

The paper is organized as follows: In Section 2, a mathematical description of the F-16 aircraft model is introduced. Section 3 describes the underlying aircraft control augmentation system (CAS) and the performance requirements imposed on it. Section 4 gives a detailed description of the RCGA. Section 5 gives a brief description of the loop shaping design procedure using H_∞ synthesis. Section 6 describes robust design using a coprime factor plant description with RCGA. Flight simulation of the closed-loop system with the proposed technique is presented in Section 7 and finally this paper concludes with a brief summary in Section 8.

2. F-16 Aircraft Modeling:

A model for an F-16 combat aircraft was used to generate the simulation results in this paper. The simulation uses the standard longitudinal equations of motion and kinematic relations found in a variety of standard references on flight dynamics (see for example [5, 6]).

$$\dot{U} = -Q\,W + \frac{F_x}{m} - g\ \sin\theta \qquad (1)$$

$$\dot{W} = Q\,U + \frac{F_z}{m} + g\ \cos\theta \qquad (2)$$

$$\dot{\theta} = Q \qquad (3)$$

$$\dot{Q} = \bar{q}\ S\ \bar{c}\,\frac{C_m}{I_y} \qquad (4)$$

$$F_x = \bar{q}SC_x + T \qquad (5)$$

$$F_z = \bar{q}SC_z \qquad (6)$$

where: U and W are the forward and downward components of the aircraft velocity V_T respectively; g is the gravitational acceleration vector; θ is the pitch angle; m is the aircraft mass; Q is the pitch-rate; S is wing area; I_y is the moment of inertia about OY axis; q is the dynamic pressure; c is the mean dynamic chord; F_x and F_z are the total forces acting along X and Z axes respectively.

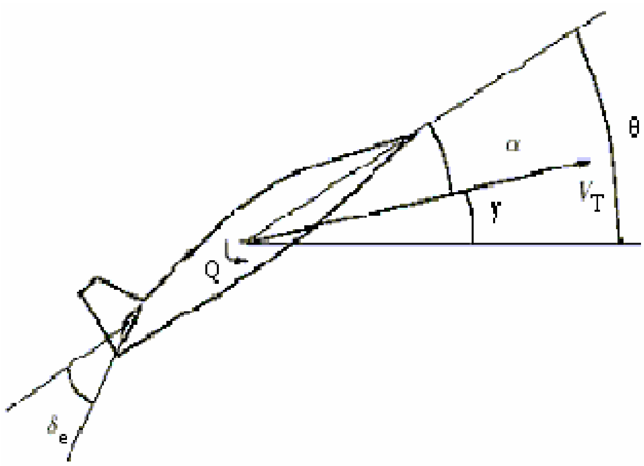

Figure (1): *Illustration of longitudinal aircraft entities*

T is the engine thrust vector (non-linear function depends on throttle setting δ_t); the non-

dimensional aerodynamic force coefficients C_x, C_z and moment coefficient C_m depend on angle of attack α and elevator deflection δ_e. The data of these coefficients are contained in lookup tables [7].
Since the aerodynamic force and moment components depend on the angle of attack and the aircraft velocity, we replace the state variables U and W in the above equations by V_T and α according to the following relations:

$$\alpha = \tan^{-1}\left(\frac{W}{U}\right) \tag{7}$$

$$V_T = \sqrt{U^2 + W^2} \tag{8}$$

By taking the derivatives of these the two equations, the state vector becomes as follows: $x = [V_T\, \alpha\, \theta\, Q]^T$.
A second-order short-period approximation is obtained by simplifying the longitudinal equations in the flight-path axes system using the usual assumptions [8]. The short-period equations with the elevator as a control input for the nominal flight condition are given as follows:

$$\frac{Q}{\delta_e} = \frac{-10.45247(s+1.0582)}{(s+1.2369 \pm j1.4961)} \tag{9}$$

The transfer function of the elevator actuator dynamics is given by:

$$\frac{\delta_e(s)}{\delta_{ec}(s)} = \frac{10}{(s+10)} \tag{10}$$

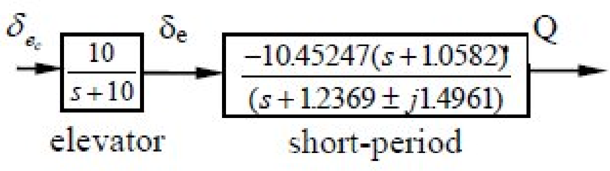

Figure (2): *Augmented plant-actuator system*

Incorporating the actuator dynamics into the aircraft state equation yields:

$$G = \frac{-104.5247s - 110.6088}{s^3 + 12.4738s^2 + 28.5058s + 37.6809} \tag{11}$$

3. Longitudinal CAS:

It is necessary to design control augmentation systems to provide the pilot with a particular type of response to the control inputs. Normally CASs are split into two control systems, to handle longitudinal and lateral problems, assuming negligible interaction. They are implemented by feedback controllers using accelerometers and rate gyros as sensors; and elevators, ailerons, or rudder as control surfaces.
In high-performance military aircraft, the pilot may have to perform tasks such as precision tracking of targets. In this situation, a suitable controlled variable is the pitch-rate (Q), which is required to follow a pilot's stick command. It has been found that a deadbeat response to pitch-rate commands is well suited to the task [7]. Therefore, a specialized control augmentation system is needed; known as a "Pitch-rate CAS". This system is conventionally designed for the longitudinal dynamics.

3.1. Controller Structure:

Most control designs use the single-Degree-Of-Freedom (SDOF) control structure. However, this structure has the disadvantage that the feedback properties cannot be attained independently to the reference tracking capability. Therefore, a compromise between good robustness and good tracking should be made.
If there are strict requirements on both set-point tracking and disturbance rejection, an acceptable compromise might not exist, i.e. we cannot achieve both of these simultaneously with a single-degree-of-freedom controller. The solution is to use a two degrees-of-freedom (TDOF) controller, by introducing an additional control block into the system, where the reference signal r and output measurement y are independently treated by the controller, rather than operating on their difference (r - y)[9].

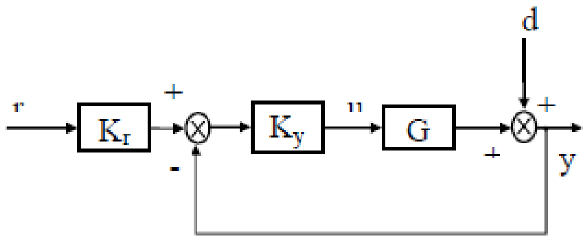

Figure (3): *Block diagram of the TDOF controller*

The controller is often split into two separate blocks as shown in Fig.3, where K_y denotes the feedback part of the controller and K_r a reference prefilter. The feedback controller K_y is used to reduce the effect of uncertainty (disturbances and modeling errors) whereas the prefilter K_r is used to shape the command r to improve tracking performance. In practice, K_y is often designed first, and then K_r is designed later.

3.2. Performance Requirements:

Since the performance specifications of aircraft are often given in terms of time-domain criteria such as the C* criterion and D* criterion [10, 11] and these criteria are close to the step response, we shall assume henceforth that the reference input is a step command. Designing for such a command will yield suitable time-response characteristics

The designed controllers must satisfy the following specifications:

1. Command Tracking: a deadbeat step response with rise time (to reach 90% of the final value) less than 0.5 sec and the overshoot less than 5% is required.
2. Disturbance Rejection: the output response to a unit step disturbance should remain within the range [-1, 1] at all times, and it should return to 0 as quickly as possible ($|y(t)|$ should at least be less than 0.1 after 3 sec, i.e. 90% is rejected within 3 sec).

4. Recessive Trait Crossover Genetic Algorithm:

In the nineteenth century, Darwin originated his theory of evolution [12]. Darwin suggested that in the universal struggle for life, nature "selects" those individuals who are best suited (fittest) for the struggle, and these individuals in turn reproduce more than those who are less fit, thus changing the composition of the population.

There are three methods of population inheritance, dominant, recessive and sex linked [13]. The sex-linked properties expressing depend on the person sex. For dominant properties, only one genetic trait is needed for this property to be expressed. However, if a genetic trait is recessive, a person needs to inherit two copies of the gene for the trait to be expressed. Thus, both parents have to be carriers of a recessive trait in order for a child to express that trait. If both parents are carriers, there is a 25% chance with each child to show the recessive trait and it becomes 100% if the both have that recessive trait.

Using the concepts taken from the recessive property inheritance, a crossover operator has been developed. Here the GA with this operator is called RCGA. The RCGA produces children by selecting the common genes between parents, and choosing the remaining genes randomly. The main difference between the traditional crossover GA and RCGA is the way of how the new population is inherited from the previous generations. To use the proposed population inheritance approach through the recessive trait crossover, we assume that the complementary of all of the chromosome parts makes its survival fitness, and the length of the chromosomes is fixed [14, 15].

The overall algorithm can be written as:

1. Create a random population of N individuals.

2. Evaluate their fitness.
3. Sort the individuals in the population according to their fitness.
4. Choose the best N/2 individuals as mating pool to generate the new population.
5. Generate four new individuals by reproducing the nearest two parents from the mating pool keeping the common genes and randomly swapping the different genes. This creates a new population of N individuals.
6. Apply mutation operation with a probability.
7. Repeat steps from 2 to 6 for the best fitness value.

According to this procedure, RCGA is different from the TCGA in six ways:

1. RCGA applies the crossover to all the mating pool solutions (i.e. the crossover rate=1), not on randomly selected pairs with certain crossover rate (usually.7).
2. RCGA mating pool is selected after sorting individuals based on their fitness rank, not randomly selected.
3. RCGA constructs the offspring by reproducing the nearest two parents in the mating pool, not randomly selected.
4. RCGA constructs the offspring by keeping the common genes without any change and select the rest randomly, not by exchanging alternate substrings.

A number of applications have been reported using the proposed RCGA in [14, 15], including active vibration control and function optimization. It was noted that the proposed RCGA offers better convergence and higher accuracy as compared to the traditional GA.

5. Normalized Coprime Factorization:

The plant model $G_S = \tilde{M}_S^{-1} N_S$ is a normalized left coprime factorization (NLCF) of G if $\tilde{M}, \tilde{N} \in RH_\infty$ there exists $V, U \in RH_\infty$ such that $\tilde{M}V + \tilde{N}U = I$, and $\tilde{M}\tilde{M}^* + \tilde{N}\tilde{N}^* = I$ where for a real rational function of s, X^* denotes $X'(-s)$. Using the notation:

$$G(s) = D + C(sI - A)^{-1} B \overset{s}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad (12)$$

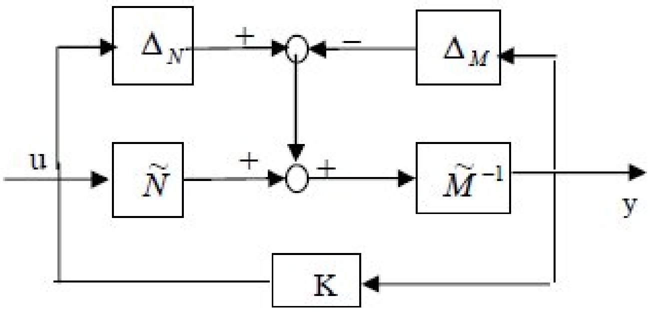

Figure (4): *Robust stabilization with respect to coprime factor uncertainty*

Then as shown in [3]:

$$[\tilde{N}\ \tilde{M}] \stackrel{s}{=} \begin{bmatrix} A+HC & B+HD & H \\ R^{-1/2}C & R^{-1/2}D & R^{-1/2} \end{bmatrix} \tag{13}$$

is a normalized coprime factorization of G, where $H = -(BD' + ZC')R^{-1}$, $R = I + DD'$, and the matrix $Z \geq 0$ is the unique stabilizing solution to the algebraic Riccati equation (ARE).

$$(A - BS^{-1}D'C)Z + Z(A - BS^{-1}D'C)' - ZC'R^{-1}CZ + BS^{-1}B' = 0 \tag{14}$$

where $S = I + D`D$

A perturbed model G_p is defined as

$$G_p = (\tilde{M} + \Delta_M)^{-1}(\tilde{N} + \Delta_N) \tag{15}$$

where Δ_M, $\Delta_N \in$RH.

To maximize the class of perturbed models defined by (15) such that the configuration of Fig.4 is stable, we need to find the controller K which stabilizes the nominal closed-loop system and which minimizes γ where

$$\gamma = \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G)^{-1}\tilde{M}^{-1} \right\| \tag{16}$$

$$(A - BS^{-1}D'C)Z + Z(A - BS^{-1}D'C)' - ZC'R^{-1}CZ + BS^{-1}B' = 0 \tag{14}$$

$$\left\| \left[\Delta_N \ \ \Delta_M \right] \right\|_\infty < \gamma^{-1} \qquad (17)$$

The minimum value of γ for all stabilizing controllers K is

$$\gamma_\circ = \inf K \ \ stablizing \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} \tilde{M}^{-1} \right\|_\infty \qquad (18)$$

It is shown in [16] that

$$\gamma_\circ = (1 + \lambda_{\max}(ZX))^{1/2} \qquad (19)$$

where λ_{max} (.) represents the maximum eigenvalue, and X 0, is the unique stabilizing solution of the ARE:

$$(A - BS^{-1}D'C')' X + X(A - BS^{-1}D'C) - XBS^{-1}B'X + C'R^{-1}C = 0 \qquad (20)$$

A controller which achievesoγ is given in [3] by

$$K \stackrel{s}{=} \begin{bmatrix} A+BF+\gamma_\circ^2(Q')^{-1}ZC(C+DF) & \gamma_\circ^2(Q')^{-1}ZC \\ B'X & -D' \end{bmatrix} \qquad (21)$$

where

$$F = -S^{-1} \ (D'C + B'X) \ \text{ and } \ Q = (1 - \gamma_\circ^2) \mathrm{I} + XZ \qquad (22)$$

From the above, the optimum controller is synthesized by the solution of two ARE's, unlike most H_∞ problems, which require an iterative search on γ to find the optimum.
The nominal plant *G* is augmented with pre compensators and post compensators W_1 and W_2, respectively, so that the augmented plant G_s is equal to W_2GW_1. The post & the pre-compensators weighting functions can be combined into compensating weighting function W_p.
Using the procedure outlined earlier, an optimum feedback controller K_{subopt} is synthesized which robustly stabilizes the NLCF of G_s given by
$(\tilde{N}_s, \tilde{M}_s)$ where $G_S = \tilde{M}_S^{-1}\tilde{N}_S$.
The final feedback controller K is then constructed by simply combining K_{subopt} with the

weights to give:

$$K_{opt} = W_1 . K_{subopt} . W_2 \tag{23}$$

Essentially with the LSDP, the weights W_1 W_2 and are the design parameters which are chosen both to give the augmented plant a "good" open-loop shape and to ensure that γ° is not too large. γ° is a design indicator of the success of the loop-shaping as well as a measure of the robustness of the stability property.

6. Robust Design Using A Coprime Factor Plant Description With RCGA:

Two aspects of design using robust stabilization of normalized coprime factor descriptions of the weighted plant make it amenable to be combined with the RCGA. First, unlike most H∞ optimization problems, the H∞ optimal controller for the weighted plant can be synthesized from the solution of just two ARE's and does not require time-consuming γ-iteration. Second in the LSDP, the weighting functions are chosen by considering the open-loop response of the weighted plant, so effectively the weights W_1 and W_2 are the design parameters to satisfy closed-loop performance.
With low-order weighting functions, high-order controllers can be synthesized which often lead to significantly better performance or robustness than if simple low-order controllers were used, Additionally, the problem of finding a stability point does not exist, because stability is guaranteed through the solution to the robust stabilization problem, provided that the weighting functions do not cause undesirable pole/zero cancellations.

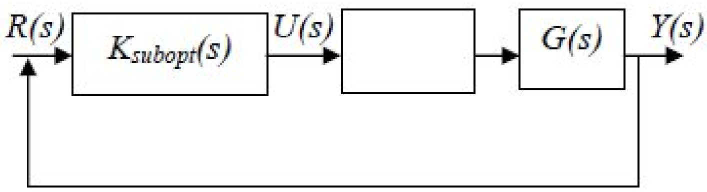

Figure (5): *The standard H_∞ loop shaping closed-loop system*

The design problem is now stated as follows:

Problem:
For the system illustrated in Fig.5, find (W_p, K_{supopt}) such that:

$$\gamma_o(\mathrm{W}) \leq \varepsilon_\gamma \tag{24}$$

and

$$\phi_i\,(W, K_{supopt}) \leq \varepsilon_i \;\; , \text{i} = 1.. \qquad (25)$$

where

$$\gamma_o = \inf K_{stablizing} \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} \tilde{M}^{-1} \right\|_\infty \qquad (26)$$

and φ_i (W, K_{subopt}) is a function of the closed loop system. ε_γ, ε_i are real numbers represent desired bounds on γ_o and φ_i respectively. W=(W_1,W_2) is a pair of fixed order-weighting function.

Design Procedure:

The design procedure to solve the above problem is:

1- Consider the plant *G*, and define the functional*i*φ.

2- Consider the values of ε_γ and ε_i.

3- Define the form and order of the weighting functions W_1 and W_2 . Bounds should be placed on the values of w_i to ensure that W_1 and W_2 are stable and minimum phase to prevent undesirable pole/zero cancellations.

4- γ_o is not fixed, but for stability robustness, it should not be too large [3], and is here taken as

$$\varepsilon_\gamma = 5.0$$

5- Implement the proposed method to find W_1 and W_2 and $K_{sup\ opt}$ which satisfies the required specifications. If the solution is not satisfactory, either increase the order of the weighting function or relax one or more of the desired bounds.

<u>7. Simulation Results:</u>

After 410 generations, 32 bit representation, 2% mutation rate, and 60 population sizes, the weighting functions obtained by the RCGA are W_1=1.999 and W_2=0.89, and the associated controller is given as:

$$K_{opt} = \frac{2.5083\text{s}^2 + 34.0092\text{s} + 26.2704}{\text{s}^2 + 31.8721\text{s} + 32.3895} \qquad (27)$$

A PID controller is used as a prefilter to enhance tracking the reference command.

Using RCGA for tuning the PID controller [17] the chosen PID controller has the transfer function as follows:

$$G_{PID} = K_p + \frac{K_i}{s} + K_D s = 20 + \frac{13}{s} + 2s \tag{28}$$

The closed-loop response to a unit step input is shown in Fig.7, where for $\gamma_o = 1.7285$, the maximum overshoot is 0.225%, the rise time is 0.2239 sec and the settling time is 1.3808 sec.

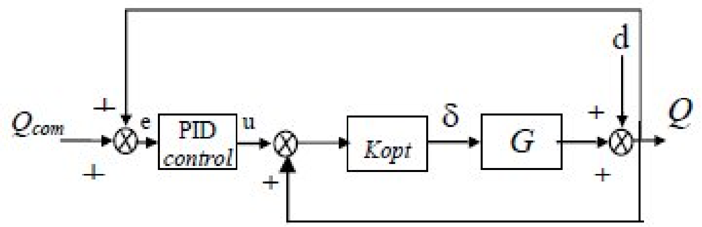

Figure (6): *A pitch-rate CAS using RCGA-LSDP*

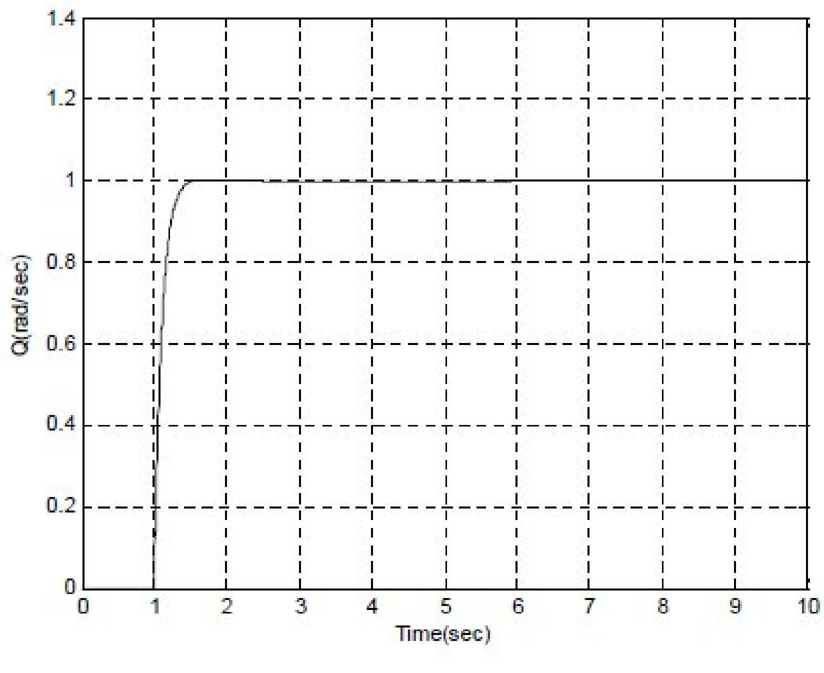

Figure (7): *Time response of the command tracking system*

The output response to a unit step change in the disturbance, applied directly at the plant output is shown in Fig. 8, where 90% is rejected within 0.1983 sec.

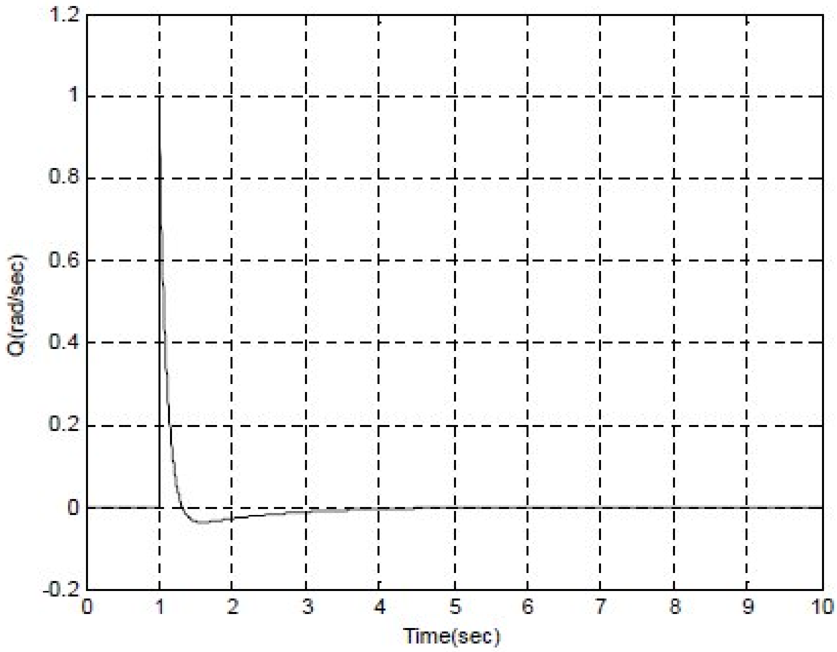

Figure (8): *Time response of disturbance unit step change*

6. Conclusions:

This paper presented the designing of a precision tracking of targets flight control system for the F-16 fighter aircraft model using a combined RCGA-H_∞ loop shaping Technique. Flight simulation showed that the proposed method satisfied all the required specifications.

The proposed method combines the flexibility of numerical optimization-type techniques with analytical optimization in an effective and practical manner. The use of RCGA to design the weighting functions is particularly suited to the NLCF approach because no γ-iteration is required.

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