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Constant Envelope Single-carrier Frequency Division Multiple Access (SC-FDMA)

By

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Abstract:

Single-carrier frequency division multiple access (SC-FDMA) has been adopted as a possible air interface for future wireless networks. It combines most of the advantages of orthogonal frequency division multiple access (OFDMA) and the low peak-to-average-power ratio (PAPR) of single-carrier transmission. In this paper, a new transceiver scheme of Discrete Cosine Transform (DCT) SC-FDMA system is suggested. The suggested system called CE DCT SC-FDMA which refers to constant envelope discrete cosine transform SC-FDMA is based on phase modulation (PM). The output from PM component has the advantage of constant envelope (i.e. 0 dB PAPR) which allows the power amplifier (PA) to operate near the saturation region, thus maximizing the power efficiency and the coverage area. The proposed system is implemented with frequency domain equalization (FDE) to obtain high diversity gains over the frequency multipath channel. The effect of modulation index is evaluated via simulation thus the optimum value can be selected.

Keywords:

SC-FDMA, PAPR, PM, DFT, DCT, M-PAM and FDE

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1. Introduction:

Broadband wireless communication systems must provide high-data-rate services to satisfy the increasing demands of the future wireless networks. As the bit rate increases, the problem of inter-symbol interference becomes more serious. Orthogonal frequency division multiple access (OFDMA) is an attractive technology to deal with the detrimental effects of multipath fading, but it has several inherent disadvantages such as the large peak-to-average-power ratio (PAPR) and the sensitivity to carrier frequency offsets [1, 2]. Recently, much attention has been focused on another broadband wireless communication system, which is the single-carrier frequency division multiple access (SC-FDMA) system [3, 4]. The SC-FDMA has two main advantages over orthogonal frequency division multiplexing (OFDM), namely, a lower PAPR and a lower sensitivity to carrier frequency errors [3]. There are two methods to choose the subcarriers in SC-FDMA systems: interleaved subcarrier mapping and localised subcarrier mapping. We will refer to the localized subcarrier mapping mode of SC-FDMA as localized frequency division multiple access (LFDMA) and the interleaved subcarrier mapping mode as interleaved frequency division multiple access (IFDMA). The LFDMA system incurs a higher PAPR compared to the IFDMA system but, compared to OFDM, it is lower, though not significantly [3]. A large peak-to-average power ratio (PAPR) brings disadvantages like an increased complexity of the analog-to-digital and digital-to-analog converters and a reduced efficiency of the RF power amplifier (PA). The high PAPR makes the system sensitive to nonlinear distortion caused by PA [5]. If there is no sufficient back-off power, the system will suffer from spectral broadening, inter-modulation distortion, and consequently performance degradation. However, these problems can be reduced by increasing the input back-off power (IBP), but this result in a reduced PA efficiency [5].
In SC-FDMA, closely spaced and overlapped subcarriers are divided into groups and assigned to multiple users for simultaneous transmissions. Unlike traditional frequency-division multiple access (FDMA), where any overlapping of the frequency spectrum of different users introduces multiple-access interference (MAI), the orthogonality of subcarriers guarantees that there is no inter-carrier interference (ICI), which prevents MAI among users in SC-FDMA systems [3,4].
Several PAPR reductions techniques for multicarrier transmissions are surveyed by [6]. These techniques can be classified into two groups, the first one which is called distortion reduction techniques, such as, windowing, clipping [7] and repeated clipping and filtering [8]. The second one called non distortion reduction techniques, which can be classified into two different types. Which are either based on multiple signal representations, such as, the partial transmit sequence (PTS) technique [9], the selective mapping (SLM) technique [10], and the interleaving technique, or technique based on signal transformation such as phase modulation (PM) [11].

The goal of this paper is to propose a new transceiver scheme called CE DCT SC-FDMA and compare its performance with the DCT SC-FDMA.
The PM based systems have several advantages. The resulted signal from PM has a constant envelope (i.e. 0 dB PAPR) which allows the PA to operate near the saturation region, thus maximizing the power efficiency. Therefore, these systems have a larger coverage area since more signal power is radiated into the channel [5]. In CE DCT SC-FDMA system, the DCT SC-FDMA signal is used to phase modulates the carrier.

The remainder of this paper is organized as follows. Section 2, provides a brief overview the DCT SC-FDMA. The proposed CE DCT SC-FDMA is presented in Section 3. Section 4, provides the simulation results and discussion. Finally, the conclusions are summarized in Section 5.

2. An Overview of DCT-SC-FDMA:

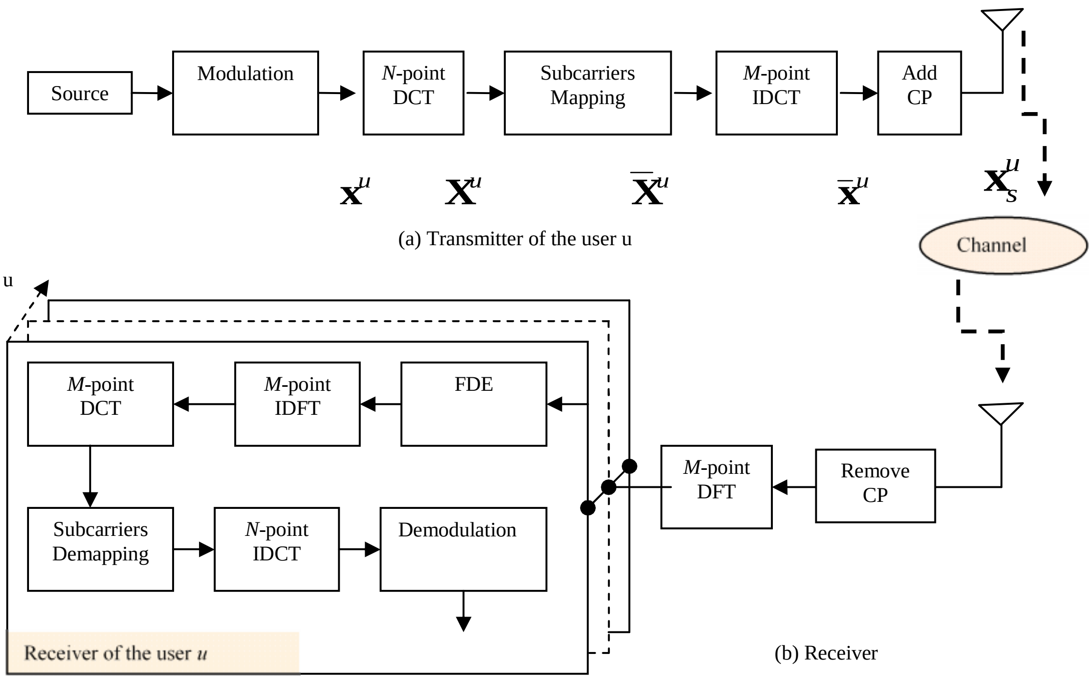
A new single carrier frequency division multiple access (SC-FDMA) system based on the discrete cosine transform (DCT) for uplink wireless transmissions is introduced and analyzed in [12]. Moreover, the time domain expressions of the DCT SC-FDMA signals are derived. The PAPR of the DCT SC-FDMA signals is compared with that of the DFT SC-FDMA and OFDMA signals in [12]. Figure 1 shows the block diagram of the DCT-SC-FDMA system. At the transmitter, the modulated symbols are grouped into blocks each containing N symbols and N-point DCT is performed. Then, subcarriers are mapped in the frequency domain. After that, the M-point IDCT is performed. The signal after the IDCT is in the time domain. Thus, adding a CP to this signal makes the frequency domain equalization at the receiver side is possible. After adding a CP of length N_c to the resulting signal, the signal is transmitted through the wireless channel. Under a frequency selective channel assumption, adding of CP is very important. The length of the CP must be greater than the maximum excess delay of the channel to combat the interblock interference. The length of the CP for DCT SC-FDMA is the same as that required for DFT SC-FDMA. Thus, the spectral efficiency of DCT SC-FDMA is equal to that of DFT SC-FDMA. At the receiver, the CP is removed from the received signal.
Unlike the conventional DFT, a single set of cosinusoidal functions $\cos(2\pi n F_{\Delta} t)$ where $n = 0, 1, N - 1$ and $0 < t < T$ is used in the DCT. F_{Δ} is the subcarrier spacing. T is the symbol period. The minimum F_{Δ}, required to satisfy the orthogonality condition

$$\int_0^T \sqrt{\frac{2}{T}}\cos(2\pi k F_{\Delta} t)\sqrt{\frac{2}{T}}\cos(2\pi n F_{\Delta} t)dt = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases} \tag{1}$$

is *1/2T* . In DFT**,** the minimum F_{Δ} , required to satisfy the orthogonality condition is 1/T.

$$\int_0^T \sqrt{\frac{1}{T}} e^{-j2\pi k F_{\Delta} t} \sqrt{\frac{1}{T}} e^{j2\pi n F_{\Delta} t} dt = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases} \quad (2)$$

Recently, the advantages of the DCT based OFDM have come to the light [13-14-15]. The bandwidth requirement of the DCT OFDM system is the same as a DFT OFDM system with the same number of subcarriers. However, for DCT OFDM systems with real-valued modulation formats, one can use single-sideband transmission technology to improve the bandwidth efficiency. In this case, the bandwidth of a DCT OFDM system can be only half that of OFDM system with the same number of subcarriers [15].



Figure(1): *Structure of the DCT SC-FDMA system over a frequency selective channel.*

The main difference of the DCT SC-FDMA system with respect to the DFT SC-FDMA system is the need for an FDE at the receiver prior to the demapping process. On the other hand, the complexity of the FDE is of *O(M)* for the DCT OFDMA system and of *O(N)* for the DFT OFDMA system. If the fast implementation algorithms are taken into consideration, the fast DCT algorithm proposed in [14] can provide fewer computational steps than DFT. This indicates that the complexity of the transmitter in the DCT OFDMA system is lower than that in the DFT OFDMA system.

3. The Proposed DCT SC-FDMA-PM:

In this section, a new system of DCT SC-FDMA based on phase modulation is presented. A block diagram for the proposed system is represented in Figure 2.
The stream of binary bits are modulated into blocks of N symbols using *M-PAM* modulator and then processed by the *N-point DCT* block. Mapping of blocks of N-symbols to M-subcarriers is then performed. During each block interval, *T-seconds*, an *M-point IDCT* calculates a block of time samples $x(n)$. Next step, a high PAPR DCT SC-FDMA sequence, $x(n)$, is passed through a phase modulator to obtain a 0 dB PAPR sequence $s(n)=exp\,(\,j\,C\,x(n)\,)$, where C is a scaling constant. Then N_g samples of cyclic prefix (CP) are added to $s(n)$. The continuous-time DCT SC-FDMA-PM signal $s(t)$ is then generated at the output of the digital-to-analog (D/A) converter. This baseband signal can be expressed as follows [11]:

$$s(t) = A\,e^{j\phi(t)} = A\,e^{j[2\pi h m(t)+\theta]}, \qquad T_g \le t < T, \tag{3}$$

where A is the signal amplitude, h is the modulation index, θ is an arbitrary phase offset used to achieve phase continuous modulation [16], T_g is the guard period, T is the block period, and $m(t)$ is a real-valued DCT-OFDMA signal and given as:

$$m(t) = C_N \sum_{k=1}^{M} X_k\, q_k(t)\ , \tag{4}$$

where C_N is normalization constant used to normalize the variance of the message signal $(\sigma^2_x=1)$ after subcarrier mapping and consequently the variance of the phase signal, $\sigma^2_\varphi=(2\pi h)^2$. This requirement is achieved by setting C_N as follows:

$$C_N = \sqrt{\frac{2}{M\,\sigma_x^2}}\ , \tag{5}$$

X_k is the resulted sequence after subcarrier mapping and $q_k(t)$ are the orthogonal subcarriers which also must be real-valued for phase modulation [16],

$$q_k(t) = \cos\left(\frac{\pi k t}{T}\right) \qquad 0 \le t < T;\ \ 1 \le k < M, \tag{6}$$

From Eqs.(3) and (4), the phase signal $\varphi(t)$ can be written as:

$$\varphi(t) = \theta + 2\pi h C_N \sum_{k=1}^{M} X_k \, q_{k(t)} \, , \tag{7}$$

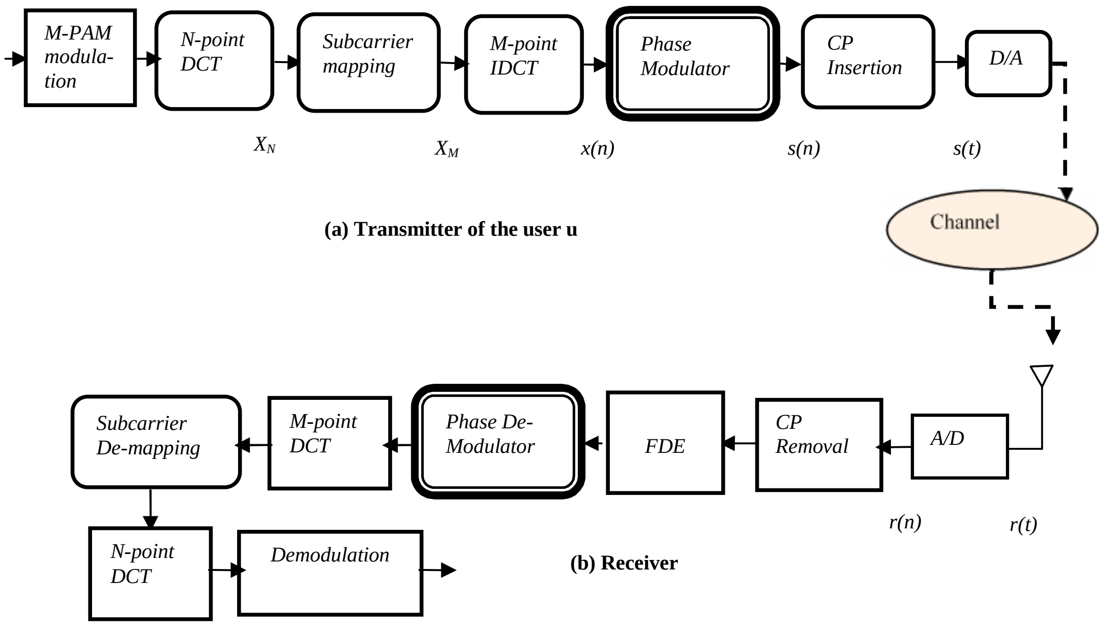


Figure (2): *A block diagram of constant envelope DCT SC-FDMA-PM system over multipath channel.*

The received signal is

$$r(t) = \int_0^{\tau_{max}} h(\tau,t) s(t-\tau) d\tau + z(t)$$
$$= \int_0^{\tau_{max}} h(\tau) s(t-\tau) d\tau + z(t), \tag{8}$$

where $h(\tau,t)$ is the channel impulse response (CIR), τ_{max} is the channel's maximum propagation delay, and $z(t)$ is the additive white Gaussian noise. The channel is assumed to be static over the block interval, and therefore $h(\tau,t) \approx h(\tau)$.
The addition of the CP makes the linear convolution with the channel impulse response equivalent to a circular convolution. Thus, by choosing $Ng \geq L$ (L is the length of the channel impulse response), the received samples $r(n)$ are represented equivalently by a linear (*) or a circular ($\otimes$) convolution,

$$r_n = s_n * h_n + z_n,$$
$$= s_n \otimes h_n + z_n = IDFT\{S_k H_k\} + z_n, \tag{9}$$

where $\{S_k, H_k\}$ are the DFTs of $\{s_n, h_n\}$ respectively. Then, after the A/D converter, the CP samples are discarded and the remaining samples are equalized by frequency domain equalization (FDE). The FDE can perfectly invert the effect of the channel. Its advantage is the relatively low complexity, and the disadvantage is the requirement of a CP overhead. The block of FDE includes three blocks, DFT, FDE, and IDFT respectively

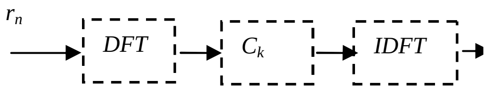


Figure (3): *Frequency Domain Equalizer*

The FDE output is:

$$\hat{s}_n = IDFT\{R_k C_k\} = r_n \otimes c_n \,, \tag{10}$$

where R_k is the DFT of the received signal r_n and C_k represents the equalizer correction term, which is computed according to the type of FDE. For MMSE equalizer the correction term is computed as follows [5]:

$$C_k = \frac{H_k^*}{|H_k|^2 + (SNR^{-1})} \,, \tag{11}$$

where *SNR* is the signal-to-noise ratio. After that, the time domain signal is applied to the phase de-modulator as shown in Figure 4.

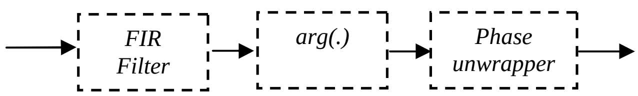


Figure (4): *Phase Demodulator.*

The phase demodulator includes three operations, a finite impulse response (FIR) filter which is used to improve the phase demodulator performance, the *arg(.)* is used to extract the phase of the signal. Finally, the phase unwrapper is used to reduce the effect of phase ambiguities and makes the receiver insensitive to phase offsets caused by the channel and the memory term.

4. Computer Simulation Results and Discussions:

This section is divided into two different subsections; the first one studies the performance of the DCT SC-FDMA system and compares it to the DFT SC-FDMA system. The second section Studies the performance of the main contribution of this paper, the CE DCT SC-FDMA. Simulation parameters used for simulation of DCT-OFDMA compared to DFT-OFDMA are listed in Table 1.

***Table (1):** Simulation Parameters for DCT SC-FDMA and DFT SC-FDMA*

	Description	Parameters
Transmitter	System bandwidth	5 MHz
	Modulation	QPSK
	CP	20 samples
	Transmitter IDFT and IDCT size	M = 256
	Number of users *'Q'*	4
	Subcarrier spacing	9.765625 kHz
	SC-FDMA input block size	64 symbols
	Subcarrier mapping	Localized and Interleaved
Channel	Channel model	Vehicular A outdoor channel
	Noise environment	AWGN
Receiver	Channel estimation	known channel
	Equalization	MMSE

Performance evaluations of the proposed scheme are provided under the vehicular A outdoor channel wireless channels. The BER is evaluated by the Monte Carlo simulation method with 10^3 run times. The conventional DFT-OFDMA scheme is also simulated for comparison purpose. One of six channel models adopted by IEEE 802.16a standard for evaluating the performance of broadband wireless systems in the 2-11 GHz band is the vehicular A outdoor channel has six Rayleigh fading taps at delays of 0, 310, 710, 1090, 1730 and 2510 ns, with relative powers of 0, 21, 29, 210, 215 and 220 dB, respectively [17]. The vehicular A outdoor channel has a mobile speed of 120 km/h. A mobile speed of 120 km/h corresponds to a Doppler spread of 223 Hz for a carrier frequency of 2 GHz.

In simulation of CE DCT SC-FDMA, the performance is studied using the same parameters as in [5] that simulate the CE-OFDM with some additional special parameters used for SC-FDMA as shown in Table 2.

Table (2): *Simulation Parameters for CE DCT SC-FDMA*

Description	Parameters
Block size for each user ‘N’	64
Number of subcarriers	256
Modulation	4-PAM
Number of users ‘Q’	4
Modulation index ‘h’	$1.0/2\pi$
Oversampling ‘J’	8
DFT size ,‘N_{DFT}’	$N*Q*J=2048$
DCT size ‘N_{DCT}’	*N*Q and N*
Block period ‘T’	128 μs
Guard period ‘T_g’	10 μs
Subcarrier mapping	Localized and Interleaved
Channel model	Vehicular A outdoor channel
Noise environment	AWGN
Channel estimation	Known channel
Equalization	MMSE
Transmission efficiency ‘η_t’	$\eta_t = T/(T+ Tg) \approx 0.93$, thus overhead due to CP is 7%.
Simulations run times	10000 runs

4.1. Performance of DCT SC-FDMA Compared to DFT SC-FDMA:

The bit error rate (BER) of the DCT SC-FDMA system is compared to the BER of the DFT SC-FDMA system in Figure 5.

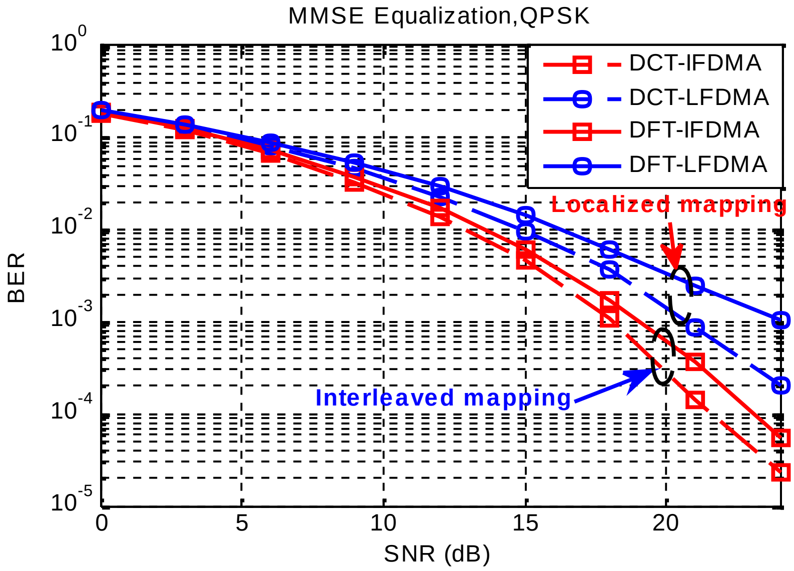


Figure (5): *BER versus SNR for DCT SC-FDMA system compared to DFT SC-FDMA system.*

It is shown from the previous figure that the DCT SC-FDMA system outperforms the DFT SC-FDMA system in terms of BER, for different types of subcarriers mapping, localized and interleaved.

4.2. Performance of Proposed CE DCT SC-FDMA System:

In this subsection, the performance of the constant envelope (i.e. phase modulated) DCT SC-FDMA is discussed using simulation. Fig.6 shows the results for the proposed scheme with interleaved mapping.

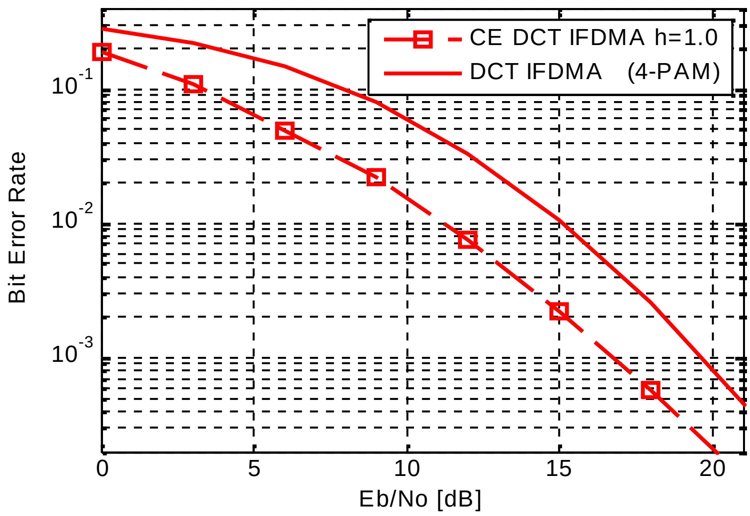


Figure (6): *BER versus Eb/No for CE DCT SC-FDMA system compared to DCT SC-FDMA system, interleaved mapping AND 4-PAM modulation.*

It is clear from previous figure that the proposed system outperforms the performance when compared to the traditional system with 4-PAM modulation used. That is due to exploiting the frequency diversity of the channel in phase modulation system. In figure 7, plots of bit error (BER) for CE DCT LFDMA and DCT LFDMA with 4-PAM modulation against signal-to-noise ratio (SNR) for localized mapping.

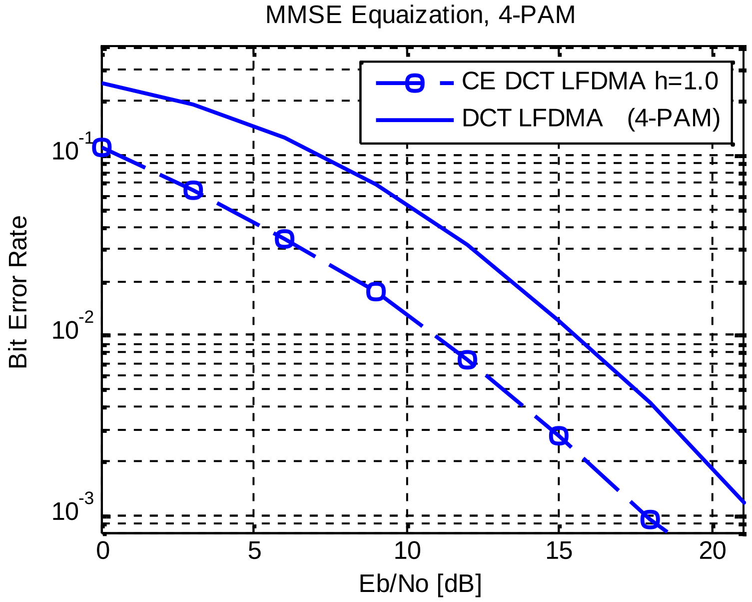


***Figure (7):** BER versus Eb/No for CE DCT IFDMA System Compared to DCT IFDMA Systems, localized mapping and 4-PAM modulation.*

It can be observed that CE DCT LFDMA system provides a significant BER performance improvement over the DCT LFDMA. At a BER=10^{-3}, the performance gain is about 4 dB for CE DCT LFDMA when compared to DCT LFDMA.

4.3. Effect of the Modulation Index:

In this subsection the effect of the modulation index on the performance of the proposed system is studied. Simulation of the proposed system for different modulation index with SNR is set to 18dB is shown in figure 11.

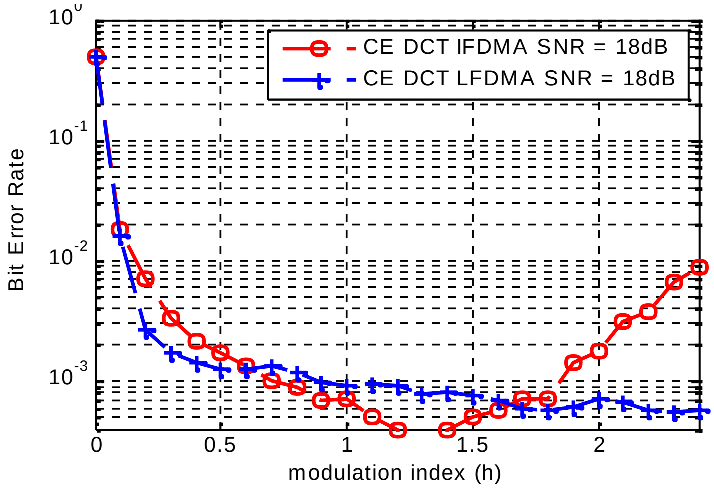


Figure (11): *BER versus modulation index for CE DCT SC-FDMA system, SNR=18 dB.*

Also, the effect of the modulation index on the BER under small value of SNR set to 6dB is simulated in figure 12.

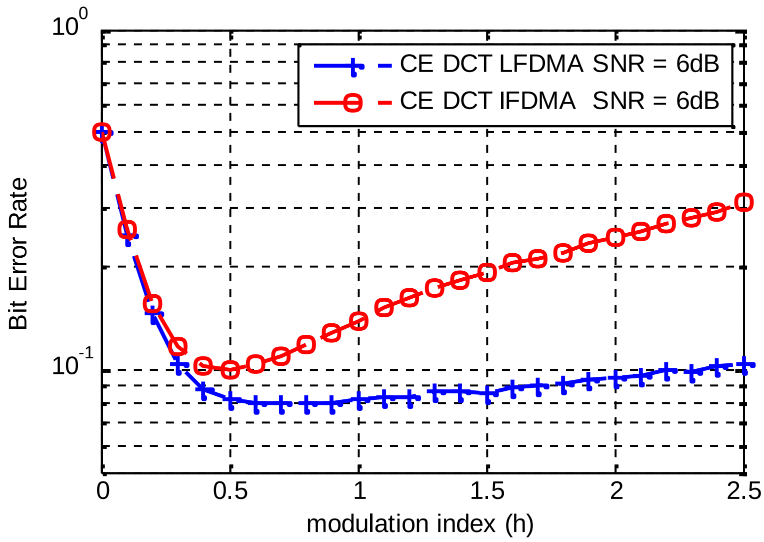


Figure (9): *BER versus modulation index for CE DCT SC-**FDM** system, SNR=6 dB.*

It is clear from the previous two figures that the BER effected by the modulation index and the SNR. As modulation index increases, the BER also decreases until it reach an approximate no decrease with modulation increase. Moreover, as SNR or modulation index increase, the exploiting of multipath diversity increases.

5. Conclusions:

In this paper, a new transceiver scheme of SC-FDMA system is proposed. This scheme is based on DCT signal processing and phase modulation. The proposed system has the advantage of constant envelope output signal (i.e. PAPR=0 dB). Due to constant envelope, the mobile terminal saves the power of the battery and can use a non linear power amplifier with lower cost. The use of M-PAM modulation and DCT signal processing before subcarriers mapping and then IDCT produce a real valued SC-FDMA signal which can be used as input to the phase modulation. A comprehensive study and simulation of the proposed system was presented by this paper. Moreover, effect of different parameters affected performance also studied and analyzed. This gives the readers and researchers good indication about the proposed system. Simulation results show the proposed system outperforms the traditional system in terms of BER and PAPR.

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