Military Technical College
Kobry El-Kobbah,
Cairo, Egypt

8th International Conference
on Electrical Engineering
ICEENG 2012

Experimental Comparison among Pileup Recovery Algorithms for Digital Gamma-Ray Spectroscopy

Imbaby I. Mahmoud[a], Mohamed S. El_Tokhy[a], and Hussein A. Konber[b]

[a]Engineering Department, NRC, Atomic Energy Authority, Inshas, Cairo, Egypt

[b]Electrical Engineering Department, Al Azhar University, Nasr City, Cairo, Egypt

engtokhy@gmail.com

Abstract

This paper presents algorithms for overcoming a common problem of gamma-ray spectroscopy, namely the peak pileup recovery problem. Three different approaches are studied and evaluated within a spectroscopy system. The first approach is a direct search based on Nelder-Mead technique without any derivatives in order to find the local minimum points. A Gaussian shape in conjunction with the peak height and its position of each pulse are used to construct the pulse. Hence, the main pulse parameters such as peak amplitude, position and width can be determined. The second algorithm is based on the nonlinear least square method. In this paper another technique is tried. This technique which is proposed as third algorithm is based on a maximum peak search method combined with the first derivative method to determine peak position of each pulse. Comparison among these approaches is conducted in terms of parameters errors and execution time. This comparison based on a reference signal from ^{137}Cs point source acquired by data acquisition system. The third approach shows the best accuracy, for determining accurate energy spectrum. Therefore, it provides better isotope identification than other algorithms.

Keywords: Peak Pileup, Tail Pileup, Least-Square Fitting, Nelder-Mead Modified Simplex.

1. Introduction

Spectroscopic gamma-ray detectors are used for many research, industrial, medical and homeland-security applications [1-4]. The advantages of a digital system for gamma-ray spectroscopy in comparison with a classical analog system are reflected in the possibilities of implementation of complex algorithms and simple and rapid modification of algorithms used for signal processing. Using these systems the highest quality of measurements is achieved at both low and high counting rates with various radiation detectors [5]. Thallium-doped sodium iodide (NaI(Tl)) scintillation crystals coupled to photomultiplier tubes provide medium resolution spectral data about the surrounding environment. These effects are hardware-dependent and have strong effects on the radioisotopic identification capability of NaI(Tl)-based systems. Pulse pileup distortion is a common problem for radiation spectroscopy measurements at high counting rates [4-6]. Moreover, pileup is one of the most delicate problems of any spectrometric method that is related to the extraction of the correct information out of the experimental spectra.

In many applications as much as 80% of information can be lost due to the effects of dead time and pulse pileup [7]. The effects of pulse pileup in applications of nuclear techniques include the following issues. Imposing a fundamental limit on detector throughput (and therefore source intensity), decreased spectral accuracy and resolution, as peaks in the energy spectrum spread, reduced peak-to-valley ratios due to false detection of pulses, and causing significant detector dead time in the system [7]. Therefore without a correction on the response function of the detector system incorrect physical data are obtained from an analysis of measured spectra [8]. The deconvolution methods are widely applied in various fields of data processing and various approaches can be employed [8]. On one hand it must decompose completely the overlapping peaks while preserving as much as possible their heights, positions, areas and widths. The objective of this paper is to study different algorithms implementing these methods. So, we discuss the performances of three different pileup recovery algorithms; direct search, least square fitting, and first derivative combined with maximum peak search algorithms. Our primary focus is the analysis of pileup signals collected from detector systems to obtain high accuracy of the spectroscopy. The influence of white Gaussian noise on the recovery performance in the direct search and least square fitting algorithms is furthermore discussed. Pileup correction of gamma-ray spectroscopy using these three different algorithms is studied. These algorithms have the advantages of decomposition of multiple overlapping events into their original peaks. This paper is organized as follows: Section 2 presents the spectroscopy system. The more interesting characteristics of pileup recovery algorithms, and registration or rejection of the peak height are represented in Section 3. Comparison between the three algorithms and discussion are summarized in Section 4 and we terminate our study by a briefly discussion and stating some important conclusions that we noted from our obtained results.

2. System Components

In this system, the components of the system for evaluation of different pileup recovery algorithms are described. It contains the following elements; ^{137}Cs point source, scintillation detector, amplifier, data acquisition system and connection to a desktop personal computer (PC). An 1.5 inches x 7.5 inches NaI(TI) scintillation detector is used to detect the radiation signal from Cs^{137} point source. This detector is connected to amplifier through coaxial cable which in turn connected to the PC through digital scope. The digital scope used for signal digitization with sampling rate of 16 MS/s. MATLAB environment is used to perform pileup correction and spectrum evaluation.

3. Evaluation of Pileup Recovery Algorithms

3.1 Pileup recovery using deconvolution [9]

The differential pulse height spectrum that is recorded from any radiation detector is the convolution of its inherent response function and the energy distribution of the incident radiation. The deconvolution is sometimes used for the process of resolving or decomposing a set of overlapping peaks into their separate components by different techniques. Pulse pileup is an important issue for radiation spectroscopy applications of scintillation detectors, since the maximum counting rate is limited by pulse pileup. Radiation particles are emitted from a source following the interval distribution as shown below [4, 9, 10]

$$f\left(\Delta t\right)dt=\lambda_n e^{-\lambda_n \Delta t}dt \tag{1}$$

where λ_n, Δt, and dt denote the true average emission rate of the radiation, the time interval between the current time pulse stamp and the relative origin of the time axis that is conveniently the occurrence time of the previous pulse, and the infinitely small time interval between Δt and Δt+dt. A certain amount of light collection time is necessary for good energy resolution.

The probability P (N) for one pulse to occur without piling up with subsequent pulses may be estimated by

$$P\left(\Delta t>\tau\right)=e^{-\lambda_n \tau} \tag{2}$$

where τ is the first pulse width. Figure 1 shows the probability curve of absence of pileup against true rate at different pulse widths. As illustrated in this figure for applications where counting rate can be kept very low, pulse pileup distortion is not a problem. Also, the probability of absence of pileup decreases with increases the width of the first pulse. From the figure, we observe the dependency of pileup probability on both pulse width and true rate. In the following, three different algorithms for the pileup recovery are investigated.

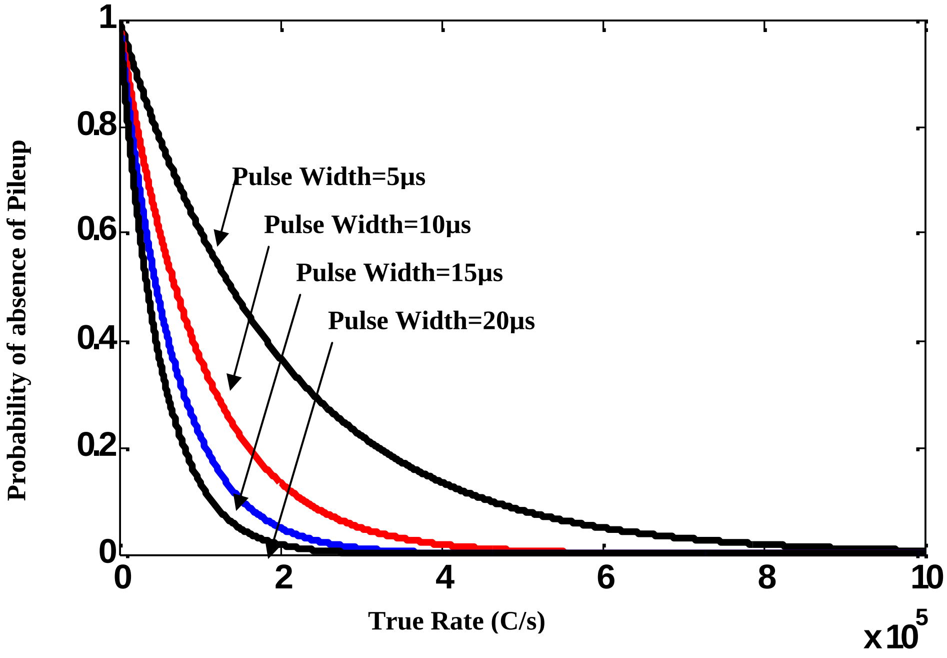

Fig. 1 Probability of absence of pileup with the true counting rate at different pulse widths

3.2 Direct search method

Here, we are interested in resolving or decomposing a set of overlapping peaks into their separate components. Nelder-Mead modified simplex technique is used for this purpose.

Direct search is a method for solving optimization problems that does not require any information about the gradient of the objective function. As opposed to more traditional optimization methods that use information about the gradient or higher derivatives to search for an optimal point, a direct search algorithm searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. We can use direct search to solve problems for which the objective function is not differentiable, or even continuous [11].

Direct search is used to describe sequential examination of trial solutions involving comparison of each trial solution with the "best" obtained up to that time together with a strategy for determining what the next trial solution will be [12]. It remains popular because of their simplicity, flexibility, and reliability. Examples of direct search methods are the Nelder-Mead Simplex method, Hooke and Jeeves' pattern search, the box method, and Dennis and Torczon`s Parallel Direct Search Algorithm (PDS) [13].

The simplex algorithm is one of the earliest and best known optimization algorithms. This algorithm solves the linear programming problems. The algorithm moves along the edges of the polyhedron defined by the constraints, from one vertex to another, while decreasing the value of the objective function at each step.

A modified simplex method for finding a local minimum of a function of several variables has been devised by Nelder and Mead [14]. The Nelder-Mead modified simplex algorithm succeeds in obtaining a good reduction in the function value using a relatively small number of function evaluations. This method finds the minimum of a function of several variables [15], starting at an initial estimate. This is generally referred to as unconstrained nonlinear optimization. Search methods that use only function evaluations (the simplex search of Nelder-Mead) are most suitable for problems that are very nonlinear or have a number of discontinuities. Since, it is a direct search method it does not use numerical or analytic gradients [11]. Despite its widespread use, essentially no theoretical results have been proved explicitly for the Nelder-Mead algorithm [16].

For two variables, a simplex is a triangle, and the method is a pattern search that compares function values at the three vertices of a triangle. The worst vertex, where the function value is largest, is rejected and replaced with a new vertex. A new triangle is formed and the search is continued. The process generates a sequence of triangles (which might have different shapes), for which the function values at the vertices get smaller and smaller. The size of the triangles is reduced and the coordinates of the minimum point are found. This algorithm is stated using one time the term *simplex* (a generalized triangle in N dimensions) and other time Nelder-Mead to find the minimum of a function of N variables. It is effective and computationally compact [14]. In other words, if n is the length of vector, x, a simplex in n-dimensional space is characterized by the n+1 distinct vectors that are its vertices. In two-space, a simplex is a triangle; in three-space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex. Usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance [11]. The MATLAB implementation of this algorithm finds both local minimum and maximum for a function without derivative [11].

Figure 2 depicts the algorithm for pileup recovery peaks using Nelder-Mead direct search method. As shown in this figure, this algorithm is applied to multiple overlapping peaks from experimental setup. Nelder-Mead algorithm is essentially a way of organizing and optimizing the changes in parameters to shorten the time required to fit function to the required degree of accuracy. The most general way of fitting any model to a set of data is the iterative method. Consequently, iterative fit is performed. Iterative methods proceed in the following general way as depicted in Fig. 3. This figure is illustrated as in the following [15]

1) Selects a model for the data;
2) First guesses of all the non-linear parameters are made;
3) A computer program computes the model and compares it to the data set, calculating a fitting error;
4) If the fitting error is greater than the required fitting accuracy, the program systematically changes one or more of the parameters and loops back around to step 3.

The pulse parameters; peak position, height and width are determined using this Nelder-Mead modified simplex algorithm. The pulse shape is constructed using its peak shape as Gaussian shape [17-20]. Consequently, the original peaks are recovered. Finally, the recovered peaks are registered as described in the following subsection. This algorithm is applied to multiple overlapping peaks from ^{137}Cs radiation point source of the experimental setup as depicted in Fig. 4. Two nonlinear parameters; peak position and width are determined using this algorithm. A Gaussian shape in conjunction with the peak and its position of each pulse are used to construct each pulse. The recovered peaks with both the input pileup peaks and sum of the recovered peaks are shown in Fig. 4. However, the recovered peaks are shown in Fig. 5.

1) Input Pileup Signal Containing Multiple Overlapping Peaks

2) Applying Nelder-Mead Technique

  A) Do Iterative Fit Routine

  B) Determine the Parameters of the Noisy Input Signal

3) Apply Recovery Peaks Routine

4) Detect Illusive Pulses (If Exist)

5) Register Actual Peaks

Fig. 2 Pileup recovery using the Nelder-Mead direct search algorithm

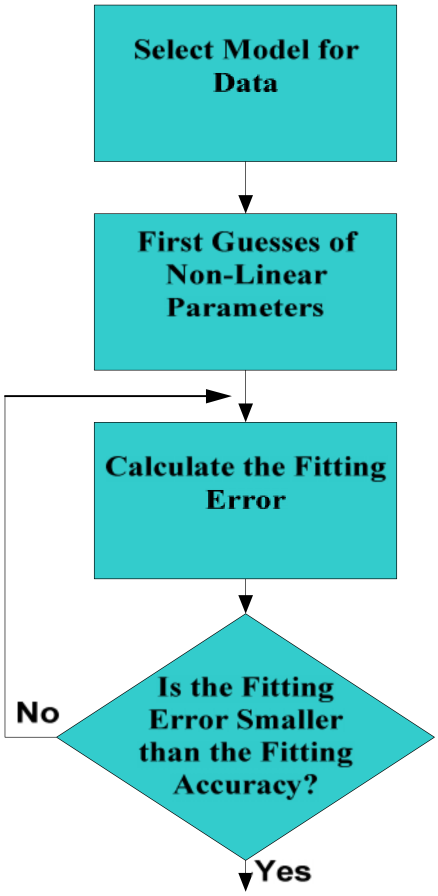

Fig. 3 Flowchart of the iterative methodology

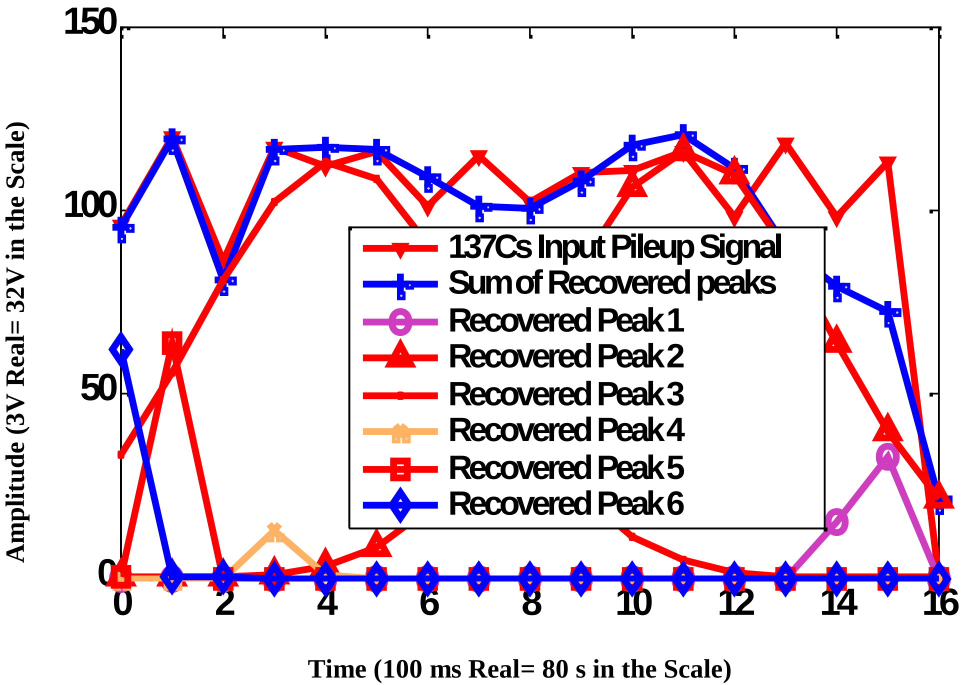

Fig. 4 ^{137}Cs input pileup signal, sum of the recovered peaks and peaks recovery using direct search algorithm.

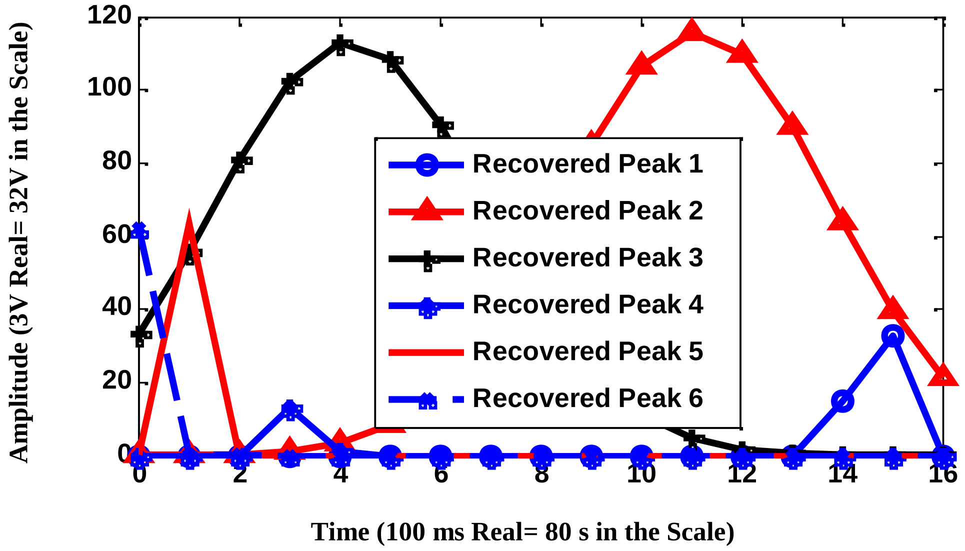

Fig. 5 Peaks recovery using direct search algorithm

3.3 Least square fitting algorithm

Traditionally, least squares curve fitting is used because it is easy to implement. It provides effective results in many fields of applications like signal processing, and noise cancellation. For resolving pileup overlapping peaks, the standard least-square technique is used [21].

The least squares analysis is used to fit a set of m observations with a model that is non-linear in n unknown parameters ($m > n$). Solution of least square fitting problem is an iterative process, whose convergence speed is problem-dependent. This technique is employed for initial guessing of peak parameters. These parameters are peak height, position, and width. The objective of curve fitting is to find a mathematical equation that describes a set of data. A simple and improved algorithm to resolve overlapped asymmetric pulses into its component peaks using nonlinear least square fitting method is reported in [10, 22, 23]. Figure 6 illustrates this algorithm. Considering voltage waveform, multiple overlapping peaks are characterized by the following equation after applying the fitting procedure

$$V(T) = \left(a_1 e^{\left(-\left(T-b1\right)/c1\right)} + a_2 e^{\left(-\left(T-b_2\right)/c_2\right)} + a_3 e^{\left(-\left(T-b3\right)/c_3\right)} + a_4 e^{\left(-\left(T-b_4\right)/c_4\right)} + a_5 e^{\left(-\left(T-b_5\right)/c_5\right)} \right) \qquad (3)$$

where V(T), $a_{1:5}$, $b_{1:5,}$ and $c_{1:5}$ denote the voltage waveform of multiple overlapping peaks as a function of time, the amplitude, position and the width of the first to the fifth recovered peaks, respectively. For signal recovery step, the overlapped peaks were assumed to be a convolution of its component peaks. It was characterized by Gaussian shape [17-20]. Therefore, the obtained coefficients are fitted to match the input signal. Consequently, the multiple overlapping peaks are recovered. As a final stage the recovered peaks can be rejected or accepted as described in the following subsection. Figure 7 shows the ^{137}Cs input overlapping peaks, sum of recovered peaks, and the recovered peaks. However, the recovered peaks are shown in Fig. 8.

1) Input Pileup Signal Containing Multiple Overlapping Peaks

2) Applying Nonlinear Least Square Fitting Technique

  A) Do Iterative Fit with Gaussian model

  B) Determine the Parameters of the Noisy Input Signal

3) Apply Recovery Peaks Routine

4) Detect Illusive Pulses (If Exist)

5) Register Actual Peaks

Fig. 6 Pileup recovery using nonlinear least square fitting algorithm

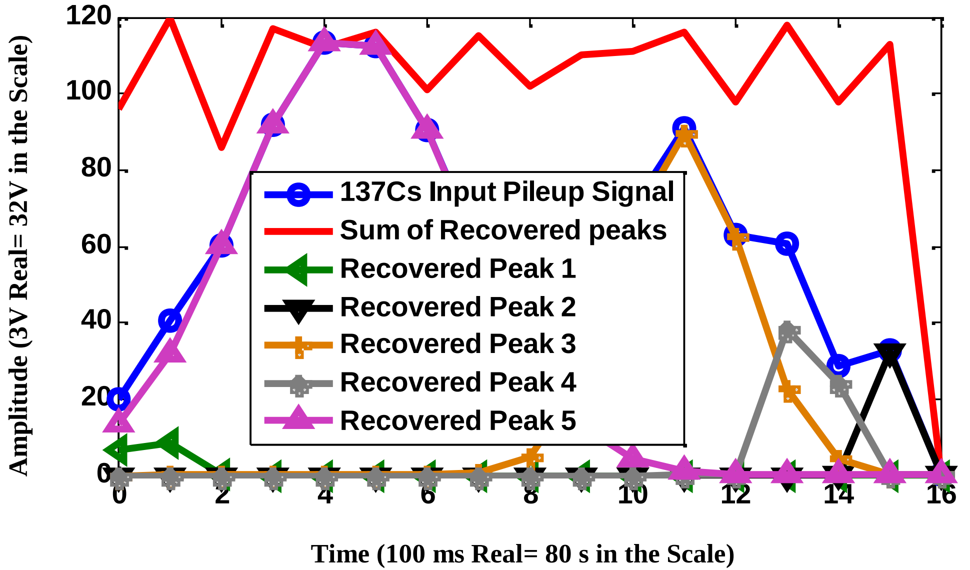

Fig. 7 ^{137}Cs input pileup signal, sum of the recovered peaks, and peaks recovery, using nonlinear least square algorithm.

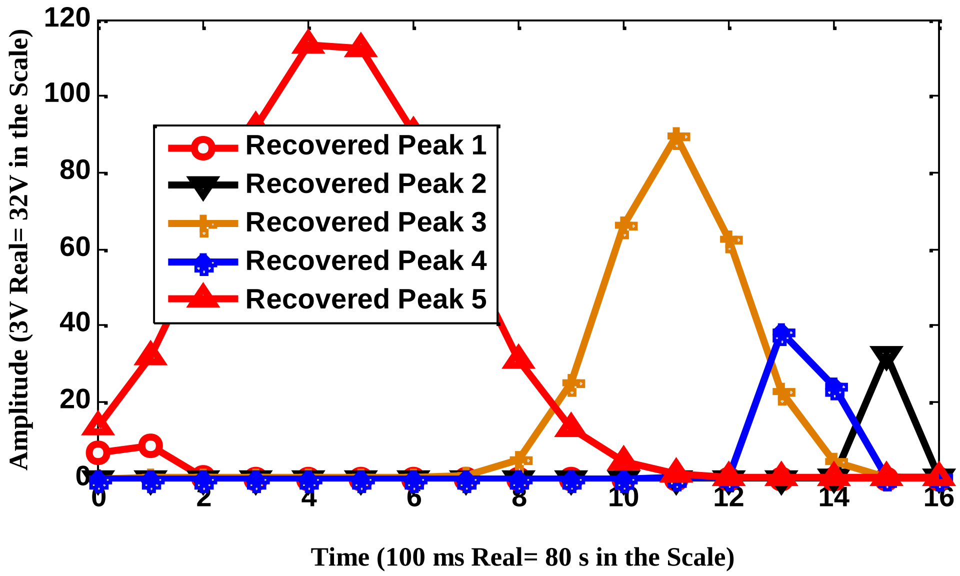

Fig. 8 Peaks recovery using nonlinear least square algorithm

3.4 First derivative and maximum peak search algorithm

The third algorithm for pileup recovery is a proposed one that is presented in Fig. 9. The detection of the peak and determination of the peak position can be realized in successive steps, independent of each other. This algorithm is based on first derivative of peak overlapping pulses, and a MATLAB routine implementing the maximum peak search algorithm [24, 25, 26].

The differentiation of signals is used to facilitate the detection and location of partially overlapped Gaussian peaks in a multi-component signal [27]. First derivative method has been used because it has the capability of spectral discrimination [28-29].

For peak search process, most peak search algorithms for γ-ray spectrometry are based on digital filters. The maximum peak is determined using the maximum peak search routine. This routine search and find local maxima in the overlapping peaks.

For signal recovery step, the overlapped peak was assumed to be a convolution of its component peaks. It was characterized by Gaussian shape as in [17-20]. Therefore, the obtained coefficients are fitted with the detected input peaks. Consequently, the overlapping peaks are recovered. As a final stage in Fig. 9, the recovered peaks can be rejected or accepted as described in the following subsection.

The ^{137}Cs input signal is shown in Fig. 10 (a). Figure 10 (b) shows the pulse derivative amplitude against time for multiple peaks. From this figure, one can predict the shape of each peak by recalling that the derivative is the slope of the original signal. The derivative of the signal is positive as the signal slopes up. However, the derivative of the signal is negative as the signal slopes down. If a signal has zero slopes, its derivative is zero. Consequently, the location of the maximum in a peak signal can be computed precisely by computing the location of the zero-crossing in its first derivative.

The peak amplitude of the signal is determined by using a maximum peak search algorithm as depicted in Fig. 11. The detected overlapping peak signals, the sum of recovered peaks, and the recovered peaks are illustrated in Fig. 12. However, these peaks are shown separately in Fig. 13.

1) **Input Pileup Signal Containing Multiple Overlapping Peaks**
2) **Do the 1st Derivative Process to Determine Peaks Positions**
3) **Do Maximum Peak Search Routine to Determine Peaks Heights**
4) **Develop a Gaussian pulses shape using the obtained parameters**
5) **Apply Recovery Peaks Routine**
6) **Detect Illusive Pulses (If Exist)**
7) **Register Actual Peaks**

Fig. 9 Pileup recovery using first derivative and maximum peak search algorithm

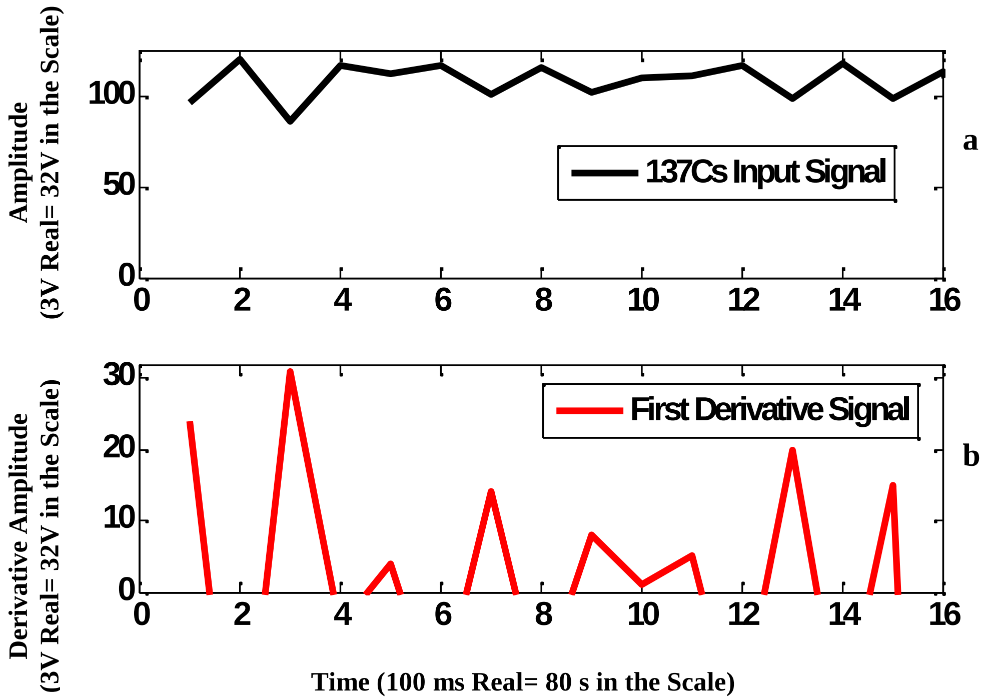

Fig. 10 a) ^{137}Cs input signal and b) First derivative result of double peak overlapping signal.

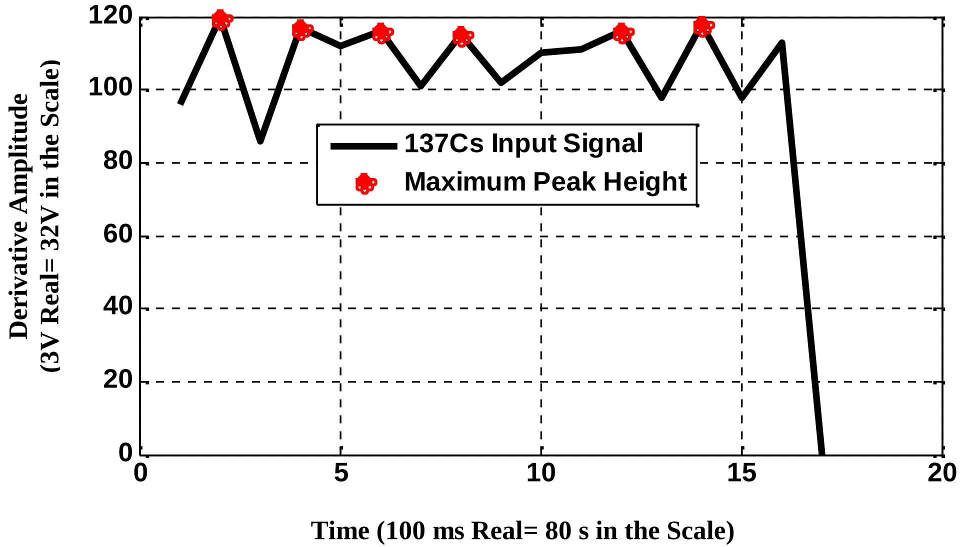

Fig. 11 ^{137}Cs input signal with maximum peak heights.

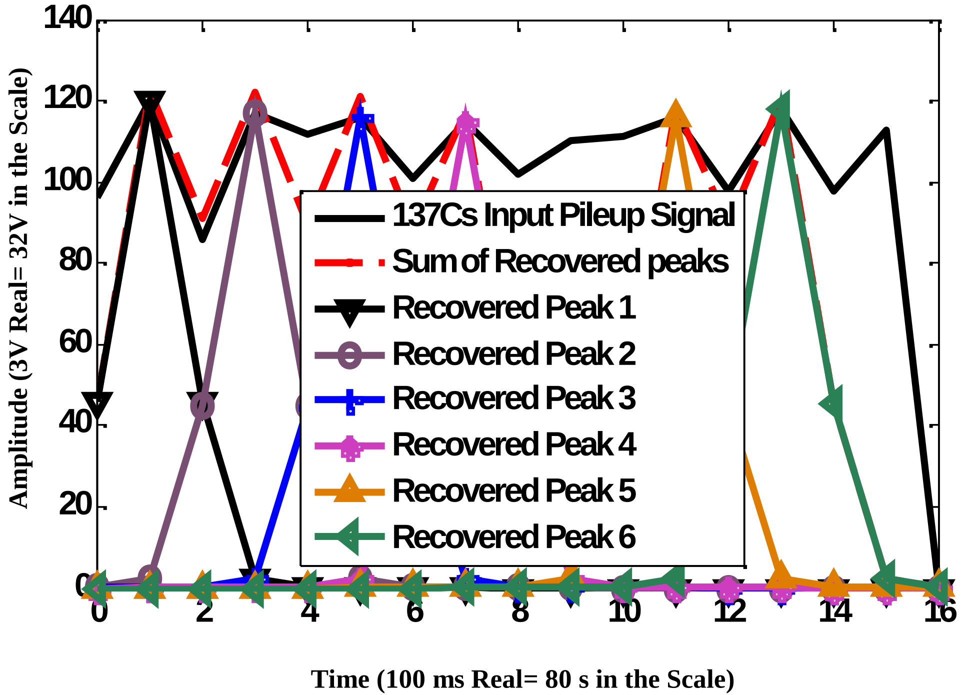

Fig. 12 ^{137}Cs input pileup signal, sum of the recovered peaks and peaks recovery using first derivative with maximum peak search method

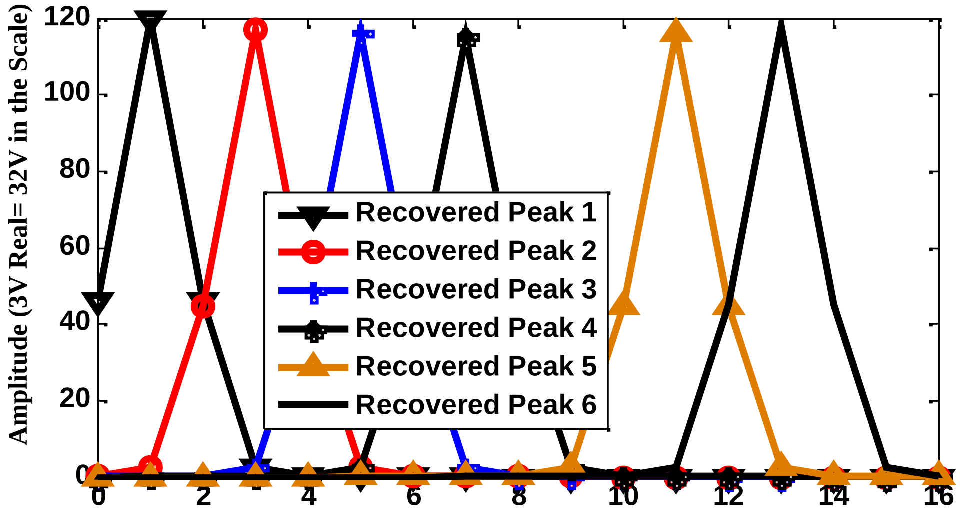

Fig. 13 Peaks recovery using first derivative with maximum peak search method

4. Comparison between the Three Algorithms and Discussion

In this section, we present comparison between the underlined algorithms. This comparison is based on the input pileup peaks and the recovered peaks. The error is found by the following relation

$$Error = O_i - S_i \tag{4}$$

where O_i, S_i, and i denote the input pileup peaks, the sum of the recovered peaks, and the set of data, respectively.

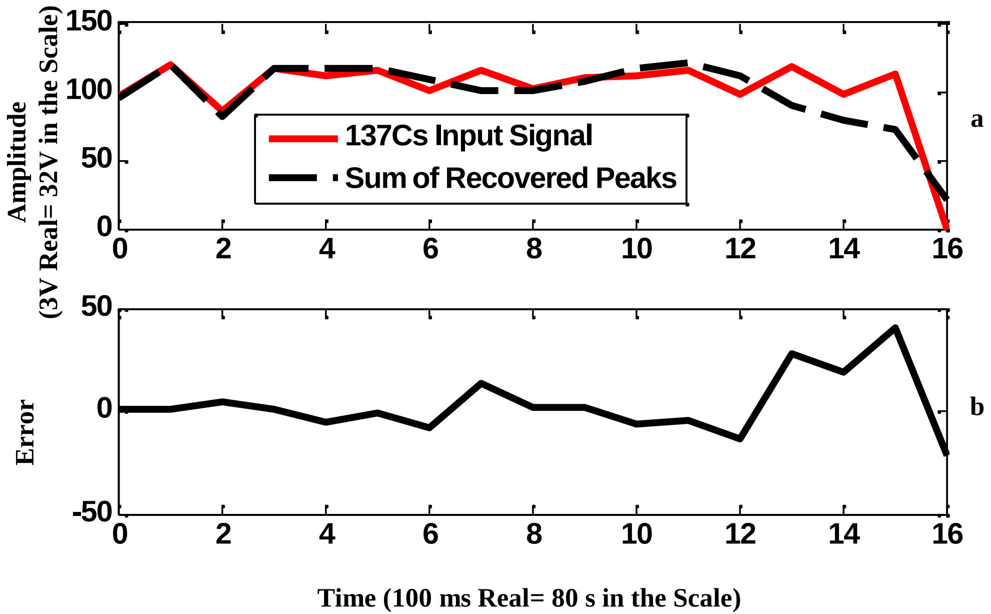

Fig. 14 Accuracy of the direct search algorithm a) The ^{137}Cs input pileup peaks and summation of the recovered overlapping peaks and b) The error that represents the difference between the original detected and recovered peaks

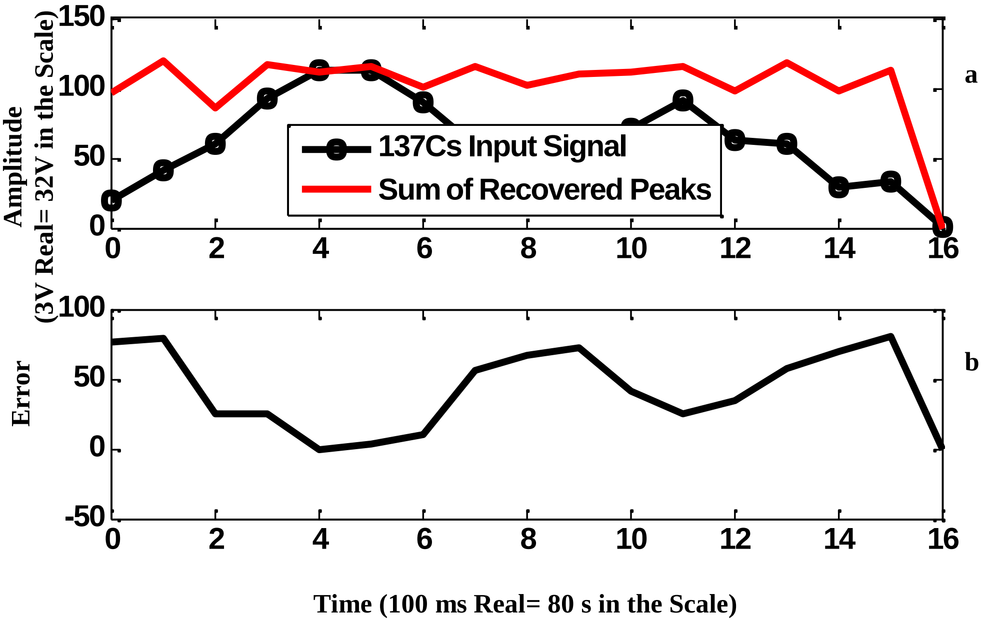

Fig. 15 Accuracy of the least square algorithm a) The ^{137}Cs input pileup peaks and summation of the recovered overlapping peaks and b) The error that represents the difference between the original detected and recovered peaks

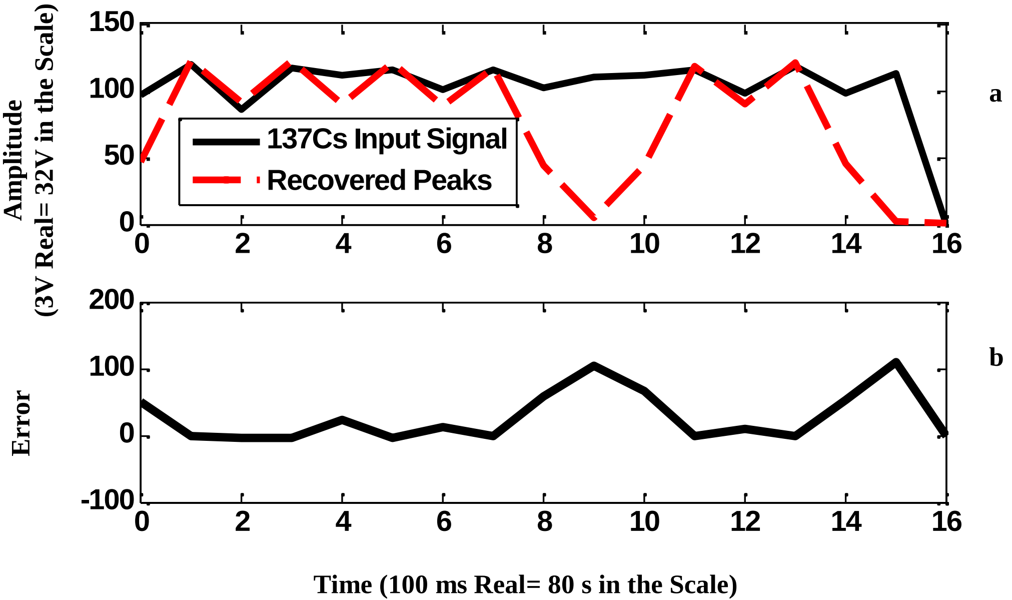

Fig. 16 Accuracy of the maximum peak search algorithm a) The ^{137}Cs input pileup peaks and summation of the recovered overlapping peaks and b) The error that represents the difference between the original detected and recovered peaks

In order to represent the accuracy and validity of these algorithms, comparison between both the ^{137}Cs input peaks and the recovered peaks are made. Figures 14-16 (a) represent the difference between the original overlapped peaks and the sum of recovered peaks for the three algorithms, respectively. On other hand, Figures 14-16 (b) show the error between the original detected overlapped peaks and the recovered peaks for the underlined three algorithms, respectively. From these results, we notice that the proposed algorithm has high accuracy for identifying both peak height and position of multiple overlapping peaks than other algorithms. Furthermore, the error signal is smaller than other two algorithms.

From computational point of view, the maximum peak search algorithm shows significant run time improvement compared with other two algorithms as shown in Table 1.

Table 1 Execution time of the underlined three algorithms

	Direct Search Algorithm	**Least Square Fitting Algorithm**	**Maximum Peak Search Algorithm**
Execution Time (s)	**1.125000**	**2.4219**	**0.1094**

4.1 Effect of pileup recovery algorithms on spectrum evaluation

Spectrums of ^{137}Cs for 4- 8- and 16 channel number are depicted in Fig. 17-19, respectively. For each spectrum, four different cases are illustrated with pileup recovery algorithms and without pileup correction. For each spectrum, the photopeak corresponds to channel number four, eight, and sixteen as shown in Figs. 17-19, respectively. From the obtained experimental results, the proposed algorithm proves superior performance for spectrum evaluation. Therefore, this algorithm demonstrates efficient solution of isotope identification. However, the worst spectrum evaluation is obtained using the least square fitting algorithm. The number of events that fall within any one channel will vary in proportion to its width. The content of a typical channel varies inversely with the total number of channels provided over the spectrum. The channels content with smaller number of channels is larger than that with larger number of channels. Moreover, from Figs. 17-19, we notice that the resolution enhances with increases the number of channels. As the number of channels increases, the FWHM decreases. Resolution represents the FWHM with respect to position peak of the centroid. Consequently, good resolution is obtained. Therefore, resolution enhancement is an essential contribution especially at small number of channels.

The effect of the pileup recovery algorithms on four, eight, and sixteen channel spectrum is depicted in Table 2. From this table, the maximum number of counts at photopeak of each algorithm is demonstrated. Also, the proposed algorithm proves superior performance of isotope identification.

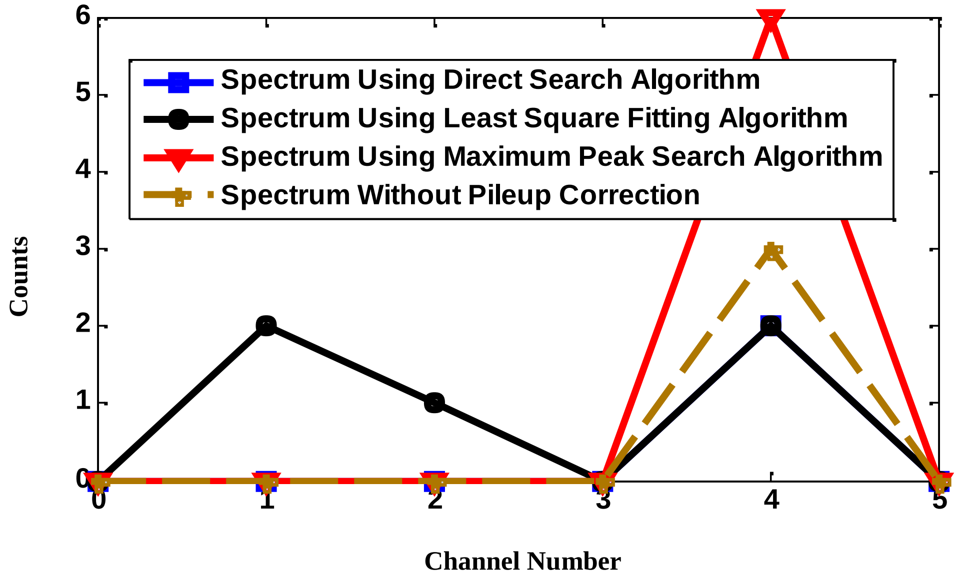

Fig. 17 Channel content versus 4-channel number of Cs^{137} source for the algorithms

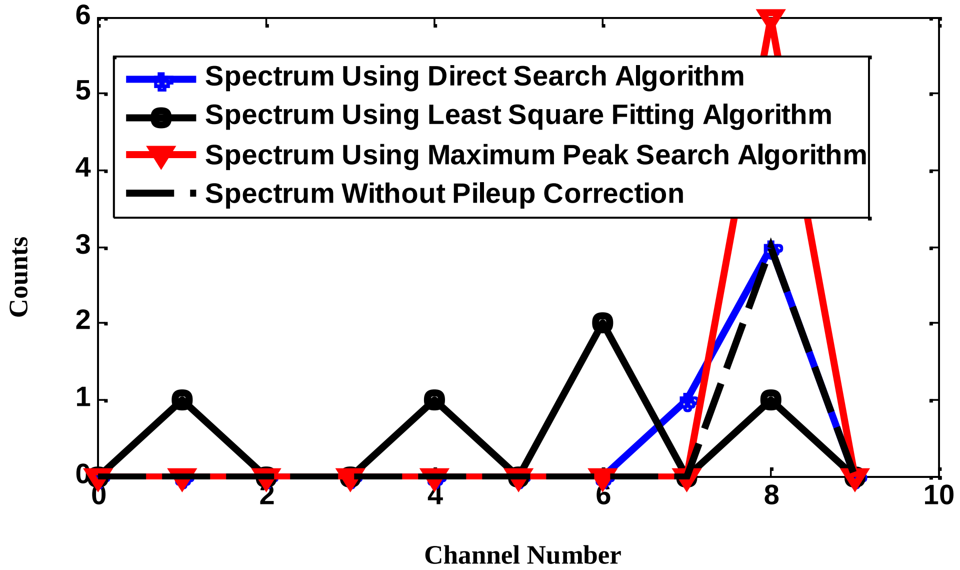

Fig. 18 Channel content versus 8-channel number of Cs^{137} source for the algorithms

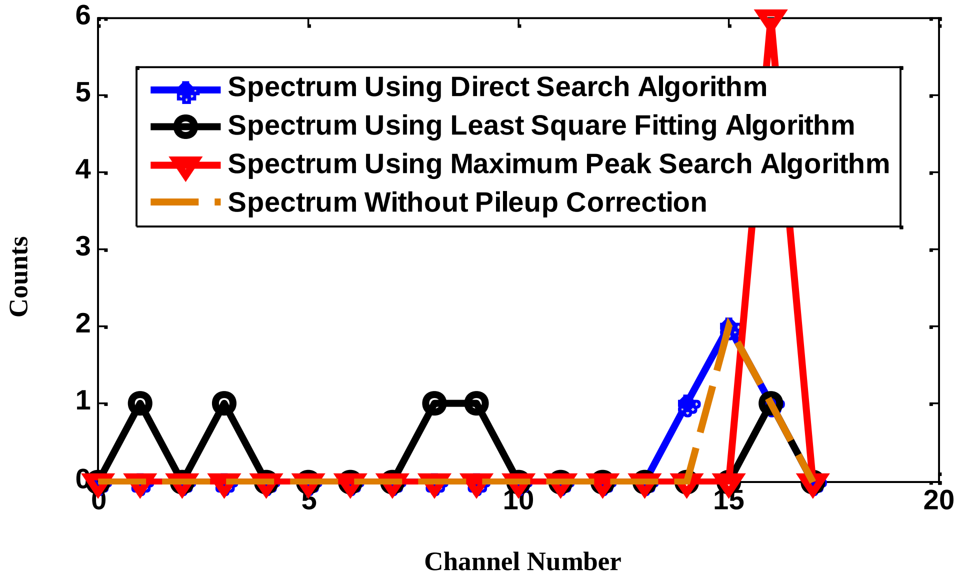

Fig. 19 Channel content versus 16-channel number of Cs^{137} source for the algorithms

Table 2 Maximum number of counts at photopeak

Number of Counts at Photopeak	Direct Search Algorithm	Least Square Fitting Algorithm	Maximum Peak Search Algorithm	Without Pileup Correction
Four Channels	2	**2**	**6**	**3**
Eight Channels	3	**1**	**6**	**3**
Sixteen Channels	2	**1**	**6**	**2**

5. Conclusion

Algorithms for overcoming the pileup problem of gamma-ray spectroscopy are presented. Three different algorithms are studied for pileup recovery. The first one is a direct search algorithm based on Nelder-Mead technique for determining both the peak position and peak amplitude. The second algorithm is the least square fitting method that is used to determine the peak height and position of the overlapping peaks. The third algorithm is a proposed one which based on the first derivative of the signal combined with maximum peak search algorithm. This approach uses first derivatives for observing peak locations of pulses. Then, a direct search method for determining the pulse parameters such as position, maximum amplitude, pulse width and area are performed. From computational point of view, the first derivative combined with maximum peak search algorithm shows significant speed improvement compared with other two algorithms. From the obtained experimental results, the third algorithm proves superior performance for spectrum evaluation. Therefore, this algorithm demonstrates efficient solution of isotope identification.

References

[1] S. M. Robinson, S. D. Kiff, E. D. Ashbaker, E. Flumerfelt, and M. Salvitti, "Effects of high count rate and gain shift on isotope-identification algorithms", Nuclear Instruments and Methods in Physics Research A, Vol. 610, pp. 509–514, 2009.

[2] C. J. Sullivan, S. E. Garner, K. B. Blagoev, and D. L. Weiss, "Generation of customized wavelets for the analysis of γ-ray spectra", Nuclear Instruments and Methods in Physics Research A, Vol. 579, pp. 275–278, 2007.

[3] W. Guo, S. H. Lee, and R. P. Gardner, "The Monte Carlo approach MCPUT for correcting pileup distorted pulse-height spectra", Nuclear Instruments and Methods in Physics Research A, Vol. 531, pp. 520–529, 2004.

[4] R. P. Gardner and S. H. Lee, "Monte Carlo simulation of pulse pileup", JCPDS-International Centre for Diffraction Data, pp. 941-950, 1999.

[5] M. Bolic, and V. Drndarevi, "Digital gamma-ray spectroscopy based on FPGA technology", Nuclear Instruments and Methods in Physics Research A, Vol. 482, pp.761–766, 2002.

[6] W. Guo, R. P. Gardner, and F. Li, "A Monte Carlo code for simulation of pulse pileup spectral distortion in pulse-height measurement", International Centre for Diffraction Data, Advances in X-ray Analysis, Vol. 48, pp. 246- 252, 2005.

[7] M. S. El_Tokhy, I. I. Mahmoud, and H. A. Konber, "Comparative analysis between different linear filtering algorithms of gamma-ray spectroscopy", Proceedings of the International Conference on Circuits, Systems, Signals, Malta, 2010.

[8] M. Morhac, and V. Matousek, "Complete positive deconvolution of spectrometric data", Digital Signal Processing Journal, Vol. 19, pp. 372–392, 2009.

[9] G. F. Knoll, "Radiation detection and measurement", Third Edition John Wiley and Sons, NY, 2000.

[10] W. Guo, R. P. Gardner, and C. W. Mayo, "A study of the real-time deconvolution of digitized waveforms with pulse pileup for digital radiation spectroscopy", Nuclear Instruments and Methods in Physics Research Section A, Vol. 544, No. 3, pp. 668-678, 2005.

[11] MATLAB Help, the Language of Technical Computing, Version 7.0.0.19920 (R14), the Math Works, Inc., Copyright 1984-2004.

[12] R. Micchael Lewis, V. Torczon, and M. W. Trosset, "Direct search methods: then and now", http://www.cs.wm.edu/~va/research/jcam.pdf.

[13] J. C. Meza and M. L. Martinez, "On the use of direct search methods for the molecular conformation problem", Journal of Computational Chemistry, Vol. 15, No. 6, pp. 627-632, 1994.

[14] Chapter 8, Numerical Optimization, "Nelder-Mead method", http://math.fullerton.edu/mathews/n2003/neldermead/NelderMeadProof.pdf.

[15] T. O'Haver, "An introduction to signal processing in chemical analysis", http://terpconnect.umd.edu/~toh/spectrum/CurveFittingC.html.

[16] Jeffrey C. Lagarias, Jeffrey C. Lagarias, James A. Reeds, James A. Reeds, Margaret H. Wright, Margaret H. Wright, Paul E. Wright, and Paul E. Wright, "Convergence properties of the Nelder-Mead simplex algorithm in low dimensions", SIAM Journal of Optimization, 1996.

[17] Emma Suckling, "Introduction to gamma-ray spectroscopy using a thallium activated sodium iodide detector", http://www.emma-suckling.co.uk/work/MSc/Lab2a.pdf.

[18] Peter J. Statham, "Pileup rejection: limitations and corrections for residual errors in energy-dispersive spectrometers", X-Ray Spectrometry, Vol. 6, No. 2, 1977.

[19] Yuntao Feng, "A Monte Carlo simulation and deconvolution study of detector response function for small field measurements", Doctorate of Philosophy (Ph.D.), Nuclear and Radiological Engineering of the College of Engineering, University of Cincinnati, 2006.

[20] New Interactive Calibration Demo, "FitzPeaks gamma analysis and calibration software", http://www.jimfitz.demon.co.uk/fitzpeak.htm.

[21] A. Likar and T. Vidmar, "A peak-search method based on spectrum convolution", J. Phys. D: Appl. Phys., Vol. 36, No. 1903, 1903, doi: 10.1088/0022-3727/36/15/323, 2003

[22] M. W. Raad , and L. Cheded, "Efficient algorithms for pulse parameter estimation, pulse peak localization and pileup reduction in gamma-ray spectroscopy", http://faculty.kfupm.edu.sa/COE/raad/publications/imtc2007.pdf.

[23] G. Ripamonti, A. Geraci, "Fast least mean squares method for time determination and pileup rejection in high resolution spectroscopy", Nuclear Science Symposium, IEEE, Vol.1, pp. 474– 477, 1996.

[24] V. B. Zlokazov, "Method for an automatic peak search in gamma-ray spectra", Nuclear Instruments and Methods in Physics Research Vol. 199, No. 3, pp. 509-519, 1982.

[25] A. Likar and T. Vidmar, "A peak-search method based on spectrum convolution", Journal of Physics D: Applied Physics, Vol.36, No. 15, 2003.

[26] Markku J. Koskelo and Mark T. Mercier, "Verification of gamma spectroscopy programs: a standardized approach", Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, Vol. 299, No. 1-3, pp. 318-321, 1990.

[27] A. H. Lawrence**,** and R. A. Goubran, **"**Experimental signal analysis in ion mobility spectrometry", International Journal of Mass Spectrometry and Ion Processes, Vol. 104, No. 3, pp. 163-178, 1991.

[28] https://ftirsearch.com/help/algo.htm#First Derivative Least Squares Search.

[29] H. Holden, and E. LeDrew, "Deconvolution of measured spectra based on principal components analysis and derivative spectroscopy", Geoscience and Remote Sensing Symposium Proceedings (IGARSS '98), IEEE International, Seattle, WA, Vol. 2, pp. 760-762, 1998.