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SMART MATERIALS USED FOR AERO ENGINE VIBRATION CONTROL

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ABSTRACT

The new generation of smart materials and structure technologies featuring at the most sophisticated level a network of sensors and actuators, real time control capabilities, computational capabilities and a host structural material. Smart materials are usually attached or embedded into structural systems to enable these structures to sense disturbances, process the information and evoke reaction at the actuators, possibly to negate the effect of the original disturbance. For active vibration reduction tasks in smart structures technology, piezoelectric ceramics are the first choice. They generate large forces, have fast response times, are commercially available as fibers, patches and stacks, and allow integration into structural components. Piezoelectric extension actuators are bonded to the surface or embedded within the structure.

Resonant vibrations of aero engine blades cause blade fatigue problems in engines, which can lead to thicker and aerodynamically lower performing blade designs, increasing engine weight, fuel burn, and maintenance costs. In order to suppress undesirable blade vibration levels, active piezoelectric vibration control are used, potentially enabling thinner blade designs for higher performing blades and minimizing blade fatigue problems. Thus, this paper gives an overview of smart materials, piezoelectric materials and their applications in vibration damping and a finite element ANSYS model analysis for piezoelectric bimorph beam including resistor capacitor (RC) circuit to study the effect of resistance (R) and capacitance (C) values change in vibration damping control.

KEY WORDS

Smart materials, piezoelectric materials, Bimorph beam, aero engine, vibration control.

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INTRODUCTION

Smart materials are materials that respond to their environments in a timely manner. The definition of smart materials has been expanded to materials that receive, transmit, or process a stimulus and respond by producing a useful effect that may include a signal that the materials are acting upon it. Some of the stimuli that may act upon these materials are strain, stress, temperature, chemicals, electric field, magnetic field, hydrostatic pressure, different types of radiation, and other forms of stimuli [1].

Numerous applications of smart structures technology are evolving to actively control vibrations, noise, and deformations. Applications range from space systems to fixed wing and rotary wing aircraft, automotive, optical systems, machine tools, medical systems, and infrastructure [2].

The vibrations induced by the airflow in turbo machines are the cause of High Cycle Fatigue (HCF) and ask for regular and unexpected maintenance needs. The minimization of the vibration responses towards the aerodynamics forces can reduce the HCF and by the way reduce significantly the deterioration phenomena and avoid high cost in repairing [3].

The piezoelectric material presents a new approach to blade vibration suppression. They can be adapted to the creation of damping or control elements. Indeed when the electric current is applied to a piezoelectric material, a corresponding strain is produced within the material, which produces an electric voltage. Vibration damping of an elastic structure can be obtained by an piezoelectric actuator embedded into it and shunted to an electric resonant circuit adjusted to the mode to be damped [3].

In the following, we shall discuss smart materials, focusing on piezoelectric materials, piezoelectric sensors and actuators and their applications in vibration suppression specifically in aero engines. Finite element simulation analysis of piezoelectric bimorph beam featuring resistor capacitor (RC) Circuit used for vibration damping using ANSYS program is demonstrated to study the effect of change resistance (R) and capacitance (C) values in RC circuit to control vibration damping.

SMART MATERIALS

Smart materials have the characteristic that they show both sensor and actuator effects. All these materials are capable of reversibly changing their mechanical properties (viscosity, stiffness and shape) due to the influence of temperature change or an electrical or magnetic field. By some of these materials the reverse effect can be used for sensor tasks meaning that a mechanical load generates an electrical or magnetic field [2].

The first recorded observation of smart material transformation was made in 1932 on gold cadmium alloy. In addition, in 1938 the phase transformation was observed in brass (copper zinc). During the 1980s and early 1990s, a number of companies

began to provide Ni-Ti materials and their components, and an increasing number of products, especially medical products, were developed to market [4].

Smart materials can be grouped into the following categories: Piezoelectric materials, electrostrictive materials, magnetostrictive materials and Shape Memory Alloys [5].

PIEZOELECTRIC THEORY

Piezoelectricity is a phenomenon observed in certain crystals, e.g., quartz and Rochelle salt, ceramic materials, and even polymers. Piezoelectric materials that are exposed to mechanical stress (pressure) generate an electric voltage between material surfaces. This is known as the direct piezoelectric effect. Conversely, if the materials are subject to an electric field the crystals show mechanical deformation (converse piezoelectric effect). This coupling between electrical and mechanical energy makes the materials very useful as transducers in many applications [6].

Piezoelectric materials exhibit an inherent coupling between electrical and mechanical properties. First discovered by Pierre and Jacques Curie in 1880, the piezoelectric, or direct, effect is the production of an electric field when subjected to an applied mechanical stress [7].

Piezoelectric Transducers

Piezoelectric transducers are used as actuators and sensors for vibration control of flexible structures. Piezoelectric materials in current use include polyvinylidene fluoride (PVDF), a semi crystalline polymer film and lead zirconate titanate (PZT), a piezoelectric ceramic material. For vibration control purposes, piezoelectric transducers are bonded to the body of a flexible structure using strong adhesive material. These piezoelectric elements can be used as sensors, actuators, or both. In a typical active control application, a piezoelectric transducer is used as an actuator, while a sensor is used to measure vibration of the base structure. A control voltage is then applied to the piezoelectric actuator to minimize the unwanted vibration of the base structure [8].

Piezoelectric sensors

These sensors are based on the piezoelectric effect observed in some materials. In this, an electrical charge change is generated when a mechanical stress is applied across the face of a piezoelectric film. For a piezoelectric disk of thickness t, the voltage (V) generated across the electrode disk, when subjected to a stress (T), is given by:

$$V = g t T$$
(1)

where g is the piezoelectric voltage coefficient, defined as the ratio of the field developed to the applied mechanical stress. The relationship between the dipole moment and the mechanical deformation is expressed by the following constitutive relationships:

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$$\sigma = C S - e E \tag{2}$$

and:
$$D = \varepsilon_0 E + e S$$
 (3)

where: σ = Mechanical stress, C = Elastic constant, S = Strain, e = Piezoelectric constant, E = Applied electric field, D = Flux density, ε_0 = Permittivity of free space. Piezoelectric devices have several advantages over other sensing mechanisms. It may be noticed that in the absence of piezoelectricity these relationships reduce to Hooke's law and the constitutive relationship for dielectric materials, respectively. The effectiveness of a piezoelectric material is best expressed in terms of its electromechanical coupling coefficient, K². By definition, this is related to other material parameters used in the above constitutive equations by the following:

$$K^2 = \frac{e^2}{C \epsilon}$$
(4)

Since this sensor generates its own voltage, it does not require power for operation. Therefore, for applications where power consumption is a significant constraint, piezoelectric devices can be used. Furthermore, the piezoelectric effect is scalable to small devices. One disadvantage of piezoelectric sensing is that it is sensitive only to time varying signals and hence static quantities such as weight cannot be measured by using this approach. Piezoelectric sensing is widely used in pressure and force sensors, accelerometers, hydrophones, microphones, etc [9].

Many different sizes and shapes of piezoelectric materials can be used in piezoelectric sensors. Acting as true precision springs, the different element configurations shown in Fig.1, offer various advantages and disadvantages. The compression design features high rigidity, making it useful for implementation in high frequency pressure and force sensors. Its disadvantage is that it is somewhat sensitive to thermal transients. The simplicity of the flexural design is offset by its narrow frequency range and low over shock survivability. The shear configuration is typically used in accelerometers as it offers a well-balanced blend of wide frequency range, low sensitivity to base strain and low sensitivity to thermal inputs [10].

After the sensing element produces a presumably desirable output, this signal must be conditioned prior to being analyzed by the oscilloscope, analyzer, recorder or other readout devices. These analog processing circuits serve the same general functions, which include: (1) Conversion to useful, low impedance, voltage signal. (2) Signal amplification / attenuation, and (3) Filtering. However, it is important to note that the location of the circuit may be critical to the proper operation of the sensing system, a detailed system schematic is shown in Fig.2.[10].

Piezoelectric actuators

The actuators in a smart material/structure provide the mechanism for the structure to adapt to its surroundings by suppressing vibrations or changing the structure's shape. Current needs are challenging since they require actuation authority over a broad bandwidth spectrum (from a few Hz for mission adaptive wings (MAW's) to hundreds of kHz for active vibration suppression) and over a wide range of displacement amplitudes (e.g. from tenths of micrometers for vibration control to millimeters/centimeters for a MAW application) [11].

Piezoelectric Transducers for Vibration Damping

When a piezoelectric transducer is attached to a base structure, it may be used as an actuator, a sensor, or both. To obtain a dynamical model of the composite system, the strain/stress properties of the piezoelectric wafer must be coupled with the dynamics of the base structure. Consider the system depicted in Fig.3, demonstrating a flexible structure which is subjected to some form of disturbance, with a collocated pair of piezoelectric transducers. In a typical active vibration control application, one transducer is used as an actuator, while the other is employed as a sensor to generate the measurement that is needed in any feedback regulator system. Therefore, in figure 3, the transducer on the left would serve as the sensor, while the one on the right hand side of the base structure would be the actuator. To delineate the underlying mechanisms, the electrical equivalent circuit of the system of figure 3, is sketched in Fig.4, The main assumption here is that both piezoelectric transducers are identical and collocated. The collocation implies that as one transducer expands, when the base structure bends, the other contracts. Therefore, considering orientations of polarization vectors of the two piezoelectric transducers, the voltages induced in them will be equal, but 180° out of phase [8].

Active damping

Two piezoelectric patches, sensor and actuator, are attached to the host structure for smart damping. Active damping in simple words is obtained by using an actuator and sensor acting in a closed loop. Active devices or controllers generally comprise a distinct set of sensor circuit, drive circuit and actuator circuit. Voltage obtained on the sensor is transferred to the actuator thus opposing the vibration of the structure and achieving active damping. When the host-structure vibrates, its motion is sensed by the piezoelectric sensor attached. This sensed vibration signal is then amplified and fed back through feedback controllers. Active damping requires complex amplifiers, complicated algorithms and associated sensing electronics which poses a limitation to its practical use in applications where low cost is a critical necessity [12].

Passive damping

Passive damping works on the concept of energy dissipation. The principle of passive damping is as simple as the following: when a mechanical disturbance is induced in the structure, it strains the piezoelectric patch attached to it, thus generating electrical energy. Electrical energy is then dissipated, or otherwise altered, through a shunting circuit before it reverts to the mechanical system. Shunting circuits may contain many electrical components in different combinations. Resistors, capacitors and inductors have already been used as passive control electrical components. The advantage of this approach is that the shunt circuit can be easily placed on the structure together with the piezoelectric patch. Generally, for broadband damping, a resistor is shunted across a direct effect mechanism, which is typically a piezo capacitor. For narrow band damping, both an inductor and a resistor must be shunted across the device to form a resonant LRC circuit (inductor, resistor and capacitor). This circuit is usually tuned to a resonant frequency of the structure to be damped. In a resistively shunted circuit, the resistance is varied until the circuit time constant is close to the modes to be damped. The primary difficulty with resistive shunted circuits is the dissipation of sufficient energy and need for large resistance values to dissipate large amounts of energy. A non-linear circuit with



different impedance designs has been proposed as one the passive damping techniques [12].

Aero Engine Vibration Damping Using Piezoelectric Materials

Vibration damping of turbine blades

In turbomachinery applications, turbine blades are subjected to high static and dynamic loads. Static loads are due to centrifugal forces and thermal strains while fluctuating gas forces are the cause of dynamic excitation which can lead to high cycle fatigue (HCF) failures. Therefore, in practical applications any further damping provided to the structure is desirable. Coupling devices like under platform dampers, lacing wires and tip shrouds are common in turbomachinery applications. The effectiveness of these damping concepts is limited to the relative vibrations of neighboring blades and therefore they are often only efficient for specific engine speeds and mode shapes. Furthermore, the aerodynamics of the balding is influenced by these coupling devices [13].

Vibration damping in aero engine axial compressor

An actuation method for the experimental investigation of compressor rotor blade vibrations is demonstrated. Three blades of the first compressor stage in a two stage low speed axial flow compressor have been equipped with piezoelectric actuators, i.e. macro fiber composites (MFC). For measuring the actuated blade vibrations, strain gages are used. The control system which has been developed within this project allows to pre set a vibration amplitude of the blade tip. The transmission of the actuation voltage of up to 1000 V into the rotor is accomplished by a slip ring. A slip ring is also used for transmitting the strain gage signals [14].

The actuation of the rotor blades is accomplished by piezoelectric actuator foils bonded to the rotor blades' suction side. As Fig.5 (a), shows, three rotor blades equipped with actuators. Fig.5 (b), shows the pressure sides of the same three blades. Each blade is equipped with one full bridge and one single strain gauge (metal foil resistive). The full bridges consist of four sensors, two of them directed in the direction of main stress and two perpendicular to it. The positions of the strain gauges on the blades have been chosen based on a numeric simulation of the eigen modes of the blades. The actuation voltage of the piezo patches as well as the strain gauge signals are transmitted into and out of the rotating system through a slip ring, respectively. The slip ring mounted on the engine shaft behind the two compressor stages is shown in Fig.6. [14].

The tests have been done with a non-rotating machine (n = 0 rpm) and at two rotational speeds of n = 2000 rpm and about 3000 rpm, respectively, in order to distinguish between the influence of electrical stray fields, mechanical unbalance, and the actual vibration frequencies caused by the actuation. Comparing the three test series results shows that the frequency of the first flap eigen mode moves from 312 Hz (for non-rotating machine) to 330 Hz (n = 2000 rpm) and 342 Hz (about 3000 rpm) while the maximum amplitude increases also. Tests using the control unit show that the vibration amplitude can be controlled [14].

FINITE ELEMENT SIMULATION OF SHUNTED PIEZOELECTRIC VIBRATION DAMPING

Materials and Approaches

Piezoelectric bimorph beam is used, it composed of two piezoelectric layers joined together with opposite polarities with resistor (R) and capacitor (C) electrical circuit. Fig.7, shows a 2-D analysis of a bimorph mounted as a cantilever.

To investigate the effect of a damping circuit on a piezoelectric structure, the finite element (FE) shunted piezoelectric circuit damping simulations were performed using ANSYS mechanical APDL 15.0 program with direct-coupled field analysis between the mechanical and piezoelectric domain together with coupled physics (RC) circuit simulation. The objectives of the modeling effort were: (a) to evaluate the electrical circuit that includes piezoelectric material, (b) to investigate the effect of a damping piezoelectric circuit on the structure, and (d) to develop a numerical modeling capability for vibration control using shunted piezoelectric circuits. The ANSYS model consists of a piezoelectric bimorph cantilever beam using PLANE13 and electrical circuit components of resistor (R) and capacitor (C) using CIRC94.

Conversion of material properties of piezoelectric materials for use in ANSYS was challenging because of differences between manufacturer-supplied data and the format required by ANSYS. The following sections provide a frame work of finite element modeling techniques for shunt circuit-fed piezoelectric vibration damping analysis. It outlines piezoelectric finite element equations, general constitutive equations of piezoelectric materials and piezoelectric circuit element equations. It also covers converting manufacturer's data to ANSYS format data for the stiffness matrix, dielectric constants, and piezoelectric constants.

Piezoelectric Finite Element Method

Piezoelectric finite element method (FEM) equations can be written in terms of nodal displacement {U} and nodal electric potential { \emptyset } for each node. The displacement and potential for each element can be expressed, respectively, as

$$\{U\} = [N_u]^T \{U_e\}$$
(5)

$$\{\phi\} = [N_{\phi}]^T \{\phi_e\}$$
(6)

where:

 $[N_u]^T$ = Displacement shape function (transposed) $[N_{\emptyset}]^T$ = Electrical potential shape function (transposed) $\{U_e\}$ = Nodal displacement $\{\emptyset_e\}$ = Nodal electric potential

The mechanical strains, {S}, are related to the nodal displacements through the derivative of the shape function $[B_{\rm u}]$

$$\{S\}=[B_u]\{U\}$$
 (7)

where $[B_u]$ is the strain-displacement matrix. Similarly, the electrical field {E} is related to the nodal potential as

$$\{E\} = - [B_{\emptyset}] \{\emptyset\}$$
(8)

Where $[B_{\phi}]$ is the electric field-displacement matrix.

The equations of motion for a piezoelectric body can be derived from the principle of minimum potential energy by means of a variation function. The resultant equations can be represented in matrix form from the assembly of all the individual finite element equations. The equations are written in terms of displacement {U} and electrical potential { \emptyset } at the nodal points. Forcing functions are expressed in terms of structural loads {F} and electrical loads {Q}, resulting in the equilibrium equations below. The equation of motion corresponding to the piezoelectric actuator and the plate structure can be assembled in a global system coordinate, which includes the degrees of freedom of the piezoelectric actuator (voltages and displacements) and the degrees of freedom of the plate structure (displacements) as follows:

$$[M_{uu}]\{\ddot{U}\} + [C_{uu}]\{\dot{U}\} + [K_{uu}]\{U\} + [K_{u\phi}]\{\emptyset\} = \{F\}$$
(9)

$$[K_{u\phi}]^{T}\{U\} + [K_{\phi\phi}] \{\phi\} = \{Q\}$$
(10)

This same expression can also be expressed in the matrix form:

$$\begin{bmatrix} M_{uu} & 0\\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{U}\\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_{uu} & 0\\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{U}\\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} [K_{uu}] & [K_{u\phi}]\\ [K_{u\phi}]^T & [K_{\phi\phi}] \end{bmatrix} \begin{bmatrix} U\\ \phi \end{bmatrix} = \begin{bmatrix} F\\ Q \end{bmatrix}$$
(11)

where:

$$[K_{uu}] = \iiint_{\Omega_{e}} [B_{u}]^{T}[C][B_{u}] dV$$
$$[K_{u\phi}] = \iiint_{\Omega_{e}} [B_{u}]^{T}[e][B_{\phi}] dV$$
$$[K_{\phi\phi}] = \iiint_{\Omega_{e}} [B_{\phi}]^{T}[\varepsilon][B_{\phi}] dV$$
$$[M_{uu}] = \rho \iiint_{\Omega_{e}} [N_{u}]^{T}[N_{u}] dV$$
$$[C_{uu}] = \beta[K_{uu}]$$

 $\begin{array}{ll} \Omega_{e} = \mbox{Finite element domain} \\ \rho = \mbox{Piezoelectric density} \\ [K_{uu}] = \mbox{Mechanical stiffness matrix} & [B_{u}], [B_{\emptyset}] = \mbox{Derivatives of FEM shape functions} \\ [K_{u\emptyset}] = \mbox{Piezoelectric coupling matrix} & [C] = \mbox{Elastic coefficients} \\ [K_{\emptyset\emptyset}] = \mbox{Dielectric stiffness matrix} & [e] = \mbox{Piezoelectric coefficients} \\ [M_{uu}] = \mbox{Mass matrix} & [\epsilon] = \mbox{Dielectric coefficients} \\ [C_{uu}] = \mbox{Mechanical damping matrix} & \beta = \mbox{Damping coefficient [15].} \end{array}$



Constitutive relationship of piezoelectric materials

The basic constitutive relationship of piezoelectric materials is outlined below. A piezoelectric model requires permittivity (or dielectric constants), the piezoelectric matrix, and the elastic coefficient matrix to be specified as material properties. The constitutive relationship usually given by manufacturers or published reports is in the following form:

$$\{S\} = [S^E]\{T\} + [d]\{E\}$$
(12)

$$\{D\} = [d]^t \{T\} + [\varepsilon^T] \{E\}$$
(13)

ANSYS requires data in the following form:

$$\{T\} = [C^E]\{S\} - [e]\{E\}$$
(14)

$$\{D\} = [e]^t \{S\} + [\varepsilon^S] \{E\}$$
(15)

where:

 $\{T\}$ = Stress vector (six components x, y, z, xy, yz, xz).

{S} = Strain vector (six components x, y, z, xy, yz, xz).

 $\{D\}$ = Electric displacement vector (three components x, y, z).

 $\{E\}$ = Electric field vector (three components x, y, z).

[C^E]= Stiffness matrix evaluated at constant electric field, i.e. short circuit.

[e] = Piezoelectric matrix relating stress/electric field.

 $[e]^{t}$ = Piezoelectric matrix relating stress/electric field (transposed).

 $[\varepsilon^{S}]$ = Dielectric matrix evaluated at constant strains, i.e. mechanically clamped.

[d] = Piezoelectric matrix relating strain/electric field.

[d]^t= Piezoelectric matrix relating strain/electric field (transposed).

 $[\varepsilon^{T}]$ =Dielectric matrix evaluated at constant stress, i.e. mechanically free.

In order to convert the manufacturer's data presented in the form of Eqs. (12) & (13) to ANSYS notation (Eqs. (14) & (15)), Eq. (12) needs to be based on stress rather than strain. Eq. (12) can be rearranged as:

$$\{T\} = [S^E]^{-1}\{S\} - [S^E]^{-1}[d]\{E\}$$
(16)

Since Eq. (13) relates electric displacement to strain rather than stress, Eq. (16) can then be plugged back into Eq. (13) to yield:

$$\{D\} = [d]^{t}[S^{E}]^{-1}\{S\} + [\varepsilon^{T}] - [d]^{t}[S^{E}]^{-1}[d]\{E\}$$
(17)

Upon comparison of Eqs. (16) & (17) with Eqs. (14) & (15), one can obtain the relationship between manufacturer-supplied data and ANSYS-required values:

$$[C^{E}] = [S^{E}]^{-1}$$
(18)

$$[\varepsilon^{\mathsf{S}}] = [\varepsilon^T] - [d]^t [S^E]^{-1} [d]$$
(19)

$$[e] = [S^E]^{-1}[d]$$
(20)



These equations form the basis of the conversion routines for converting manufacturer's data to ANSYS data for stiffness matrices, dielectric constants, and piezoelectric constants. Note that the manufacturer's data has mechanical vectors in the form {x, y, z, yz, xz, xy} whereas ANSYS's mechanical vectors are in the form {x, y, z, xy, yz, xz}.

Permittivity matrix (dielectric constants)

Dielectric constants represent the diagonal components and ε_{11} , ε_{22} , ε_{33} respectively of the permittivity matrix [ε^{S}] (The superscript "s" indicates that the constants are evaluated at constant strain.) The permittivity matrix evaluated at constant strain is input into ANSYS. Typically, manufacturers' data has permittivity evaluated at constant stress, so conversion is necessary.

After evaluating Eq. (19), the permittivity matrix has only diagonal terms:

$$[\varepsilon^{S}] = \begin{bmatrix} \varepsilon_{11}^{S} & 0 & 0 \\ & \varepsilon_{11}^{S} & 0 \\ & & & \varepsilon_{33}^{S} \end{bmatrix} = \varepsilon_{0} \begin{bmatrix} K_{11}^{S} & 0 & 0 \\ & K_{11}^{S} & 0 \\ & & & K_{33}^{S} \end{bmatrix}$$
(21)

Where $K_{11}^S = \frac{\epsilon_{11}^S}{\epsilon_0}$ is relative permittivity.

Although there is a choice of inputting permittivity as an absolute value ϵ_{33}^T or relative value K_{33}^T , the relative value is recommended.

Piezoelectric matrix [e]

Usually, manufacturers' data gives [*d*], which relates mechanical strain to electric field. However, ANSYS requires [*e*], relating mechanical stress to electric field, so conversion is required. Note from Eq. (20), a relationship between [*e*] and [*d*] is established as follows where assuming polarization in the 3-axis (z-direction) and symmetry in the un polarized directions ($d_{32} = d_{31}$ and $d_{24} = d_{15}$).

$$[d]^{t} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & d_{15} \\ 0 & 0 & 0 & 0 & d_{15} & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}$$
(22)

This matrix can be used with $[S^E]^{-1} = [C^E]$ to evaluate [e] which will become,

$$[e]^{t} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & e_{15} \\ 0 & 0 & 0 & e_{15} & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$
(23)

This matrix relates the electric field to stress.

Stiffness Matrix [C]

The stiffness matrix is a symmetric matrix that specifies the stiffness coefficients:

$$[C]^{E} = \begin{pmatrix} C_{11} & & & \\ C_{21} & C_{22} & & \\ C_{31} & C_{32} & C_{33} & \\ C_{61} & C_{62} & C_{63} & C_{66} & \\ C_{41} & C_{42} & C_{43} & C_{46} & C_{44} & \\ C_{51} & C_{52} & C_{53} & C_{56} & C_{54} & C_{55} \end{pmatrix}$$
(24)

Piezoelectric circuit elements

The piezoelectric circuit element in ANSYS, CIRCU94, simulates basic linear electric circuit components that can be directly connected to the piezoelectric finite element domain. It is suitable for the simulation of circuit-fed piezoelectric dampers for vibration control under the influence of harmonic forces, currents, displacements, and voltages. CIRCU94 elements were used to model resistor (R), capacitor (C) and inductor (L) elements connected to each electrode of the piezoelectric patch. Any number of combinations of loadings is permitted and they need not be in phase, but they must be at the same frequency. The finite element equations for the resistor, inductor, capacitor and current source were derived using the nodal analysis method. To be compatible with the system of piezoelectric finite element equations, the nodal analysis method was adapted to maintain the charge balance at each node:

$$[K]{V} = {Q}$$
(25)

[K]= Stiffness (capacitance) matrix.

{V}= Vector of nodal voltages (to be determined).

 $\{Q\}$ = Load vector of nodal charges.

In a harmonic analysis, the stiffness matrix becomes:

$$C\begin{bmatrix} 1 & -1\\ -1 & 1 \end{bmatrix} \begin{pmatrix} V_1\\ V_2 \end{pmatrix} = \begin{pmatrix} 0\\ 0 \end{pmatrix} \quad \text{for capacitor,}$$
(26)

$$j\omega \left(-\frac{1}{\omega^{2}R}\right) \begin{pmatrix} 1 & -1\\ -1 & 1 \end{pmatrix} \begin{cases} V_{1}\\ V_{2} \end{cases} = \begin{cases} 0\\ 0 \end{cases} \quad \text{for a resistor,}$$
(27)

$$\begin{pmatrix} -\frac{1}{\omega^{2}L} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{cases} V_{1} \\ V_{2} \end{cases} = \begin{cases} 0 \\ 0 \end{cases} \quad \text{for an inductor}$$
(28)

where j = imaginary unit, $j^2 = -1$, ω = driving frequency, V_1 and V_2 are the shunt voltage as degrees of freedom at the two nodes for R, C and L elements [15].

Geometric Data of Piezoelectric Bimorph Beam

Piezoelectric bimorph cantilever beam was studded using ANSYS simulation, It is composed of two piezoelectric layers joined together with opposite polarities. The beam is constructed as a bimorph with the elements along the top polarized in the negative Y direction while the elements along the bottom of the beam are polarized in the positive Y direction; the bimorph beam material is Polyvinylidene Fluoride (PVDF).



The dimensions and the physical properties are: Length of the beam L=50×10⁻³ m, one layer thickness of piezoelectric material H=0.5×10⁻³ m, Electrode voltage V=100 Volt, piezoelectric Material properties (PVDF): Young's modulus E_1 =2.0×10⁹ N/m^2, Poisson's ratio NU_{12} =0.29, shear modulus G_{12} =0.775×10⁹ N/m^2, Piezoelectric strain coefficients d_{31} =2.2×10⁻¹¹C/N, d_{32} =0.3×10⁻¹¹C/N and d_{33} =-3.0×10⁻¹¹C/N, and Relative permittivity at constant stresses ept_{33} =12. Fig.8, shows the finite element model of the piezoelectric bimorph beam.

Static Analysis

Piezoelectric bimorph beam is widely used for actuation and sensing. In the actuation mode, on the application of an electric field across the beam thickness, one layer contracts while the other expands. This results in the bending of the entire structure and tip deflection. In the sensing mode, the bimorph is used to measure an external load by monitoring the piezoelectrically induced electrode voltages.

Actuator mode

For applied voltage of 100 volts along the top surface, deflection is determined in actuator mode. The deflection obtained from theoretical solution is given by formula,

$$U_{y} = \frac{-3 \, (d_{31}) V L^{2}}{8 H^{2}} \tag{29}$$

Substituting by, $d_{31}=2.2 \times 10^{-11}$ C/N, V=100, L=0.05 m, H=0.5 $\times 10^{-3}$ m in the above equation, the result is as follows:

$$U_v = -8.25 \times 10^{-6} \text{ m}$$

From ANSYS, deflection of -8.88×10^{-6} m is obtained at the tip of the beam for applied 100 Volts. The result of the analysis is shown in Fig.9. This simulation made to validate the model.

Sensor mode

In sensor mode, the piezoelectric bimorph is used to measure an external load by monitoring the piezoelectrically induced electrode voltages. The induced piezoelectric voltages are determined by giving a tip deflection of 10×10^{-3} m. Fig.10. shows the static analysis of piezoelectric bimorph beam for the sensor mode with different voltages at different beam sections.

Transient Analysis

The first transient analysis of piezoelectric bimorph beam is conducted in ANSYS with natural damping only. The potential of 100 volts are applied to the electrode at the top of the beam while the bottom electrode was grounded and the beam was allowed to deform. After the two initial load steps apply the potential, subsequent steps are used to determine the transient response of the beam to 0.5 seconds. The result of the analysis is shown in Fig.11. this figure shows that the beam oscillates 15 cycles in 0.45 seconds which gives a natural period of $\tau = \frac{0.45}{15} = 0.03$ seconds, and gives a natural frequency $\omega_n = 1/0.03 = 33.3$ Hz.

The results also raise a concern that the voltage applied to the top electrode is not as expected and that the response of the piezoelectric beam is influencing that voltage. To verify that the applied voltage is ramping monotonically and not oscillating with the beam, the voltage response at the top electrode was plotted in Fig.12.

If the oscillations in Fig.11 are considered excessive, then additional damping will be required. One method of applying that damping would be to limit the application of the voltage potential with a resistor capacitor (RC) circuit. With this type of circuit, the full voltage potential will be mitigated initially and the full will be applied when the capacitor is charged. The time required to charge the capacitor is known as the circuit time constant and is dependent upon the values of the resistor and the capacitor. The time constant for an RC circuit is given as,

$$\tau = RC \tag{30}$$

We can use the standard rule of thumb that 5 time constants are required to fully charge the RC circuit. Suppose we choose 0.03 seconds for the time constant of the circuit. Then the time for the circuit to be fully changed would be 0.15 seconds.

RESULTS AND DISCUSSIONS

The second transient analysis of piezoelectric bimorph beam is conducted in ANSYS by applying damping with a resistor capacitor (RC) shunt circuit. At constant voltage 100 V, the capacitor (C) value is held constant at C=0.5 μ F (microfarad) and the resistor (R) permitted to vary with values R = 10,000, 20,000, 40,000, 60,000 and 100,000, ohms and observe beam response for each value. The results in Fig.13, show that for (RC) circuit at constant capacitance (C), the vibration response of the beam at resistor (R) value R=10,000 ohms has settled time of 1 second while resistor (R) value R=100,000 ohms has settled time of 0.2 second, this result shows that vibration damping increased 80% by increasing resistor (R) value 10 times. This result help in designing of (RC) shunt circuit for piezoelectrically vibration damping.

The third transient analysis of piezoelectric bimorph beam is conducted in ANSYS model by applying damping with a resistor capacitor (RC) shunt circuit. At constant voltage 100 V, the resistor (R) value is held constant at R=10,000 ohms, and the capacitor (C) permitted to vary with values C= 1×10^{-7} , 3×10^{-7} , 5×10^{-7} , 9×10^{-7} and 3×10^{-6} Farad, and observe beam response for each value. The results in Fig. 14 show that for (RC) circuit at constant resistor (R) value, the vibration response of the beam at capacitor (C) value C= 1×10^{-7} Farad has settled time of 1 second while capacitor value C= 3×10^{-6} Farad has settled time of 0.1 second, this result shows that vibration damping increased 90% by increasing the capacitor (C) value 30 times. This result also help in designing of (RC) shunt circuit for piezoelectrically vibration damping.

CONCLUSIONS

Resonant vibrations of aero engine blades cause blade fatigue problems in engines.

Piezoelectric transducers have found extensive applications in vibration control systems. A shunted piezoelectric vibration damping technique is used to reduce vibrations in aircraft engine blades. In active vibration control problems, these transducers are used as actuators and sensors in feedback control loops designed for vibration suppression.

An effective, adaptable and real time Piezoelectric bimorph cantilever beam has been tested using finite element analysis using ANSYS program with natural damping only and with applying piezoelectric damping with a resistor capacitor (RC) shunt circuit. The results show that this model could be used for vibration suppression. The results also show that, for Piezoelectric (RC) shunt circuit, vibration damping increased 80% by increasing the resistor value (R) 10 times, and also vibration damping increased 90% by increasing the capacitor value (C) 30 times. These results help for getting optimum values for (R) and (C) in designing piezoelectric (RC) shunt circuit for vibration damping.

Piezoelectric bimorph cantilever beam with (RC) shunt circuit model results could be used for real applications in aero engines, by bonding piezoelectric with (RC) shunt circuit into propeller blades, fan blades, compressor blades or turbine blades for vibration suppression tasks, which could be modeled as some cantilever beams.

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Fig. 1. Material Configurations.



Fig. 2. Sensor System.



Fig. 3. Flexible structure with a collocated pair of piezoelectric transducers.



Fig. 4. Electrical equivalent circuit of the system in figure 3.



(a) Suction side

(b) pressure side

Fig. 5. Piezoelectric actuators bond on compressor blade.



Fig. 6. Test rig with slip ring, from the pressure side against the flow direction.



Fig. 7. Piezoelectric Bimorph Beam.



Fig. 8. Finite element model of a piezoelectric bimorph.







Fig.10. Static analysis for Bimorph beam in sensor mode.



Fig. 11. Transient analysis of piezoelectric bimorph without electrical damping.



Fig. 12. Voltage applied to the top electrode of beam.



Fig. 13. Bimorph beam response with variable resistance R and C = $0.5 \ \mu F$.

DV 100



Fig. 14. Bimorph beam response with variable capacitance, R=10,000 ohm.