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MODIFIED ANALYSIS AND OPTIMIZATION TECHNIQUE FOR COMPOUND CYLINDERS USED IN ARTILLERY PRACTICE

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ABSTRACT

This paper represents a computer aided study of the induced stresses in compound open ended cylinders subjected to high internal pressures. A novel optimization procedure has been herein proposed with the objective of assigning compound cylinders wall thickness strength for maximum material utilization. Design charts are, thus, provided to help in optimizing the selection of cylinders dimentions and material under a wide range of operating pressures.

INTRODUCTION

Several theories have been developed to analyse the distribution of induced stresses in compound cylinders subjected to high internal or external pressures. Assuming a full elastic behaviour, Lamé 1 in his theoretical analysis of thick walled cylinders could formulate the basic well known stress equation in axial, radial and tangential directions. Due to the assumptions involved in his analysis, the attained stress governing equations are claimed to be most applicaple, for design purposes, to ductile materials 2 . However, it should be noted that Lamé equations are theoretically correct, whereas other thick cylinders formulæ in common use are either semiemperical or already induce some yield criterion and/or some end conditions.

In a subsequent modification of Lamé'equations by introducing the possible effect of lateral strains, Clavarino and Birnie 2 could derive a fairly accurate solution. Although Clavarino's equations are bound to closed end cylinders, the Birnie's equations which assume an open end cylinder, render higher wall thickness than that which would be attained if Clavarino's solution is adopted. So it is recommended to use Birnie's equations for safe design of pipes whereas Clavarino's equations would be more applicable in closed vessels applications.

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In a further development to simplify the stress distribution equations, Barlow 2 tried to rearrange Lamé's equations for internally pressurised cylinders by neglecting terms of low relative values to reach a fairly simplified description of the tangential stresses. His formula is similar to the thin walled cylinder equation except that the outer cylinder diameter replaces the inner. There are some claims to indicate that the formula is much useful in practice especially in computing the wall thickness for high pressure and gas pipes.

The basic modes of yield criteria proposed for cylinders design are : the maximum normal stress criterion, the maximum shear stress criterion and the maximum energy of distortion yield criterion 2,3 . The latter failure criterion is found to be difficult to apply especially in case of compound cylinders 4. However, in another design approach an elastic plastic boundary has been assumed to take account of the partial plastic range (restricted to plane strain) of multy compound cylinders made by shrinkage fit 5 .

Of interest to note that an increase in the cylinder wall thickness, cannot always ensure the required strength. In case of an infintely large cylinder thickness, the maximum induced stresses either in radial or tangential directions become equal to the internal pressure. Hence, in many engineering and military applications e.g. in nuclear reactors, in artillery practice for strengthening barrels of heavy guns and in other applications where the pressure may reach thousands of atmosphere, a very high strength material is supposed to be in use. Such material has not been developed yet. Consequently, the use of compound cylinders gives a direct and practical solution.

Although the fundamental concepts and the equations governing the use of compound cylinders are well known 2,3 , a rational solution to predict the cylinders materials strength requirements, dimensions and class of shrinkage fit has not been fully attained on optimal conditions yet. In a simple solution 3 adopting Lamé's equations for thick cylinders 1 and assuming the modulus of elasticity and Poisson's ratio to be the same for both the cylinders, an optimum interface diameter in case of two compound cylinders has been shown to have a value equals to the square root of the product of outer and inner diameters. Such solution is referred to as Gadolin's condition. However, in a more general solution 4 , a theory for elastic design of a compound cylinder made by shrinking together many concentric cylinders is described. Each cylinder has been dealt with taking into account that each may have entirely different strength and operates at any temperature and under any pressure. The only assumptions, in addition to the usual homoginity and isotropy in each member, are that they have the same modulus of elasticity and are not so thin that stability becomes a problem. Eventhough, the case of different elastic modulii can be in this theory 4 easily accommodated.

In the present work it has been decided to rationalise compound cylinders design by introducing an optimization technique for maximum material utilization.

OPTIMIZATION

For the problem under consideration, the main independent variables can be grouped as follows:

a. Parameters to describe the cylinders limiting dimensions; these are mainly the inner, contact (interface) and outer diameters,

$$U_d = \begin{bmatrix} d_i \\ d_c \\ d_o \end{bmatrix} \tag{1}$$

where U_d is the geometric parameter vector.

b. Parameters to describe assembly and operating conditions; these are mainly the interference pressure, internal pressure and operating temperature,

$$U_o = \begin{bmatrix} p_c \\ p_i \\ T \end{bmatrix} \tag{2}$$

where U_o is the operating parameters vector.

c. Parameters to describe cylinders materials ; these are the modulii of elasticity and Poisson's ratios,

$$U_m = \begin{bmatrix} E_1 \\ E_2 \\ \nu_1 \\ \nu_2 \end{bmatrix} \tag{3}$$

where U_m is the materials parameters vector.

Hence, the total independent parameters will be

$$U = \begin{bmatrix} U_d \\ U_o \\ U_m \end{bmatrix} \tag{4}$$

where U is the general parameter vector.

In formulating the optimization problem, it will be assumed that the compound cylinder inner diameter d_i, operating pressure p_i and the working temperature T with the mechanical properties of the cylinders materials are given input data. Hence, the design parameters will be defined by the design vector U_{des}

$$U_{des} = \begin{bmatrix} p_c \\ d_c \\ d_o \end{bmatrix} \tag{5}$$

which will read in nondimensional form

$$U^*_{des} = \begin{bmatrix} p_c/p_i \\ d_{ic} \\ d_{io} \end{bmatrix} \tag{6}$$

where U^*_{des} is the design vector, d_{ic} is the ratio between inner and contact (interface) diameter, and d_{io} is the ratio between inner and outer diameter.

Let the objective of the optimization procedure be to maximize the material utilization i.e. the maximum induced tangential stresses should be minimum and comparable with the yield strength of the cylinders' materials. Hence, the objective function will be,

$$\emptyset = \sigma = fn(p_c/p_i,\ d_{ic},\ d_{io}, \nu_1, \nu_2) \quad (7)$$

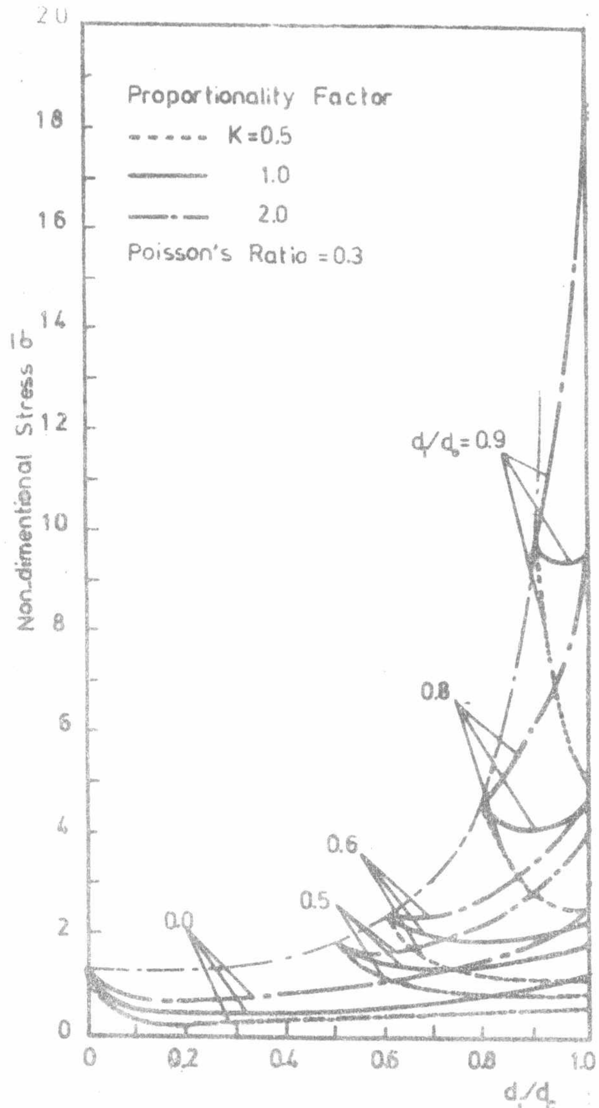
which should be minimum.

In the general case where the materials forming the compound cylinder are of different strength properties, the condition:

$$\emptyset = \sigma_{t1} = K\ \sigma_{t2} \quad (8)$$

becomes a requirement to satisfy maximum material utilization, where K being a proportionality factor which equals to

$$K = \sigma_{y1}/\ \sigma_{y2}$$



FIG(3) Effect of compound cylinder dimensions on inner cylinder maximum tangential stress

and can be equal to unity in case of using materials of identical strength properties. Recalling the basic equations that predict the maximum tangential stress in thick cylinders 2 , and rearranging terms and adapting equations to suit the general case of multi-cylinders concentrically fitted together at boundary diameters d_i, d_a, d_b, ..., d_n, the maximum tangential stresses will read as such:

$$\sigma_{ti} = p_i \left\{ \frac{1+ d_{ia}^2}{1- d_{ia}^2} + \nu \right\} - \frac{2p_a}{1- d_{ia}^2} \quad (9.1)$$

$$\sigma_{ta} = p_a \left\{ \frac{1+ d_{ab}^2}{1- d_{ab}^2} + \nu \right\} - \frac{2p_b}{1- d_{ab}^2} \quad (9.2)$$

$$\sigma_{tn} = p_{n-1}\left\{\frac{1+ d_{n-1,n}^2}{1- d_{n-1,n}^2} + \nu_n\right\} - \frac{2p_n}{1- d_{n-1,n}^2} \qquad (9.n)$$

where the sufixes a,b,...,n represent subsequent outer cylinders respectively with i being the sufix of inner cylinder.

The maximum tangential stresses σ_t can thus be obtained by substituting the interference pressures each time from the preceeding equations in the terms of internal and external pressure at the inner and outer boundary diameters as previously studied by some investigators 4,5 .

Substituting σ_{t1} and σ_{t2} of equation 8 from equations 9, we get the locus for the inner to interface diameters ratios as a function of the inner to outer diameters ratios with the inner to contact pressure ratio as a parameter. The resultant locus will thus represent the situations at which the induced stresses are minimum and an optimum utilization of the material could be secured. Graphical representation of this locus is given, for equistrength materials as well as for different cylinders materials, in Figure 1. However, optimum cylinders dimensions and interference pressure can also be determined analytically by differentiating equation 8 after being rearranged and then equating it to zero. For quick reach of design solution, the optimum design values have been provided in a chart form, Figure 2, with trajectories for direct prediction of the contact pressure and consequently the interference fit for optimum design criterion.

DESIGN PROCEDURE

The design procedure of a compound cylinder composed of two (or more) cylinders can be concluded in the following steps:

1. Assign the basic data of the problem, namely, the cylinder internal diameter di, the internal operating pressure p_i and working temperature T. Number of cylinders (if other than two) should be specified.
2. Select most convenient materials to be used and then specify their elastic and strength properties, namely, modulii of elasticity E_1 and E_2, Poisson's ratios ν_1 and ν_2 and yield strength σ_1 and σ_2 .
3. Assume inner to outer cylinder diameters ratio d_i/d_o appropriate to the

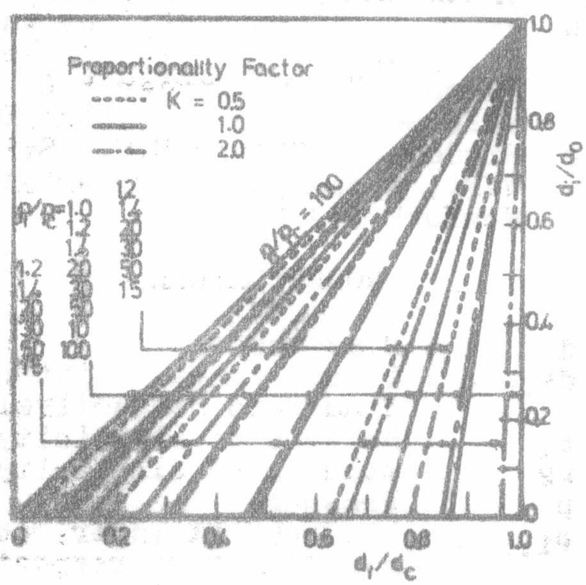


FIG (5) Interference (contact) pressure values as a function of compound cylinder dimensions application requirements; this could be altered throughout the design procedure to reach a final optimum ratio.

4. Consult the science of material technology to adopt a proper theory of material failure according to materials' behaviour whether brittle or ductile. Let us assume by way of example that the cylinders' failure criterion will depend on maximum normal stress theory 6-8 , hence:

$$\sigma_{y1} = K\,\sigma_{y2} \quad , \quad \sigma_{y1} \geqslant \sigma_{t1} \quad \text{and} \quad \sigma_{y2} \geqslant \sigma_{t2}$$

5. Get interference diameter d_c and pressure p_c by making use of charts in Figures 1 and 2. Once the ratio d_i/d_c has been evaluated from optimum design chart, Figure 1, interpolate the pressure ratio p_i/p_c fron chart in Figure 2.

6. Calculate the corresponding interference fit 2 to give the predicted design contact pressure.

7. At raised operating temperatures the materials properties should be taken to correspond to this particular temperature, also the interference fit should be modified to take account of materials expansion.

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NOMENCLATURE

d	:	Diameter
$d_{i,c,o}$	:	Inner, contact (interference) and outer cylinder diameters respectively.
E_1	:	Modulus of elasticity of inner cylinder material
E_2	:	Modulus of elasticity of outer cylinder material
p	:	Pressure
$p_{i,c,o}$	:	Pressure at inner, contact and outer cylinder boundary diameters respectively.
σ_t	:	Tangential stress
$\bar{\sigma}_t$	:	Nondimensional tangential stress ($\bar{\sigma}_t = \sigma_t/p_i$)
$\sigma_{y1,2}$	:	Yield strength of inner and outer cylinders respectively
ν_1	:	Poisson's ratio of inner cylinder material
ν_2	:	Poisson's ratio of outer cylinder material