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Military Technical College, Kobry Elkobbah, Cairo, Egypt
Tel: +(202) 24025292 – 24036138, Fax: +(202) 22621908

Modeling and Simulation of Capacitive Gravitational Accelerometer Based Tilt Sensor

{Mohammed I. Hosni*, Ayman M. Mokhtar†, Samy Ghoniemy}‡

Abstract: Physical quantities can be simulated using Matlab Simulink and Matlab Simscape. In this paper the behavioral analysis of capacitive gravitational accelerometer based tilt sensor for both mechanical transducer and interface circuits is modeled and simulated. The output signal from each stage of the proposed tilt sensor circuit was measured and displayed for an input tilt angles ranging from -15° to 15°. The simulated results showed a good agreement with the published results

Keywords: Tilt sensor, capacitive accelerometer, Matlab Simulink, Matlab Simscape.

1. Introduction

Tilt sensors play an important role for measuring the angle of inclination of an object with respect to a reference axis. These sensors have a significant role in many applications such as avionics, automobiles, robot control, navigation system, phones, and military applications .Tilt sensors can be used in measuring tilt angles with respect to single or multi axes [1, 2]. In this paper measuring of inclination angle with respect to single axis reference is considered. Accelerometer can be used as a tilt sensor "that is when the gravity is the measured acceleration over all the time". When tilt occurs in a single axis with certain angle, the output voltage from accelerometer is changed corresponding to that angle as shown in figure 1[3].

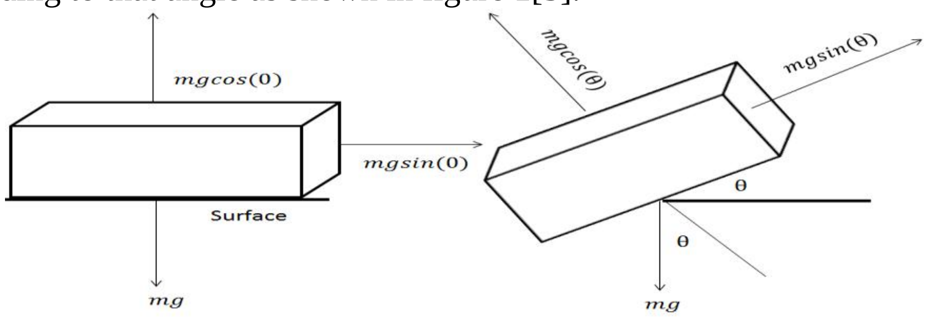

Figure 1 Single axis used for tilt sensing

There are two types of accelerometer based tilt sensors piezoresistive and capacitive accelerometers. In piezoresistive accelerometer the resistance is modified according to the induced forces that are occurred due to the change in acceleration. While in capacitive accelerometer the capacitance is changed according to the change in acceleration. The piezoresistive tilt sensor has a nonlinear behavior at higher temperatures[4].

In this paper we will present modeling and simulation of capacitive accelerometer.

* mohamed.hosni@mtc.edu.eg
† ayman.mokhtar@mtc.edu.eg
‡‡ Egyptian Armed Forces, Egypt.

2. Proposed Tilt Sensor Model

Our proposed tilt sensor model was introduced based on accelerometer that is presented in [3-5] and is shown in figure 2. Brief description for each block will be introduced later in the paper.

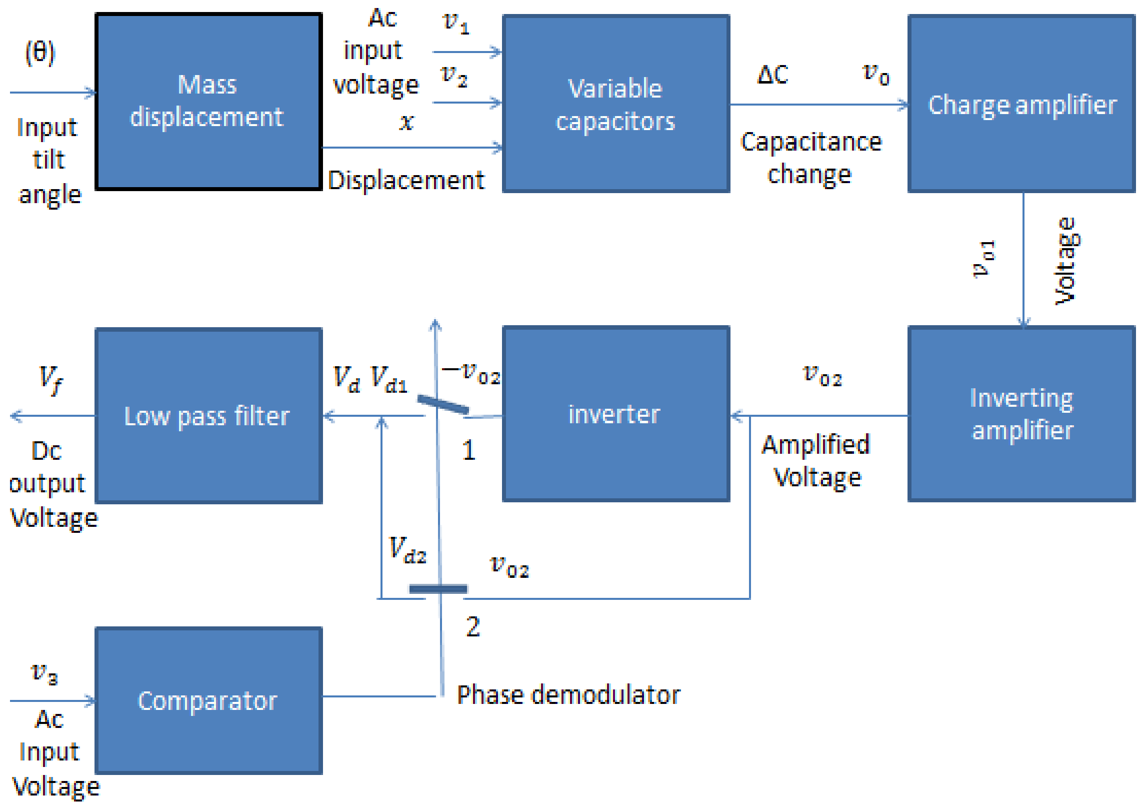

Figure 2 Proposed tilt sensor block diagram

The mechanical part of the tilt sensor consists of a spring and a dumper that are attached to a specific mass with a movable plate that is placed between two fixed plates constituting two variable capacitors as shown in figure 3 [6].

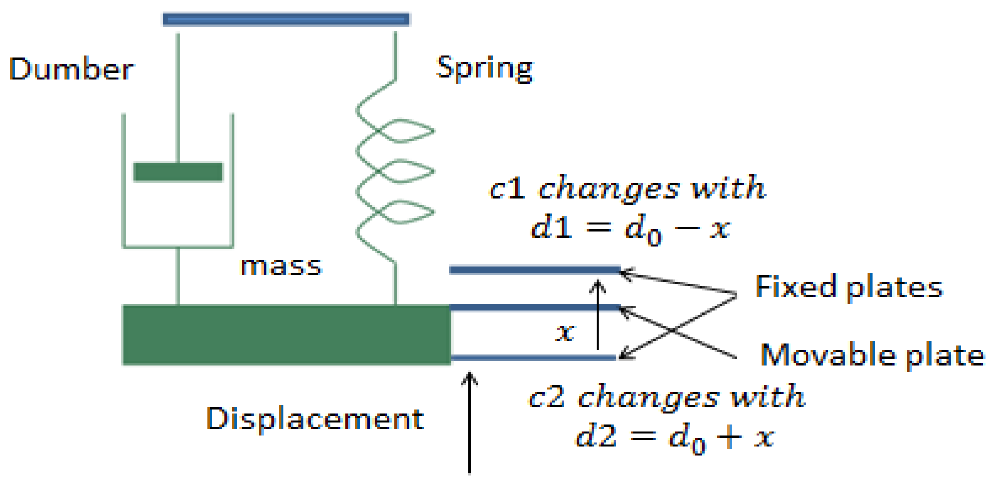

Figure 3 Tilt sensor structure

When applying tilt angle, the transfer function of mass displacement can be expressed according to equation (1)[4, 6]. This equation was used in matlab Simulink to model the mechanical part of the tilt sensor as shown in figure 4.

$$\frac{\mathbf{x(s)}}{\mathbf{g \sin\theta}} = \frac{\mathbf{1}}{\mathbf{s^2 + \frac{b}{m}s + (\frac{k}{m} - Fel)}} = \frac{\mathbf{1}}{\mathbf{s^2 + \frac{w_n}{Q}s + {w_n}^2}} = \frac{\mathbf{1}}{\mathbf{s^2 + 2\xi w_n s + {w_n}^2}} \quad \textbf{(1)}$$

Where: $\mathbf{x}$ is mass displacement output in $\mathbf{m}$.
$g = 9.8\ m/s^2$ is earth gravity, θ is tilt angle in degree, b is dumper coeffient in Nsm^{-1}, m is mass weight in Kg, k is spring constant in Nm^{-1}, Fel is elastic force difference according to AC source magnitude voltage $Fel = \frac{\mathcal{E}_0 AV^2}{{d_0}^3}x$ in N, w_n is resonant frequency in rad/sec, Q is quality factor and ξ is damping factor.

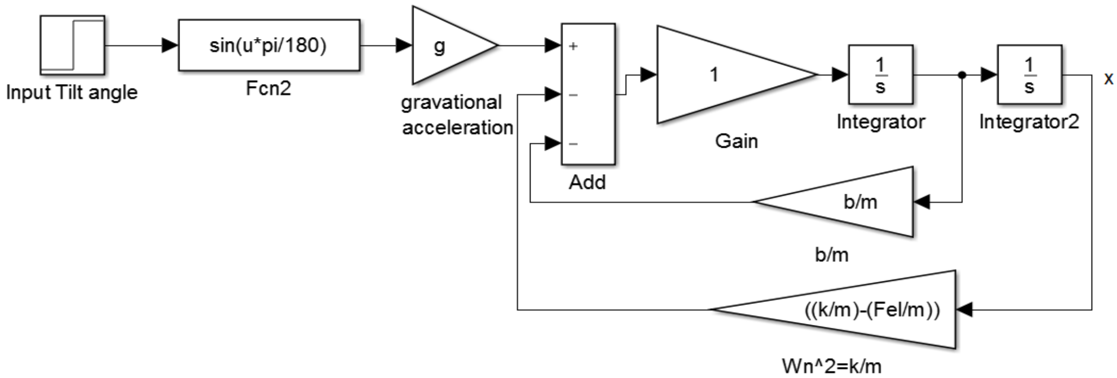

Figure 4 Tilt sensor mechanical part model

When: $\xi = 0$ system is undamped, $0 < \xi < 1$ system is over damped, $\xi = 1$ system is critical damped and $\xi > 1$ system is under damped.
When mass displacement occurs, a change in the position of mass plate produced with displacement x causing a change in the separation d_0 between the fixed plates of capacitor to $d_0 \pm x$, hence the capacitance of the two capacitors will be changed according to equation (2)[4, 5].

$$C_1 = \frac{\mathcal{E}_0 \mathcal{E}_r A}{d_0 - x} \ \& \ C_2 = \frac{\mathcal{E}_0 \mathcal{E}_r A}{d_0 + x} \quad (2)$$

Where: $\boldsymbol{\mathcal{E}_0}$ is air permittivity, $\boldsymbol{\mathcal{E}_r}$ Capacitor material permittivity, **A** capacitor plate area in $\mathbf{mm^2}$, $\mathbf{d_0}$ initial distance between each capacitor plates in $\mathbf{mm}$ and $\mathbf{x}$ is displacement according to tilt angle.
Using these variable capacitors in electronic circuit shown in figure 5 with two input ac voltages $v_1 = V\sin(wt) \& v_2 = -V\sin(wt)$ the output voltage v_0 from this circuit can be expressed as in equation (3) [5].

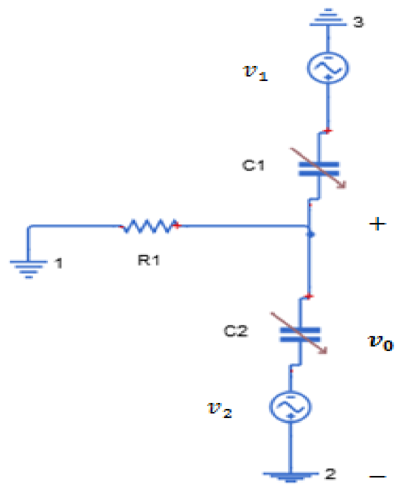

Figure 5 Variable capacitors for tilt sensor

$$v_0(s) = \frac{v_1(s)sC_1 + v_2(s)sC_2}{s(C_1 + C_2) + {}^{1}/_{R_1}} \qquad \textbf{(3)}$$

The charge amplifier is used to convert the capacitive signal into electronic signal as shown in figure 6. The output voltage from charge amplifier $\mathbf{v_{01}}$ can be expressed in equation (4)[4, 5].

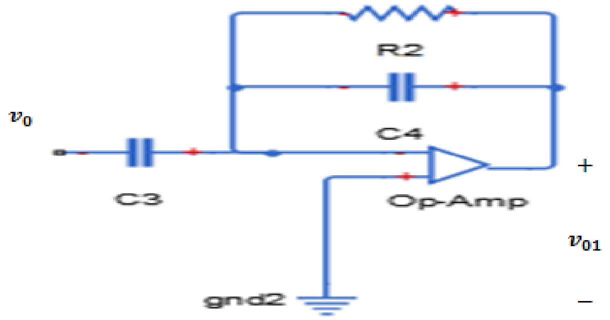

Figure 6 Charge amplifier

$$v_{01}(s) = -v_0(s)\frac{sC_3R_2}{1 + SC_4R_2} \qquad \textbf{(4)}$$

The inverting amplifier is used to amplify and invert the applied input signal to it as shown in figure 7.The output voltage $v_{01}(s)$ is inverted and amplified with gain $\frac{R_4}{R_3}$ as shown in equation (5)[5].

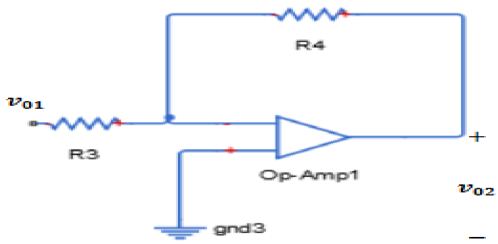

Figure 7 Inverting amplifier

$$v_{02} = -v_{01}\frac{R_4}{R_3} \qquad \textbf{(5)}$$

The inverter is used to invert the applied input signal to it as shown in figure 8. The output voltage from it is $\mathbf{v_{03}(s) = -v_{02}(s)}$ **[5].**

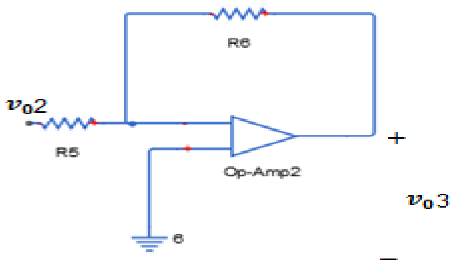

Figure 8 Inverter

The phase demodulator circuit is used to convert the AC voltage to DC voltage by using comparator with input AC voltage $v_3 = V\sin(wt)$ that compares the output voltage from the inverting amplifier and inverter with voltage v_3, the result is used to switch on/off the switches 1 and 2 as to generate voltages V_{d1} and V_{d2} forming voltage V_d as shown in figure 2.This low pass filter circuit shown in figure 9 is used to filter any AC voltage from phase demodulator generating final DC voltage V_f.

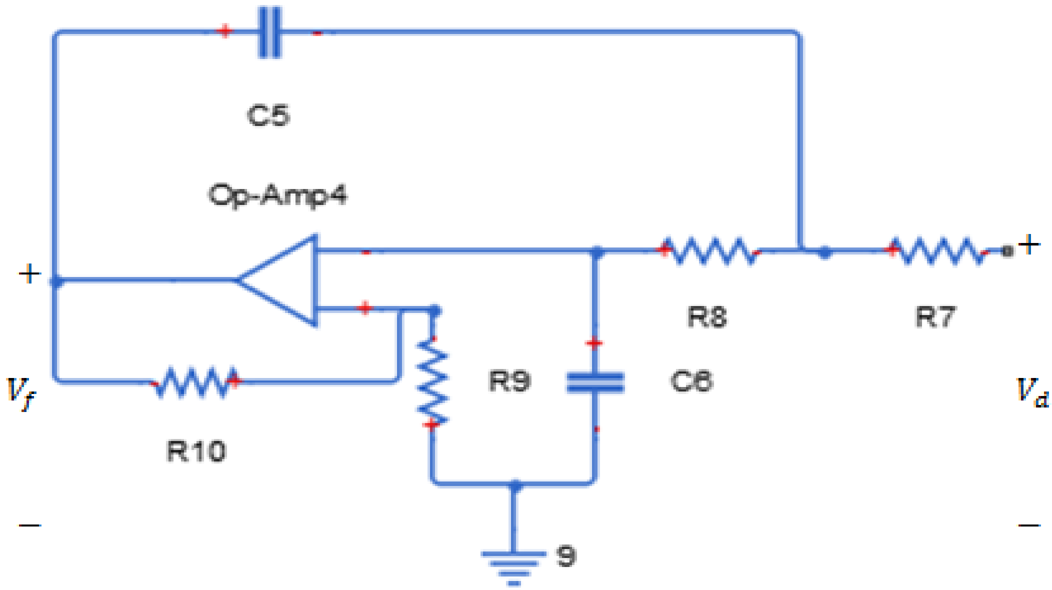

Figure 9 Low pass filter circuit

$$\mathbf{V_f = 2\pi k_a k_A \frac{\mathcal{E}_0 \mathcal{E}_r AV}{C_4 {d_0}^2} x} \qquad (6)$$

Where: $\boldsymbol{k_a = -\frac{R_4}{R_3}}$ & $\boldsymbol{k_A = 1 + \frac{R_{10}}{R_9}}$

The proposed tilt sensor circuit shown in figure 10 is simulated using matlab simscape depending on the parameters shown in table 1[4,5].

Table 1 Tilt sensor circuit parameters

m	k	b	A	d_0	$\mathcal{E}_0$	$\mathcal{E}_r$	V	w	R_1	R_2	R_3
4.313 mg	149.94 Nm^{-1}	0.05545 NSm^{-1}	12 μm	10 μm	8.85 pFm^{-1}	1 Fm^{-1}	0.5 V	100 KHz	250 $M\,\Omega$	820 $K\,\Omega$	1 $K\,\Omega$
R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	C_3	C_4	C_5	C_6	
10 $K\Omega$	10 $K\Omega$	10 $K\Omega$	180 $K\Omega$	180 $K\Omega$	10 $K\Omega$	5.6 $K\Omega$	1 nF	22 pF	1.5 nF	1.5 nF	

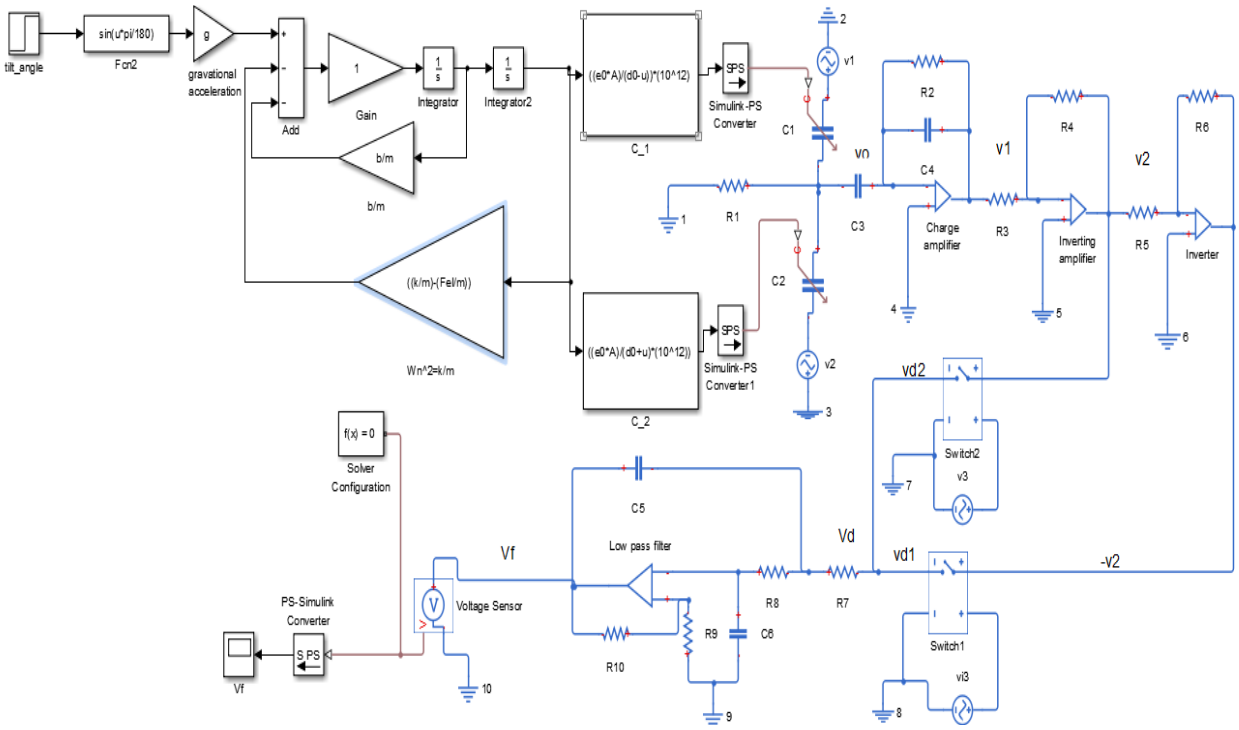

Figure 10 Proposed integrated tilt sensor circuit

3. Results and Discussion

The output signals from each above stages of the proposed tilt sensor circuit for $\pm 10°$ tilt angle are shown in figure 11

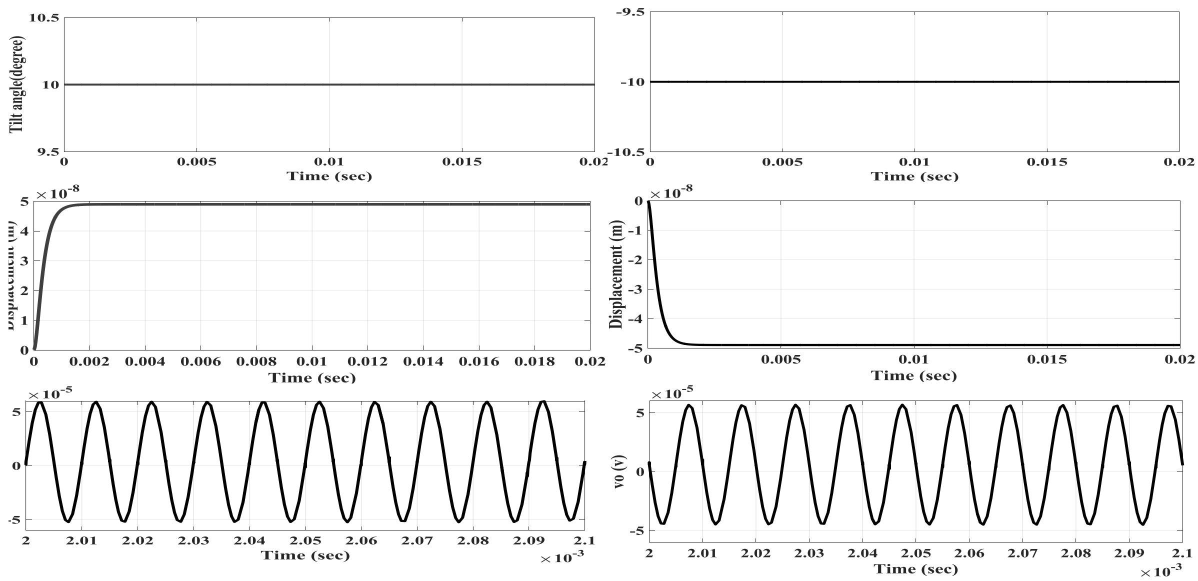

Figure 11-a Output signals from different stages from tilt sensor circuit for ±10

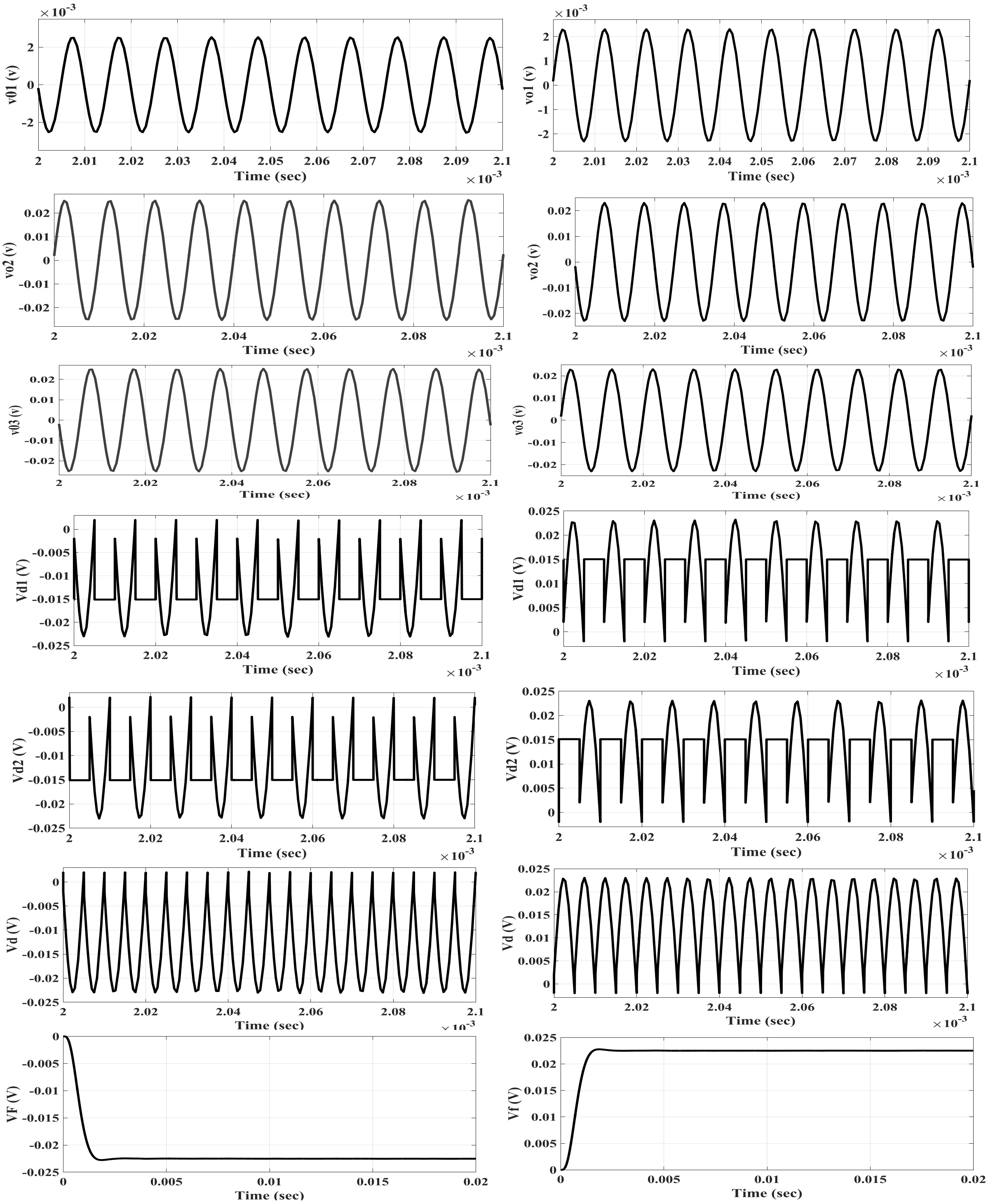

Figure 12-b Output signals from different stages from tilt sensor circuit for ±10°

When applying different steps input tilt angles ranging from $\mathbf{-15}°$ to, the output voltage decreases linearly as shown in figure 12. The noise for this sensor was calculated at room temperature $\mathbf{27}$℃ according to equation (7) [6]. Figure 13 shows the output voltage with calculated noise which is equal to $\mathbf{7.0259\ \mu g/\sqrt{Hz}}$ and bias voltage which is equal to $\mathbf{0.1\ V}$ are added to the voltage in figure 11.

$$\mathbf{Noise} = \sqrt{\frac{\mathbf{4K_b T\ w_r}}{\mathbf{mQ}}} \qquad \mathbf{(7)}$$

Where: $\boldsymbol{K_b}$ is Boltzmann constant ($\mathbf{1.38 * 10^{-23} J/k}$) and $\boldsymbol{T}$is room temperature in$\boldsymbol{K}$.

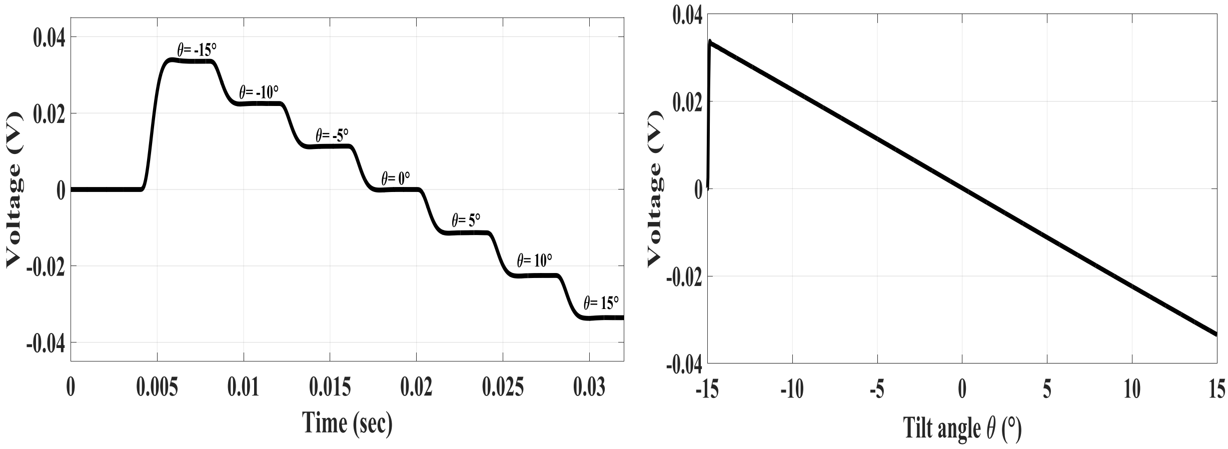

Figure 13 Output voltages from proposed tilt sensor circuit

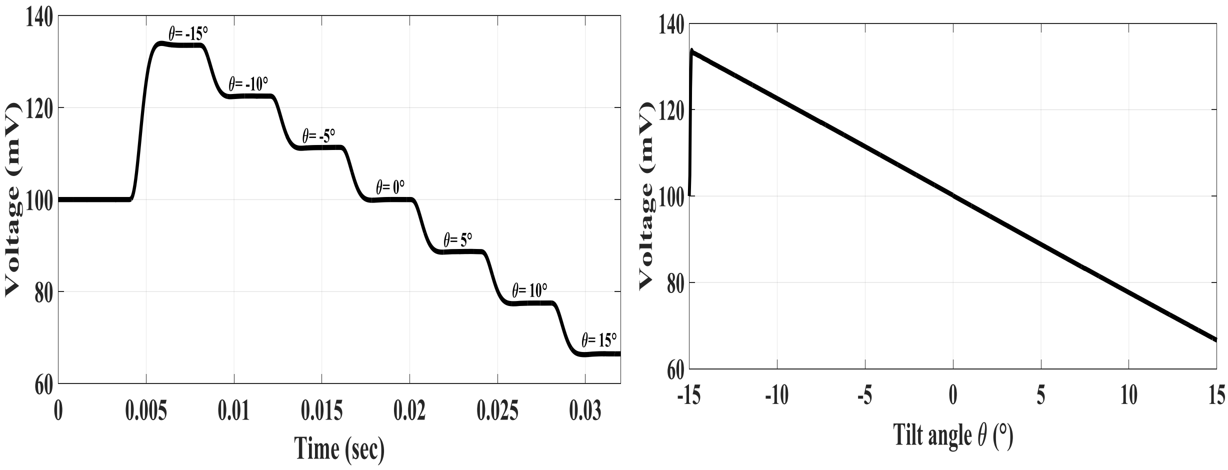

Figure14 Tilt sensor output voltages after adding noise and bias voltage

4. Conclusions

In this paper single axis gravitational capacitive accelerometer based tilt sensor is presented. The proposed tilt sensor circuit consists of two parts mechanical part and capacitive electronic circuit part. The mechanical part is modeled and simulated using matlab Simulink. The capacitive electronic circuit with different stages is modeled and simulated using matlab Simscape. The results from the overall tilt sensor circuit show a good agreement with the results presented in reference [5].

5. References

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