A Comparative Study of Bayesian and Non-Bayesian Estimations for New Generalized Inverted Weibull Distribution

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الملخص

يقترح هذا البحث امتدادًا جديدًا لتوزيع معكوس وايبل (IW) لتقديم توزيع أكثر قابلية وتهيئة لنمذجة بيانات العمر، تسمى فئة توزيع معكوس ويبل اكس جاما (Weibull X-Gamma) الجديدة .(NIWXG) تم تضمين كل من توزيع معكوس ويبل، وتوزيع معكوس رايلي، ومعكوس الأسي، ومعكوس اكس جاما(X-Gamma) ، وتوزيعات معكوس X-Gamma الأسية كنماذج فرعية خاصة في التوزيع المقترح. يتم رسم منحنيات دالة كثافة الاحتمال ومعدل الخطر لجميع التكوينات الممكنة. الهدف الرئيسي هو تقدير معلمات التوزيع المقترح غير المعروفة باستخدام تقنيات التقدير غير البايزية مثل تقدير الامكان الأكبر وتقدير كرامر - فون ميزس (Cramér-von-Mises) وطريقة تقدير المربعات الصغرى وطريقة المربعات الصغرى الموزونة وتقدير بايزي في ظل دالة خسارة الخطأ التربيعية ودالة خسارة لينكس(LINEX) ، وكذلك اشتقاق بعض خصائصه الرياضية. استخدم لتحقيق هذا الهدف عمليات محاكاة عددية مكثفة لكل من العينات الصغيرة والكبيرة لمقارنة أداء المقدر غير البايزي والمقدر البايزي. بناءً على معيار التحيز النسبي المطلق ومعيار جذر متوسط مربعات الخطأ، تظهر النتائج أن التقدير البايزي هو أفضل تقدير.

الكلمات المفتاحية: توزيع معكوس وايبل، تقدير غير بايزي، دراسة محاكاة، تقدير بايزي، جذر متوسط مربعات الخطأ، الدالة الربيعية.

Abstract

This research proposes a new extension of the inverted Weibull distribution (IW) to provide a more adaptable distribution for modelling lifetime data, called the new inverted Weibull X-Gamma (NIWXG) distribution class. Inverted Weibull, inverted Rayleigh, inverted exponential, X-Gamma inverted Rayleigh, and X-Gamma inverted exponential distributions are included as special sub-models in the suggested distribution. For certain values of the parameters, the probability density and hazard rate curves are sketched in all feasible configurations. Our main goal is to estimate its unknown parameters using non-Bayesian estimation techniques such as maximum likelihood estimation, Cramér-von-Mises estimation, least squares estimation, weighted least squares estimation, and Bayesian estimation under squared error loss function and LINEX loss function, as well as to derive some of its mathematical properties. We utilize extensive numerical simulations for both small and big samples to compare the performances of the non-Bayesian and Bayesian estimators. Based on the relative absolute bias and root mean squared error criterion, the results demonstrate that the Bayesian estimation is the best estimation.

Keywords: Inverted Weibull distribution, Non-Bayesian estimation, Simulation study, Bayesian estimation, Root mean squared error, Quantile function.

1. Introduction

Numerous new families of lifetime distributions are produced and frequently used in statistics literature to represent phenomena that occur in the actual world. It is commonly known that the statistical distribution theory frequently involves adding a new parameter to an existing family of distributions. A class of probability distributions can frequently be made more flexible by adding an additional parameter, which can be very helpful for data analysis. These new types of distributions all have the trait of having additional parameters, and the new generalized distribution outperforms the baseline distribution in terms of distribution adequacy, see Nelson (2003).

Due to their versatility, inverted distributions are important in a variety of domains, including biological sciences, life test issues, real sciences, and more. Regarding density and hazard ratio, inverted conformation distributions differ from non-inverted conformation distributions in their structural makeup. An important probability distribution that can be used to evaluate life-time data with some monotone failure rates is the inverted Weibull (IW) distribution. The IW distribution has been applied in a variety of real-life contexts, including engineering, reliability, ecology, and medical. Moreover, the IW distribution is a suitable distribution for a number of failure characteristics, including useful life, wear-out period, and infant mortality. The forms of the density and failure rate functions for the fundamental inverted distribution were studied by Khan and King (2012).

The probability density function (PDF) and cumulative distribution function (CDF) of the IW distribution, respectively, are as follows;

$$f(x;\alpha,\beta) = \alpha\beta x^{-\beta-1}e^{-\alpha x^{-\beta}}, \quad x \geq 0 \quad ;\alpha,\beta > 0. \qquad (1.1)$$

$$F(x;\alpha,\beta) = e^{-\alpha x^{-\beta}}, \quad x \geq 0 \quad ;\alpha,\beta > 0. \qquad (1.2)$$

where β and α are the shape and scale parameters, respectively. From Equation (1.2), we study the sub-models of inverted Weibull distribution as follows;

- If $\alpha = 1$, the Fréchet distribution function is available.
- If $\beta = 1$, the PDF of IW becomes inverted exponential distribution.
- If $\beta = 2$, the PDF of IW is referred to inverted Raleigh distribution.

Also, the quantile function for IW distribution takes the following form;

$$Q_F(x) = \left(\frac{-\log(y)}{\alpha}\right)^{-1/\beta}. \qquad (1.3)$$

Also, the PDF of IW distribution satisfies:

$$xf(x,\beta,\alpha) = \beta F(x,\beta,\alpha)(-\log(F(x,\beta,\alpha)), \quad x \geq 0 \quad ;\beta,\alpha > 0. \qquad (1.4)$$

Many generalized classes of distributions have been put forth in recent years for the distribution of real-life dataset, offering a great deal of flexibility in the distribution of data in a number of applied fields like reliability, research of biological, economics, real sciences, and finance. El-Batal and Muhammad (2014) developed an exponentiated generalized inverted Weibull distribution. Using the beta family, Baharith et al. (2014) studied the beta generalized inverted Weibull distribution. By utilizing exponentiated generalized families. The Alpha power inverted Weibull distribution was first presented by Basheer (2019). Inverted Weibull with Marshall-Olkin alpha power was first introduced by Basheer et al. (2021). On the other hand, Fuzzy Bayesian estimation for the Kumaraswamy distribution was proposed by Alharbi and Kamel (2022), along with a comparison of Bayesian and non-Bayesian estimations. Finally, by utilizing Markov Chain Monte Carlo (MCMC) techniques. Moreover, Kamel et al. (2022) investigated the type I Gumbel extreme value (GEV) distribution with Bayesian estimators of the parameters.

The IW extended distribution offer a wide range of behavior then basic distribution from which they are derived. We believe this is the first time the new class has been considered and the issue specifically seen in the auto industry has been explored, see Kamel and Alqarni (2020). By adopting the IW distributions as a reference model, we create a new class of distributions known as the new inverted Weibull- XG-Generator family and investigate one of its unique sub-models. The studied model is called the new inverted Weibull-XG-Generator (NIWXG) distribution. The NIWXG distribution's reliability and hazard rate properties are given. Additionally, the estimate of the NIWXG distribution’s parameters using Bayesian and non-Bayesian approaches is examined. The performance of the estimators is evaluated through a thorough simulation study. Along with various other well-known distributions, our NIWXG model. Some well-known distributions can’t match data as well as the NIWXG distribution; see Khan and King (2016).

This research was organized as follows. Section 2 is devoted to defining our proposed model, and presenting its special cases. In Section 3, the NIWXG distribution's mathematical quantities (reliability and hazard rate properties) are discussed. While the estimation of the parameters of this distribution by the non-Bayesian and Bayesian estimation method is discussed in Section 4. In Section 5, the simulation results to compare the performance of the estimation methods for the NIWXG model were provided. Finally, Section 7 provides a few conclusionary notes.

2. The New Inverted Weibull-XG Class

We will introduce the NIWXG distribution and some of its sub-distributions in this section. The NIWXG distribution was created using the XG-Class and IW distribution. Sen et al. (2016) provide the X-Gamma distribution, a probability distribution with a single shape parameter. A probability distribution called the X-Gamma distribution may be useful for distributing lifespan data from a variety of scientific fields. Moreover, Cordeiro et al. (2020) presented the XG-Generator class to include any distribution into a larger class by using the XG-Generator class. The shape parameter

λ adds to the CDF of the XG-Generator class, which is provided by;

$$F(x;\lambda,\omega) = 1 - \frac{[1-G(x;\omega)]^{\lambda}}{\lambda+1}\{1+\lambda-\lambda\, ln(1-G(x;\omega)) + 0.5\,\lambda^2\,[ln(1-G(x;\omega))]^2\}, \quad (2.1)$$

where $G(x;\omega)$ is a baseline CDF with a parameter vector of ω. XG-Generator class PDF can be stated as;

$$f(x;\lambda,\omega) = \frac{\lambda}{\lambda+1}\, g(x;\omega)[1-G(x;\omega)]^{\lambda-1}\,\{\lambda + 0.5\,\lambda^2\,[ln(1-G(x;\omega))]^2\}. \quad (2.2)$$

Where $g(x;\omega) = dG(x;\omega)\,/\,dx$.

The NIWXG distribution represented by the random variable $X \sim NIWXG\ (\alpha,\lambda,\beta)$. Equations (1.1) to (2.2) are used to create the following CDF for the three-parameter NIWXG distribution:

$$F(x;\alpha,\lambda,\beta) = 1 - \frac{\left[1-e^{-\alpha x^{-\beta}}\right]^{\lambda}}{\lambda+1}\left\{1+\lambda-\lambda\, ln(1-e^{-\alpha x^{-\beta}}) + 0.5\,\lambda^2\left[ln(1-e^{-\alpha x^{-\beta}})\right]^2\right\}, \quad (2.3)$$

where $x > 0$ and $\alpha,\lambda,\beta > 0$. The PDF of NIWXG distribution is provided as:

$$f(x,\alpha,\lambda,\beta) = \frac{\alpha\lambda\beta}{\lambda+1}\, x^{-\beta-1}\, e^{-\alpha x^{-\beta}}\,[1-e^{-\alpha x^{-\beta}}]^{\lambda-1}\left\{\lambda + 0.5\,\lambda^2\left[ln\left(1-e^{-\alpha x^{-\beta}}\right)\right]^2\right\}, \quad (2.4)$$

Figure 1 displays the PDF graphically for various values of α,λ and β.

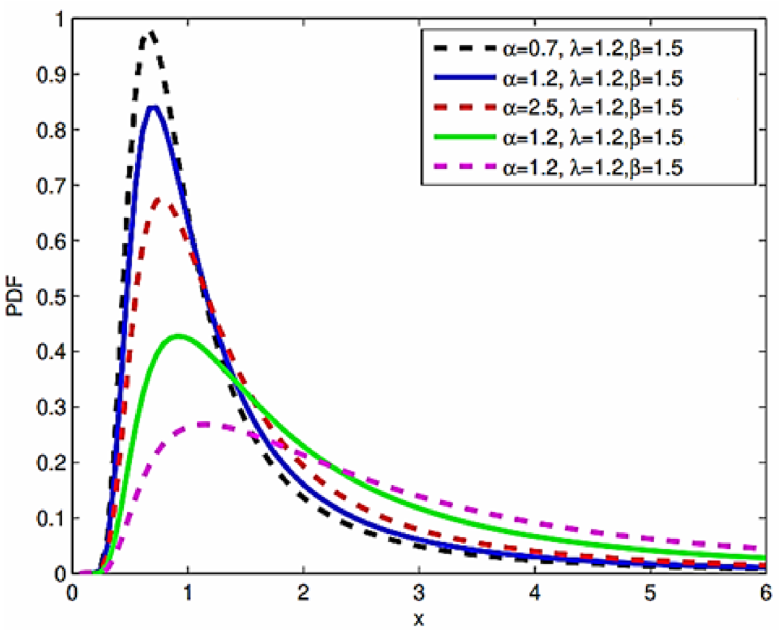


Figure 1: PDF of the NIWXG distribution.

A particularly adaptable model, the NIWXG distribution with three parameters (α, λ, β) and PDF in Equation (2.4) takes a special sub-model-based approach to many distributions. If $\alpha \to 1$, the new model, known as the X-Gamma inverted exponential (XGIE) distribution, reduces to the two-parameter distribution. While, If $\alpha \to 2$, the new model, known as the X-Gamma inverted Rayleigh (XGIR) distribution, reduces to the two-parameter distribution.

3. Mathematical Quantities

The survival function of NIWXG distribution is given by:

$$S(x;\alpha,\lambda,\beta) = \frac{[1-e^{-\alpha x^{-\beta}}]^{\lambda}}{\lambda+1}\left\{1+\lambda\ -\lambda\ ln\left(1-e^{-\alpha x^{-\beta}}\right)+0.5\ \lambda^2\left[ln\left(1-\ e^{-\alpha x^{-\beta}}\right)\right]^2\right\}, \quad (3.1)$$

The following formula represents the hazard rate function (HRF) of a random variable X with NIWXG distribution

$$h(x;\alpha,\lambda,\beta) = \frac{\alpha\,\lambda\,\beta\,x^{-\beta-1}\,e^{-\alpha x^{-\beta}}\left\{\lambda+0.5\,\lambda^2\left[ln\left(1-e^{-\alpha x^{-\beta}}\right)\right]^2\right\}}{\left(1-e^{-\alpha x^{-\beta}}\right)\left\{1+\vartheta-\vartheta\ ln(1-e^{-\alpha x^{-\beta}})+0.5\ \lambda^2\left[ln\left(1-e^{-\alpha x^{-\beta}}\right)\right]^2\right\}}, \quad (3.2)$$

Plots of the hazard rate function (HRF) of the NIWXG distribution are shown in Figure 2, for the following parameter values (α,λ,β).

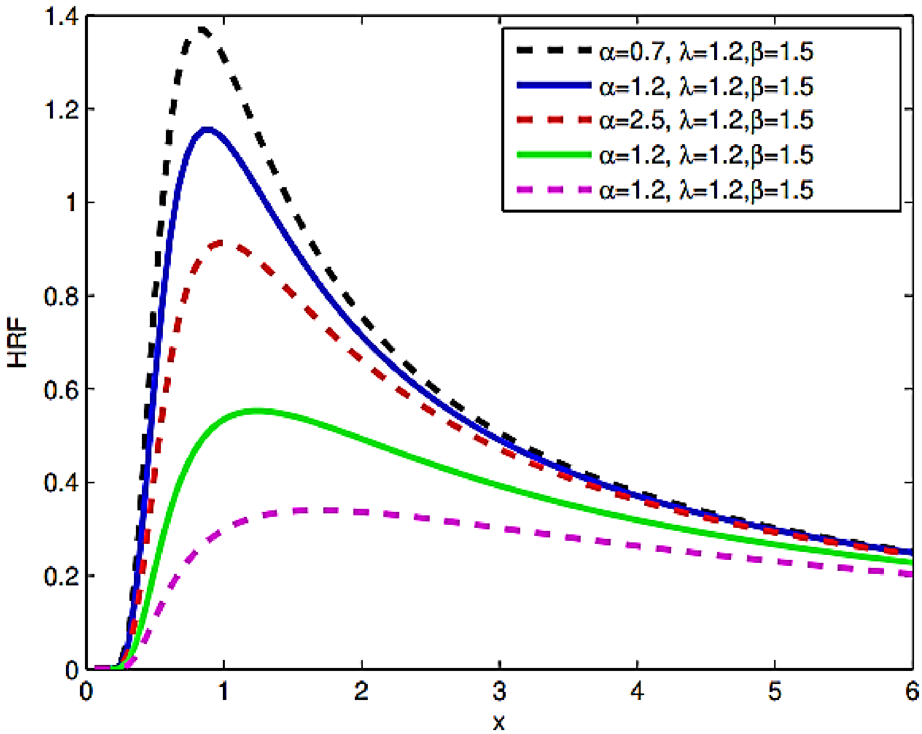


Figure 2: HRF of the NIWXG distribution.

4. Non-Bayesian and Bayesian Estimation Methods

In this section, the estimate of the NIWXG parameters (α, λ, β) using non-Bayesian methods is examined using four estimation techniques: maximum likelihood (ML) estimation, Cramér-von-Mises (CVM) estimation, least squares (LS) estimation, weighted least squares (WLS) estimation and Bayesian estimation under squared error loss function and LINEX loss function in the presence of a complete sample.

4.1. ML Estimation Method

Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n from NIWXG model. The log-likelihood function for parameter vector $\delta = (\lambda, \alpha, \beta)^T$ is given by:

$$\ell(\alpha,\lambda,\beta) = n\,ln\left(\frac{\lambda}{\lambda+1}\right) + n[ln(\beta)+ln(\alpha)] - (\beta+1)\sum_{i=1}^{n} ln(x_i) - \alpha\sum_{i=1}^{n} x_i^{-\beta}$$

$$+(\lambda-1)\sum_{i=1}^{n} ln\left(1-e^{-\alpha x_i^{-\beta}}\right) + \sum_{i=1}^{n} ln\left\{\lambda + 0.5\,\lambda^2\left[ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2\right\}, \quad (4.1)$$

Using the log-likelihood function's initial partial derivatives with respect to the three parameters (α, λ, β) we have;

$$U_n(\delta) = \left(\frac{\partial \ell}{\partial \alpha}, \frac{\partial l}{\partial \lambda}, \frac{\partial \ell}{\partial \beta}\right)^T,$$

are given by;

$$\frac{\partial \ell}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} x_i^{-\beta} + (\lambda - 1)\sum_{i=1}^{n} \frac{e^{-\alpha x_i^{-\beta}} x_i^{-\beta}}{1 - e^{-\alpha x_i^{-\beta}}} + \lambda^2 \sum_{i=1}^{n} \frac{ln\left(1 - e^{-\alpha x_i^{-\beta}}\right) \frac{e^{-\alpha x_i^{-\beta}} x_i^{-\beta}}{1 - e^{-\alpha x_i^{-\beta}}}}{\lambda + 0.5\,\lambda^2 [ln\left(1 - e^{-\alpha x_i^{-\beta}}\right)]^2},$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{n}{\lambda(\lambda + 1)} + \sum_{i=1}^{n} ln\left(1 - e^{-\alpha x_i^{-\beta}}\right) + \sum_{i=1}^{n} \frac{1 + \lambda [ln\left(1 - e^{-\alpha x_i^{-\beta}}\right)]^2}{\lambda + 0.5\,\lambda^2 [ln\left(1 - e^{-\alpha x_i^{-\beta}}\right)]^2},$$

$$\frac{\partial \ell}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^{n} ln(x_i) + \alpha \sum_{i=1}^{n} x_i^{-\beta}\, ln(x_i) - (\lambda - 1)\alpha \sum_{i=1}^{n} \frac{e^{-\alpha x_i^{-\beta}}\, ln(x_i)}{1 - e^{-\alpha x_i^{-\beta}}}$$
$$-\lambda^2 \sum_{i=1}^{n} \frac{ln\left(1 - e^{-\alpha x_i^{-\beta}}\right) \frac{e^{-\alpha x_i^{-\beta}}\, ln(x_i)}{1 - e^{-\alpha x_i^{-\beta}}}}{\lambda + 0.5\,\lambda^2 [ln\left(1 - e^{-\alpha x_i^{-\beta}}\right)]^2},$$

The solutions to the aforementioned equations can be found using any iterative procedural approaches, such as Newton–Raphson algorithm or conjugate gradient algorithms, see Jennrich and Robinson (1969).

4.2 LS and W LS Estimation Methods

By minimizing the following function, the ordinary least squares and weighted least square estimators of the NIWXG parameters can be produced.

$$LS(\delta) = \sum_{i=1}^{n} \omega_i \left\{\epsilon(i,n) - \frac{\log[\psi(x_i;\delta)]}{\log(\alpha)}\right\}^2, \qquad (4.2)$$

where $\epsilon(i,n) = \frac{n+1-i}{n+1}$ and $\omega_i = 1$ in the case of LS estimation method and $\omega_i = \frac{(n+1)^2(n+2)}{i(n-i+1)}$ in the case of WLS estimation method.

4.3 Cramér–von-Mises Estimation Method

By minimizing the following function, the CVM estimations of the NIWXG parameters are obtained.

$$C(\delta) = \frac{1}{12n} + \sum_{i=1}^{n} \left\{ \varepsilon_i - \frac{\log[\psi(x_i;\theta)]}{\log(\alpha)} \right\}^2, \qquad (4.3)$$

with respect to α, λ and β, where $\varepsilon_i = \frac{n-i+0.5}{n}$. See Azaïs et al. (2022).

4.4 Bayesian Estimation

For Bayesian estimation, the unknown parameters α, λ and β of NIWXG distribution is assumed to be independent and have their own conjugate prior distributions. The loss function, on the other hand, is crucial to Bayesian techniques. The majority of Bayesian inference techniques were created using symmetric and asymmetric loss functions. The squared error loss (SEL) function is one of the most often used symmetric loss functions, whereas the most widely used asymmetric loss function is the LINEX loss function. Thus the suggested prior for $, \lambda$ and β are;

$$f_1(\alpha) \propto \alpha^{a_1-1}e^{-b_1\alpha}, f_2(\beta) \propto \beta^{a_2-1}e^{-b_2\beta}, f_3(\lambda) \propto \lambda^{a_3-1}e^{-b_2\lambda}$$

Respectively, where a_1, a_2, a_3, b_1, b_2 and b_3 are the hyper parameters of prior distributions, see Eliwa et al. (2021).

Based on the likelihood function in Equation (4.1), the joint posterior density function of (δ) can be written as:

$$\pi(\delta \mid \underline{x}) = \frac{\ell(\underline{x}|\delta)\cdot\pi(\delta)}{\int \delta\,\ell(\underline{x}|\delta)\cdot\pi(\delta)}, \qquad (4.4)$$

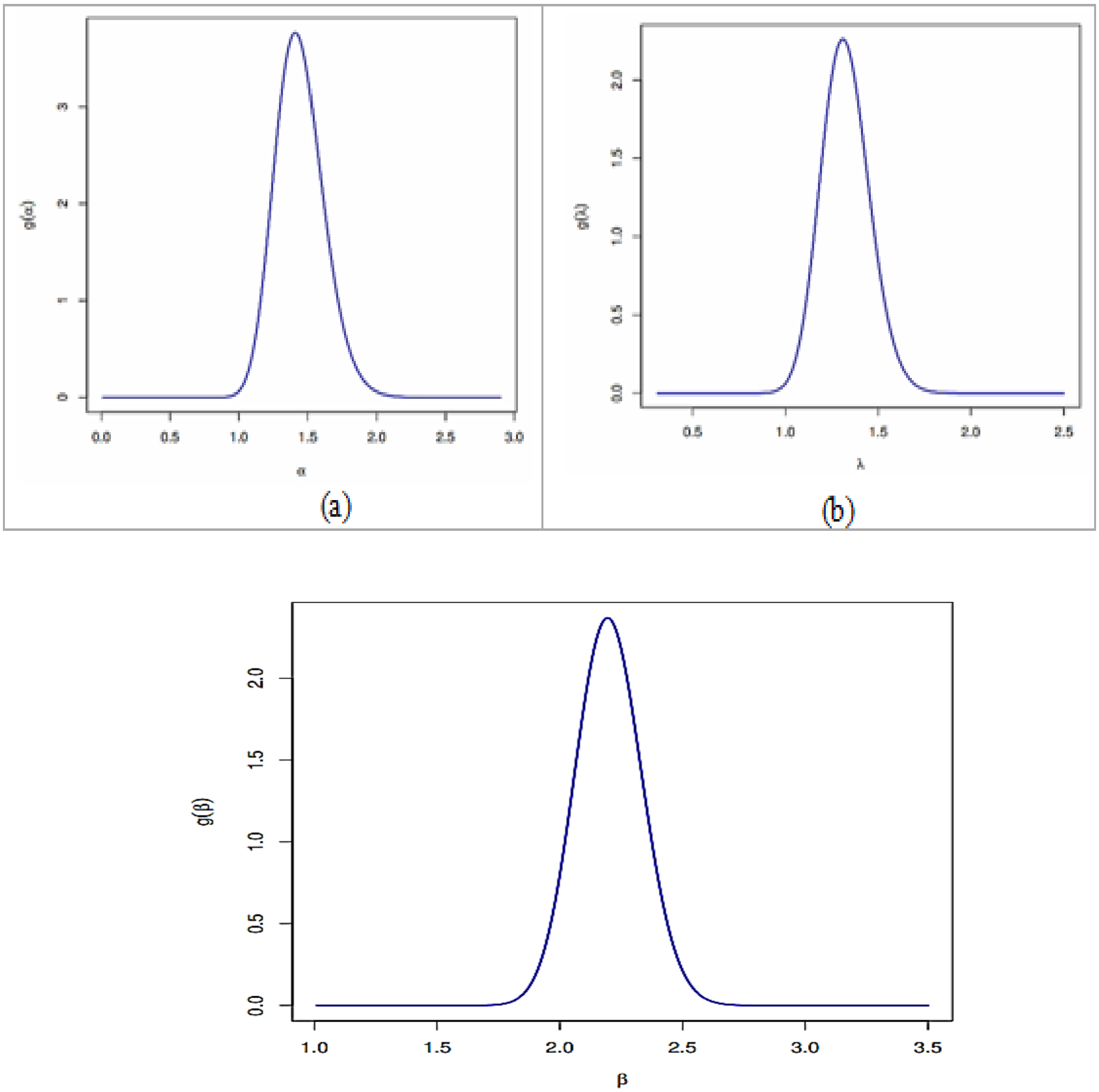
Using SEL function, the Bayesian Estimate of δ, say $p(\alpha, \lambda, \beta)$ is provided by;

$$\begin{aligned}\hat{p}_{B-SEL}(\alpha,\lambda,\beta) &= E_{(\alpha,\lambda,\beta|\underline{x})}[p(\alpha,\lambda,\beta)] \\ &= \int_0^\infty \int_0^\infty \int_0^\infty p(\alpha,\lambda,\beta) \times \pi(\delta \mid \underline{x})d\alpha\, d\lambda\, d\beta.\end{aligned}$$

Using LINEX loss function, the Bayesian estimate of δ, say $p(\alpha, \lambda, \beta)$ is provided by;

$$\begin{aligned}\hat{p}_{B-LINEX}(\alpha,\lambda,\beta) &= \frac{-1}{\varphi}\log\left[E_{(\alpha,\lambda,\beta|x)}\left[e^{-\varphi p(\alpha,\lambda,\beta)}\right]\right] \\ &= \frac{-1}{\varphi}\log\left[\int_0^\infty \int_0^\infty \int_0^\infty e^{-\varphi p(\alpha,\lambda,\beta)} \times \pi(\delta \mid \underline{x})d\alpha\, d\lambda\, d\beta\right], \ \varphi \neq 0.\end{aligned}$$

To handle this kind of problem, there are numerous approximation techniques accessible in the literature. The Monte Carlo Markov Chain (MCMC) approximation method is taken into consideration here. Although the aforementioned distributions are unknown, Figure 3 demonstrates that their graphs resemble the normal distribution. Therefore, we create random samples from these distributions using the Metropolis-Hastings procedures, see Metropolis et al. (1953) and Almetwally et al. (2018).



(c)

Figure 3: Conditional posterior distributions of NIWXG parameters (α, λ, β).

5. Simulation Study and Results

The parameters of the NIWXG distribution are determined using the R-programming environment "version 4.1.2", implementation of Monte-Carlo numerical simulation study to compare the non-Bayesian and Bayesian estimation approaches, for more details about simulation study in R software, see Robert et al. (2009).

The data were produced using the NIWXG Distribution for the lifetime of various parameters α, λ and β such as:

- Case 1: $\alpha = 0.5, \beta = 1.6$ and $\lambda = 1.5$
- Case 2: $\alpha = 1.5, \beta = 0.8$ and $\lambda = 1.5$
- Case 3: $\alpha = 0.5, \beta = 1.5$ and $\lambda = 0.6$
- Case 4: $\alpha = 1.5, \beta = 1.2$ and $\lambda = 2.5$
- Case 5: $\alpha = 0.9, \beta = 2.5$ and $\lambda = 3.5$
- Case 6: $\alpha = 1.7, \beta = 3.5$ and $\lambda = 2.5$

By creating 1000 replications for different sample sizes n = 30, 80, 150, 200, and 300 using the NIWXG distribution, the relative absolute bias (RAB) and root mean squared errors (RMSE) are calculated to compare the performances of non-Bayesian and Bayesian estimation methods.

The results of the numerical simulation have been provided in Tables 1 to 6. We can see from Tables 1 through 6 that as sample size n increase, the RMSE for each approach decrease. The Bayesian estimation is also the most accurate estimation

Table 1: RAB and RMSE of NIWXG parameters in Case 1

n	Estimate	ML		LS		CVM		WLS		SEL		LINEX $\varphi = 0.6$		LINEX $\varphi = 1.3$	
		RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE
30	$\hat{\alpha}$	0.2716	0.6937	0.2536	0.6019	0.2257	0.5243	0.1949	0.4176	0.1528	0.2142	0.1719	0.2907	0.1891	0.3035
	$\hat{\beta}$	0.1817	0.3645	0.1582	0.2775	0.1579	0.2319	0.1417	0.1964	0.1046	0.1207	0.1146	0.1371	0.1271	0.1695
	$\hat{\lambda}$	0.1869	0.4209	0.1681	0.3792	0.1379	0.3162	0.1149	0.2634	0.0913	0.1178	0.0996	0.1812	0.1115	0.2247
80	$\hat{\alpha}$	0.1783	0.4554	0.1665	0.3952	0.1482	0.3442	0.1280	0.2742	0.1242	0.1993	0.1003	0.1406	0.1129	0.1909
	$\hat{\beta}$	0.1039	0.1822	0.1193	0.2393	0.1037	0.1523	0.0930	0.1289	0.0834	0.1113	0.0752	0.0900	0.0687	0.0793
	$\hat{\lambda}$	0.1227	0.2763	0.1104	0.2490	0.0905	0.2076	0.0754	0.1729	0.0732	0.1475	0.0599	0.0773	0.0654	0.1190
150	$\hat{\alpha}$	0.1171	0.2990	0.1093	0.2595	0.0973	0.2260	0.0840	0.1800	0.0815	0.1308	0.0741	0.1253	0.0659	0.0923
	$\hat{\beta}$	0.0783	0.1571	0.0682	0.1196	0.0681	0.1000	0.0611	0.0846	0.0451	0.0520	0.0494	0.0591	0.0548	0.0731
	$\hat{\lambda}$	0.0806	0.1814	0.0725	0.1635	0.0594	0.1363	0.0495	0.1135	0.0481	0.0969	0.0394	0.0578	0.0429	0.0781
200	$\hat{\alpha}$	0.0769	0.1963	0.0718	0.1704	0.0639	0.1484	0.0552	0.1182	0.0486	0.0823	0.0535	0.0859	0.0432	0.0606
	$\hat{\beta}$	0.0514	0.1032	0.0448	0.0785	0.0447	0.0656	0.0401	0.0556	0.0360	0.0480	0.0296	0.0342	0.0324	0.0388
	$\hat{\lambda}$	0.0529	0.1191	0.0476	0.1073	0.0390	0.0895	0.0325	0.0745	0.0258	0.0333	0.0282	0.0513	0.0316	0.0636
300	$\hat{\alpha}$	0.0505	0.1289	0.0471	0.1118	0.0419	0.0974	0.0362	0.0776	0.0351	0.0564	0.0284	0.0398	0.0319	0.0540
	$\hat{\beta}$	0.0338	0.0677	0.0294	0.0516	0.0293	0.0431	0.0263	0.0365	0.0213	0.0255	0.0236	0.0315	0.0194	0.0224
	$\hat{\lambda}$	0.0347	0.0782	0.0312	0.0705	0.0256	0.0587	0.0213	0.0489	0.0170	0.0219	0.0207	0.0417	0.0185	0.0337

Table 2: RAB and RMSE of NIWXG parameters in Case 2

n	Estimate	ML		LS		CVM		WLS		SEL		LINEX $\varphi = 0.6$		LINEX $\varphi = 1.3$	
		RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE
30	$\hat{\alpha}$	0.7871	2.0103	0.7349	1.7444	0.6541	1.5194	0.5648	1.2102	0.4428	0.6208	0.4982	0.8424	0.5480	0.8795
	$\hat{\beta}$	0.5266	1.0563	0.4585	0.8042	0.4576	0.6720	0.4106	0.5691	0.3031	0.3499	0.3321	0.3973	0.3683	0.4912
	$\hat{\lambda}$	0.5416	1.2198	0.4872	1.0989	0.3996	0.9162	0.3330	0.7634	0.2646	0.3414	0.2886	0.5251	0.3231	0.6512
80	$\hat{\alpha}$	0.5168	1.3199	0.4825	1.1453	0.4294	0.9976	0.3708	0.7946	0.3598	0.5775	0.3271	0.5531	0.2907	0.4076
	$\hat{\beta}$	0.3010	0.5280	0.3457	0.6935	0.3004	0.4412	0.2696	0.3736	0.2418	0.3225	0.1990	0.2297	0.2180	0.2609
	$\hat{\lambda}$	0.3556	0.8008	0.3198	0.7215	0.2624	0.6015	0.2186	0.5012	0.2121	0.4275	0.1895	0.3448	0.1737	0.2241
150	$\hat{\alpha}$	0.3393	0.8666	0.3168	0.7519	0.2819	0.6549	0.2435	0.5217	0.2147	0.3631	0.2362	0.3791	0.1909	0.2676
	$\hat{\beta}$	0.2270	0.4553	0.1976	0.3466	0.1972	0.2897	0.1770	0.2453	0.1432	0.1713	0.1307	0.1508	0.1588	0.2117
	$\hat{\lambda}$	0.2335	0.5258	0.2100	0.4737	0.1723	0.3949	0.1435	0.3290	0.1141	0.1674	0.1393	0.2807	0.1244	0.2264
200	$\hat{\alpha}$	0.2227	0.5689	0.2080	0.4937	0.1851	0.4300	0.1598	0.3425	0.1410	0.2384	0.1253	0.1757	0.1551	0.2489
	$\hat{\beta}$	0.1490	0.2989	0.1297	0.2276	0.1295	0.1902	0.1162	0.1611	0.1042	0.1390	0.0940	0.1124	0.0858	0.0990
	$\hat{\lambda}$	0.1533	0.3452	0.1379	0.3110	0.1131	0.2593	0.0942	0.2160	0.0749	0.0966	0.0914	0.1843	0.0817	0.1486
300	$\hat{\alpha}$	0.1462	0.3735	0.1366	0.3241	0.1215	0.2823	0.1049	0.2249	0.0823	0.1153	0.1018	0.1634	0.0926	0.1565
	$\hat{\beta}$	0.0978	0.1963	0.0852	0.1494	0.0850	0.1249	0.0763	0.1057	0.0684	0.0913	0.0617	0.0738	0.0563	0.0650
	$\hat{\lambda}$	0.1006	0.2266	0.0905	0.2042	0.0743	0.1702	0.0619	0.1418	0.0600	0.1210	0.0492	0.0634	0.0536	0.0976

Table 3: RAB and RMSE of NIWXG parameters in Case 3

n	Estimate	ML		LS		CVM		WLS		SEL		LINEX $\varphi = 0.6$		LINEX $\varphi = 1.3$	
		RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE
30	$\hat{\alpha}$	0.0980	0.2503	0.0915	0.2172	0.0814	0.1892	0.0703	0.1507	0.0551	0.0773	0.0620	0.1049	0.0682	0.1095
	$\hat{\beta}$	0.0656	0.1315	0.0571	0.1001	0.0570	0.0837	0.0511	0.0709	0.0377	0.0436	0.0414	0.0495	0.0459	0.0612
	$\hat{\lambda}$	0.0674	0.1519	0.0607	0.1368	0.0498	0.1141	0.0415	0.0951	0.0329	0.0425	0.0359	0.0654	0.0402	0.0811
80	$\hat{\alpha}$	0.0643	0.1644	0.0601	0.1426	0.0535	0.1242	0.0462	0.0989	0.0448	0.0719	0.0362	0.0507	0.0407	0.0689
	$\hat{\beta}$	0.0375	0.0657	0.0430	0.0864	0.0374	0.0549	0.0336	0.0465	0.0301	0.0402	0.0272	0.0325	0.0248	0.0286
	$\hat{\lambda}$	0.0443	0.0997	0.0398	0.0898	0.0327	0.0749	0.0272	0.0624	0.0264	0.0532	0.0216	0.0279	0.0236	0.0429
150	$\hat{\alpha}$	0.0422	0.1079	0.0394	0.0936	0.0351	0.0816	0.0303	0.0650	0.0294	0.0472	0.0267	0.0452	0.0238	0.0333
	$\hat{\beta}$	0.0283	0.0567	0.0246	0.0432	0.0246	0.0361	0.0220	0.0305	0.0163	0.0188	0.0178	0.0213	0.0198	0.0264
	$\hat{\lambda}$	0.0291	0.0655	0.0261	0.0590	0.0215	0.0492	0.0179	0.0410	0.0173	0.0350	0.0142	0.0208	0.0155	0.0282
200	$\hat{\alpha}$	0.0277	0.0708	0.0259	0.0615	0.0231	0.0535	0.0199	0.0426	0.0176	0.0297	0.0193	0.0310	0.0156	0.0219
	$\hat{\beta}$	0.0186	0.0372	0.0162	0.0283	0.0161	0.0237	0.0145	0.0201	0.0130	0.0173	0.0107	0.0123	0.0117	0.0140
	$\hat{\lambda}$	0.0191	0.0430	0.0172	0.0387	0.0141	0.0323	0.0117	0.0269	0.0093	0.0120	0.0102	0.0185	0.0114	0.0229
300	$\hat{\alpha}$	0.0182	0.0465	0.0170	0.0404	0.0151	0.0352	0.0131	0.0280	0.0127	0.0204	0.0102	0.0144	0.0115	0.0195
	$\hat{\beta}$	0.0122	0.0244	0.0106	0.0186	0.0106	0.0155	0.0095	0.0132	0.0077	0.0092	0.0085	0.0114	0.0070	0.0081
	$\hat{\lambda}$	0.0125	0.0282	0.0113	0.0254	0.0092	0.0212	0.0077	0.0177	0.0061	0.0079	0.0075	0.0151	0.0067	0.0121

Table 4: RAB and RMSE of NIWXG parameters in Case 4

n	Estimate	ML		LS		CVM		WLS		SEL		LINEX $\varphi = 0.6$		LINEX $\varphi = 1.3$	
		RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE
30	$\hat{\alpha}$	0.1866	0.4767	0.1743	0.4137	0.1551	0.3603	0.1339	0.2870	0.1050	0.1472	0.1181	0.1998	0.1299	0.2086
	$\hat{\beta}$	0.1249	0.2505	0.1087	0.1907	0.1085	0.1594	0.0974	0.1350	0.0719	0.0830	0.0788	0.0942	0.0873	0.1165
	$\hat{\lambda}$	0.1284	0.2892	0.1155	0.2606	0.0948	0.2173	0.0790	0.1810	0.0627	0.0810	0.0684	0.1245	0.0766	0.1544
80	$\hat{\alpha}$	0.1225	0.3130	0.1144	0.2716	0.1018	0.2366	0.0879	0.1884	0.0853	0.1369	0.0689	0.0966	0.0776	0.1312
	$\hat{\beta}$	0.0714	0.1252	0.0820	0.1645	0.0712	0.1046	0.0639	0.0886	0.0573	0.0765	0.0517	0.0619	0.0472	0.0545
	$\hat{\lambda}$	0.0843	0.1899	0.0758	0.1711	0.0622	0.1426	0.0518	0.1188	0.0503	0.1014	0.0412	0.0531	0.0449	0.0818
150	$\hat{\alpha}$	0.0805	0.2055	0.0751	0.1783	0.0669	0.1553	0.0577	0.1237	0.0560	0.0899	0.0509	0.0861	0.0453	0.0634
	$\hat{\beta}$	0.0538	0.1080	0.0469	0.0822	0.0468	0.0687	0.0420	0.0582	0.0310	0.0358	0.0339	0.0406	0.0376	0.0502
	$\hat{\lambda}$	0.0554	0.1247	0.0498	0.1123	0.0408	0.0937	0.0340	0.0780	0.0330	0.0666	0.0270	0.0397	0.0295	0.0537
200	$\hat{\alpha}$	0.0528	0.1349	0.0493	0.1171	0.0439	0.1020	0.0379	0.0812	0.0334	0.0565	0.0368	0.0590	0.0297	0.0417
	$\hat{\beta}$	0.0353	0.0709	0.0308	0.0540	0.0307	0.0451	0.0276	0.0382	0.0247	0.0330	0.0203	0.0235	0.0223	0.0267
	$\hat{\lambda}$	0.0363	0.0819	0.0327	0.0737	0.0268	0.0615	0.0223	0.0512	0.0178	0.0229	0.0194	0.0352	0.0217	0.0437
300	$\hat{\alpha}$	0.0347	0.0886	0.0324	0.0769	0.0288	0.0669	0.0249	0.0533	0.0241	0.0388	0.0195	0.0273	0.0219	0.0371
	$\hat{\beta}$	0.0232	0.0465	0.0202	0.0354	0.0202	0.0296	0.0181	0.0251	0.0146	0.0175	0.0162	0.0216	0.0134	0.0154
	$\hat{\lambda}$	0.0239	0.0537	0.0215	0.0484	0.0176	0.0404	0.0147	0.0336	0.0117	0.0150	0.0142	0.0287	0.0127	0.0231

Table 5: RAB and RMSE of NIWXG parameters in Case 5

n	Estimate	ML		LS		CVM		WLS		SEL		LINEX $\varphi = 0.6$		LINEX $\varphi = 1.3$	
		RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE
30	$\hat{\alpha}$	0.3780	0.9655	0.3530	0.8378	0.3141	0.7298	0.2713	0.5812	0.2127	0.2981	0.2393	0.4046	0.2632	0.4224
	$\hat{\beta}$	0.2529	0.5073	0.2202	0.3862	0.2198	0.3228	0.1972	0.2733	0.1456	0.1680	0.1595	0.1908	0.1769	0.2359
	$\hat{\lambda}$	0.2601	0.5858	0.2340	0.5278	0.1919	0.4401	0.1599	0.3666	0.1271	0.1640	0.1386	0.2522	0.1552	0.3128
80	$\hat{\alpha}$	0.2482	0.6339	0.2317	0.5501	0.2063	0.4791	0.1781	0.3816	0.1728	0.2773	0.1396	0.1957	0.1571	0.2656
	$\hat{\beta}$	0.1446	0.2536	0.1660	0.3331	0.1443	0.2119	0.1295	0.1795	0.1161	0.1549	0.1047	0.1253	0.0956	0.1103
	$\hat{\lambda}$	0.1708	0.3846	0.1536	0.3465	0.1260	0.2889	0.1050	0.2407	0.1019	0.2053	0.0834	0.1076	0.0910	0.1656
150	$\hat{\alpha}$	0.1630	0.4162	0.1522	0.3611	0.1354	0.3146	0.1169	0.2505	0.1135	0.1821	0.1031	0.1744	0.0917	0.1285
	$\hat{\beta}$	0.1090	0.2187	0.0949	0.1665	0.0947	0.1391	0.0850	0.1178	0.0628	0.0724	0.0688	0.0823	0.0763	0.1017
	$\hat{\lambda}$	0.1121	0.2525	0.1009	0.2275	0.0827	0.1897	0.0689	0.1580	0.0669	0.1348	0.0548	0.0804	0.0598	0.1087
200	$\hat{\alpha}$	0.1070	0.2733	0.0999	0.2371	0.0889	0.2065	0.0768	0.1645	0.0677	0.1145	0.0745	0.1195	0.0602	0.0844
	$\hat{\beta}$	0.0716	0.1436	0.0623	0.1093	0.0622	0.0913	0.0558	0.0774	0.0501	0.0668	0.0412	0.0476	0.0451	0.0540
	$\hat{\lambda}$	0.0736	0.1658	0.0662	0.1494	0.0543	0.1245	0.0453	0.1038	0.0360	0.0464	0.0392	0.0714	0.0439	0.0885
300	$\hat{\alpha}$	0.0702	0.1794	0.0656	0.1557	0.0584	0.1356	0.0504	0.1080	0.0489	0.0785	0.0395	0.0554	0.0445	0.0752
	$\hat{\beta}$	0.0470	0.0943	0.0409	0.0718	0.0408	0.0600	0.0366	0.0508	0.0296	0.0355	0.0329	0.0438	0.0271	0.0312
	$\hat{\lambda}$	0.0483	0.1089	0.0435	0.0981	0.0357	0.0818	0.0297	0.0681	0.0236	0.0305	0.0288	0.0581	0.0258	0.0469

Table 6: RAB and RMSE of NIWXG parameters in Case 6

n	Estimate	ML		LS		CVM		WLS		SEL		LINEX $\varphi = 0.6$		LINEX $\varphi = 1.3$	
		RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE	RAB	RMSE
30	$\hat{\alpha}$	0.6212	1.5866	0.5800	1.3768	0.5162	1.1992	0.4458	0.9551	0.3495	0.4899	0.3932	0.6649	0.4325	0.6942
	$\hat{\beta}$	0.4156	0.8337	0.3618	0.6347	0.3611	0.5304	0.3241	0.4492	0.2392	0.2761	0.2621	0.3136	0.2907	0.3877
	$\hat{\lambda}$	0.4275	0.9627	0.3845	0.8673	0.3154	0.7231	0.2628	0.6025	0.2088	0.2694	0.2278	0.4144	0.2550	0.5139
80	$\hat{\alpha}$	0.4078	1.0417	0.3808	0.9039	0.3389	0.7873	0.2927	0.6271	0.2840	0.4557	0.2295	0.3217	0.2581	0.4365
	$\hat{\beta}$	0.2376	0.4167	0.2728	0.5473	0.2371	0.3482	0.2128	0.2949	0.1909	0.2545	0.1721	0.2059	0.1571	0.1813
	$\hat{\lambda}$	0.2807	0.6320	0.2524	0.5694	0.2071	0.4748	0.1725	0.3955	0.1674	0.3374	0.1371	0.1769	0.1496	0.2721
150	$\hat{\alpha}$	0.2678	0.6839	0.2500	0.5934	0.2225	0.5169	0.1922	0.4117	0.1864	0.2992	0.1695	0.2866	0.1506	0.2112
	$\hat{\beta}$	0.1791	0.3594	0.1560	0.2736	0.1557	0.2286	0.1397	0.1936	0.1031	0.1190	0.1130	0.1352	0.1253	0.1671
	$\hat{\lambda}$	0.1843	0.4150	0.1657	0.3739	0.1360	0.3117	0.1133	0.2597	0.1099	0.2215	0.0900	0.1321	0.0982	0.1786
200	$\hat{\alpha}$	0.1758	0.4490	0.1642	0.3896	0.1461	0.3394	0.1262	0.2703	0.1113	0.1882	0.1224	0.1965	0.0989	0.1386
	$\hat{\beta}$	0.1176	0.2359	0.1024	0.1796	0.1022	0.1501	0.0917	0.1271	0.0823	0.1097	0.0677	0.0781	0.0742	0.0887
	$\hat{\lambda}$	0.1210	0.2724	0.1088	0.2454	0.0893	0.2046	0.0744	0.1705	0.0591	0.0762	0.0645	0.1173	0.0722	0.1454
300	$\hat{\alpha}$	0.1154	0.2948	0.1078	0.2558	0.0959	0.2228	0.0828	0.1775	0.0804	0.1290	0.0649	0.0910	0.0731	0.1235
	$\hat{\beta}$	0.0772	0.1549	0.0672	0.1179	0.0671	0.0986	0.0602	0.0835	0.0487	0.0583	0.0540	0.0720	0.0445	0.0513
	$\hat{\lambda}$	0.0794	0.1789	0.0714	0.1611	0.0586	0.1344	0.0488	0.1119	0.0388	0.0501	0.0474	0.0955	0.0423	0.0770

6. Conclusionary Notes

This research aims to develop a new inverted Weibull X-Gamma (NIWXG) distribution class, which produces the best univariate continuous models. There are discussions on several statistical and reliability traits. The density and hazard functions of the NIWXG distribution have adjustable shapes as well. We used the four non-Bayesian accredited approaches of ML, LS, WLS, CVM, and Bayesian estimation under squared error loss function and LINEX loss function to estimate the NIWXG parameters. Based on the relative absolute bias and RMSE criterion, we came to the conclusion that Bayesian estimation methods are more efficient than non-Bayesian estimation methods ML, LS, WLS, and CVM for the majority of NIWXG distribution parameter values.

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