Military Technical College
Kobry El-Kobbah,
Cairo, Egypt.

15th International Conference
on Applied Mechanics and
Mechanical Engineering.

PROBABILISTIC ANALYSIS OF GLASS EPOXY COMPOSITE BEAMS FOR DAMAGE INITIATION DUE TO HIGH VELOCITY IMPACT

S. D. Patel*, S. Ahmad** and P. Mahajan**

ABSTRACT

A numerical 3D dynamic finite element approach was adopted to study damage in composite beam subjected to high velocity impact. The contact force between the impactor and the target depends on the impactor mass, velocity and the elastic properties and other characteristic of composite beams. Fiber reinforced composite beams are susceptible to damage due to impact by foreign objects and in plane loading. In order to assess the safe load carrying capacity and the probability of failure under impact, dynamic analysis of composite beam subjected to high velocity impact is carried out. Finite element method is used to study the impact. During high velocity impact the out-of-plane damage modes such as matrix cracking and fiber failure are modeled using a failure criterion. The limit state functions for the composite beam under impact are derived from Chang-Chang [9] failure model.

The uncertainties associated with the properties and their inherent scatter in the geometric and material properties and input load are modeled in a probabilistic fashion. Random parameters represent various characteristics appearing in the limit state function. The probabilistic analysis and reliability prediction of the system is carried out using the first order reliability method (FORM) and validity of method is established using Monte Carlo simulation (MCS) procedure .The results show that for the given system and respective scatter, first order reliability method yields satisfactory level of accuracy. Sensitivity analysis of probability of failure with respect to random parameters considered is an important study for design optimization. The safety level quantification is achieved in terms of reliability level targeted. The mean and standard deviations of random variables show an appreciable influence on the probabilistic failure. Systematic changes in the input parameters are governed by probabilistic sensitivity tools to achieve target reliability.

KEY WORDS

Probabilistic Analysis, High Velocity Impact, Damage Initiation, Composite Beams, Reliability.

---

* Research Scholar, Department of Applied Mechanics, Indian Institute of Technology Delhi, New Delhi, India.
** Professor, Department of Applied Mechanics, Indian Institute of Technology Delhi, New Delhi, India.

INTRODUCTION

The Fiber reinforced composite materials are used increasingly in many military, aerospace, marine and civil applications due to their excellent mechanical properties like high specific strength, specific stiffness, resistance to corrosion, increased fatigue life among others. A prediction of the failure under impact loads is an important topic of research for reliability assurance of composites. Considerable work on the reliability assessment of composite components has been reported. Cederbaum et al. [1] and Miki et al. [2] used the first-order reliability method and Hashin's failure criteria to derive the first-ply failure reliability of laminated composite plates subjected to in-plane random loads.

Engelstad and Reddy [3] developed a probabilistic finite element analysis procedure for the prediction of the first-ply failure reliability of composite laminates with the use of the first-order reliability method using Tsai–Wu criterion. Kam and Chang [4] used a phenomenological failure criterion to establish the limit state equation of laminated composite plates for the first-ply failure reliability assessment. Guedes Soares [5] gave a state of the art survey of reliability of composite components and also reviewed different practical reliability approaches to assess the reliability of composite structures and discussed various failure criteria. Jeong and Shenoi [6] presented a direct simulation approach to assess the first-ply failure reliability of composite structures. They used Monte Carlo simulation technique to estimate probability of failure. Ahmad and Gupta [7] used the first order reliability method and 2D Hashin's failure criteria to determine the probability of failure of the circular composite plate. Patel et al. [8] used to the Monte Carlo simulation and quadratic failure criteria estimate the probability of failure of the square composite plate due to low velocity impact. However, some aspects of probabilistic analysis of composite under high velocity impact have an ample scope of further research.

The objective of the present study is to obtain system reliability involving the initiation of matrix cracking and fiber breakage process. It is followed by the deterministic and probabilistic analysis of composite beam under for classes of material and impact loads. It employs the finite element analysis of composite beam under high velocity impact using ABAQUS. Impact behavior is further explored for varied velocity and impactor mass. The magnitude of the contact force rises sharply from zero to its maximum value. In order to simulate the high velocity impact under study, contact force, residual velocity, strain and energy etc. time histories model are employed. The contact force between the impactor and the target depends on the impactor mass, velocity and the elastic properties and other characteristics of composite laminate beam. Its lateral dimensions, thickness, arrangement of plies, the elastic properties, edge conditions, curvature, greatly influence the response. The beam after impact undergoes deformation, both in the vicinity of the point of contact and in the form of the overall structural deformation. The high velocity impact produces a small indentation. A region of high stresses develops in a small zone around the point of impact in very short time duration. It is, in fact, responsible for the complex failure events which take place in the laminate. Interface of ABAQUS with reliability analysis is established and the application of Monte Carlo simulation technique is adopted for reliability assessment using appropriate probabilistic distribution of material properties. The Chang-Chang [9] failure model for matrix cracking and fiber breakage failure mode criterion for composites is adopted to establish the limit state function.

DEFINITION OF THE PROBLEM

Composite beams under study contains 24 layers of glass epoxy placed at different orientations $[0_3{}^0/90_3{}^0]_{2S}$.The overall beam thickness is 6.35 mm. The dimensions of the composite beam are 254mm x 25.4mm x 6.35mm.The dimensions of impactor is shown in figure (1). The boundary conditions applied to the model. The ends of the beam are fixed (u = v = w = 0 and also $\theta_x = \theta_y = \theta_z = 0$). Here, u is the displacement along x-axis, v is displacement along y-axis and w is displacement along z-axis. Similarly, θ_x is rotation about x-axis, θ_y is rotation about y-axis and θ_z is rotation about z-axis. Experimental values and respective statistical properties of glass epoxy have been adopted as reported in the literature [6]. Element type chosen is first order ABAQUS explicit solid element C3D8 (fully integrated).The full integration avoids the inaccuracies due to hour glassing as observed in preliminary analysis. The general contact model is employed to simulate the impact force in ABAQUS. Chang-Chang [9] failure model, is adopted here to model the matrix cracking and fiber breakage more accurately under high velocity impact energy. Impact induced damage of composite beam is a probabilistic phenomenon due to wide range of uncertainties arising in material properties and loading condition. Composite beam probabilistic analysis is involving multiple limit state functions (Matrix cracking and fiber breakage). The stresses obtained by the finite element software are used in the failure criteria to determine the probability of failure or reliability of the beam under impact induced damage. Controlling parameters such as impactor mass and velocity are used to determine the impact response for probabilistic analysis. The intensity of impact is measured by the magnitude of contact force developed. Damaged predicted by the failure criteria conform to the assigned target reliability.

PROBABILISTIC ANALYSIS

Response under impact essentially accounts for the worst possible damage initiation occurring at the worst possible location in the composite beam while it is subjected to the highest conceivable loads. In reality, the probability of occurrence of such a scenario involves large uncertainties. Hence, a probabilistic approach is a realistic solution that considers the stochastic variability and distribution of material's characteristic data. It is needed to account for the uncertainties in composite design and manufacturing. A beam under impact can't be guaranteed as absolutely safe because of the unpredictability of the loads, uncertainties in the material properties, the use of simplified assumptions in the analysis (which include limitations of the numerical methods used), and human factors (errors and omissions). Nevertheless, the probability of failure is usually required to be within a specified acceptable range for the specific application. Therefore, the estimation of probability of failure and risk assessment is an important task for the analysis, design and optimization of a component.

The basic concept of the classical theory of structural reliability and risk-based design starts with the identification of relevant load and resistance parameters, called random variables Xi (such as loads, material properties etc.) and the functional relationship between the response variable Z (e.g., stress at a point, deflection, frequency, etc.) and the random variables. Mathematically, it can be described as:

$$Z(x) = Z(X1, X2, X3\ldots\ldots\ldots XN) \tag{1}$$

A limit state function (sometimes referred to as performance function) is defined as:

$$g(x) = Z(x) - Z0 \tag{2}$$

where Z0 is a particular value of Z. It is a function of random variables and is divided in such a way that g(x) = 0 is a boundary between the failure region [g(x) < 0] and safe region [g(x) > 0]. A limit state function could be implicit or explicit in nature. It should be noted that since the cumulative distribution function (CDF) of Z at Z0 equals the probability that [g(x) <0], the CDF can be computed by varying the Z (0) and computing the joint probability as follows,

$$P_f = \iiint f(X_1, X_2, X_3 \ldots\ldots\ldots\ldots X_N) dX_1 dX_2 dX_3 .. dX_N \tag{3}$$

The probability of failure, Pf, is given by the integral in which $f(X_1,X_2 \ldots X_N)$ is the joint probability density function for the random variables $X_1,X_2 \ldots X_N$, and the integration is performed over the failure region X where g(x)< 0. If the random variables are statistically independent, then the joint probability density function can be replaced by individual density functions. This integral can be computed by the standard Monte Carlo procedure presently employed. Depending upon the number of random variables involved and the level of Pf (usually very small), this integration must be repeated thousands of times to accurately build the response variable's stochastic characteristics. Although the method is inherently simple, the large number of output sets that must be generated to build an accurate cumulative distribution function of the output variable becomes computationally expensive. A large size problem in terms of random variables and degrees of freedom, this method is an obvious disadvantage. Hence, the literature offers a wide range of simplified procedures, recommended according to the nature of the problem. Furthermore, if the deterministic computation of the response is complicated (e.g., need for a large nonlinear finite element analysis), the computation becomes prohibitive.

For the present problem following steps are adopted to carry out the probabilistic analysis:

1) A set of input random variables (of composite beam and load) are identified, and the corresponding probabilistic distributions are obtained. For a given set of random variables, a deterministic finite element analysis is carried out using the code ABAQUS. The response results at failure locations are collected from the finite element analysis output.
2) The above process is repeated a number of times to generate a table of response variable values that correspond to the perturbed set of values of the selected input random variables.

3) The FPI (Fast Probability Integration) analysis then uses the previously generated table to compute the CDF and corresponding sensitivities of the response.

In addition to the CDF of the response, the FPI (Fast Probability Integration) technique provides additional information regarding the sensitivity of the response to the random variables. The magnitude of the sensitivity factor provides a way to rank the random variables that have the major influence on the uncertainty of the response variable. This helps the user to prioritize the data collection resources. Also, by controlling the scatter in the more significant variables, the reliability can be improved. In a Monte Carlo simulation, a random value is selected for each of the tasks, based on the range of estimates. The model is calculated based on this random value. The result of the model is recorded and the process is repeated. A typical Monte Carlo simulation calculates the model hundreds or thousands of times, each time using different randomly-selected values. When the simulation is complete, we have a large number of results from the model, each based on random input values. These results are used to describe the likelihood or probability of reaching various results in the model. The stresses in an individual lamina are fundamental to control the failure initiation and propagation in the laminate. The strength of each individual lamina is assessed separately by considering the stresses acting on it along material axes. The initial failure of a lamina (first ply failure) is governed by exceeding the maximum limit prescribed by a failure criterion. The determination of first ply failure load is very essential in understanding the failure process as well as the reliability of the beam under impact.

DETERMINISTIC RELIABILITY ANALYSIS

It is carried out in order to identify the most critical node or point using the mean values of random variables. Therefore, deterministic values of ‘g’ function or the limit state function are calculated at nodes around the central nodes where the values of stresses are higher than the other nodes in the beam. The values of maximum shear stresses are obtained layer wise. Because it is observed that under application of transverse point loads the cracks develop on the most critical location depending on the material properties and orientation of the laminas. It is observed that the lowest value of limit state function appears at the node, which is referred to as the most critical point. This value of limit state functions Fib(x) and Mat(x) helps in assessing the influence of random variables in probabilistic analysis leading to an improved design.

The Limit State Function

The limit state for the composite beam under impact is derived from Chang-Chang [9] failure model. This is interacting failure criteria where more than one stress components have been used to evaluate the different failure modes. Usually Chang-Chang failure criteria is implemented within two dimensional classical lamination approaches for point stress calculations with ply discounting as the material degradation model. Failure indices for Chang-Chang [9] failure are related to fiber breakage, matrix cracking and compressive failure and involve three failure modes.

The Chang-Chang failure is extended to problems such as the in-plane and out-of-plane matrix cracking and fiber breakage failure.

The out-of-plane fiber damage initiation failure tension and compression are determined from the following Chang-Chang failures [9],

$$\mathrm{Fib}(x_T) = 1 - \left(\frac{S_{11t}}{T_{11t}}\right)^2 - \left(\frac{S_{13}}{T_{13}}\right)^2 \le 0 \tag{4}$$

$$\mathrm{Fib}(x_C) = 1 - \left(\frac{S_{11C}}{T_{11C}}\right)^2 \le 0 \tag{5}$$

where S_{11t} and T_{11t} are the tensile stress and associated strength in the longitudinal (fiber) direction and S_{11c} and T_{11c} are the compressive stress and associated strength in the longitudinal (fiber) direction whereas S_{13} and T_{13} are the out-of-plane shear stress and strength between fibers and matrix, respectively. The out-of-plane matrix damage initiation tension and compressive failure are determined from the following Chang-Chang failure [9],

$$\mathrm{Mat}(x_T) = 1 - \left(\frac{S_{33t}}{T_{33t}}\right)^2 - \left(\frac{S_{13}}{T_{13}}\right)^2 \le 0 \tag{6}$$

$$\mathrm{Mat}(x_C) = 1 - \left(\frac{S_{33t}}{2T_{13}}\right)^2 - \left(\frac{S_{13}}{T_{13}}\right)^2 - \left[\left(\frac{T_{33c}}{2T_{13}}\right)^2 - 1\right]\frac{S_{33}}{T_{33}} \le 0 \tag{7}$$

where S_{33t} and T_{33t} are the tensile stress and associated strength in the transverse (matrix) direction and S_{33c} and T_{33c} are the compressive stress and associated strength in the transverse (matrix) direction whereas S_{13} and T_{13} are the out-of-plane shear stress and strength between fibers and matrix, respectively. Experimentally obtained statistical scatter is also assumed as reported in the literature [6].

A limit state is a function of random variables or their functions. It is defined as:

$$g(x) = Z(x) - Z_0 = 0 \tag{8}$$

where Z-function is a response or a performance function such as stress, displacement, natural frequency, fatigue life… etc.

$$Z(X) = Z(X_1, X_2, X_3, \ldots\ldots\ldots\ldots X_N) \tag{9}$$

And Z_0 is a particular value of Z. The g-function is defined such that g(x))=0 is a boundary that divides the random variable space in to regions , for Failure [g(x)≤0] and safe [g(x)>0] .Because the CDF of Z at Z_0 equals the probability that [g (x)≤0], the CDF can be computed by varying Z_0 and computing the point probability.

FAULT TREE ANALYSIS

System reliability analysis involves multiple g-functions. Reliability software uses a fault tree structure to define the system failure. A fault tree has three major characteristics; bottom events, combination gates and the connectivity between the bottom events and gates. Reliability software is presently limited to AND and OR gates. Conditional gates can be simulated using the AND gates with appropriately defined conditional performance functions.

Reliability analysis uses a graphical approach to define the fault tree as shown in the figure (2) below. First the Pf block is selected to begin definition of the fault tree. Next gates and bottom events are added to define the fault tree.

Fault Tree Analysis of Parallel System Damage Initiation Model

The two modes of failure namely matrix cracking and fiber breakage failure are linked in either in series or parallel. If the failure of the system occurs due to matrix crack initiation together with fiber breakage the system failure is termed as parallel failure.

$$Pf = P\ \{Fib(x)\ AND\ Mat(x)\}$$

CUMULATIVE DISTRIBUTION FUNCTION

Due to an impact caused by an impactor at a velocity 100 m/s, failures are initiated in plies of the composite beam. Theses plies nos. are 1, 2, 3, 7, 8, 9, and 16 oriented at an angle zero degree. However, no significant failures are initiated in other plies oriented at angles of 0 and 90 degree. Cumulative density function plots (Fig.4-5) show the estimation of Cumulative probability distribution against variation of response (Z) using first order reliability method (FORM) and Monte Carlo simulation method respectively. The comparison shows that the accuracy of FORM is significantly close to Monte Carlo Method. Monte Carlo simulation required 10,000 cycles to achieve this accuracy. It is computationally very expensive in comparison to FORM. As per parallel system, probability of failure under impact is obtained as Pf =9.3E-3 (17^{th} ply) for the given statistics of random variables. The mass of the impactor is taken as 3 g with velocity of impact as 100 m/s. Hence, properties of composite material may be optimized to achieve target reliability. Cumulative probability distribution is an important property of the system to optimize with respect to statistical properties of random variables to achieve the required reliability level.

SENSITIVITY ANALYSIS

Sensitivity analysis of Limit State Function with respect to the changes in the random variables involved in the mathematical model can be apportioned, qualitatively or quantitatively. It is important because of the different sources of variation in the statistical input of a performance model. It is a technique for systematically changing the strength and resistance parameters in a model to optimize the design to achieve target reliability. Sensitivity levels (ψi) indicate the influence of mean and deviation on probability of failure "p". The following equations state the same in general form,

$$\psi_1 = \left(\frac{\partial p}{\partial \mu}\right)(\sigma / p) \tag{10}$$

$$\psi_2 = \left(\frac{\partial p}{\partial \sigma}\right)(\sigma / p) \tag{11}$$

Uncertainty and Reliability Sensitivity against Ultimate Limit State: The mean (μ) and standard deviation (σ) are the two basic statistical parameters that characterize the levels of reliability and uncertainty respectively. These random parameters form the limit state function or performance function. Probabilistic sensitivity factors in terms of bar chart show partial derivatives of probability of failure (Pf =1-Re) with respect to mean (μ) and standard deviation (σ) are shown at a glance in figures (5-6). It shows that the reliability increases with an increase in mean value (μ) of ultimate and interlaminar strengths, T_{11t}, T_{33t} and T_{13}. However, the reliability is inversely proportional to the scatter (σ) in ultimate and shear strengths. Higher deviation causes reduction in reliability. The longitudinal, transverse and shear stresses S11t, S_{33t}, and S_{13} are obtained by using finite element package, ABAQUS. Their influences in terms of means and standard deviations are shown in figures (5-6). The reliability reduces with the increase in mean stresses as well as with the increase in its scatters (σ). Parallel system 90 degree fiber sensitivity levels shown in fig.5, random parameter shear strength (T_{13}) is most significant but the 0 degree fiber sensitivity levels shown in fig.4, random parameter transverse tensile stress and strength are least significant. The rate of change in probability of failure "Pf" with respect to the standard deviation (σ) against the response or limit state functions (matrix cracking and fiber breakage) is shown in figures (5-6). The probabilistic sensitivity factors are plotted for all the random variables of the response functions. The variations diminish to negligibly small value at the target response. One can select the response as per the required reliability. The change in probability of failure with respect to mean values (μ) of the random variables against the response is shown in figures (5-6). The plot helps in selecting the values of target reliability. The rate of change of probability of failure for all the random variables diminishes at the required value of the limit state functions. It is also an optimization tool.

CONCLUSIONS

The reliability analysis of two sided clamped mid-plane symmetric composite beams subjected to high velocity impact load performed. Probability of failure is estimated with respect to each plies in beam. The composite beam failures are initiated in plies 1, 2, 3, 7, 8, 9, and 16 oriented at an angle of zero degree in the laminate. However, no significant failures are initiated in other plies oriented at angles of $0^0$and 90^0. It is an important input for probabilistic design. The limit state functions formulated using Chang-Chang [9] failure initiation model is appropriate because it considers the uncertainties in transverse shear stresses. The mean and standard deviations of random variables show an appreciable influence on the probabilistic failure. Strengths in longitudinal and shear direction in tension is the most sensitive parameter to influence the product reliability. Other parameters like laminate thickness, ratio of modulus of elasticity in longitudinal and transverse direction also have significant influence on probability of failure. The cumulative distribution functions for loads applied in static and dynamic fashion provided a cumulative measure of probability due to impact. The reliability approach has the ability to assess the contribution of statistical properties of various random variables on limit state functions. Hence, the analysis and design of the product accounts for the uncertainties associated with the basic properties of the material, loads and the boundary conditions. The reliable and economic design of composite beams, it is important to narrow the dimension of the basic variables corresponding to material properties, ultimate strength and geometric properties. Safety and reliability of the overall system is ensured. Probabilistic sensitivity factors provide insight in to the extent of influence of random variables on limit state function. Systematic changes in the input parameters are governed by probabilistic sensitivity tools to achieve target reliability.

REFERENCES

[1] Cederbaum, G., Elishakoff, I. and Librescu, L., "Reliability of Laminated Plates via the First-Order-Moment Mewthod", J. Composite Structures, No. 15, pp. 161-167 (1990).

[2] Miki, M., Murotsu, Y., Tanaka, T. and Shao S., “Reliability of Unidirectional Fibrous Composites,” J. AIAA, No. 28, (1980–1986).

[3] Engelstad, S.P. and Reddy, J.N., “Probabilistic Nonlinear Finite Element Analysis of Composite Structures,” J. AIAA, No.31, pp 362–369,(1993).

[4] Kam, T.Y. and Chang, E.S., “Reliability Formulation for Composite Laminates Subjected to First-Ply Failure,” J. Composite Structures, No. 38, pp 447–452, (1997).

[5] Guedes Soares C., “Reliability of Components in Composite Materials,” J. Reliability Engineering and System Safety, No. 55, pp 171–177, (1997).

[6] Jeong, H.K. and Shenoi, R.A., “Reliability Analysis of Mid-Plane Symmetric Laminated Plates Using Direct Simulation Method,” J. Composite Structures, No. 43, pp 1–13, (1998).

[7] Ahmad Suhail, and Gupta N.K., “Probabilistic Analysis of Composite Panels under Low Velocity Impact,” Proc. 10th International IMPLAST Conference, Society of Experimental Mechanics, USA, (2010).

[8] Patel S.D., Mahajan P., and Ahmad Suhail, “Probabilistic Analysis of Anisotropic Composite Plate Damage due to Low Velocity Impact,” ISAMPE National Conference on Composites, R & D E , Pune, India, Nov. 18-19, (2011).

[9] Sevkat Ercan, Liaw B., Delale F., Raju B.B., “A Combined Experimental and Numerical Approach to Study Ballistic Impact Response of S2 Glass Fiber/Toghened Epoxy Composite Beams,” Composites Science and Technology, No. 69, pp. 965-982, (2009).

FIGURES AND TABLES

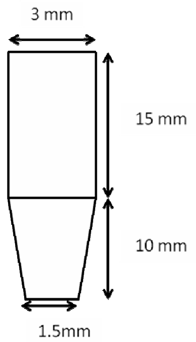

Fig.1. Impactor Geometry.

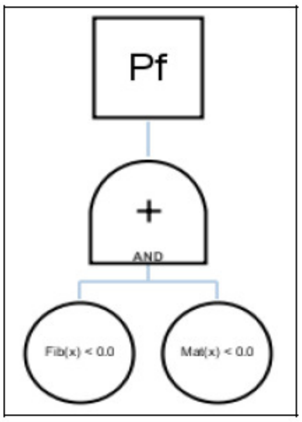

Fig.2. Fault tree used.

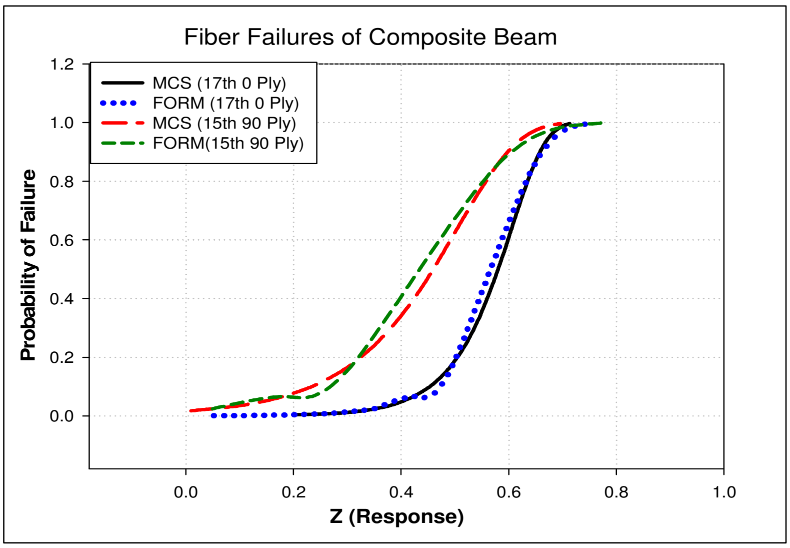

Fig. 3. Comparison of probability of fiber failures at Monte Carlo Simulation (MCS) and first order reliability method (FORM).

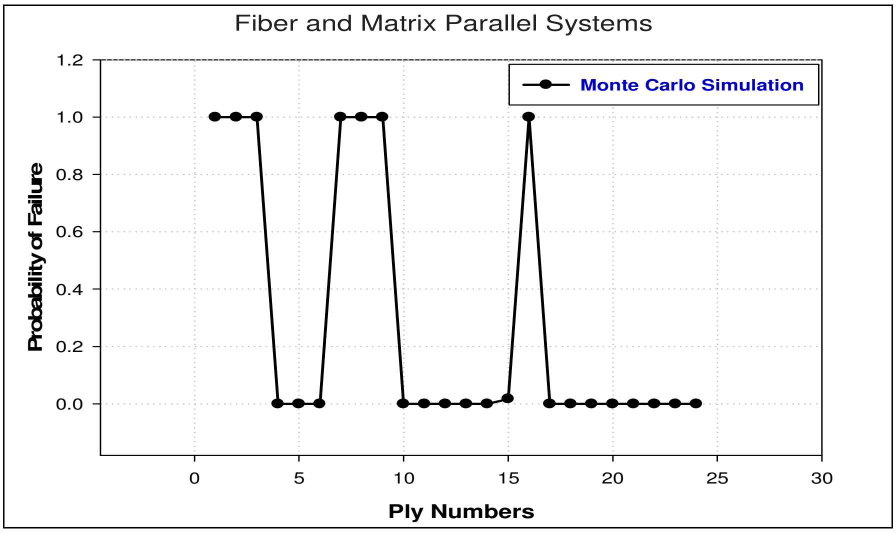

Fig .4. Probability of failure composite beams.

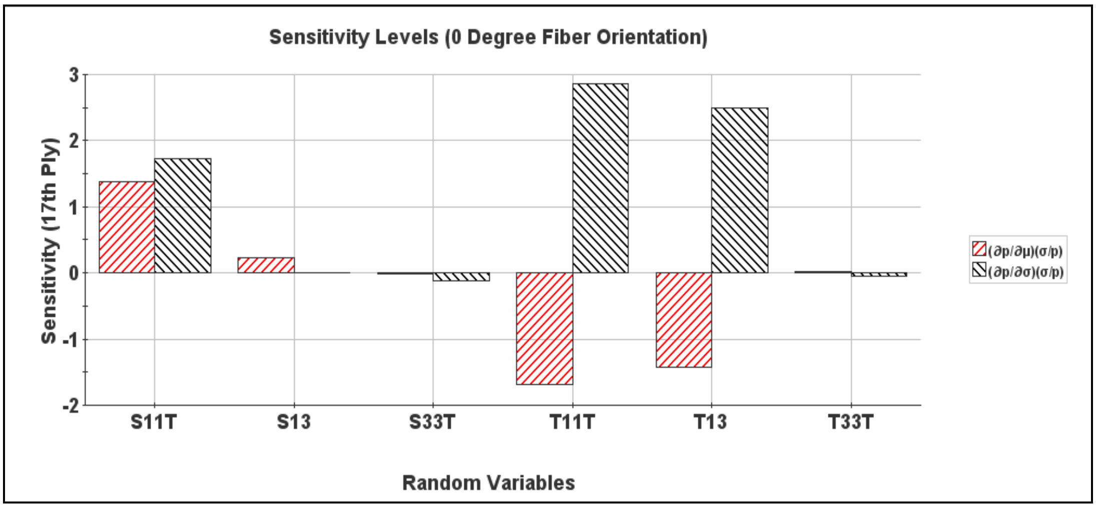

Fig. 5. Sensitivity analysis of random variables at 0^0 fiber orientation.

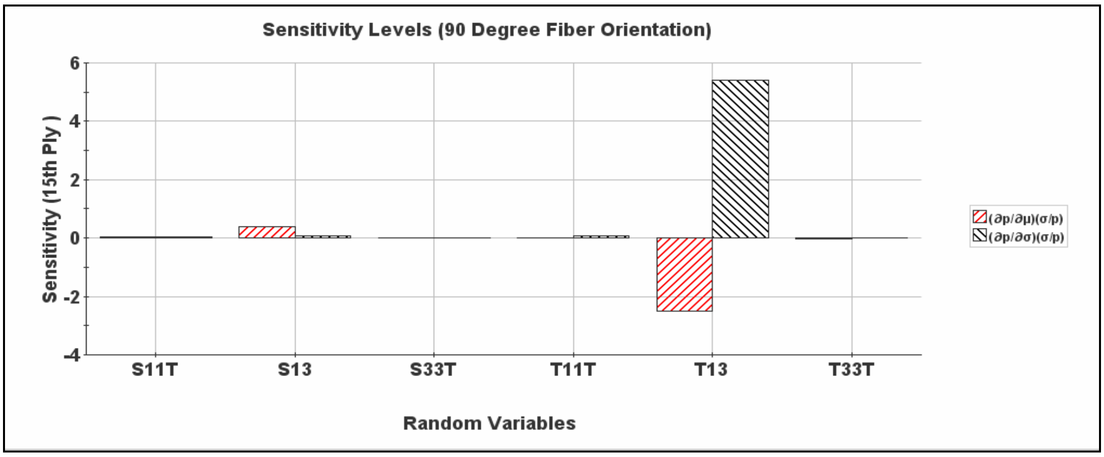

Fig.6. Sensitivity analysis of random variables at 90^0 fiber orientation.

Table 1. Probability of failure composite beams at Monte Carlo simulation. (Velocity = 100 m/s and impactor mass = 3g.)

Ply No.	S_{11}(GPa)	S_{33}(GPa)	S_{13}(GPa)	Orientations Angle (Degree)	Limit State Functions (G(X)<0)	MCS (P_F)
1	2.2794	-0.918	0.642	0	NO	-
2	0.992	0.769	0.427	0	NO	-
3	5.08	0.909	0.2934	0	NO	-
4	-0.969	1.415	0.189	90	YES	0
5	-0.508	0.964	0.27	90	YES	0
6	-0.3144	0.867	0.298	90	YES	0
7	0.94	0.759	0.256	0	NO	-
8	0.378	0.591	0.128	0	NO	-
9	0.279	0.5011	0.089	0	NO	-
10	-3.43	0.5392	0.082	90	YES	0
11	-0.15	0.4499	0.0916	90	YES	0
12	-0.07725	0.3699	0.0836	90	YES	0
13	-0.0422	0.2793	0.074	90	YES	0
14	-0.00908	0.2016	0.175	90	YES	0
15	0.03105	0.1908	0.0445	90	YES	0.144E-1
16	0.499	0.1576	0.0776	0	NO	-
17	0.344	0.1297	0.0330	0	YES	0.93E-2
18	0.2957	0.1067	0.0274	0	YES	0.10E-4
19	0.05186	0.0823	0.0269	90	YES	0.10E-4
20	0.0886	0.0589	0.0235	90	YES	0.10E-4
21	0.1443	0.0279	0.0170	90	YES	0
22	0.581	0.0276	0.0193	0	YES	0
23	0.647	0.00398	0.0128	0	YES	0
24	0.803	0.000158	0.00217	0	YES	0