



Investigating Metamaterial Properties Using Transmission Line Matrix Method

{H. Hamza, M.A. Abdalla, M.H. Azeem and A.A. Mitkees}*

Abstract: Split Ring Resonators (SRRs) are widely used to design metamaterial structures. These structures when excited by suitable electromagnetic fields have resonance behavior and show unusual properties near the resonance frequency region. In this paper, Transmission Line Matrix Method (TLM) is used as a new, efficient and accurate time domain method using the 3D Symmetrical Condensed Node (SCN) to study, design and simulate SRRs as a metamaterial structure. This approach is done by using a modified written FORTRAN code. Results of TLM verified with those obtained by measured data of the fabricated structure and comparison showed great matching between results.

Key words: Transmission Line Matrix Method, 3D SCN, Metamaterial

1. Introduction:

Numerical electromagnetic computation is the theory and practice of solving electromagnetic field problems using computers. While experimental and analytical (exact) solutions remain the two traditional pillars of science and engineering; numerical modeling and simulation represent the third pillar that support, complement, and sometimes replaces them. The experimental techniques are expensive and time consuming but still widely used. The analytical solutions are not applicable in a general case, while numerical simulators offer the key to comprehensive solutions of Maxwell's equations using different approaches with the availability of high performance computers. Development of electromagnetic field simulators makes considerable efforts to ensure that users can solve electromagnetic problems without prior knowledge of the numerical method used in their tool. Nevertheless, a user who knows the fundamental properties and characteristics of the method implemented in a simulation tool will be better prepared to exploit its full capabilities, achieve reliable results more quickly, and avoid errors and pitfalls that occur when the limitations of a particular method are ignored.

Split Ring Resonators are well-known artificial magnetic metamaterial that are widely used to design metamaterial structures. SRR consists of two concentric metal rings separated by a gap, each with a split at opposite sides placed over a ground plane, which was introduced by Pendry et al. [1] and experimentally confirmed by Smith et al. [2].

In this paper, TLM was proven to be a powerful method in modeling such SRR geometries. The paper explores the capability of an own written FORTRAN code extracted from the solution of TLM [3-5] to deal with such new technology [6, 7] that is; the metamaterial structures.

* Egyptian Armed Forces, Egypt

SRR acts as Radar Absorbing Material (RAM) that is used for absorbing incident electromagnetic waves and reducing the Radar Cross Section (RCS). The structure proposed here was designed, simulated using TLM, manufactured and measured in two cases; normal and oblique electromagnetic wave incidence.

2. Implementation of Metamaterial using 3-D SCN TLM

TLM method is a numerical technique for solving field problems using circuit equivalence. It is based on the analogy between propagation of electromagnetic waves described by Maxwell's equations and the equations describe electric impulses traveling in a network of transmission lines.

By this technique, a medium with its electromagnetic properties are modeled by substituting the field space by interconnecting unitary circuits or cells of transmission lines forming a so called TLM node. Voltage pulses propagate from node to node in the mesh as a substitution for the actual electric and magnetic fields in the original medium. It is worth noting that 3D-SCN TLM used in this computer code has an advantage over other methods, like Finite Difference Time Domain (FDTD), for example, which defines each component at slightly different points and even at a different time. So the 3D-SCN is especially interesting for modeling non-homogeneous media in which the position of precise transitions between different homogeneous materials is required.

A 3D-SCN in its simplest form consists of three shunt nodes connected with three series nodes Fig. 1. The voltages at the three shunt nodes represent the three components of the electric field while the currents of the series nodes represent the three components of the magnetic field. To simulate the electromagnetic properties of the media, the node will have another (6) additional ports (stubs), where open circuit stub add capacitance to the node (to represent permittivity) while short circuit stub add inductance to the node (to represent permeability). Finally lossy stubs are present to simulate losses in the three coordinates. Thus the scattering matrix takes the form: $V^r = SV^i$, where, S is a 18x18) matrix for lossless case.

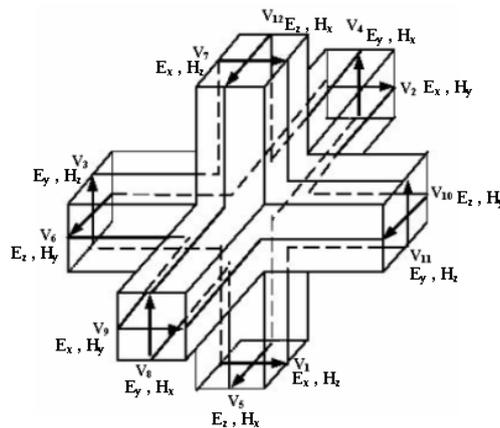


Fig. 1 Ports & field distribution in homogenous, lossless SCN

Applying the TLM simulation technique for an infinite structure consists of infinite periodic arrays of SRR above a ground plane of substrate, acting as an electromagnetic absorber, the scattered incident impulse along the structure has been traced as in (1).

$$S = \begin{bmatrix}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
 1 & a_{yx} & b_{yx} & d_{yx} & 0 & 0 & 0 & 0 & 0 & b_{yx} & 0 & -d_{yx} & c_{yx} & g_x & 0 & 0 & 0 & i_{yx} \\
 2 & b_{zx} & a_{zx} & 0 & 0 & 0 & d_{zx} & 0 & 0 & c_{zx} & -d_{zx} & 0 & b_{zx} & g_x & 0 & 0 & 0 & -i_{xz} \\
 3 & d_{xy} & 0 & a_{xy} & b_{xy} & 0 & 0 & 0 & b_{xy} & 0 & 0 & c_{xy} & -d_{xy} & 0 & g_y & 0 & 0 & -i_{xy} \\
 4 & 0 & 0 & b_{zy} & a_{zy} & d_{zy} & 0 & -d_{zy} & c_{zy} & 0 & 0 & b_{zy} & 0 & 0 & g_y & 0 & i_{zy} & 0 \\
 5 & 0 & 0 & 0 & d_{yz} & a_{yz} & b_{yz} & c_{yz} & -d_{yz} & 0 & b_{yz} & 0 & 0 & 0 & 0 & g_z & -i_{zy} & 0 \\
 6 & 0 & d_{xz} & 0 & 0 & b_{xz} & a_{xz} & b_{xz} & 0 & -d_{xz} & c_{xz} & 0 & 0 & 0 & 0 & g_z & 0 & i_{xz} \\
 7 & 0 & 0 & 0 & -d_{yz} & c_{yz} & b_{yz} & a_{yz} & d_{yz} & 0 & b_{yz} & 0 & 0 & 0 & 0 & g_z & i_{zy} & 0 \\
 8 & 0 & 0 & b_{zy} & c_{zy} & -d_{zy} & 0 & d_{zy} & a_{zy} & 0 & 0 & b_{zy} & b_{zy} & 0 & g_y & 0 & -i_{zy} & 0 \\
 9 & b_{zx} & c_{zx} & 0 & 0 & 0 & -d_{zx} & 0 & 0 & a_{zx} & d_{zx} & 0 & 0 & g_x & 0 & 0 & 0 & i_{xz} \\
 10 & 0 & -d_{xz} & 0 & 0 & b_{xz} & c_{xz} & b_{xz} & 0 & d_{xz} & a_{xz} & 0 & 0 & 0 & 0 & g_z & 0 & -i_{xz} \\
 11 & -d_{xy} & 0 & c_{xy} & b_{xy} & 0 & 0 & 0 & b_{xy} & 0 & 0 & a_{xy} & d_{xy} & 0 & g_y & 0 & 0 & i_{xy} \\
 12 & c_{yx} & b_{yx} & -d_{yx} & 0 & 0 & 0 & 0 & 0 & b_{yx} & 0 & d_{yx} & a_{yx} & g_x & 0 & 0 & 0 & -i_{yx} \\
 13 & e_{yx} & e_{zx} & 0 & 0 & 0 & 0 & 0 & 0 & e_{zx} & 0 & 0 & e_{yx} & h_x & 0 & 0 & 0 & 0 \\
 14 & 0 & 0 & e_{xy} & e_{zy} & 0 & 0 & 0 & 0 & e_{zy} & 0 & 0 & e_{xy} & 0 & 0 & h_y & 0 & 0 \\
 15 & 0 & 0 & 0 & 0 & e_{yz} & e_{xz} & e_{yz} & 0 & 0 & 0 & 0 & e_{xz} & 0 & 0 & 0 & h_z & 0 \\
 16 & 0 & 0 & 0 & f_x & -f_x & 0 & f_x & -f_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & j_x & 0 \\
 17 & 0 & -f_y & 0 & 0 & 0 & f_y & 0 & 0 & f_y & -f_y & 0 & 0 & 0 & 0 & 0 & 0 & j_y \\
 18 & f_z & 0 & -f_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & f_z & -f_z & 0 & 0 & 0 & 0 & j_z
 \end{bmatrix} \quad (1)$$

The variables in the scattering matrix in (1), (a through j), are the scattering coefficients around the 3D SCN TLM and can be calculated after derivation as:

$$\begin{aligned}
 a_{pq} &= \frac{-y_p}{2(4+y_p)} + \frac{z_q}{2(4+z_q)} & b_p &= \frac{4}{2(4+y_p)} & c_{pq} &= \frac{-y_p}{2(4+y_p)} - \frac{z_q}{2(4+z_q)} \\
 d_q &= \frac{4}{2(4+z_p)} & e_p &= b_p & f_q &= z_q d_q \\
 g_p &= y_p b_p & h_p &= \frac{y_p - 4}{y_p + 4} & i_q &= d_q & j_q &= \frac{4 - z_q}{4 + z_q}
 \end{aligned} \quad (2)$$

3. Modeling and Results

The metamaterial radar absorber was designed using periodic array of rectangular shaped SRR. The circuit is printed over a lossy FR4 substrate with relative dielectric constant $\epsilon_r = 4.4$, a dielectric loss tangent, $\tan \delta = 0.02$, and thickness, $h = 1.6$ mm.

3.1 SRR Model

The designed model is a symmetric absorber of periodic distance 5 mm. The inner rectangular has an inner diameter ($2a=1$ mm), the two rings separation is ($d=0.3$ mm), all the lines thickness are ($w=0.25$ mm), and the gap in both inner and outer ring is ($g=0.5$ mm). The structure dimensions were selected to achieve radar absorption at X- band, Fig. 2.

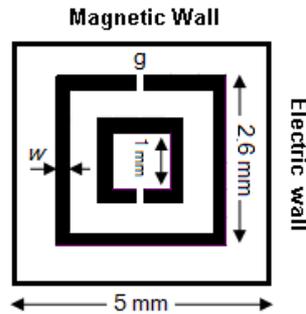


Fig. 2 A unit sample of the periodic SRR radar absorber

The periodicity in TLM was simulated by inserting the metamaterial structure into a 3D model of perfect electric and magnetic conductors and surrounded in direction of propagation by Perfect Matching Layer as shown in Fig. 3.

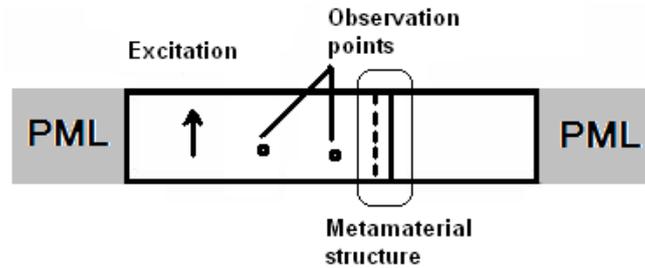


Fig. 3 SRR inside a 3-D TLM structure

3.2 Results

Numerical simulations based on iterations; which is done in this paper using the FORTRAN code for 80,000 times to ensure stabilization and accuracy of the solution. In order to test the manufactured RAM, the simulation and measured data are done mainly for three cases; normal incidence ($\theta_{inc}=0^\circ$), oblique incidence of electromagnetic wave at ($\theta_{inc} = 15^\circ$) and ($\theta_{inc} = 30^\circ$). Fig. (4-a through c) illustrate the comparison of simulation and measured data for normal, 15° and 30° incidence angles, respectively.

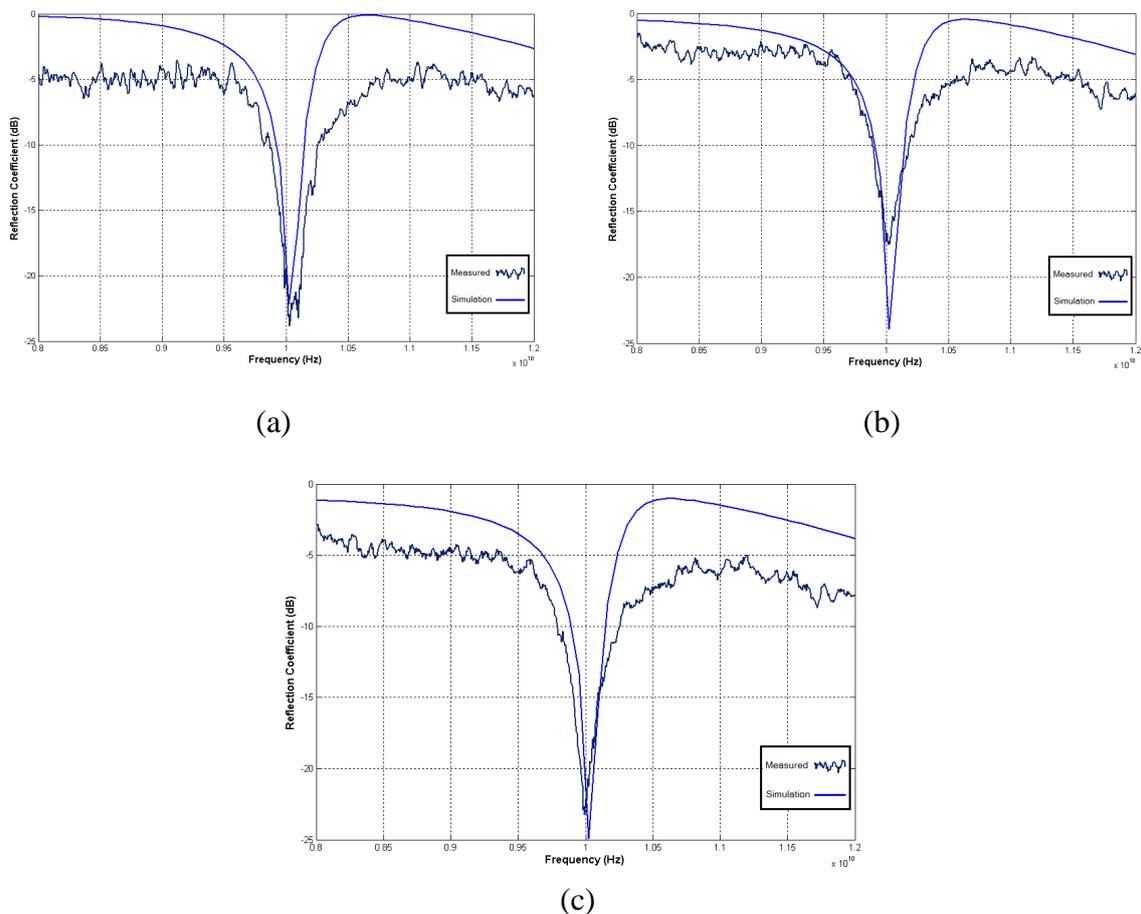


Fig. 4 Reflection coefficient for three cases: a) normal incidence, b) 15° oblique incidence, c) 30° oblique incidence

From comparison point of view, we notice that the measured results can fulfill better than (-20 dB) reflection coefficient which has great matching with the simulated results at 10 GHz, approximately, in the three cases with absorption capability over a 10 dB bandwidth equals 5.6%. The difference in level between the measured and simulated results can be explained due to the various reflection and diffractions which could not avoided in measurements. Also, due to the coupling between the two used horns. This may be minimized by using absorber material laminates within the measurement setting.

5. Conclusion

TLM method is used in this paper as an efficient and accurate time domain method to study, design and simulate SRRs. Comparisons showed perfect matching in results, such as estimating the resonance frequency, bandwidth and even the reflection coefficient amplitudes. The desired SRR also showed good absorption coefficient not less than (-20) dB in either cases of normal or oblique incidence.

6. References

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