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ORIGINAL ARTICLE

Generalized sequence spaces defined by a sequence of moduli

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Abstract In the present paper we introduce the sequence spaces defined by a sequence of modulus function $F = (f_k)$. We study some topological properties and inclusion relations between these spaces.

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1. Introduction and preliminaries

Mursaleen and Noman [1] introduced the notion of λ-convergent and λ-bounded sequences as follows :

Let $\lambda = (\lambda_k)_{k=1}^{\infty}$ be a strictly increasing sequence of positive real numbers tending to infinity i.e.

$$0 < \lambda_0 < \lambda_1 < \cdots \text{ and } \lambda_k \to \infty \text{ as } k \to \infty$$

and said that a sequence $x = (x_k) \in w$ is λ-convergent to the number L, called the λ-limit of x if $\Lambda_m(x) \longrightarrow L$ as $m \to \infty$, where

$$\lambda_m(x) = \frac{1}{\lambda_m}\sum_{k=1}^{m}(\lambda_k - \lambda_{k-1})x_k.$$

The sequence $x = (x_k) \in w$ is λ-bounded if $\sup_m|\Lambda_m(x)| < \infty$. It is well known [1] that if $\lim_m x_m = a$ in the ordinary sense of convergence, then

$$\lim_m\left(\frac{1}{\lambda_m}\left(\sum_{k=1}^{m}(\lambda_k - \lambda_{k-1})|x_k - a|\right)\right) = 0.$$

This implies that

$$\lim_m|\Lambda_m(x) - a| = \lim_m\left|\frac{1}{\lambda_m}\sum_{k=1}^{m}(\lambda_k - \lambda_{k-1})(x_k - a)\right| = 0$$

which yields that $\lim_m\Lambda_m(x) = a$ and hence $x = (x_k) \in w$ is λ-convergent to a.

Let w be the set of all sequences, real or complex numbers and l_∞, c and c_0 be respectively the Banach spaces of bounded, convergent and null sequences $x = (x_k)$, normed by $\|x\| = \sup_k|x_k|$, where $k \in \mathbb{N}$, the set of positive integers.

A modulus function is a function $f : [0, \infty) \to [0, \infty)$ such that

(1) $f(x) = 0$ if and only if $x = 0$,
(2) $f(x + y) \leqslant f(x) + f(y)$ for all $x \geqslant 0, y \geqslant 0$,

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(3) f is increasing
(4) f is continuous from right at 0.

It follows that $f$must be continuous everywhere on $[0,\infty)$. The modulus function may be bounded or unbounded. For example, if we take $f(x)=\frac{x}{x+1}$, then $f(x)$ is bounded. If $f(x)=x^p$, $0<p<1$, then the modulus $f(x)$ is unbounded. Subsequently, modulus function has been discussed in ([2–15]) and many others.

Let X be a linear metric space. A function $p: X\to\mathbb{R}$ is called paranorm, if

(1) $p(x)\geqslant 0$, for all $x\in X$,
(2) $p(-x)=p(x)$, for all $x\in X$,
(3) $p(x+y)\leqslant p(x)+p(y)$, for all $x,y\in X$,
(4) if (λ_n) is a sequence of scalars with $\lambda_n\to\lambda$ *as* $n\to\infty$ and (x_n) is a sequence of vectors with $p(x_n-x)\to 0$ *as* $n\to\infty$, then $p(\lambda_n x_n-\lambda x)\to 0$ *as* $n\to\infty$.

A paranorm p for which $p(x)=0$ implies $x=0$ is called total paranorm and the pair (X,p) is called a total paranormed space. It is well known that the metric of any linear metric space is given by some total paranorm (see [16], Theorem 10.4.2, P-183).

Let $F=(f_k)$ be a sequence of modulus function, X be a locally convex Hausdorff topological linear spaces whose topology is determined by a set Q of continuous seminorm q, $p=(p_k)$ be a bounded sequence of positive real numbers. By $w(X)$ be denotes the spaces of all sequences defined over X. Now, we define the following sequence spaces in the present paper:

$$w(\Lambda,F,p,q)=\left\{x\in w(X):\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(x)-L))]^{p_k}\to 0,\right.$$
$$\left.\text{as } n\to\infty \text{ for some } L\right\},$$

$$w_0(\Lambda,F,p,q)=\left\{x\in w(X):\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(x)))]^{p_k}\to 0,\ \text{as } n\to\infty\right\}$$

and

$$w_\infty(\Lambda,F,p,q)=\left\{x\in w(X):\sup_n\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(x)))]^{p_k}<\infty\right\}.$$

If $F(x)=x$, we have

$$w(\Lambda,p,q)=\left\{x\in w(X):\frac{1}{n}\sum_{k=1}^{n}(q(\Lambda_k(x)-L))^{p_k}\to 0,\right.$$
$$\left.\text{as } n\to\infty \text{ for some } L\right\},$$

$$w_0(\Lambda,p,q)=\left\{x\in w(X):\frac{1}{n}\sum_{k=1}^{n}(q(\Lambda_k(x)))^{p_k}\to 0,\ \text{as } n\to\infty\right\}$$

and

$$w_\infty(\Lambda,p,q)=\left\{x\in w(X):\sup_n\frac{1}{n}\sum_{k=1}^{n}(q(\Lambda_k(x)))^{p_k}<\infty\right\}.$$

If $p=(p_k)=1$, for all $k\in\mathbb{N}$, we shall write above spaces as

$$w(\Lambda,F,q)=\left\{x\in w(X):\frac{1}{n}\sum_{k=1}^{n}f_k(q(\Lambda_k(x)-L))\to 0,\right.$$
$$\left.\text{as } n\to\infty \text{ for some } L\right\},$$

$$w_0(\Lambda,F,q)=\left\{x\in w(X):\frac{1}{n}\sum_{k=1}^{n}f_k(q(\Lambda_k(x)))\to 0,\ \text{as } n\to\infty\right\}$$

and

$$w_\infty(\Lambda,F,q)=\left\{x\in w(X):\sup_n\frac{1}{n}\sum_{k=1}^{n}f_k(q(\Lambda_k(x)))<\infty\right\}.$$

The following inequality will be used throughout the paper. If $0<h=\inf p_k\leqslant p_k\leqslant \sup p_k=H$, $D=\max(1,2^{H-1})$ then

$$|a_k+b_k|^{p_k}\leqslant D\{|a_k|^{p_k}+|b_k|^{p_k}\} \tag{1.1}$$

for all k and a_k, $b_k\in\mathbb{C}$. Also $|a|^{p_k}\leqslant\max(1,|a|^H)$ for all $a\in\mathbb{C}$.

The main purpose of this paper is to introduce the sequence spaces defined by a sequence of modulus function $F=(f_k)$. We study some topological properties and prove some inclusion relations between these spaces.

2. Main results

Theorem 2.1. *Let $F=(f_k)$ be a sequence of modulus function, $p=(p_k)$ be a bounded sequence of positive real numbers. Then $w(\Lambda,F,p,q)$, $w_0(\Lambda,F,p,q)$ and $w_\infty(\Lambda,F,p,q)$ are linear spaces over the field of complex numbers $\mathbb{C}$.*

Proof. Let x, $y\in w_0(\Lambda,F,p,q)$ and α, $\beta\in\mathbb{C}$, there exists M_α and N_β integers such that $|\alpha|\leqslant M_\alpha$ and $|\beta|\leqslant N_\beta$. Since F is subadditive and q is a seminorm. Therefore

$$\begin{aligned}\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(\alpha x+\beta y)))]^{p_k}&\leqslant\frac{1}{n}\sum_{k=1}^{n}[(f_k|\alpha|q(\Lambda_k(x))+f_k|\beta|q(\Lambda_k(y)))]^{p_k}\\&\leqslant D(M_\alpha)^H\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(x)))]^{p_k}\\&\quad+D(N_\beta)^H\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(y)))]^{p_k}\to 0.\end{aligned}$$

This proves that $w_0(\Lambda,F,p,q)$ is a linear space. Similarly, we can prove that $w(\Lambda,F,p,q)$ and $w_\infty(\Lambda,F,p,q)$ are linear spaces. □

Theorem 2.2. *Let $F=(f_k)$ be a sequence of modulus function, $p=(p_k)$ be a bounded sequence of positive real numbers. Then $w_0(\Lambda,F,p,q)$ is a paranormed space with paranorm*

$$g(x)=\sup_n\left\{\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(x)))]^{p_k}\right\}^{\frac{1}{M}},$$

where $H=\sup p_k<\infty$ and $M=\max(1,H)$.

Proof. Clearly, $g(x)=g(-x)$, $x=\theta$ implies $\Lambda_k(x)=\theta$ and such that $q(\theta)=0$ and $f_k(0)=0$, where θ is the zero sequence. Therefore $g(\theta)=0$. Since $p_k/M\leqslant 1$ and $M\geqslant 1$, using the Minkowski's inequality and definition of $F=(f_k)$ for each n,

$$\left\{\frac{1}{n}\sum_{k=1}^{n}\left[f_k(q(\Lambda_k(x)+\Lambda_k(y)))\right]^{p_k}\right\}^{\frac{1}{M}}$$
$$\leqslant \left\{\frac{1}{n}\sum_{k=1}^{n}\left[f_k(q(\Lambda_k(x))+(\Lambda_k(y)))\right]^{p_k}\right\}^{\frac{1}{M}}$$
$$\leqslant \left\{\frac{1}{n}\sum_{k=1}^{n}\left[f_k(q(\Lambda_k(x)))\right]^{p_k}\right\}^{\frac{1}{M}}+\left\{\frac{1}{n}\sum_{k=1}^{n}\left[f_k(q(\Lambda_k(y)))\right]^{p_k}\right\}^{\frac{1}{M}}.$$

Now it follows that g is subadditive. Finally, to check the continuity of multiplication, let us take any complex number μ. By definition of $F=(f_k)$, we have

$$g(\mu x)=\sup_n\left\{\frac{1}{n}\sum_{k=1}^{n}\left[f_k(q(\Lambda_k(\mu x)))\right]^{p_k}\right\}^{\frac{1}{M}}\leqslant K_\mu^{H/M}g(x),$$

where K_μ is an integer such that $|\mu|<K_\mu$. Now, let $\mu\to 0$ for any fixed x with $g(x)\neq 0$. By definition of f for $|\mu|<1$, we have

$$\frac{1}{n}\sum_{k=1}^{n}\left[f_k(q(\Lambda_k(\mu x)))\right]^{p_k}<\epsilon \tag{2.1}$$

Also, for $1\leqslant n\leqslant N$, taking λ small enough, since $F=(f_k)$ is continuous, we have

$$\frac{1}{n}\sum_{k=1}^{n}\left[f_k(q(\Lambda_k(x)))\right]^{p_k}<\epsilon. \tag{2.2}$$

Eqs. (2.1) and (2.2) together imply that $g(\mu x)\to 0$ as $\mu\to 0$. This completes the proof of the theorem. □

Theorem 2.3. *Let $F=(f_k), F'=(f'_k), F''=(f''_k)$ are modulus functions and $0<h=\inf p_k\leqslant p_k\leqslant \sup_k p_k=H<\infty$. Then*

(i) $w_0(\Lambda,F',p,q)\subseteq w_0(\Lambda,F\circ F',p,q)$,
(ii) $w_0(\Lambda,F',p,q)\cap w_0(\Lambda,F'',p,q)\subseteq w_0(\Lambda,F'+F'',p,q)$.

Proof. (i) Let $x\in w_0(\Lambda,F',p,q)$. Let $\epsilon>0$ and choose δ with $0<\delta<1$ such that $f_k(t)<\epsilon$ for $0\leqslant t\leqslant\delta$. Write $y=f'_k(q(\Lambda_k(x)))$ and consider

$$\sum_{k=1}^{n}[f_k(y)]^{p_k}=\sum_1[f_k(y)]^{p_k}+\sum_2[f_k(y)]^{p_k}$$

where the first summation is over $y\leqslant\delta$ and the second over $y>\delta$. Since $F=(f_k)$ is continuous, we have

$$\sum_1[f_k(y)]^{p_k}<n\max(\epsilon^h,\epsilon^H) \tag{2.3}$$

and for $y>\delta$ we use the fact that

$$y<\frac{y}{\delta}<1+\frac{y}{\delta}.$$

By the definition of $F=(f_k)$, we have for $y>\delta$,

$$f_k(y)\leqslant f_k(1)\left[1+\left(\frac{y}{\delta}\right)\right]\leqslant 2f_k(1)\frac{y}{\delta}.$$

Hence

$$\frac{1}{n}\sum_2[f_k(y)]^{p_k}\leqslant \max\left(1,\left(\frac{2f_k(1)}{\delta}\right)^H\right)\frac{1}{n}\sum_{k=1}^{n}[y]^{p_k}\to 0. \tag{2.4}$$

By (2.3) and (2.4), we have $w_0(\Lambda,F',p,q)\subseteq w_0(\Lambda,F\circ F',p,q)$.

(ii) Let $x\in w_0(\Lambda,F',p,q)\cap w_0(\Lambda,F'',p,q)$. Then using inequality (1.1) it can be shown that $x\in w_0(\Lambda,F'+F'',p,q)$. Hence $w_0(\Lambda,F',p,q)\cap w_0(\Lambda,F'',p,q)\subseteq w_0(\Lambda,F'+F'',p,q)$. □

Corollary 2.4. *Let $F=(f_k)$, $F'=(f'_k)$, $F''=(f''_k)$ are modulus functions. Then*

(i) $w(\Lambda,F',p,q)\subseteq w(\Lambda,F\circ F',p,q)$;
(ii) $w(\Lambda,F',p,q)\cap w(\Lambda,F'',p,q)\subseteq w(\Lambda,F'+F'',p,q)$;
(iii) $w_\infty(\Lambda,F',p,q)\subseteq w_\infty(\Lambda,F\circ F',p,q)$;
(iv) $w_\infty(\Lambda,F',p,q)\cap w_\infty(\Lambda,F'',p,q)\subseteq w_\infty(\Lambda,F'+F'',p,q)$.

Proof. It is easy to prove by using Theorem 2.3, so we omit the details. □

Theorem 2.5. *Let $F=(f_k)$ be a sequence of modulus functions. For any two sequences $p=(p_k)$ and $t=(t_k)$ of positive real numbers and q_1,q_2 be any two seminorms. Then, we have*

(i) $w_0(\Lambda,F,p,q_1)\cap w_0(\Lambda,F,t,q_2)\neq\phi$;
(ii) $w(\Lambda,F,p,q_1)\cap w(\Lambda,F,t,q_2)\neq\phi$;
(iii) $w_\infty(\Lambda,F,p,q_1)\cap w_\infty(\Lambda,F,t,q_2)\neq\phi$.

Proof. (i) Since the zero element belongs to $w_0(\Lambda,F,p,q_1)$ and $w_0(\Lambda,F,t,q_2)$, thus the intersection is non-empty. Similarly, we can prove (ii) and (iii) in view of (i). □

Proposition 2.6. *Let $F=(f_k)$ be a sequence of modulus functions. Then*

(i) $w_0(\Lambda,p,q)\subseteq w_0(\Lambda,F,p,q)$;
(ii) $w(\Lambda,p,q)\subseteq w(\Lambda,F,p,q)$;
(iii) $w_\infty(\Lambda,p,q)\subseteq w_\infty(\Lambda,F,p,q)$.

Proof. It is easy to prove, so we omit it. □

Theorem 2.7. *Let $0<p_k\leqslant r_k$ and $\left(\frac{r_k}{p_k}\right)$ be bounded, then $w(\Lambda,F,r,q)\subseteq w(\Lambda,F,p,q)$.*

Proof. Let $x\in w(\Lambda,F,r,q)$. Let $t_k=[f_k(q(\Lambda_k(x)-L))]^{r_k}$ and $\mu_k=\left(\frac{p_k}{r_k}\right)$ for all $k\in\mathbb{N}$ so that $0<\mu\leqslant\mu_k\leqslant 1$. Define the sequence (u_k) and (v_k) as follows:

For $t_k\geqslant 1$, let $u_k=t_k$ and $v_k=0$ and for $t_k<1$, let $u_k=0$ and $v_k=t_k$.

Then clearly for all $k\in\mathbb{N}$, we have $t_k=u_k+v_k$, $t_k^{\mu_k}=u_k^{\mu_k}+v_k^{\mu_k}$, $u_k^{\mu_k}\leqslant u_k\leqslant t_k$ and $v_k^{\mu_k}\leqslant v_k^{\mu}$. Therefore

$$\frac{1}{n}\sum_{k=1}^{n}t_k^{\mu_k}\leqslant\frac{1}{n}\sum_{k=1}^{n}t_k+\left[\frac{1}{n}\sum_{k=1}^{n}v_k\right]^{\mu}.$$

Hence $x\in w(\Lambda,F,p,q)$. Thus $w(\Lambda,F,r,q)\subseteq w(\Lambda,F,p,q)$.

This completes the proof of the theorem. □

3. Statistical convergence

The notion of statistical convergence of sequences was introduced by Fast [17], Buck [18], and Schoenberg [19] independently. It is also found in Zygmund [20], later on it was studied from sequence space point of view and linked with summability theory by Fridy [21], Connor [22], Salat [23], Maddox [24], Kolk [25], Rath and Tripathy [26], Tripathy [27] and many others. The notion depends on the density of subsets of the set N of natural numbers. A subset E of N is said to have density $\delta(E)$ if $\delta(E) = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}\chi_E(k)$ exists, where χ_E is the characteristic function of E.

A sequence x is said to be statistically convergent to L(i.e $x \in \mathbb{C}$) if for every $\epsilon > 0, \delta(\{k \in N : |x| \geqslant \epsilon\}) = 0$. We write $x \xrightarrow{\text{stat}} L$ or stat-lim $x = L$.

A sequence x is said to be Λ-statistically convergent to $L \in X$ if for all $q \in Q$ and any $\epsilon > 0$, $\lim_{n\to\infty} \frac{1}{n}|\{k \leqslant n : q(\Lambda_k(x) - L) \leqslant \epsilon\}| = 0$, where the vertical bars indicate the number of elements in the closed set. In this case we write $x \to L(S(\Lambda))$. The set of Λ-statistical convergent sequences will be denoted by $S(\Lambda)$.

Theorem 3.1. *Let $F = (f_k)$ be a sequence of modulus function and $\sup_k p_k = H < \infty$. Then $w(\Lambda, F, p, q) \subset S(\Lambda)$.*

Proof. Let $x \in w(\Lambda, F, p, q)$. Take $q \in Q, \epsilon > 0$ and $\sum_1$ denote the sum over $k \leqslant n$ with $q(\Lambda_k(x) - L) \geqslant \epsilon$ and $\sum_2$ denote the sum over $k \leqslant n$ with $q(\Lambda_k(x) - L) < \epsilon$. Then

$$\begin{aligned}
&\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(x) - L))]^{p_k} \\
&= \frac{1}{n}\left(\sum_1 [f_k(q(\Lambda_k(x) - L))]^{p_k} + \frac{1}{n}\sum_2 [f_k(q(\Lambda_k(x) - L))]^{p_k}\right) \\
&\geqslant \frac{1}{n}\sum_1 [f_k(q(\Lambda_k(x) - L))]^{p_k} \geqslant \frac{1}{n}\sum_1 [f_k(\epsilon)]^{p_k} \\
&\geqslant \frac{1}{n}\sum_1 \min([f_k(\epsilon)]^h, [f_k(\epsilon)]^H) = \frac{1}{n}|\{k \leqslant n : q(\Lambda_k(x) - L) \\
&\geqslant \epsilon\}| \min([f_k(\epsilon)]^h, [f_k(\epsilon)]^H)
\end{aligned}$$

Hence $x \in S(\Lambda)$. $\square$

Theorem 3.2. *Let $F = (f_k)$ be bounded and $0 < h = \inf p_k \leqslant p_k \leqslant \sup_k = H < \infty$. Then $S(\Lambda) \subset w(\Lambda, F, p, q)$.*

Proof. Suppose that $F = (f_k)$ be bounded. Let $q \in Q, \epsilon > 0$ and $\sum_1$ and $\sum_2$ was denoted in Theorem 3.1. Since $F = (f_k)$ is bounded there exists an integer K such that $f_k(x) < K$ for all $x \geqslant 0$. Then

$$\begin{aligned}
&\frac{1}{n}\sum_{k=1}^{n}[f_k(q(\Lambda_k(x) - L))]^{p_k} \\
&\leqslant \frac{1}{n}\left(\sum_1 [f_k(q(\Lambda_k(x) - L))]^{p_k} + \frac{1}{n}\sum_2 [f_k(q(\Lambda_k(x) - L))]^{p_k}\right) \\
&\leqslant \frac{1}{n}\sum_1 \max(K^h, K^H) + \frac{1}{n}\sum_2 [f_k(\epsilon)]^{p_k} \\
&\leqslant \max(K^h, K^H)\frac{1}{n}|\{k \leqslant n : q(\Lambda_k(x) - L) \geqslant \epsilon\}| + \max([f_k(\epsilon)]^h, [f_k(\epsilon)]^H).
\end{aligned}$$

Hence $x \in w(\Lambda, F, p, q)$. $\square$

Theorem 3.3. *$S(\Lambda) = w(\Lambda, F, q)$ if and only if $F = (f_k)$ is bounded.*

Proof. Let $F = (f_k)$ be bounded. By Theorems 3.1 and 3.2, we have $S(\Lambda) = w(\Lambda, F, q)$.

Conversely, suppose that $S(\Lambda) = w(\Lambda, F, q)$ and $F = (f_k)$ is unbounded, then there exists a positive sequence (t_n) with $f(t_n) = n^2$, $n = 1, 2, 3, \ldots$. If we choose

$$\Lambda_k(x) = \begin{cases} t_n, & k = n^2,\ n = 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

Then we have

$$\frac{1}{n}|\{k \leqslant n : q(\Lambda_k(x)) \geqslant \epsilon\}| \leqslant \frac{\sqrt{n}}{n} \to 0,\ n \to \infty.$$

Hence $x \to 0(S(\Lambda))$, but $x \notin w(\Lambda, F, q)$ for $q = |x|$ and $X = \mathbb{C}$. Indeed let $q = |x|$ and $X = \mathbb{C}$. Then

$$\begin{aligned}
\Lambda_k(x) &= (\Lambda_k(x_1), \Lambda_k(x_2), \Lambda_k(x_3), \Lambda_k(x_4), \ldots, \\
&\quad \Lambda_k(x_8), \Lambda_k(x_9), \Lambda_k(x_{10}), \ldots, \Lambda_k(x_{16}), \ldots) \\
&= (t_1, 0, 0, t_2, 0, \ldots, 0, t_3, 0, \ldots, 0, t_4, 0, \ldots, 0, t_5, 0, \ldots.)
\end{aligned}$$

Let

$$s_n = \frac{1}{n}\sum_{k=1}^{n} f_k(|\Lambda_k(x)|).$$

Then

$$s_{n^2} = \frac{1}{n^2}(1^2 + 2^2 + 3^2 + \ldots + n^2) = \frac{n(n+1)(2n+1)}{6n^2}.$$

Now the subsequence (s_{n^2}) of (s_n) is unbounded. Therefore $(s_n) \notin w_\infty(\Lambda, F, q)$ and hence $(s_n) \notin w(\Lambda, F, q)$. This contradicts $S(\Lambda) = w(\Lambda, F, q)$. This completes the proof of the theorem. $\square$

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