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ORIGINAL ARTICLE

Slightly double fuzzy continuous functions

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Abstract In this paper, we introduce the concepts of slightly double fuzzy continuous functions and slightly generalized double fuzzy semicontinuous functions in double fuzzy topological spaces. Several interesting properties and characterizations are introduced and discussed. Furthermore, the relationships among the new concepts are introduced and established with some interesting counter examples.

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1. Introduction

The concept of fuzzy topological spaces was introduced by Chang [1]. In Chang's fuzzy topological spaces, each fuzzy set is either open or closed. These spaces and its generalization are later developed by Goguen [2], who replaced the closed interval $[0,1]$ by more general lattice L. On the other hand, Kubiak and Šostak's [3,4] by the independent and parallel generalization, made topology itself fuzzy besides their dependence on fuzzy set in 1985.

As a generalization to fuzzy sets, the notion of intuitionistic fuzzy sets was introduced by Atanassove [5–10]. After that Çoker [11] defined intuitionistic fuzzy topology in Chang's sense. Later, Samanta and Mondal [12] introduced the notion of intuitionistic gradation of openness of fuzzy sets. The term "intuitionistic" is still used in literature until 2005, when Gutierrez Garcia and Rodabaugh [13] concluded that the most appropriate work under the name "double".

In 1980, Jain [14] introduced the notion of slightly continuous functions. Recently, Nour [15] defined slightly semicontinuous functions as a weak form of slight continuity and investigated its properties. On the other hand, Takashi Noiri [16] introduced the concept of slightly β-continuous functions. M. Sudha et al. [17] introduced slightly fuzzy ω-continuous functions. Also in 2004, Ekici and Caldas [18] introduced the notion of slightly γ-continuity (slightly b-continuity). After that slightly fuzzy continuous functions are introduced by Sudha et al. [19].

In this paper, the concepts of slightly double fuzzy continuous functions and slightly generalized double fuzzy semicontinuous functions are introduced. Several interesting properties and characterizations are introduced and discussed.

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Furthermore, the relationships among the concepts are introduced and established with some interesting counter examples.

2. Preliminaries

Throughout this paper, X will be a non-empty set, I is the closed unit interval $[0,1], I_0=(0,1]$ and $I_1=[0,1)$. The set of all fuzzy sets on X is denoted by I^X. $Pt(X)$ is the family of all fuzzy points in X. By $\underline{0}$ and $\underline{1}$, we denote the smallest and the greatest fuzzy sets on X. For a fuzzy set $\lambda \in I^X, \underline{1}-\lambda$ denotes its complement. Given a function $f: X \longrightarrow Y, f(\lambda)$ and $f^{-1}(\lambda)$ defined the direct image and the inverse image of f, are defined by $f(\lambda)(y)=\bigvee_{f(x)=y}\lambda(x)$ and $f^{-1}(\mu)(x)=\mu(f(x))$ for each $\lambda \in I^X, \mu \in I^Y$ and $x \in X$, respectively. All other notations are standard notations of fuzzy set theory.

Definition 2.1. *[12,13]*. A double fuzzy topology (τ,τ^*) on X is a pair of maps $\tau,\tau^*: I^X \to I$, which satisfies the following properties:

(O1) $\tau(\lambda) \leqslant \underline{1}-\tau^*(\lambda)$ for each $\lambda \in I^X$.
(O2) $\tau(\lambda_1 \wedge \lambda_2) \geqslant \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \leqslant \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$.
(O3) $\tau(\bigvee_{i\in\Gamma}\lambda_i) \geqslant \bigwedge_{i\in\Gamma}\tau(\lambda_i)$ and $\tau^*(\bigvee_{i\in\Gamma}\lambda_i) \leqslant \bigvee_{i\in\Gamma}\tau^*(\lambda_i)$ for each $\lambda_i \in I^X, i \in \Gamma$.

The triplet (X,τ,τ^*) is called a double fuzzy topological spaces (dfts, for short). A fuzzy set λ is called an (r,s)-fuzzy open ((r,s)-fo, for short) if $\tau(\lambda) \geqslant r$ and $\tau^*(\lambda) \leqslant s$. A fuzzy set λ is called an (r,s)-fuzzy closed ((r,s)-fc, for short) set iff $\underline{1}-\lambda$ is an (r,s)-fo set. Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be two dfts's. A function $f: X \to Y$ is said to be a double fuzzy continuous iff $\tau_1(f^{-1}(v)) \geqslant \tau_2(v)$ and $\tau_1^*(f^{-1}(v)) \leqslant \tau_2^*(v)$ for each $v \in I^Y$.

Theorem 2.1. *[20,21]. Let (X,τ,τ^*) be a dfts. Then for each $r \in I_0, s \in I_1$ and $\lambda \in I^X$, we define an operator $C_{\tau,\tau^*}: I^X \times I_0 \times I_1 \to I^X$ as follows:*

$$C_{\tau,\tau^*}(\lambda,r,s)=\bigwedge\{\mu \in I^X | \lambda \leqslant \mu, \tau(\underline{1}-\mu) \geqslant r, \tau^*(\underline{1}-\mu) \leqslant s\}.$$

For $\lambda,\mu \in I^X, r_1,r_2 \in I_0$ and $s_1,s_2 \in I_1$, the operator C_{τ,τ^*} satisfies the following statements:

(C1) $C_{\tau,\tau^*}(\underline{0},r,s)=\underline{0}$,
(C2) $\lambda \leqslant C_{\tau,\tau^*}(\lambda,r,s)$,
(C3) $C_{\tau,\tau^*}(\lambda,r,s) \vee C_{\tau,\tau^*}(\mu,r,s)=C_{\tau,\tau^*}(\lambda \vee \mu,r,s)$,
(C4) $C_{\tau,\tau^*}(\lambda,r_1,s_1) \leqslant C_{\tau,\tau^*}(\lambda,r_2,s_2)$ if $r_1 \leqslant r_2$ and $s_1 \geqslant s_2$,
(C5) $C_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda,r,s),r,s)=C_{\tau,\tau^*}(\lambda,r,s)$.

Theorem 2.2. *[20,21]. Let (X,τ,τ^*) be a dfts. Then for each $r \in I_0, s \in I_1$ and $\lambda \in I^X$, we define an operator $I_{\tau,\tau^*}: I^X \times I_0 \times I_1 \to I^X$ as follows:*

$$I_{\tau,\tau^*}(\lambda,r,s)=\bigvee\{\mu \in I^X | \mu \leqslant \lambda, \tau(\mu) \geqslant r, \tau^*(\mu) \leqslant s\}.$$

For $\lambda,\mu \in I^X, r,r_1,r_2 \in I_0$ and $s,s_1,s_2 \in I_1$, the operator I_{τ,τ^*} satisfies the following statements:

(I1) $I_{\tau,\tau^*}(\underline{1}-\lambda,r,s)=\underline{1}-C_{\tau,\tau^*}(\lambda,r,s)$,
(I2) $I_{\tau,\tau^*}(\underline{1},r,s)=\underline{1}$,
(I3) $I_{\tau,\tau^*}(\lambda,r,s) \leqslant \lambda$,
(I4) $I_{\tau,\tau^*}(\lambda,r,s) \wedge I_{\tau,\tau^*}(\mu,r,s)=I_{\tau,\tau^*}(\lambda \wedge \mu,r,s)$,
(I5) $I_{\tau,\tau^*}(\lambda,r_1,s_1) \geqslant I_{\tau,\tau^*}(\lambda,r_2,s_2)$ if $r_1 \leqslant r_2$ and $s_1 \geqslant s_2$,
(I6) $I_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda,r,s),r,s)=I_{\tau,\tau^*}(\lambda,r,s)$,
(I7) If $I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda,r,s),r,s)=\lambda$, then $C_{\tau,\tau^*}(I_{\tau,\tau^*}(\underline{1}-\lambda,r,s),r,s)=\underline{1}-\lambda$.

Definition 2.2. Let (X,τ,τ^*) be a dfts. For each $\lambda \in I^X, r \in I_0$ and $s \in I_1$.

(1) A fuzzy set λ is called an (r,s)-fuzzy semi open [22] (briefly, (r,s)-fso) if $\lambda \leqslant C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda,r,s),r,s)$. λ is called an (r,s)-fuzzy semi closed (briefly, (r,s)-fsc) iff $\underline{1}-\lambda$ is an (r,s)-fuzzy semi open set.
(2) An (r,s)-fuzzy semi closure of λ [23] is defined by $SC_{\tau,\tau^*}(\lambda,r,s)=\bigwedge\{\mu \in I^X | \lambda \leqslant \mu$ and μ is (r,s)-fsc$\}$.
(3) A fuzzy set λ is called an (r,s)-generalized fuzzy closed [24] (briefly, (r,s)-gfc) if $C_{\tau,\tau^*}(\lambda,r,s) \leqslant \mu, \lambda \leqslant \mu, \tau(\mu) \geqslant r$ and $\tau^*(\mu) \leqslant s$. λ is called an (r,s)-generalized fuzzy open (briefly, (r,s)-gfo) iff $\underline{1}-\lambda$ is (r,s)-gfc set.
(4) A fuzzy set λ is called an (r,s)-generalized fuzzy semi closed [23] (briefly, (r,s)-gfsc) if $C_{\tau,\tau^*}(\lambda,r,s) \leqslant \mu, \lambda \leqslant \mu$ and μ is (r,s)-fso set. λ is called an (r,s)-generalized fuzzy semi open (briefly, (r,s)-gfso) iff $\underline{1}-\lambda$ is (r,s)-gfsc set.
(5) An (r,s)-generalized fuzzy semi-closure of λ [23] is defined by $GSC_{\tau,\tau^*}(\lambda,r,s)=\bigwedge\{\mu \in I^X | \lambda \leqslant \mu$ and μ is (r,s)-gfsc$\}$.

Remark 2.1. Let (X,τ,τ^*) be a dfts, $\lambda \in I^X, r \in I_0$ and $s \in I_1$. A fuzzy set λ is called:

(1) (r,s)-fuzzy semi clopen set (briefly, (r,s)-fsco) iff λ is (r,s)-fso set and (r,s)-fsc set.
(2) (r,s)-generalized fuzzy semi clopen set (briefly, (r,s)-gfsco) iff λ is (r,s)-gfso set and (r,s)-gfsc set.

Definition 2.3. Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be a dfts. A function $f: (X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is called:

(1) double fuzzy open [12] (briefly, dfo) if $\tau_2(f(\lambda)) \geqslant \tau_1(\lambda)$ and $\tau_2^*(f(\lambda)) \leqslant \tau_1^*(\lambda)$ for each $\lambda \in I^X, r \in I_0$ and $s \in I_1$.
(2) generalized double fuzzy continuous [24] (briefly, gdfc) iff $f^{-1}(\mu)$ is (r,s)-gfc set for each $\mu \in I^Y, r \in I_0$ and $s \in I_1$ with $\tau_2(\underline{1}-\mu) \geqslant r$ and $\tau_2^*(\underline{1}-\mu) \leqslant s$.
(3) generalized double fuzzy semicontinuous [23] (briefly, gdfsc) iff $f^{-1}(\mu)$ is (r,s)-gfsc set for each $\mu \in I^Y$ such that $\tau_2(\underline{1}-\mu) \geqslant r$ and $\tau_2^*(\underline{1}-\mu) \leqslant s$.
(4) double fuzzy irresolute [25] (briefly, dfir) if $f^{-1}(\mu)$ is (r,s)-fso set for each (r,s)-fso set $\mu \in I^Y, r \in I_0$ and $s \in I_1$.

Definition 2.4 [22]. Let (X,τ,τ^*) be a dfts. For $\lambda,\mu,\rho \in I^X, \lambda$ and μ are called an (r,s)-fuzzy separated iff for $r \in I_0$ and $s \in I_1$,

$$C_{\tau,\tau^*}(\lambda,r,s) \wedge \mu = C_{\tau,\tau^*}(\mu,r,s) \wedge \lambda = \underline{0}.$$

A fuzzy set λ is called an (r,s)-fuzzy connected if there is not exist (r,s)-fuzzy separated fuzzy sets $\lambda,\mu \in I^X - \{\underline{0}\}$ such that $\rho = \lambda \vee \mu$. A fuzzy set λ is called double fuzzy connected if it is (r,s)-fuzzy connected for all $r \in I_0$ and $s \in I_1$. A triplet (X,τ,τ^*) is called (r,s)-fuzzy connected if $\underline{1}$ is (r,s)-fuzzy connected.

Definition 2.5 [26]. A dfts (X,τ,τ^*) is said to be an (r,s)-fuzzy extremelly disconnected if $\tau(C_{\tau,\tau^*}(\lambda,r,s)) \geqslant r$ and $\tau^*(C_{\tau,\tau^*}(\lambda,r,s)) \leqslant s$, for every $\lambda \in I^X$ with $\tau(\lambda) \geqslant r, \tau^*(\lambda) \leqslant s$.

3. Properties and characterizations of slightly double fuzzy continuous functions

Definition 3.1. Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's. A function $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ is called:

(1) almost ★-double fuzzy continuous function (briefly, a★dfc) if for every $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that $\tau_2(\mu) \geqslant r, \tau_2^*(\mu) \leqslant s$ and $f(\lambda) \leqslant \mu$, there exists $\nu \in I^X$ such that $\tau_1(\nu) \geqslant r, \tau_1^*(\nu) \leqslant s, \lambda \leqslant \nu$ and

$$f(\nu) \leqslant I_{\tau_2,\tau_2^*}\left(C_{\tau_2,\tau_2^*}(\mu,r,s),r,s\right).$$

(2) θ★-double fuzzy continuous function (briefly, θ★dfc) if for every $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that $\tau_2(\mu) \geqslant r, \tau_2^*(\mu) \leqslant s$ and $f(\lambda) \leqslant \mu$, there exists $\nu \in I^X$ such that $\tau_1(\nu) \geqslant r, \tau_1^*(\nu) \leqslant s, \lambda \leqslant \nu$ and

$$f\left(C_{\tau_1,\tau_1^*}(\nu,r,s)\right) \leqslant C_{\tau_2,\tau_2^*}(\mu,r,s).$$

(3) weakly ★-double fuzzy continuous function (briefly, w★dfc) if for every $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that $\tau_2(\mu) \geqslant r, \tau_2^*(\mu) \leqslant s$ and $f(\lambda) \leqslant \mu$, there exists $\nu \in I^X$ such that $\tau_1(\nu) \geqslant r, \tau_1^*(\nu) \leqslant s, \lambda \leqslant \nu$ and

$$f(\nu) \leqslant C_{\tau_2,\tau_2^*}(\mu,r,s).$$

(4) slightly double fuzzy continuous function (briefly, sdfc) if for every $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that μ is (r,s)-fco set and $f(\lambda) \leqslant \mu$, there exists $\nu \in I^X$ such that $\tau_1(\nu) \geqslant r, \tau_1^*(\nu) \leqslant s, \lambda \leqslant \nu$ and

$$f(\nu) \leqslant \mu.$$

Definition 3.2 [19]. Let $(D,\geqslant)$ be a directed set. Let X be an ordinary set and f be the collection of all fuzzy points in X. The function $S: D \to f$ is called a fuzzy net in X. In other words, a fuzzy net is a pair $(S,\geqslant)$ such that S is a function $: D \to f$ and $\geqslant$ directs the domain of S. For $n \in D, S(n)$ is often denoted by S_n and hence a net S is often denoted by $\{S_n : n \in D\}$.

Proposition 3.1. *Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's. For the function $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$, the following statements are equivalent:*

(1) *f is sdfc function.*
(2) $\tau_1(f^{-1}(\nu)) \geqslant r$ and $\tau_1^*(f^{-1}(\nu)) \leqslant s$ for each $\nu \in I^Y, r \in I_0$ and $s \in I_1$ such that ν is (r,s)-fco set.
(3) $f^{-1}(\nu)$ is (r,s)-fco set for each $\nu \in I^Y, r \in I_0$ and $s \in I_1$ such that ν is (r,s)-fco set.
(4) *For each fuzzy set $\lambda \in I^X, r \in I_0, s \in I_1$ and for every fuzzy net $\{S_n : n \in D\}$ which converges to λ, the fuzzy net $\{f(S_n) : n \in D\}$ is eventually in each (r,s)-fco set μ with $f(\lambda) \leqslant \mu$.*

Proof. (1)⇒(2): Let $\nu \in I^Y, r \in I_0$ and $s \in I_1$ such that ν is an (r,s)-fco set and let $\lambda \in I^X$ such that $\lambda \leqslant f^{-1}(\nu)$. Since ν is an (r,s)-fco set with $f(\lambda) \leqslant \nu$. By (1), there exists $\mu \in I^X$ such that $\tau_1(\mu) \geqslant r, \tau_1^*(\mu) \leqslant s, \lambda \leqslant \mu$ and $f(\mu) \leqslant \nu$. Hence $\tau_1(f^{-1}(\nu)) \geqslant r$ and $\tau_1^*(\nu) \leqslant s$.

(2)⇒(3): Clear.

(3)⇒(4): Let $\{S_n : n \in D\}$ be a fuzzy net converging to $\lambda \in I^X$ and let $\mu \in I^Y$ be an (r,s)-fco set such that $f(\lambda) \leqslant \mu$. By using (3), $f^{-1}(\mu)$ is an (r,s)-fco set. Since $\tau_1(f^{-1}(\mu)) \geqslant r$ and $\tau_1^*(f^{-1}(\mu)) \leqslant s$, there exists $\nu \in I^X$ such that $\tau_1(\nu) \geqslant r, \tau_1^*(\nu) \leqslant s, \lambda \leqslant \nu$ and $f(\nu) \leqslant \mu$. Since the fuzzy net $\{S_n : n \in D\}$ converges to $\lambda, S_n \leqslant \lambda$. Now, $S_n \leqslant \lambda \leqslant \nu$. Thus $f(S_n) \leqslant f(\nu) \leqslant \mu$. Hence $\{f(S_n) : n \in D\}$ is eventually in μ.

(4)⇒(1): Suppose that f is not sdfc function. Then for every $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that μ is an (r,s)-fco set and $f(\lambda) \leqslant \mu$, there doesn't exists $\nu \in I^X$ such that $\tau_1(\nu) \geqslant r, \tau_1^*(\nu) \leqslant s, \lambda \leqslant \nu$ and $f(\nu) \leqslant \mu$. Hence $f(S_n) \leqslant \mu$. That is, the fuzzy net $\{f(S_n) : n \in D\}$ isn't eventually in an (r,s)-fco set μ with $f(\lambda) \leqslant \mu$, which is a contradiction. Hence, f is sdfc function. □

Proposition 3.2. *Let $(X,\tau_1,\tau_1^*), (Y,\tau_2,\tau_2^*)$ and (Z,τ_3,τ_3^*) be dfts's. For the functions $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ and $g:(Y,\tau_2,\tau_2^*) \to (Z,\tau_3,\tau_3^*)$, the following statements are satisfied:*

(1) If f and g are sdfc functions, then so is $g \circ f$.
(2) If f is a surjective double fuzzy irresolute and double fuzzy open function and g be any function, then $g \circ f$ is sdfc function iff g is sdfc.

Proof.

(1) Clear.
(2) Suppose that $g \circ f$ is sdfc function, $\lambda \in I^Z, r \in I_0$ and $s \in I_1$ such that λ an (r,s)-fco set. By using Proposition 3.1(2), $\tau_1(f^{-1}(g^{-1}((\lambda)))) = \tau_1((g \circ f)^{-1}(\lambda)) \geqslant r$ and $\tau_1^*(f^{-1}(g^{-1}((\lambda)))) = \tau_1^*((g \circ f)^{-1}(\lambda)) \leqslant s$. Since f is double fuzzy open, $\tau_2(g^{-1}(\lambda)) = \tau_2(f(f^{-1}(g^{-1}(\lambda)))) \geqslant r$ and $\tau_2^*(g^{-1}(\lambda)) = \tau_2^*(f(f^{-1}(g^{-1}(\lambda)))) \leqslant s$. Therefore by Proposition 3.1, g is sdfc function.

Conversely, let $\nu \in I^Z, r \in I_0$ and $s \in I_1$ such that ν an (r,s)-fco set. Since g is sdfc function, $\tau_2(g^{-1}(\nu)) \geqslant r$ and $\tau_2^*(g^{-1}(\nu)) \leqslant s$. Since f is double fuzzy irresolute function, $\tau_1(f^{-1}(g^{-1}(\nu))) = \tau_1((g \circ f)^{-1}(\nu)) \geqslant r$ and $\tau_1^*(f^{-1}(g^{-1}(\nu))) = \tau_1^*((g \circ f)^{-1}(\nu)) \geqslant s$. Therefore by Proposition 3.1, $g \circ f$ is sdfc. □

Proposition 3.3. *Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's and $f:(X,\tau_1,\tau_1^*) \to (Y,\tau_2,\tau_2^*)$ be a function. If (Y,τ_2,τ_2^*) is an (r,s)-fuzzy connected, then f is sdfc function.*

Proof. Let (Y, τ_2, τ_2^*) be an (r,s)-fuzzy connected spaces. Then $\underline{0}$ and $\underline{1}$ are the only (r,s)-fco sets. Since $\tau_1(f^{-1}(\underline{0})) = \tau_1(\underline{0}) \geqslant r, \tau_1^*(f^{-1}(\underline{0})) = \tau_1^*(\underline{0}) \leqslant s$ and $\tau_1(f^{-1}(\underline{1})) = \tau_1(\underline{1}) \geqslant r, \tau_1^*(f^{-1}(\underline{1})) = \tau_1^*(\underline{1}) \leqslant s$, then f is sdfc function (by using Proposition 3.1). □

Proposition 3.4. *Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's and $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be sdfc function. If (X, τ_1, τ_1^*) is (r,s)-fuzzy connected, then so is (Y, τ_2, τ_2^*).*

Proof. Suppose that (Y, τ_2, τ_2^*) be an (r,s)-fuzzy disconnected space and $\nu \in I^Y - \{\underline{0}, \underline{1}\}$ be an (r,s)-fco set. Since f is sdfc function, $f^{-1}(\nu) \in I^X - \{\underline{0}, \underline{1}\}$ is (r,s)-fco set which is contradiction. Hence (Y, τ_2, τ_2^*) is (r,s)-fuzzy connected. □

Proposition 3.5. *Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's. If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be sdfc function and (Y, τ_2, τ_2^*) be an (r,s)-fuzzy extremally disconnected, then f is a★dfc function.*

Proof. Let $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that $\tau_2(\mu) \geqslant r, \tau_2^*(\mu) \leqslant s$ and $f(\lambda) \leqslant \mu$. Since (Y, τ_2, τ_2) is an (r,s)-fuzzy extermally disconnected, $C_{\tau_2,\tau_2^*}(\mu, r, s)$ is (r,s)-fco set. Now, $f(\lambda) \leqslant C_{\tau_2,\tau_2^*}(\mu, r, s)$ and since f is sdfc function, there exists $\nu \in I^X$ such $\tau_1(\nu) \geqslant r, \tau_1(\nu) \leqslant s, \lambda \leqslant \nu$ and $f(\nu) \leqslant C_{\tau_2,\tau_2^*}(\mu, r, s)$. Since $\tau_2(C_{\tau_2,\tau_2^*}(\mu, r, s)) \geqslant r$ and $\tau_2^*(C_{\tau_2,\tau_2^*}(\mu, r, s)) \leqslant s$, then

$$f(\nu) \leqslant I_{\tau_2,\tau_2^*}\Big(C_{\tau_2,\tau_2^*}(\mu, r, s), r, s\Big).$$

Hence f is a★dfc function. □

4. Slightly generalized double fuzzy semicontinuous functions

Definition 4.1. Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's. A function $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is called:

(1) almost ★-generalized double fuzzy semicontinuous (a★gdfsc, for short) if for each $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that $\tau_2(\mu) \geqslant r, \tau_2^*(\mu) \leqslant s$ and $f(\lambda) \leqslant \mu$, there exists an (r,s)-gfso set $\nu \in I^X$ such that $\lambda \leqslant \nu$ and

$$f(\nu) \leqslant I_{\tau_2,\tau_2^*}\Big(C_{\tau_2,\tau_2^*}(\mu, r, s), r, s\Big).$$

(2) θ★-generalized double fuzzy semicontinuous (θ★gdfsc, for short) if for each $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that $\tau_2(\mu) \geqslant r, \tau_2^*(\mu) \leqslant s$ and $f(\lambda) \leqslant \mu$, there exists an (r,s)-gfso set $\nu \in I^X$ such that $\lambda \leqslant \nu$ and

$$f\Big(C_{\tau_1,\tau_1^*}(\nu, r, s)\Big) \leqslant C_{\tau_2,\tau_2^*}(\mu, r, s).$$

(3) weakly ★-generalized double fuzzy semicontinuous (w★gdfsc, for short) if for each $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that $\tau_2(\mu) \geqslant r, \tau_2^*(\mu) \leqslant s$ and $f(\lambda) \leqslant \mu$, there exists an (r,s)-gfso set $\nu \in I^X$ and $\lambda \leqslant \nu$ such that

$$f(\nu) \leqslant C_{\tau_2,\tau_2^*}(\mu, r, s).$$

(4) slightly generalized double fuzzy semicontinuous (sgdfsc, for short) if for each $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that μ is an (r,s)-fco set and $f(\lambda) \leqslant \mu$, there exists an (r,s)-gfso set $\nu \in I^X$ such that $\lambda \leqslant \nu$ and $f(\nu) \leqslant \mu$.

Proposition 4.1. *Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's. For the function $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$, the following statements are equivalent:*

(1) f is sgdfsc function.
(2) $f^{-1}(\nu)$ is an (r,s)-gfso set for each $\nu \in I^Y, r \in I_0$ and $s \in I_1$ such that ν is (r,s)-gfsco set.
(3) $f^{-1}(\nu)$ is an (r,s)-gfsco set for each $\nu \in I^Y, r \in I_0$ and $s \in I_1$ such that ν is (r,s)-gfsco set.
(4) For each fuzzy set $\lambda \in I^X, r \in I_0, s \in I_1$ and for every fuzzy net $\{S_n : n \in D\}$ with converges to λ, the fuzzy net $\{f(S_n) : n \in D\}$ is evantually in each (r,s)-gfsco set μ with $f(\lambda) \leqslant \mu$.

Proof. (1) ⇒ (2): Let $\nu \in I^Y, r \in I_0$ and $s \in I_1$ such that ν is (r,s)-gfsco set and let $\lambda \in I^X$ such that $\lambda \leqslant f^{-1}(\nu)$. Since ν is an (r,s)-gfsco set with $f(\lambda) \leqslant \nu$. By (1), there exists $\mu \in I^X$ such that μ is an (r,s)-gfso, $\lambda \leqslant \mu$ and $f(\mu) \leqslant \nu$. Hence $f^{-1}(\nu)$ is an (r,s)-gfso set.

(2) ⇒ (3): Clear.

(3) ⇒ (4): Let $\{S_n : n \in D\}$ be a fuzzy net converges to the (r,s)-gfsco set $\lambda \in I^X$ and let $\mu \in I^Y$ be an (r,s)-gfsco set such that $f(\lambda) \leqslant \mu$. By using (3), there exist an (r,s)-gfso set $\nu \in I^X$ such that $\lambda \leqslant \nu$ and $f(\nu) \leqslant \mu$. Since the fuzzy net $\{S_n : n \in D\}$ converges to $\lambda, S_n \leqslant \lambda \leqslant \nu$. Thus $\{f(S_n) : n \in D\}$ is eventually in each (r,s)-gfsco set μ.

(4) ⇒ (1): Suppose that f is not sgdfsc function. Then for every $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that μ is an (r,s)-gfso set and $f(\lambda) \leqslant \mu$, there does not exist $\nu \in I^X$ such that $\lambda \leqslant \nu$ and $f(\nu) \leqslant \mu$. Hence $f(S_n) \leqslant \mu$. That is the fuzzy net $\{f(S_n) : n \in D\}$ is not eventually in an (r,s)-gfsco set μ with $f(\lambda) \leqslant \mu$, which is a contradiction. Hence f is sgdfsc function. □

Definition 4.2. Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's. A function $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is said to be generalized double fuzzy semi irresolute (gdfsi, for short) if $f^{-1}(\lambda)$ is an (r,s)-gfsc set, for each (r,s)-gfsc set $\lambda \in I^Y$ $r \in_0$ and $s \in I_1$.

Proposition 4.2. *Let $(X, \tau_1, \tau_1^*), (Y, \tau_2, \tau_2^*)$ and (Z, τ_3, τ_3^*) be dfts's. For the functions $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ and $g : (X, \tau_2, \tau_2^*) \to (Y, \tau_3, \tau_3^*)$, the following statements are satisfied:*

(1) If f and g are sgdfsc functions, then so is $g \circ f$.
(2) If f is a surjective gdfsi, gdfso function and g be any function, then $g \circ f$ is sgdfsc function iff g is sgdfsc.

Proof. (1): clear.

(2): Suppose that $g \circ f$ is sgdfsc function, $\lambda \in I^Z$ is an (r,s)-gfsco set. By using Proposition 4.1 (2), $f^{-1}(g^1(\nu)) = (g \circ f)^{-1}(\nu)$ is an (r,s)-gfso set in I^X. Since f is gdfso, $g^{-1}(\lambda) = f(f^1(g^{-1}(\lambda)))$ is an (r,s)-gfso set. Therefore by Proposition 4.1, g is sgdfsc function.

Conversely, let $\nu \in I^Z$ be an (r,s)-gfsco set where $r \in I_0$ and $s \in I_1$. Since g is sgdfsc function, $g^{-1}(\nu)$ is an (r,s)-gfso set $\in I^Y$ and f is gdfsi function, $f^{-1}(g^1(\nu)) = (g \circ f)^{-1}(\nu)$ is an (r,s)-gfso set $\in I^X$. Therefore by Proposition 4.1, $(g \circ f)$ is sgdfsc function. $\square$

Definition 4.3. A dfts (X, τ, τ^*) is said to be an (r, s)-generalized fuzzy semi-connected iff $\underline{0}$ and $\underline{1}$ are the only fuzzy sets which are both (r, s)-gfso and (r, s)-gfsc.

Proposition 4.3. *Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's and let $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be a function. If (Y, τ_2, τ_2^*) is an (r, s)-generalized fuzzy semi-connected, then f is sgdfsc function.*

Proof. Let (Y, τ_2, τ_2^*) be an (r, s)-generalized fuzzy semi-connected space. Then $\underline{0}$ and $\underline{1}$ are the only (r, s)-gfsco sets. Since $f^{-1}(\underline{0})$ and $f^{-1}(\underline{1})$ are both (r, s)-gfso in I^X. Hence by Proposition 4.1, f is sgdfsc function. $\square$

Proposition 4.4. *Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's and let $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be sgdfsc function. If (X, τ_1, τ_1^*) is an (r, s)-generalized fuzzy semi-connected, then so is (Y, τ_2, τ_2^*).*

Proof. Suppose that (Y, τ_2, τ_2^*) be an (r, s)-generalized fuzzy semi-disconnected space and $\nu \in I^Y$-$\{\underline{0}, \underline{1}\}$ be an (r, s)-gfsco set. Since $f^{-1}(\nu)$ is an (r, s)-gfsco set which is contradiction. Hence (Y, τ_2, τ_2^*) is an (r, s)-generalized fuzzy semi-connected function. $\square$

Definition 4.4. A dfts (X, τ, τ^*) is said to be an (r, s)-generalized fuzzy semi-extremely disconnected if $GSC_{\tau,\tau^*}(\lambda, r, s)$ is an (r, s)-gfso set for each $\lambda \in I^X, r \in I_0$ and $s \in I_1$ such that λ is an (r, s)-gfso set.

Proposition 4.5. *Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's. If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is sgdfsc function and (Y, τ_2, τ_2^*) is an (r, s)-generalized fuzzy semi-extremely disconnected, then f is a★gdfsc function.*

Proof. Let $\lambda \in I^X, \mu \in I^Y, r \in I_0$ and $s \in I_1$ such that λ and μ are (r, s)-gfso sets. Since (Y, τ_2, τ_2^*) is an (r, s)-generalized fuzzy semi-extremely disconnected, $GSC_{\tau_2,\tau_2^*}(\mu, r, s)$ is an (r, s)-gfsco set. Now, $f(\lambda) \leqslant GSC_{\tau_2,\tau_2^*}(\mu, r, s)$ and since f is sgdfsc function, there exists an (r, s)-gfso set $\nu \in I^X$ such that $\lambda \leqslant \nu$ and $f(\nu) \leqslant C_{\tau_2,\tau_2^*}(\mu, r, s)$. Therefor, f is a★gdfsc function. $\square$

5. Interrelations

The following implication illustrates the relationships between different functions:

$$\text{dfc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \text{a}\star\text{dfc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \theta\star\text{dfc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \text{w}\star\text{dfc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \text{sdfc}$$

$$\text{gdfsc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \text{a}\star\text{gdfsc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \theta\star\text{gdfsc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \text{w}\star\text{gdfsc} \;\underset{\not\longleftarrow}{\longrightarrow}\; \text{sgdfsc}$$

where $A \longrightarrow B$ represents A implies B and $A \not\longleftarrow B$ means the reverse implication is not true.

Example 5.1. Let $X = \{a, b, c\}$ and $f : (X, \tau_1, \tau_1^*) \to (X, \tau_2, \tau_2^*)$ be a function defined by:

$$f(a) = b,\; f(b) = a,\; f(c) = c.$$

(1) Define μ, ν, γ and δ as follows:

$$\mu(a) = 0.5,\; \mu(b) = 0.5,\; \mu(c) = 0.5,$$
$$\nu(a) = 0.5,\; \nu(b) = 0.5,\; \nu(c) = 0.5,$$
$$\gamma(a) = 1.0,\; \gamma(b) = 0.5,\; \gamma(c) = 0.5,$$
$$\delta(a) = 0.0,\; \delta(b) = 0.0,\; \delta(c) = 0.3,$$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \nu, \\ \frac{1}{3}, & \text{if } \lambda = \gamma, \\ \frac{1}{4}, & \text{if } \lambda = \delta, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \nu, \\ \frac{2}{3}, & \text{if } \lambda = \gamma, \\ \frac{3}{4}, & \text{if } \lambda = \delta, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is sdfc function but not w★dfc.

(2) Define μ and ν as follows:

$$\mu(a) = 0.3,\; \mu(b) = 0.0,\; \mu(c) = 0.6,$$
$$\nu(a) = 0.0,\; \nu(b) = 0.2,\; \nu(c) = 0.3,$$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \nu, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \nu, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is w★dfc function but not θ★dfc.

(3) Define μ, v, γ and δ as follows:

$\mu(a) = 0.6,\ \mu(b) = 1.0,\ \mu(c) = 0.5,$
$v(a) = 0.4,\ v(b) = 0.0,\ v(c) = 0.5,$
$\gamma(a) = 0.0,\ \gamma(b) = 0.0,\ \gamma(c) = 0.5,$
$\delta(a) = 1.0,\ \delta(b) = 0.5,\ \delta(c) = 0.5.$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ \frac{1}{3}, & \text{if } \lambda = v, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{3}, & \text{if } \lambda = \mu, \\ \frac{1}{2}, & \text{if } \lambda = v, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ \frac{1}{3}, & \text{if } \lambda = v, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{3}, & \text{if } \lambda = \mu, \\ \frac{1}{2}, & \text{if } \lambda = v, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is $\theta\bigstar$dfc function but not a$\bigstar$dfc.

(4) Define μ and v as follows:

$\mu(a) = 1.0,\ \mu(b) = 1.0,\ \mu(c) = 0.6,$
$v(a) = 1.0,\ v(b) = 0.5,\ v(c) = 0.5,$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = v, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = v, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is a$\bigstar$dfc function but not dfc.

(5) Define μ, v, γ, δ and η as follows:

$\mu(a) = 0.3,\ \mu(b) = 0.4,\ \mu(c) = 0.5,$
$v(a) = 0.7,\ v(b) = 0.6,\ v(c) = 0.5,$
$\gamma(a) = 0.5,\ \gamma(b) = 0.5,\ \gamma(c) = 0.5,$
$\delta(a) = 1.0,\ \delta(b) = 0.5,\ \delta(c) = 0.5,$
$\eta(a) = 0.0,\ \delta(b) = 0.0,\ \delta(c) = 0.3,$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{3}, & \text{if } \lambda = \mu, \\ \frac{1}{4}, & \text{if } \lambda = v, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{4}, & \text{if } \lambda = \mu, \\ \frac{1}{3}, & \text{if } \lambda = v, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \gamma, \\ \frac{1}{3}, & \text{if } \lambda = \delta, \\ \frac{1}{4}, & \text{if } \lambda = \eta, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \gamma, \\ \frac{2}{3}, & \text{if } \lambda = \delta, \\ \frac{3}{4}, & \text{if } \lambda = \eta, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is sgdfsc function but not w$\bigstar$gdfsc.

(6) Define μ, v, γ and δ as follows:

$\mu(a) = 0.3,\ \mu(b) = 0.0,\ \mu(c) = 0.5,$
$v(a) = 0.7,\ v(b) = 1.0,\ v(c) = 0.5,$
$\gamma(a) = 0.3,\ \gamma(b) = 0.3,\ \gamma(c) = 0.3,$
$\delta(a) = 1.0,\ \delta(b) = 0.5,\ \delta(c) = 0.5.$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{3}, & \text{if } \lambda = \mu, \\ \frac{1}{4}, & \text{if } \lambda = v, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{4}, & \text{if } \lambda = \mu, \\ \frac{1}{3}, & \text{if } \lambda = v, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \gamma, \\ \frac{1}{3}, & \text{if } \lambda = \delta, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \gamma, \\ \frac{2}{3}, & \text{if } \lambda = \delta, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is w$\bigstar$gdfsc function but not $\theta\bigstar$gdfsc.

(7) Define μ, v and γ as follows:

$\mu(a) = 0.3,\ \mu(b) = 0.4,\ \mu(c) = 0.5,$
$v(a) = 0.7,\ v(b) = 0.6,\ v(c) = 0.5,$
$\gamma(a) = 0.0,\ \gamma(b) = 0.0,\ \gamma(c) = 0.5.$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ \frac{1}{4}, & \text{if } \lambda = v, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{3}, & \text{if } \lambda = \mu, \\ \frac{1}{2}, & \text{if } \lambda = v, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \gamma, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \gamma, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is $\theta\bigstar$gdfsc function but not a$\bigstar$gdfsc.

Example 5.2. Let $X = \{a, b\}$ and $f : (X, \tau_1, \tau_1^*) \to (X, \tau_2, \tau_2^*)$ be the identity function. Define μ, v, γ and δ as follows:

$\mu(a) = 0.1,\ \mu(b) = 0.2,$
$v(a) = 0.9,\ v(b) = 0.8,$
$\gamma(a) = 0.1,\ \gamma(b) = 0.1,$
$\delta(a) = 0.9,\ \delta(b) = 0.9.$

and define (τ_1, τ_1^*) and (τ_2, τ_2^*) as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{4}, & \text{if } \lambda = \mu, \\ \frac{1}{8}, & \text{if } \lambda = \nu, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_1^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{8}, & \text{if } \lambda = \mu, \\ \frac{1}{4}, & \text{if } \lambda = \nu, \\ 1, & \text{otherwise.} \end{cases}$$

and

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{4}, & \text{if } \lambda = \gamma, \\ \frac{1}{8}, & \text{if } \lambda = \delta, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau_2^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{8}, & \text{if } \lambda = \gamma, \\ \frac{1}{4}, & \text{if } \lambda = \delta, \\ 1, & \text{otherwise.} \end{cases}$$

Then f is a★gdfsc function but not gdfsc.

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