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THE SINGLE-MACHINE MULTI-PRODUCT PROBLEM: NEAR OPTIMAL SCHEDULING

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ABSTRACT

An iterative procedure is presented for determining near optimal production schedule in the single-machine multi-product lot sizing problem. The approach does not require equal lot sizes, and based on finding a feasible solution (lower bound).

Two examples from the previous literature indicated that cancelling the restriction of equal lot sizes may provide a chance for producing better solutions.

INTRODUCTION

The classical problem of scheduling lot sizes for n products on one machine is treated by many contributers considering equal lot sizes (see,e.g., [1] , [3] , [7] and [9]). The equal lot sizes restriction provides a possibility of designing systematic procedures for achieving the optimum of a restricted version of the original problem (see [5]). But the original problem has not been solved and there is no available procedure for achieving the "true optimum".

The main contribution of this paper is to indicate that the optimum solut-on does not require equal lot sizes.

REFORMULATION OF THE PROBLEM

Standard assumptions are made with regard to demand and production characteristics (see [1]) and the following notation is assumed.

i = product designation ($i = 1,2,3,\ldots,n$)
A = setup cost per production lot
h = unit inventory holding cost per unit time

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p = production rate per unit time
r = demand rate per unti time
s = setup time per production lot
T = cycle time

For later convenience let $\rho=r/p;\tau=\rho T$, processing time per lot, $\sigma=s+\tau$, the total production time for lot; and $B=hr(1-\rho)$.

The average cost per unit time when product i is produced in equal cycles of length T_i is given by

$$C_i=A_i/T_i + B_iT_i/2 \quad (1)$$

which immediately yields that the min-cost cycle is given by

$$T_i^* = \text{SQRT}\ (2A_i/B_i) \quad (2)$$

corresponding to the minimum cost

$$C_i^* = \text{SQRT}\ (2A_i\ B_i) \quad (3)$$

Evidently, the total cost, given by $\sum C_i^*$, is a lower bound on the optimal value of any feasible solution.
We refer to this solution as the Independent solution (IS).

Let T_t be the (unknown) total production cycle which repeats over time. In the IS, product i is produced k_i^* times per T_t (ie, $T_t= k_i^*\ T_i^*$). If k_i^* is restricted to the nearest integer or even number, for feasibility considerations, and product i is produced in equal cycles every T_t (i.e., $T_t=k_iT_i$) then, the corresponding average cost per unit time is given by

$$C_i=C_i^*(T_i/T_i^*+T_i^*/T_i)/2=(B_i/2T_i)(T_i^{*2}+T_i^2) \quad (4)$$

For escaping from infeasibility, if there is, product i can be preoduced in k_i unequal cycles per T_t

$$\text{i.e.} \quad T_t= \sum_{j=1}^{j=k_i} T_{ij}).$$

In this case the average cost per unti time is given by

$$C_i' =(B_i/2T_t)(k_iT_i^{*2}+ \sum_{j=1}^{j=k_i} T_{ij}^2) \quad (5)$$

FORMING A FEASIBLE SCHEDULE

In the following we are going to explore some rules for guiding the trials of achieving a production schedule with associated low cost, as near as possible to the lower bound solution.

(1) If one increases the min-cost cycle by X days then, the Deviation Factor (DF)

$$DF = (T_i^* + X) / T_i^* = 1 + X/T_i^* \qquad (6)$$

Expression (6) demostrates that the short cycles is more sensitive than the long ones.

(2) One can easily show that $T_i^*/(T_i^* - X) > (T_i^* + X)/T_i^*$ which means that incresing the min-cost cycle time by X days, for feasibility considerations, produces lower deviation than that produced if the cycle time is decreased by X days.

The followingthree rules are improtant for obtaining lower cost schedule and escaping from infeasibility:

(a) begin by the products of the higher frequencies (i.e., short cycles), because they have higher sensitivities [11].

(b) comapre the values of $T_i^*/(T_i^* - X_1)$ and $(T_i^* + X_2)/T_i^*$ before decreasing or increasing T_i^*, for escaping from infeasibility,

(c) place the runs for each product at equal intervals as possible and give the priority for the products of associated higher values of B_i.

In the light of the above rules the following steps can be designed, for achieving the near optimal solution.

(1) Determine the values of (B_i, C_i^* and T_i^*) for each product independently.

(2) Generate a set of multiples (k_i) by dividing the largest T_i^* by the set (T_i^*) and rounding the resultant fractions to the nearest integer, or to the nearest even number, if required, for feasibility considerations.

(3) Compute the total production time (σ_i) for each product.

(4) Considering the rules mentioned above place the runs for each product at equal intervals on a time scale. Interference can be treated by increasing and/or decreasing the cycle times. This step is the most difficult one and applying it may necessitate unequal slots on the time scale. The summation of the slots defines T_t. The smallest and the largest values of T_i^* are the initial values of the slots and T_t respectively.

No systeimatic manner for applying this step and msut be applied by a knowledgeable person because the problem is data dependent and some judgement is required. For all the judged schedules and sequences the final refree is the cost indicator.

TWO EXAMPLES

Applying the proposed procedure to two published problems had achieved lower cost than that reached by the published procedures. The two exaples wre taken from [1] and [12].

A 4-products problem

The data of the problem are given in [12] . The resultant solution is summarized in Table 1 , and illustrated by Fig. 1. A comparative results are shown in Table 2.

Table 1. Summary of the solution for 4- products example

product	B_i	T_{ij} (Days)	T_i^* (Days)	C_i' ($)
1	31.47	87	79.73	2518.66
2	103.40	40.3 , 46.7	43.98	4559.98
3	195.55	29, 29, 29,	31.98	6283.63
4	280.80	29, 30, 28	26.70	7526.20
sum				20888.47

Table 2. Comparative results.

Solution Method	Common cycle [8]	Saipe [12]	Makroum	Lower Bound
Cost ($)	22,112.84	21,157.88	20,888.47	20,804.10

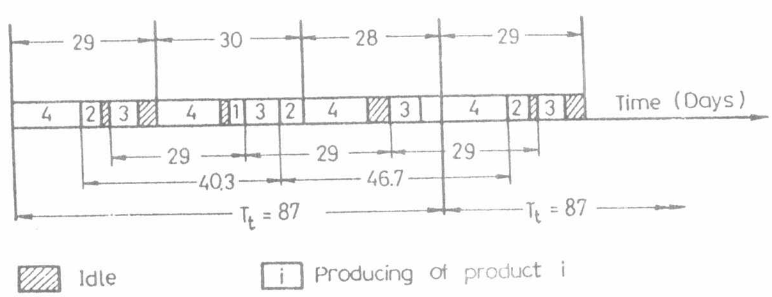

Fig.1 A four products schedule.

A 10-products example

The data of Bomberger's problem are given in many sources (e.g, [1] and [9]) .The proposed approach is applied to the probelm (with factor α =4). The resultant schedule is shown in Fig.2 and the corresponding solution is summarized in Table 3.

Table 3. Summary of the solution for 10-products example.

product	B_i	T_i^*	C_i^* ($)	C_i' ($)
1	0.2565	167.5	42.98	44.16
2	6.7450	37.7	254.46	263.19
3	9.3410	39.3	366.76	374.67
4	12.5867	19.5	245.80	246.61
5	21.3888	49.7	1062.70	1066.21
6	2.1115	106.6	225.11	225.09
7	3.5640	204.3	728.23	728.62
8	148.1354	20.5	3040.34	3041.16
9	25.3980	61.5	1561.48	1599.48
10	1.5573	39.3	61.14	63.39
sums			7589.00	7652.58

Table 4. Comparative results

Solution Method		Cost ($)
Common Cycle	[8]	9880
Bomberger	[1]	8796
Stankard, Gupta	[13]	8698
Elmaghraby	[5]	8383
Madigan	[9]	8145
Goyal	[6]	7704
Delporte, Thomas	[2]	7699
Doll, Whybark	[3]	7699
Haessler, Hogue	[7]	7699
Makroum		7653
Lower Bound (IS)		7589

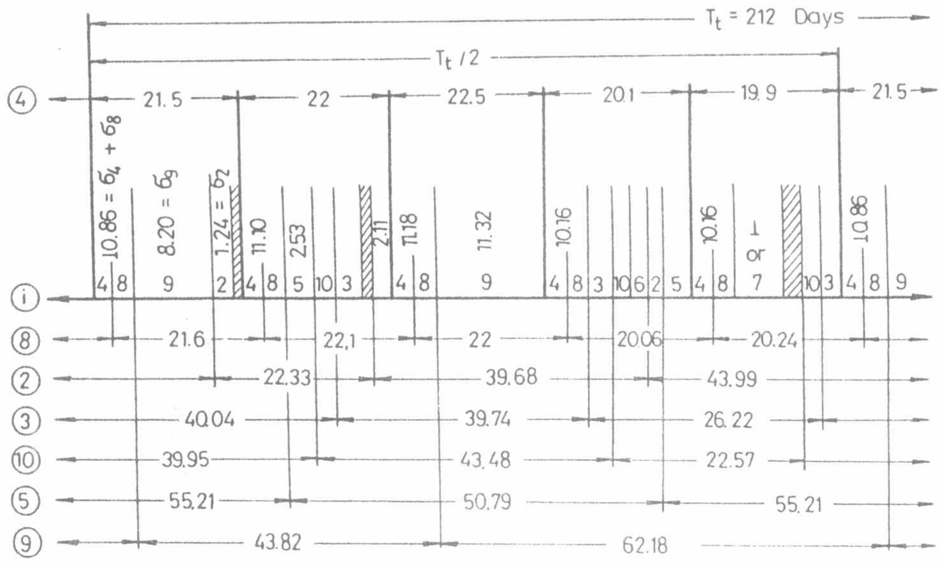

Fig.2 A ten products schedule.

Comparing the resultant cost (7652.58 \$) with the results of the o her solutions (as shown in Table 4) indicates the advantage ofthe proposed approach. Very small modifications may be available by additional tedious trials, but the cost of the trials may exceed the savings it offer over the resultant solution or over the systematic solution of Haessler and Hogue [7].

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