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PI and PID Power system Controller: Eigenspectrum-Based Design and Sensitivities

By

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Abstract:

A proposed design procedure for PI and PID type controllers is presented in the present paper utilizing eigenspectrum assignment as a design tool. As it is well known that the right eigenvector gives the mode shape, i.e. the relative activity of the state variables when a particular mode is excited, while the left eigenvector identifies the weights for the contribution of the activity to the mode. In the analysis and operation of power systems, one needs to assess the influence of some parameter variation that may be a source of oscillations and deteriorate the frequency and tie-line power regulation. This includes what might be considered "natural" modes of oscillations that are due to the inherent system characteristics as well as "forced" modes of oscillations that are driven by a particular system. A comprehensive treatment and evaluation for the sensitivities of eigenvalues and eigenvectors are presented.

Keywords: Eigenvalues assignment, Eigenvectors assignment, PI controller, PID controller, eigenvalues/eigenvectors sensitivities, power system control.

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1. Introduction:

One of the principal tasks in power system analysis is to carry out small signal stability analysis to assess the power system under the specified operating conditions. Power system analysis is a fundamental issue in planning, design, and operating stages. The effect of harmonics on power system is a dynamic and evolving field. Load changes and predicted load demand problems are treated through automatic gain controller (AGC) so as to maintain frequency at scheduled value (frequency control), maintain the net power interchanges with neighbouring control areas at their scheduled values (tie-line control), besides maintaining power allocation among the units in accordance with area dispatching needs. In some systems, the role of (AGC) may be restricted to one or two of the above mentioned objectives. Low frequency oscillations have harmful effects to the goals of maximum power transfer and optimal power system operation. A contemporary solution to this problem is the addition of power system stabilizer (PSS) to the automatic voltage regulator (AVR) on the generator [1-3] to enhance the damping of low frequency oscillations by providing positive damping to overcome the undamped electromechanical modes. A practical stabilizer has its input either a generator speed, terminal voltage frequency or electrical power. Its output is normally a signal to the reference input of the (AVR). Without loss of generality, (PSS) is one of the most cost-effective methods for enhancing power system stability, adding supplementary control loops to the generator (AVR) is one of the most common ways of enhancing both steady-state (small signal) stability related to small disturbance and transient (large signal) stability related to severe disturbance. The main idea of power system stabilization is to recognize that in the steady-state, that is when the speed deviation is nearly zero, the voltage controller should be driven by the voltage or state error only. Whereas in the transient state, the generator speed is not constant that's why rotor swings and voltage or state error undergoes oscillations caused by the change in rotor angle. The application of high voltage direct current (HVDC) and flexible alternating current transmission system (FACTS) added new control measures to electrical power systems, and have increased power transmission capacity, enhanced control capability, and improved operating characteristics. Improper designed (PSS) can become the source of a variety of undesired oscillations. For historical, comprehensive review of electrical power system stability problems with classification, analysis, improvement, modeling, and existing solutions, reader can refer to [4].

Although several control structures have been proposed for (PSS) design, the one initially proposed design methodology is the proportional integral (PI) and the proportional integral derivative (PID) is still quite used in nowadays power systems [5]. Such a type of controllers effectively reduces the effect of constant disturbance in the input channel and parameter perturbation. A proposed design procedure for PI and PID type controllers is presented in the present paper utilizing eigenspectrum assignment as a design tool. As it is well known that the right eigenvector gives the mode shape, i.e. the relative activity of the state variables when a particular mode is excited, while the left eigenvector identifies the weights for the contribution of the activity to the mode. In the analysis and operation of power systems, one needs to assess the influence of some parameter variation that may be a source of oscillations and deteriorate the frequency and tie-line power regulation. This includes what might be considered "natural" modes of oscillations that are due to the inherent system characteristics as well as "forced" modes of oscillations that are driven by a particular system. A comprehensive treatment and evaluation for the sensitivities of eigenvalues and eigenvectors are presented.

The paper is organized as follows; the PI controller design procedure is presented in section two followed by an illustrative example for a realistic power system [6]. Section three extends the design to

introduce a PID controller scheme which is validated by the same adopted power system illustrative example. Section four addresses the first order eigenvalue sensitivity to parameter perturbation in the system matrix. Finally, first order eigenvector sensitivity is introduced.

2- Proportional Integral (PI) Controller Design Methodology

To present the design procedure, consider the linearized differential equation model of a continuous time controllable dynamical electrical power system governed by the state equations of the form:

$$\begin{aligned}\dot{x}(t) &= A x(t) + B u(t) + L w(t) \\ y(t) &= C x(t)\end{aligned}\tag{1}$$

where $x(t) \in \mathfrak{R}^n$ is the state vector, $A \in \mathfrak{R}^{n \times n}$ is a constant system matrix, $B \in \mathfrak{R}^{m \times n}$ is the constant control (input) matrix, $C \in \mathfrak{R}^{q \times n}$ is a constant measurement (output) matrix while $L \in \mathfrak{R}^{m \times n}$ is a constant disturbance matrix; in addition matrices B and C are of full rank (m, q) respectively. For a derivation of the most commonly used electric power system models, the interested reader can follow [4,7]. The control vector $u(t) \in \mathfrak{R}^m$, $y(t)$ is the output vector $\in \mathfrak{R}^q$ whereas $w(t)$ is the load disturbance.

As power systems cannot operate satisfactorily without proper control, therefore to achieve the highest control strategy with least variability, a state error is defined as $e(t) = x(t) - x_r(t)$, where $x_r(t)$ is the reference state vector “which will be dropped later” as the deviation of state vector relative to its reference values. It should be mentioned that throughout this paper, the state vector $x_r(t)$ will be assumed to be a slowly varying quantity.

It is frequently necessary to introduce integral as well as proportional feedback for all state variables of the power system in order to achieve an acceptable steady-state error with a desirable transient response.

Let the control which derives the dynamical system (1) from initial state and assigning a prescribed set of complex conjugate eigenvalues be composed of two terms as:

$$u(t) = K_p x(t) + K_I \int_0^{\infty} y(t) dt\tag{2}$$

Where $K_p \in \mathfrak{R}^{m \times n}$ represents the proportional feedback gain matrix, while $K_I \in \mathfrak{R}^{m \times q}$ represents the integral part of the feedback gain matrix. Feedback gains K_p and K_I are to be designed based on eigenvalue assignment as a tool. Such a type of controllers represents the best possible trade-offs among robustness, stability, and performance criteria both in transient and steady-state [8]. It is logical to assume that a step change of load is often the case. In order to reject the effects of finite constant disturbance $w(t)$, equation (1) is differentiated to get:

$$\dot{z}(t) = A z(t) + B \dot{u}(t)\tag{3}$$

Where $z(t) = \frac{dx}{dt}$. Defining an augmented state vector $x_a(t) = [z^T(t) \quad y^T(t)]^T$, where $x_a(t) \in \mathfrak{R}^{n+q}$.

Therefore, the augmented system representation becomes as:

$$\dot{x}_a(t) = A_a x_a(t) + B_a \dot{u}(t)\tag{4.a}$$

$$y_a(t) = C_a x_a(t)\tag{4.b}$$

$$\text{where } A_a = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}_{(n+q) \times (n+q)}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}_{(n+q) \times m}, C_a = [C \quad 0]_{q \times (q+n)} \quad (5)$$

The objective is to design the nonunique feedback gain matrices K_P and K_I .

As long as the pair (A, B) is controllable and to have complete control over the system dynamics, the following necessary and sufficient conditions must be satisfied for the existence of state feedback which are:

$$(i) \text{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + \min(m, q)$$

(ii) The pair (A_a, B_a) is a controllable pair [9].

The properties of these conditions can be used to design the appropriate proportional and integral feedback gains. Once these conditions are satisfied, it is possible to design a proportional plus integral controller of the form (2) by assigning a set of self conjugate eigenvalues (electromechanical modes) for fast regulation coupled with system stability. Recalling that the real part represents oscillation damping whereas the imaginary part represents the frequency of oscillations. Differentiating equation (2) results in:

$$\dot{u}(t) = K_P z + K_I C x = K_a x_a(t) \quad (6)$$

where $K_a = [K_P \quad K_I] \in \mathbb{R}^{m \times (n+q)}$

Substituting in (4.a), then the modified state equation can be expressed as:

$$\dot{x}_a(t) = (A_a + B_a K_a) x_a(t) \quad (7)$$

The resulting augmented system will consist of $(n + q) \times (n + q)$ dimensions and obtained as:

$$\begin{bmatrix} \dot{z}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} A + BK_P & BK_I \\ C & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} \quad (8)$$

There is a need to deal with various modes and frequencies that occur in such systems. The performance of the system can be examined by evaluating the eigenvalues for speed of response and stability; besides the eigenvectors for the distribution of eigenvalues within states thus achieving robustness and mode shaping for meeting performance specifications [10-12]. For some complex eigenvalue $\lambda_i \{i = 1, 2, \dots, (n + q)\}$ and the corresponding $(n + q)$ dimensional right eigenvector V_i that shapes the response, the following relation is satisfied:

$$\begin{bmatrix} A + BK_P - \lambda_i I & BK_I \\ C & -\lambda_i I \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \end{bmatrix} = 0 \quad (9)$$

where the vector V_i is partitioned to $v_{i1} \in \mathbb{R}^{n \times 1}$ and $v_{i2} \in \mathbb{R}^{q \times 1}$ according to the state $z(t) \in \mathbb{R}^n$ and the output $y(t) \in \mathbb{R}^q$. From which the lower portion of the eigenvector is obtained as:

$$v_{i2} = \lambda_i^{-1} C v_{i1} \text{ where } i = 1, 2, \dots, (n+q) \quad (10)$$

Manipulating equation (9), the compact form is obtained as:

$$[(A - \lambda_i I) \quad B] \begin{bmatrix} v_{i1} \\ (K_P + \lambda_i^{-1} K_I C) v_{i1} \end{bmatrix} = 0 \quad (11)$$

In compact form, equation (11) can be expressed as:

$$\begin{bmatrix} (A - \lambda_i I) & B \end{bmatrix} \begin{bmatrix} v_{i1} \\ w_i \end{bmatrix} = 0 \quad (12)$$

where the m -dimensional vector w_i equals to $(K_p + \lambda^{-1} K_I C) v_{i1}$. Inspecting equation (12) reveals that it represents (n) linear equations in $(n + m)$ unknowns representing the associated vector. In order to satisfy equation (12), the vector $[v_{i1}^T, w_i^T]^T$ must lie in the null space of the $(n \times (n + m))$ matrix $\begin{bmatrix} (A - \lambda_i I) & B \end{bmatrix}$ which is of m -dimensional admissible space. This allows a free selection of (m) unknowns, those forming the vector w_i , consequently this reflects the freedom in determining feedback gains, in order to get a solution for the upper vector v_{i1} of dimension (n) . Iterating equation (12) for the prescribed set of $(n + q)$ eigenvalues, the following expression is obtained:

$$W = \begin{bmatrix} w_1 & w_2 & \dots & w_{n+q} \end{bmatrix} = \begin{bmatrix} K_p & K_I \end{bmatrix} \begin{bmatrix} v_{11} & \dots & v_{(n+q)1} \\ \lambda_1^{-1} C v_{11} & \dots & (\lambda_{n+q})^{-1} C v_{(n+q)1} \end{bmatrix} \quad (13)$$

The right most matrix V is square of dimension $(n + q) \times (n + q)$, as long as the eigenvalues are distinct and the matrix is nonsingular, hence the solution of equation (13) is guaranteed as:

$$W V^{-1} = \begin{bmatrix} K_p & K_I \end{bmatrix} \quad (14)$$

Effectiveness of the proposed design procedure is summarized in the following steps where it is implemented using MATLAB control toolbox software package:

- Consider equation (12), arbitrarily select the lower (m) elements forming the vector w_i (except zeros), then construct $W = \begin{bmatrix} w_1 & w_2 & \dots & w_{n+q} \end{bmatrix}$
- Apply the relation $v_{i1} = -(A - \lambda_i I)^{-1} B w_i$ to get v_{i1} , then calculate $\lambda_i^{-1} C v_{i1}$ as equation (10)
- Construct the $(n + q)$ dimensional vector as $V_i = \begin{bmatrix} v_{i1} \\ \lambda_i^{-1} C v_{i1} \end{bmatrix}$
- Repeat for $i = 1, 2, \dots, (n + q)$ to get $V = \begin{bmatrix} V_1 & V_2 & \dots & V_{n+q} \end{bmatrix}$
- Apply equation (14) to get the gains of K_p and K_I

Interestingly once v_{i1} is obtained, the next steps are easily obtained. Effectiveness of the proposed design is illustrated taking the advantage of MATLAB control system toolbox software package. A numerical example for a realistic power system extracted from [6] where the matrices are:

$$A = \begin{bmatrix} -0.05 & 6 & 0 & -6 & 0 & 0 & 0 \\ 0 & -3.33 & 3.33 & 0 & 0 & 0 & 0 \\ -5.21 & 0 & -12.5 & 0 & 0 & 0 & 0 \\ 0.45 & 0 & 0 & 0 & -0.545 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.05 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.33 & 3.33 \\ 0 & 0 & 0 & 0 & -5.21 & 0 & -12.5 \end{bmatrix} \quad (15.a)$$

$$B = \begin{bmatrix} 0 & 0 & 12.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12.5 \end{bmatrix}^T \quad (15.b)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 1 \end{bmatrix} \quad (15.c)$$

The open loop system eigenvalues (electromechanical modes)

are $(-0.8312 \pm j 2.8855, -0.9386, -1.2479 \pm j 2.4743, -13.2789, -13.2843)$.

To illustrate the procedure, let the desired $(n + q) = 9$; such that the arbitrary eigenvalues be $(-0.14 \pm j 2.47, -0.2 \pm j 0.3, -0.3 \pm j 0.8, -0.7 \pm j 0.7, -13)$ which are chosen to achieve various damping ratios and frequencies.

Applying the aforementioned steps, the proportional and integral feedback gains are obtained as:

$$K_p = \begin{bmatrix} 0.3649 & -0.3013 & 0.8947 & 0.3501 & 0.0093 & 0.1114 & -0.0549 \\ -0.1521 & -0.3441 & -1.1121 & 1.2496 & 0.4219 & 1 & 0.3917 \end{bmatrix}$$

$$K_I = \begin{bmatrix} -0.057 & 0.0303 \\ -0.1577 & 0.0817 \end{bmatrix}$$

3- Proportional plus Integral plus Derivative (PID) Controller Design [13]

Derivative action is used for providing phase lead, which offsets phase lag caused by integral term. The action is also helpful in hastening system recovery from disturbances.

Consider the system equations are described as:

$$\begin{aligned} \dot{x}(t) &= A x(t) + B u(t) \\ y(t) &= C x(t) \end{aligned} \quad (16)$$

Let the control vector be composed of three components, namely

$$u(t) = K_p x(t) + K_I \int_0^{\infty} y(t) dt + K_D \dot{x}(t) \quad (17)$$

Where $K_p \in \mathbb{R}^{m \times n}$ represents the proportional feedback gain matrix, while $K_I \in \mathbb{R}^{m \times q}$ represents the integral part of the feedback gain matrix; whereas the derivative feedback gain $K_D \in \mathbb{R}^{n \times n}$.

$$\text{Let } \int_0^\infty y(t) dt = z(t), \text{ then } \dot{z}(t) = C x(t) \quad (18)$$

Substituting in equation (16) with some manipulation, an augmented representation is obtained as:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} (I_n - BK_D)^{-1} (A + BK_P) & (I_n - BK_D)^{-1} BK_I \\ C & 0_{q \times q} \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \quad (19)$$

For some eigenvalue λ_i , the corresponding eigenvector satisfies the equation:

$$\begin{bmatrix} (I_n - BK_D)^{-1} (A + BK_P) - \lambda_i I & (I_n - BK_D)^{-1} BK_I \\ C & -\lambda_i I_q \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \end{bmatrix} = 0 \quad (20)$$

$$\text{From which } v_{i2} = \lambda_i^{-1} C v_{i1} \quad (21)$$

Where v_{i1} is $(n \times 1)$ vector while v_{i2} is $(q \times 1)$ vector.

Expanding equation (20), results in:

$$[(I_n - BK_D)^{-1} (A + BK_P) - \lambda_i I] v_{i1} + (I_n - BK_D)^{-1} BK_I v_{i2} = 0 \quad (22)$$

Premultiplying by $(I_n - BK_D)$ and arranging up the terms, it can be written in the form of:

$$[(A - \lambda_i I) \quad B] \begin{bmatrix} v_{i1} \\ (K_P + \lambda_i^{-1} K_I C + \lambda_i K_D) v_{i1} \end{bmatrix} = 0 \quad (23)$$

Let the lower (m) element of the above vector be w_i which is arbitrarily chosen, such that

$$w_i = (K_P + \lambda_i^{-1} K_I C + \lambda_i K_D) v_{i1} \quad (24)$$

Iterating equation (23) for $i = 1, 2, \dots, (2n + q)$, where the additional (n) eigenvalue are due to introducing a derivative term in equation (17); the following expression is obtained:

$$W = [w_1 \quad w_2 \quad \dots \quad w_{2n+q}] = [K_P \quad K_I \quad K_D] \begin{bmatrix} v_{11} & \dots & v_{2n+q} \\ \lambda_1^{-1} C v_1 & \dots & \lambda_{2n+q}^{-1} C v_{2n+q} \\ \lambda_1 v_1 & \dots & \lambda_{2n+q} v_{2n+q} \end{bmatrix} \quad (25)$$

The compact form of the above equation can be written as:

$$W = K V,$$

$$\text{Hence } K = [K_P \quad K_I \quad K_D] = W V^{-1} \quad (26)$$

To illustrate the last procedure, using the system equations of (16), let the desired arbitrary chosen closed-loop eigenvalues are accounted as:

$(-0.14 \pm j 1.47, -0.2 \pm j 0.7, -0.3 \pm j 0.4, -0.3 \pm j 0.8, -0.7 \pm j 0.7, -1.4 \pm j 2.47, -2.4 \pm j 6.5, -4, -15)$ The MATLAB control system toolbox software package has been used to calculate the design steps where the following gains are obtained:

$$K_P = \begin{bmatrix} 0.1836 & 0.2656 & -2.6563 & 3.3125 & 0.1797 & -24.5 & 8 \\ 0.7188 & -1.0156 & -18.3125 & 19.3125 & 0.2188 & -193.5 & 89.8125 \end{bmatrix}$$

$$K_I = \begin{bmatrix} -0.0740 & -0.0059 \\ -0.0882 & 0.0148 \end{bmatrix}$$

$$K_D = \begin{bmatrix} 0.3594 & 1.1094 & 0.0676 & 0.4141 & 2.9375 & -2.2813 & -0.0068 \\ 3.0625 & 5.5078 & -0.0218 & -1.0703 & 17.3125 & -26.75 & 0.0727 \end{bmatrix}$$

4- Eigenvalues/Eigenvectors First Order Sensitivities

Expressions for eigenvalue/eigenvector sensitivity coefficients due to perturbation in system parameters have been given in various forms and from different points of view, whether numerical analysis, perturbation theory, and as problem in linear system theory [14].

The influence of certain component's parameters such as governor, turbine, load frequency time constant should be investigated in power system analysis, design, and operation in order to achieve an adequate and satisfactory performance. Badly designed and/or improper parameters selection may be a source of oscillations and can deteriorate the frequency and the tie line power regulation process. The linearized system state matrix $A \in \mathbb{R}^{n \times n}$ contains several parameters, thus the eigenvalues λ_i 's are also functions of these parameters, consequently the right and left eigenvectors will vary since the components of the right eigenvector measure the relative activity of each state variable in the i -th mode, while components of the left eigenvector weight the initial conditions in the i -th mode [14]. Variation of the eigenvalues (λ_i 's) with respect to parameter represent the influence of parameter variation on power system stability.

First Order Eigenvalue Sensitivity

Method (i)

Let the system matrix A be an $(n \times n)$ matrix $[a_{kl}]$ ($a, k=1, 2, \dots, n$) with distinct eigenvalues λ_i ($i=1, 2, \dots, n$) that are also functions of those parameters.

The corresponding linearly independent right eigenvectors v_i 's ($n \times 1$) satisfy the relation

$$A v_i = \lambda_i v_i \quad (27.a)$$

and the corresponding linearly independent left eigenvectors u_i 's ($n \times 1$) satisfy the relation

$$A' u_i = \lambda_i u_i \quad (27.b)$$

The right-left eigenvectors satisfy the relation $v_i' u_j = u_j' v_i = \delta_{ij}$ ($i, j=1, 2, \dots, n$) (27.c)

where δ_{ij} is the Kronecker delta. If the element a_{kl} is perturbed due to changes in system parameters, differentiating equation (27.a) with respect to element a_{kl} yields:

$$\frac{\partial A}{\partial a_{kl}} v_i + A \frac{\partial v_i}{\partial a_{kl}} = \frac{\partial \lambda_i}{\partial a_{kl}} v_i + \lambda_i \frac{\partial v_i}{\partial a_{kl}} \quad (28)$$

Premultiplying by u_i' and noting that $u_i' v_i = 1$, all elements of $\frac{\partial A}{\partial a_{kl}}$ are zeros (except for the element in the k -th row and l -th column which is equal to 1) reduces equation (28) to the set of scalar equations

$$\frac{\partial \lambda_i}{\partial a_{kl}} = u_i^k v_i^l \quad (i, j, k=1, 2, \dots, n) \quad (29)$$

Where u_i^k is the k -th element of the left eigenvector u_i while v_i^l is the l -th element of the right eigenvector v_i . Equation (29) describes the first order eigenvalue sensitivity coefficients which relate changes in the i -th eigenvalue of the system matrix A to changes in the element a_{kl} . Combining the coefficients for $(i, j, k=1, 2, \dots, n)$, a set of n eigenvalue sensitivity matrices are obtained as:

$$S_i = \left[\frac{\partial \lambda_i}{\partial a_{kl}} \right] = U_i V_i^T \quad (30)$$

which are of unit rank, and therefore singular. The sensitivity matrices have an interesting properties which are useful in both theoretical work and in checking numerical calculations.

Accordingly, the first order estimates of the eigenvalue λ_i where the element a_{kl} is perturbed is

$$\hat{\lambda}_i = \lambda_i + \frac{\partial \lambda_i}{\partial a_{kl}} a_{kl} \quad (i=1,2,\dots,n) \quad (31)$$

The calculation procedure of the first order eigenvalue sensitivity is summarized as:

- Calculate the eigenvalue λ_i from equations (27.a,b), and the corresponding right and left eigenvectors (v_i, u_i') respectively then realize their orthogonality.
- Calculate $\frac{\partial A}{\partial a_{kl}}$, then $\frac{\partial \lambda_i}{\partial a_{kl}} = u_i^k v_i^l$
- The perturbed eigenvalue of the element a_{kl} is obtained as $\hat{\lambda}_i$ due to perturbation δa_{kl}
 $\hat{\lambda}_i = \lambda_i + u_i^k v_i^l \delta a_{kl}$ where the deviation from the nominal value is $u_i^k v_i^l \delta a_{kl}$
 $(i,j,k=1,2,\dots,n).$

Method (ii) [7]

Instead of dealing with the right and left eigenvectors of the system matrix, consider Taylor series expansion, assuming the parameter a_{kl} changes to $(a_{kl} + \delta a_{kl})$, the corresponding change in the i -th eigenvalue is from $\lambda_i(a_{kl})$ to $\lambda_i(a_{kl} + \delta a_{kl})$. Taylor expansion of $\lambda_i(a_{kl} + \delta a_{kl})$ at a_{kl} is obtained as:

$$\lambda_i(a_{kl} + \delta a_{kl}) = \lambda_i(a_{kl}) + \left. \frac{\partial \lambda_i(a_{kl})}{\partial a_{kl}} \right|_{a_{kl}} \delta a_{kl} + \frac{1}{2} \left. \frac{\partial^2 \lambda_i(a_{kl})}{\partial^2 a_{kl}} \right|_{a_{kl}} (\delta a_{kl})^2 + \dots \quad (32)$$

since δa_{kl} is very small, the change can be approximated as:

$$\lambda_i(a_{kl} + \delta a_{kl}) = \lambda_i(a_{kl}) + \left. \frac{\partial \lambda_i(a_{kl})}{\partial a_{kl}} \right|_{a_{kl}} \delta a_{kl} \quad (33)$$

The calculation procedure is summarized as:

- Set (a_{kl}) to the state matrix $A = [a_{kl}]$
- Calculate the eigenvalue $\lambda_i(a_{kl})$, and the corresponding right and left eigenvectors (v_i, u_i') respectively from equations (27.a,b)
- Evaluate $\left. \frac{\partial A}{\partial a_{kl}} \right|_{a_{kl}}$ which is all zeros except for the element a_{kl} which is equal to 1

- The first order eigenvalue sensitivity is $\frac{\partial \lambda_i}{\partial a_{kl}} = u_i^T \frac{\partial A}{\partial a_{kl}} \bigg|_{a_{kl}} v_i$

First Order Eigenvalue Sensitivity

Let the eigenvector (right and left) changes from perturbation of the element a_{kl} of the system matrix A , equation (28) reduces to

$$(A - \lambda_i I) \frac{\partial v_i}{\partial a_{kl}} = \frac{\partial \lambda_i}{\partial a_{kl}} v_i - \frac{\partial A}{\partial a_{kl}} v_i \quad (34)$$

Consequently, equation (27.b) can be written as $u_j^T A = \lambda_j u_j^T$, by partial differentiation with respect

$$\text{to } a_{kl} \text{ reduces to } \frac{\partial u_j^T}{\partial a_{kl}} (A - \lambda_j I) = \frac{\partial \lambda_j}{\partial a_{kl}} u_j^T - u_j^T \frac{\partial A}{\partial a_{kl}} \quad (35)$$

since λ_i and λ_j are eigenvalues of the system matrix A , equations (34,35) cannot be solved for the eigenvectors sensitivity coefficients $\frac{\partial v_i}{\partial a_{kl}}$, $\frac{\partial u_j^T}{\partial a_{kl}}$.

Premultiplying equation (34) by u_j^T ($j \neq i$) and equation (35) is post multiplied by v_i ($j \neq i$), it yields:

$$(\lambda_j - \lambda_i) u_j^T \frac{\partial v_i}{\partial a_{kl}} = -u_j^k v_i^l \quad (i \neq j, j = 1, 2, \dots, n) \quad (36)$$

$$\text{and } \frac{\partial u_j^T}{\partial a_{kl}} v_i (\lambda_i - \lambda_j) = -u_j^k v_i^l \quad (i \neq j, j = 1, 2, \dots, n) \quad (37)$$

Premultiplying equation (36) by v_j ($j \neq i$) and equation (37) is post multiplied by u_i^T , it will be as:

$$S'_j \frac{\partial v_i}{\partial a_{kl}} = - \left(\frac{u_j^k v_i^l}{(\lambda_j - \lambda_i)} \right) v_j \quad (38)$$

$$\text{and } \frac{\partial u_j^T}{\partial a_{kl}} S'_i = - \left(\frac{u_j^k v_i^l}{(\lambda_j - \lambda_i)} \right) u_i^T \quad (39)$$

These last two equations can be written compactly introducing the $(n \times n)$ matrix H as:

$$H = [h_{ij}^{kl}] = \frac{u_j^k v_i^l}{\lambda_i - \lambda_j} \quad (40)$$

Therefore equations (38,39) can be written as:

$$S'_j \frac{\partial v_i}{\partial a_{kl}} = h_{ji}^{kl} v_j \quad (41)$$

$$\frac{\partial u_j^T}{\partial a_{kl}} S'_i = -h_{ji}^{kl} u_i^T \quad (42)$$

Since the eigenvectors of the matrices A and A' are linearly independent due to the assumption of distinct eigenvalues, the needed vectors $\frac{\partial v_i}{\partial a_{kl}}$ and $\frac{\partial u'_j}{\partial a_{kl}}$ give the required expression for the first order eigenvector sensitivity coefficients.

The derived coefficients can be used in the proposed PI controller design via an arbitrary selection of the vector w_i {equation (13)} then calculating the vector v_{il} in equation (12). The entire

$(n+q)$ eigenvector $\begin{bmatrix} v_{il} \\ \lambda_i^{-1} C v_{il} \end{bmatrix}$ ($i=1,2,\dots,n+q$) can now be formed. The arbitrary selection process can

be achieved such that the eigenvectors are as insensitive as possible to changes. For the case of PID controller design, same concept can be implemented for equation (24) and equation (25) to get the

$$(2n+q) \text{ vector } \begin{bmatrix} v_i \\ \lambda_i^{-1} C v_i \\ \lambda_i v_i \end{bmatrix}.$$

5. Conclusions

Eigenspectrum assignment is one of the central problems in control system design. Different versions of feedback were considered as a means for response shaping and ensuring stability for power systems. Eigenspectrum analysis is attractive since it provides the frequencies and the damping at each frequency for the entire system in a single calculation. Two versions that are commonly used for power system stabilizers are presented; namely proportional integral (PI) and the proportional plus integral plus derivative (PID) have been designed based on eigenspectrum assignment as an effective tool. Two illustrative numerical examples for a typical power system are presented. Moreover, in order to investigate and assess the influence of element perturbation of the linearized system matrix on eigenvalues and eigenvectors, first order sensitivities were presented in terms of (i) right and left eigenvectors of the system matrix (ii) Taylor series expansion. The sensitivity assessment helps to take account of the range of operating conditions and to deal with interarea oscillatory electromechanical modes.

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