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AN IMPROVED GEOIDAL MODEL FOR EGYPT

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نموذج جيودى متطور لمصر

الخلاصة: يعتبر استخدام أنظمة الأقمار الصناعية العالمية للملاحة ذات فائدة كبيرة للمشروعات القومية والتى تختص بتسجيل الأراضى الزراعية والتى تعاظمت فى الآونة الأخيرة بهدف تحديد ملكية هذه الأراضى، إلا أن الكثير منها والتى تعتمد على طرق رى مختلفة بالإضافة إلى ارتفاعاتها المتباينة مما يستلزم نموذجاً جيودياً خاصاً لتحويل الارتفاعات فى مناسيب الأراضى إلى منسوب عالمى هو سطح البحر.

وقد حققت النماذج الجيودية فى مصر تقدماً ملموساً خاصة فيما يتعلق بالبيانات الناتجة عن الجاذبية والأقمار الصناعية والتى شملت النموذج المعروف بـ AGM 96 حيث بلغت دقة القياس (RMS) ٠,٤٩ متراً إلا أن هذا الرقم لم يكن كافياً خاصة بالنسبة للأراضى المنخفضة. وقد أجريت مؤخراً تحسينات من خلال قياسات لـ ٤٠٨ كيلو مترات على امتداد دلتا النيل باستخدام المساحة المستوية وأسلوب النظام العالمى لتحديد المواقع (GPS) حيث وصلت الدقة إلى ٠,٠٤ متر كما أنه كانت هناك تطورات أخرى أحدث أدت إلى إيجاد نماذج أكثر تطوراً منها نموذج الجاذبية الأرضية EGM 2008.

وهذا البحث هو دراسة مقارنة بين البيانات الواردة من التطورات الحديثة فى هذا المجال والأعمال السابقة التى تناولت النماذج الجيودية لمصر.

ABSTRACT: *Use of Global Navigation Satellite Systems (GNSS) are of considerable national benefit to projects such as that of land registration, which has been ongoing in Egypt for a number of years. Its particular advantage lies in delineation of title in the agricultural sector where land provides the primary means of sustenance to the community. However, much of the agricultural lands depend on irrigation, and the application of heights within the engineering design for such schemes require the use of a geoidal model to convert heights obtained from GNSS to orthometric levels. Geoidal models for Egypt have progressed as gravity and satellite data have been acquired with the latest model at national level being derived from a combination of the Earth Gravity Model of 1996 (EGM96) together with gravity data. The suggested RMS accuracy is 0.49 m, which is acknowledged as being insufficient for more detailed applications across large low lying agricultural regions. Improvement through a 408 km stretch of the Nile delta has been obtained through levelling and GPS observations to obtain a surface that is suggested as being accurate to better than 0.04 m. This is in use for hydrographic work within the delta. More recent developments from the Gravity Recovery and Climate (GRACE) experiment have provided higher resolution models of the earth gravity field, for example GGM02, and the data is now being used to develop an Earth Gravity Model of 2008 (EGM2008). This research compares the output from the recent developments in the field with previous work undertaken in providing geoidal models for Egypt..*

INTRODUCTION

In comparison with many states, a significant amount of effort has been expended in development of geoidal models in Egypt. The primary purpose of such a model is to facilitate users of Global Navigation Satellite Systems (GNSS) in determining height. It is important to retain the reference level for heighting conventions as some potential surface, and Mean Sea Level (MSL) has been traditionally adopted. Thus, orthometric heights determined using levelling techniques are designed to provide distances above MSL, and the geoid is defined as a surface that would represent MSL in the absence of land. There are still issues in levelling whereby potential surfaces above the geoid are not necessarily parallel to the geoid. For high precision work over long distances gravity data is acquired in conjunction with levels to determine dynamic heights whereby equivalent values will have the same potential. Dynamic heights would only be used over extensive distances where drainage or irrigation schemes require accurate determination of potential to predict water flow. This problem is then particularly relevant in countries such as Egypt.

Modern techniques in space geodesy can provide positioning to high precision. The height component is always least accurate due to geometry of the satellite constellation observable, and heights are provided relative to a mathematical rather than a physical surface. The geoidal model is designed to provide a conversion between the two surfaces, thus enabling high precision satellite positioning methods to be used in orthometric heighting.

Considering astro-geodetic methods as being redundant, there are two types of data used in the development of local geoidal models. Firstly, the disturbance from Keplerian motion to Earth orbiting satellites under the gravitational attraction of the Earth is used to develop models of the geopotential. Secondly, gravity data acquired on the surface of the Earth is normally incorporated into the local model derivation.

The primary purpose here is to investigate the differences between various geopotential models and their applicability to producing an improved geoidal model for Egypt. As new satellite missions have been launched, further data on the geopotential is acquired, and advanced models released.

The Form of Geopotential Models

Potential at some point with polar coordinates (φ, λ, r) external to a spherical mass of non-uniform density can be represented by a spherical harmonic series as shown by Hofmann-Wellenhof and Moritz (2005). Then, with an adjustment to an ellipsoid with semi-major axis (a) this becomes:

$$V(\varphi,\lambda,r) = \frac{GM}{r}\left\{1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n \left[\overline{C}_{nm}\cos(m\lambda) + \overline{S}_{nm}\sin(m\lambda)\right]\overline{P}_n(\cos\varphi)\right\}$$

Where GM is the gravitational constant, $\overline{C}$ and $\overline{S}$ are fully normalised harmonic coefficients, which are provided to some finite degree (n) and order (m). Normalisation is undertaken as detailed in Torge (1991) and $\overline{P}_n(\cos\varphi)$ is the Legendre polynomial. To determine the disturbing potential relative to that of some reference ellipsoid the potential of the ellipsoid is removed to provide:

$$T(\varphi,\lambda,r) = \frac{GM}{r}\left\{\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n \left[\delta\overline{C}_{nm}\cos(m\lambda) + \overline{S}_{nm}\sin(m\lambda)\right]\overline{P}_n(\cos\varphi)\right\}$$

As gravitational acceleration on the ellipsoid (γ_0) is defined by rate of change in potential with distance normal to the potential surface, the distance (N) that separates point (φ, λ, r) from the ellipsoid is provided by Bruns formula (Hofmann-Wellenhof and Moritz, 2005):

$$N = \frac{T(\varphi,\lambda,r)}{\gamma_0}$$

However, the potential of the reference ellipsoid and that of the geoid are not necessarily equal, so further correction is required and the geoid spheroid separation becomes:

$$N = \frac{T(\varphi,\lambda,r)}{\gamma_0} - \frac{(W_0 - U_0)}{\gamma_0}$$

The problem is further complicated when the reference ellipsoid used in generating spherical harmonic coefficients differs from that used in defining the reference surface for *N*. Now, in determining $T(\varphi, \lambda, r)$ the reference surface for the coefficients is adopted, while values for a different ellipsoid are used to define U_0 and hence also γ_0. For work undertaken using GNSS systems, the standards adopted here are the WGS84 spheroid with values for U_0 being provided by the National Imagery and Mapping Agency, (NIMA, 2000) as 62636851.7146 m^2s^{-2}.

Determination of potential on the geoid has been difficult due to different estimates being available, and often a value for the potential on the reference ellipsoid adopted in determining T is used. However, based on a review undertaken by Groten (1999), McCarthy and Petit (2004) published a value of 62636856.0±0.5 m^2s^{-2} through International Earth Rotation Service (IERS) to provide a numerical standard that is now more widely accepted.

The geoidal reference surface will be disturbed beneath the land mass by the density of material contained above it, and adjustments that are defined by Rapp (1997) are used here to correct for topographic effects. Topographic data for Egypt is extracted from the GTOPO30 model that is readily available on a 30”x30” grid from the United States Geological Survey (USGS, 2008) and shown as a contour diagram in Figure 1.

Available Geopotential Models

During the last 40 years there have been in excess of 100 geopotential models produced and made readily available. One of the first to be widely accepted and used was OSU91A, released in 1991 by the Ohio State University in conjunction with the Goddard Space Flight Centre (GSDC). This was further developed jointly with NIMA using data acquired from six different satellite systems. In addition to satellite tracking data and satellite altimetry, ground based gravity data was also used to provide EGM96, the first geopotential model to an order of 360. When applied correctly, the coefficients can be used to provide a geoidal model to an accuracy of 1.0 m, particularly where ground based gravity data is also available. An accompanying global geoidal model for WGS84 is also provided at 15’x15’ grid. However, Smith (1998) also provides full details of methods to perform computations, with relevant numerical constants being provided by NIMA (2000).

The Gravity Recovery And Climate Change Experiment (GRACE) satellite system was launched in 2002 with the specific aim of mapping the gravity field of the Earth. The Grace Gravity Model (GGM02) was the primary output from the experiment, and was released in two forms. GGM02S provides harmonic coefficients derived from satellite tracks only, while GGM02C also incorporates terrestrial gravity data.

Tapley *et al.,* (2005) show that this model performs better than EGM96 at longer wavelengths, and coefficients to an order of 200 are provided. It is suggested that the model is directly compatible with EGM96 in all other respects, and that higher order coefficients may be directly substituted to give a 360 degree model.

The EIGEN-GL04C model has been developed jointly by GeoForschungsZentrum Potsdam (GPZ) and Groupe de Recherche de Gèodésie Spatiale (GRGS) using data from GRACE and LAGEOS. Complete to order 360, it is independent from EGM96 in terms of satellite data used and institutions responsible for model development. Although, much of the terrestrial gravity data is likely to have been used in both EGM96 and EIGEN-GL04C, the latter will have benefited from data acquired after 1996. Förste *et al.,* (2008) document the European research, and compare the results obtained with those from EGM96. Comparing root mean square values of gravity anomaly across four regions (USA, Canada, Europe and Australia) shows the most significant improvement of 16 cm in Europe with that in the USA being just 4 cm. To draw a comparison between the performance of the two models at higher and lower wavelengths, EGM96 coefficients of order higher than 200 were substituted into the EIGEN-GL04 model, and over Europe the EGM96 model was seen to perform slightly better with RMS values 2 cm lower. Results are identical in the USA, and the EIGEN-GL04C model gives values of 1 cm lower in both Canada and Australia.

In the development of geopotential models, the satellite based data dominates in the production of low frequency components while terrestrial data is the primary contributor at the higher frequencies. It is apparent that while advancement in satellite systems has improved the data acquisition, there is little development in terrestrial data. A review of changes that have taken place in the models across Egypt, and the effect on geoidal modelling will be of interest for comparison with the newly derived values obtained from the EGM2008 geopotential model.

Geoidal Modelling in Egypt

A contour map showing a geoidal model for Egypt was published by Dawod and Ismail (2005), and this is reproduced here as Figure 2. This employed the EGM96 model as a base, and incorporated two networks of GPS points, but no gravity data. The difficulty with using GPS points is that orthometric heights are also required, and leveling becomes inaccurate over the spacing of 200 km used here. This may explain the obvious anomaly within the model that exists at latitude of 26.5° and longitude of 33.5º. There is no further evidence of a geological feature that would cause such an anomaly, and it is more likely to be due to an error in the level data point that is located there. This error distorts a significant part of the model. Consider the path of the River Nile, which would be expected to follow the shortest path across the potential surface and cross contours at right angles. At the bend in the river close to the point of peculiarity, it is seen to flow across the contours first in one direction and then in the other. In one of these states it must be flowing toward increasing potential.

The AGP2003 model produced by Merry (2003) to cover the whole of Africa is available in digital format on a 5’x5’ grid. The component that covers Egypt has been extracted and is presented in Figure 3. Again, the model is based on EGM96, but incorporates gravity data as well as terrain corrections. Nine leveled GPS stations are used for comparison, and these show a substantial bias of 1.24 m with a standard deviation of 0.8 m. Otherwise the model conforms with expectations given the topographic features with contours following fault lines both sides of Sinai, the Nile crosses contours at right angles, and the bend at 26º North flows around a small gravity anomaly. Finally, in the delta, the two branches of the river are seen to flow either side of a feature in the potential surface. However, numerically there are significant differences from the Dawod and Ismail (2005) model, particularly in the South East where the level of the contours in the AGP 2003 model are some 2.0 m lower.

Further Developments

Prior to illustrating the grid produced from the new EGM2008 geopotential model, it will be useful for comparison purposes to show equivalent results from previous models, and three geopotential models are considered. EGM96 has been widely used, but as new satellite data has been acquired, the GGM02C model has been adopted in other regions, and most recently the EIGEN-GL04C model has been released. Computing the geoid with reference to the WGS84 spheroid gives the results that follow. In each case the geoidal model is produced from the geopotential to order 360 with the application of terrain corrections using topography from the GTOPO30 terrain model. Values for the geoid/spheroid separation are computed on a 5’x5’ grid and contoured.

The EGM96 model (Figure 4) provides the underlying trend to AGP2003 with levels in the south east in agreement with those of the Dawod and Ismail (2005) model. However, without the incorporation of terrestrial gravity data there is insufficient detail to represent the geoid in areas of rapid change as are provided in AGP2003. This is particularly notable in the contour pattern in the region of the Sinai peninsula, which is also the location where the greatest differences occur between EGM96 and the result obtained from the GGM02C model shown in Figure 5. There is also variation in south east Egypt with GGM02C being some 0.5 m above EGM96.

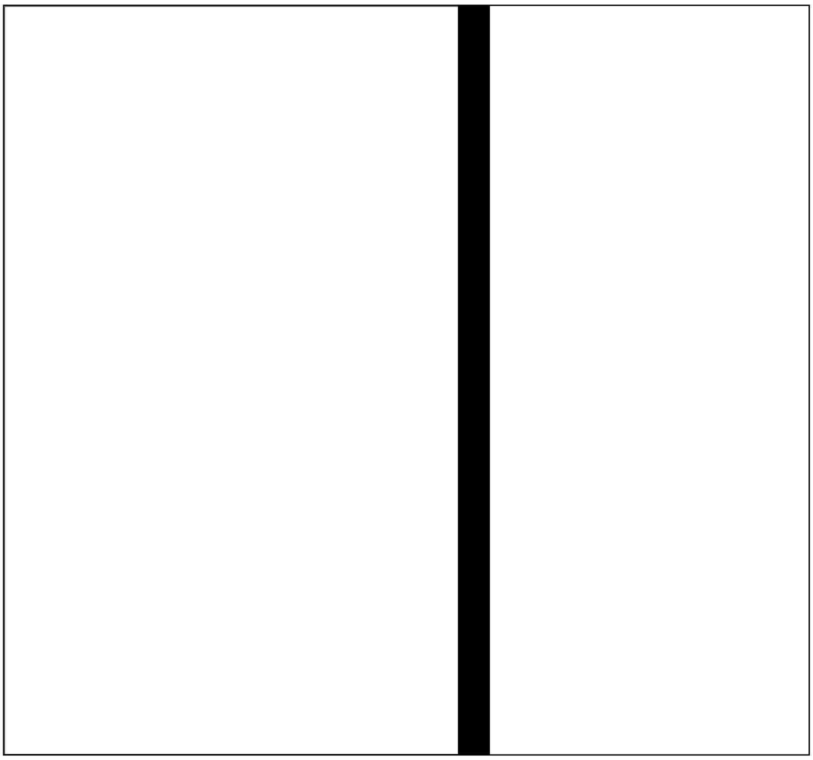

Figure 1. Topography of Egypt from GTOPO30 data. (USGS, 2008)

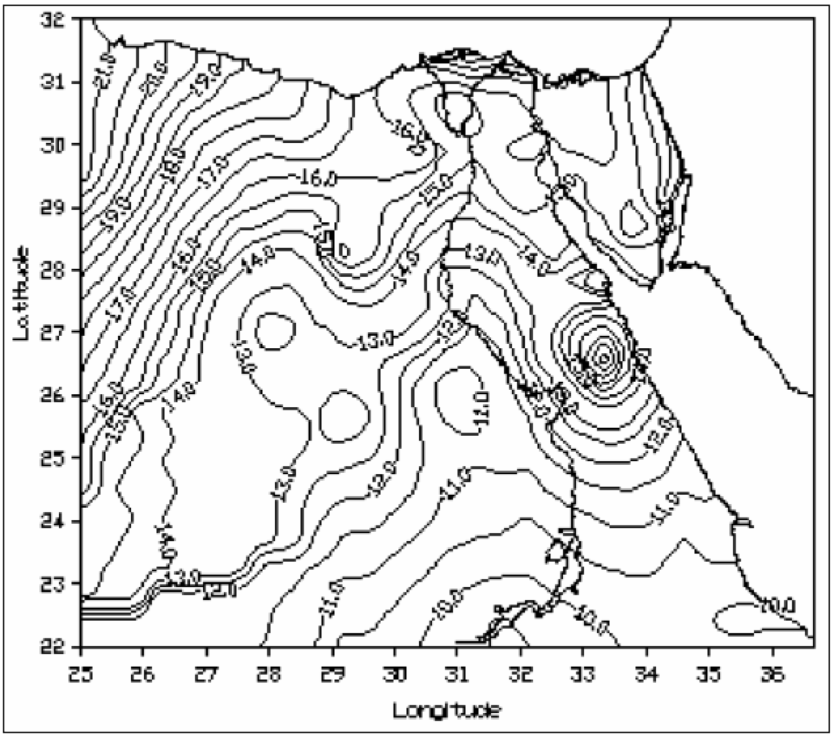

Figure 2. Geoidal model for Egypt with contours of geoid/spheroid separation in metres at 0.5 m intervals. (Dawod and Ismail, 2005).

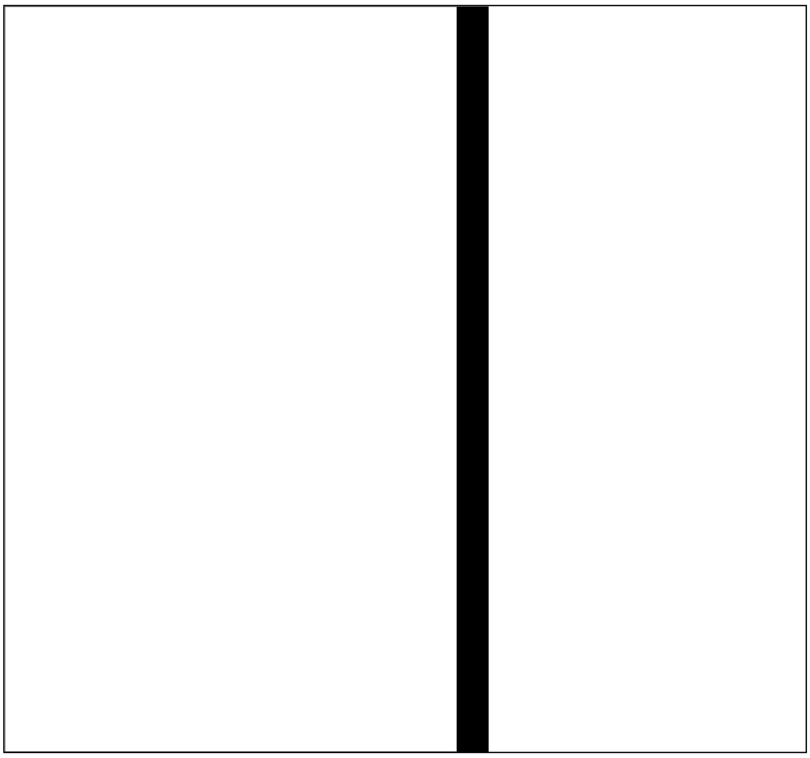

Figure 3. The Geoid across Egypt extracted from AGP2003, Merry (2003).

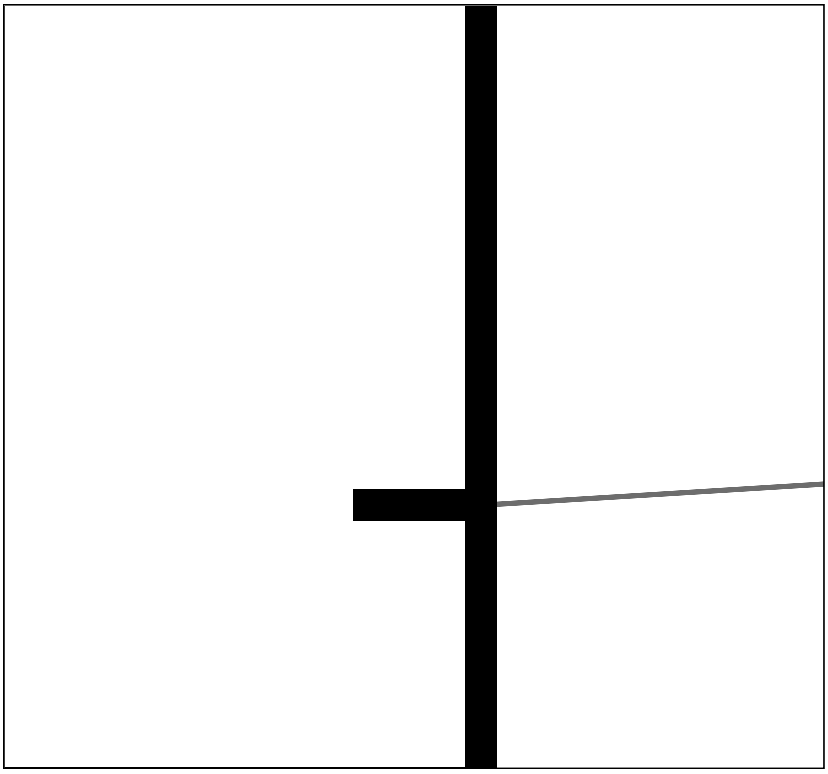

Figure 4. Geoid across Egypt generated from EGM96.

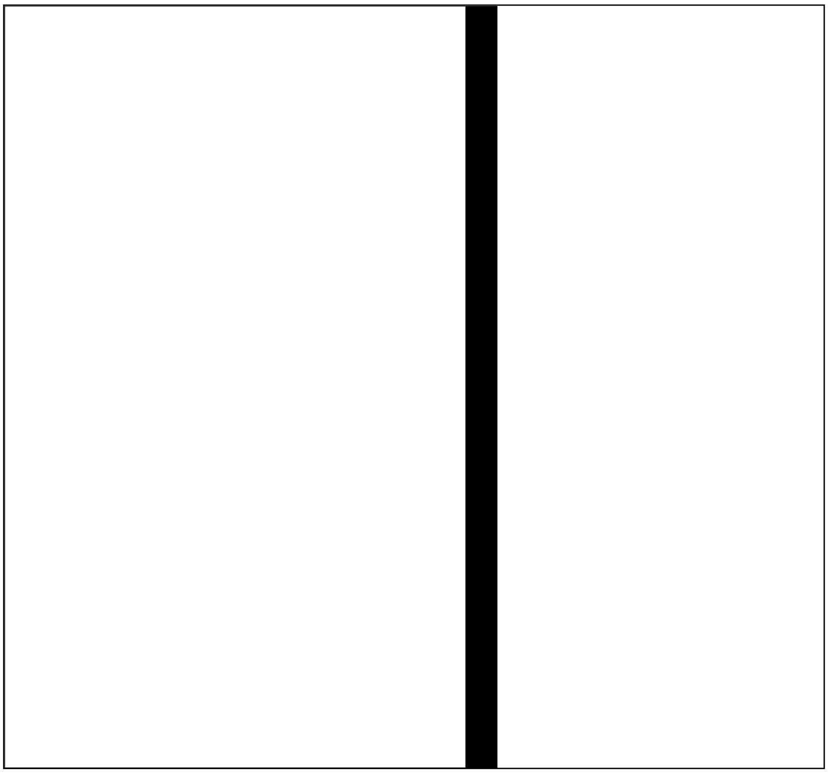

Figure 5. Geoid across Egypt generated from GGM02C.

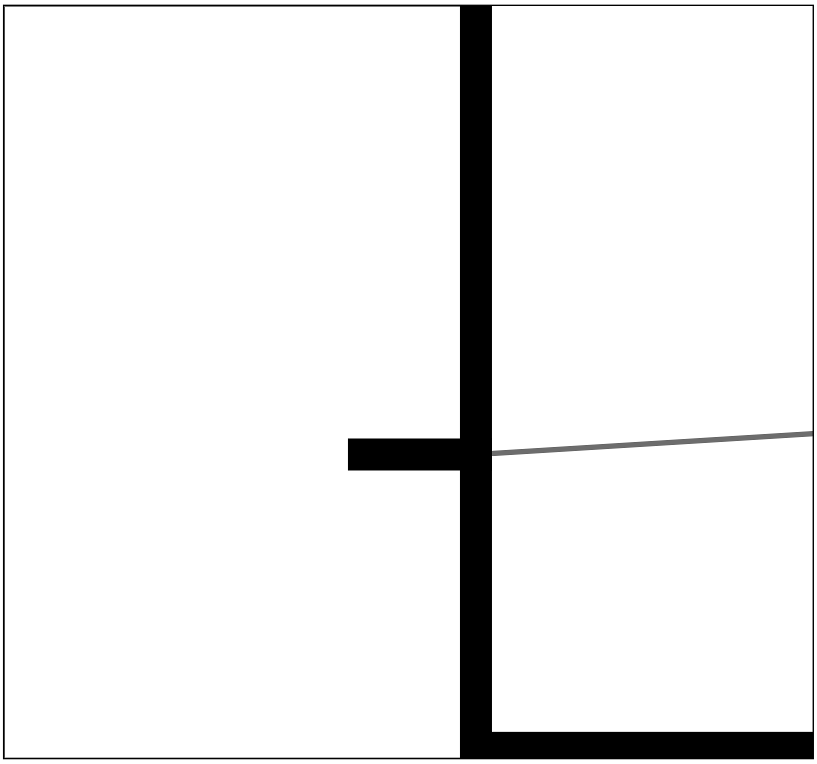

Figure 6. Geoid across Egypt generated from EIGEN-GL04.

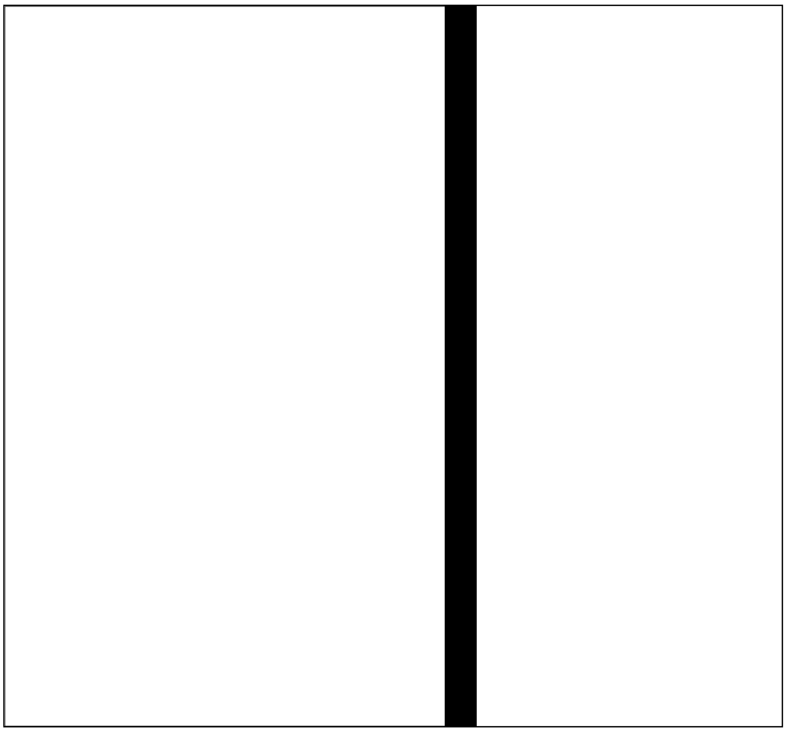

Figure 7. Geoid across Egypt generated from EGM2008.

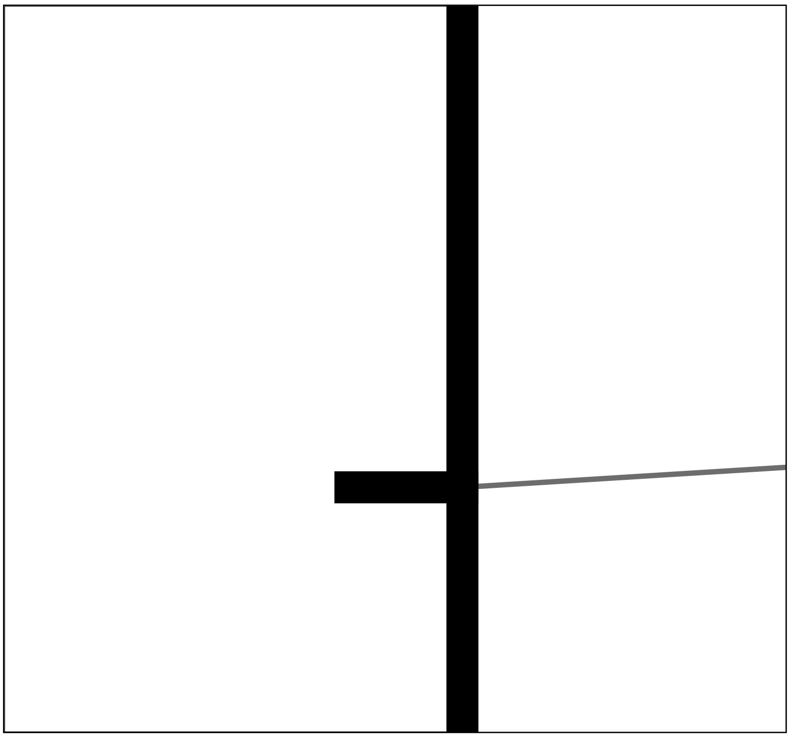

Figure 8: Difference between the geoidal model from EGM2008 and AGP2003.

In both form and numerical values the EIGEN-GL04 model is very similar to that of GGM02C, which suggests that research in data processing methods in both the USA and in Europe are parallel. Similar data was used in this independent work to provide results that are comparable, and that have been validated in regions where quality terrestrial gravity data exists.

EGM2008 improves of previous models by increasing the order from 360 to degree 2190 and order 2160, effectively reducing the minimum wavelength that can be incorporated from 1° (about 100 km) to 10' (about 18 km). A difference in resulting detail is clearly visible in Figure 7 when compared with Figure 6. The model is made available in a number of forms; in addition to the full set of over 2.4 million spherical harmonic coefficients, values for geoidal undulations relative to the WGS84 spheroid are also provided on a 1'x1' grid. It is the grid values that have been used here and contoured to produce Figure 7. Pavlis *et al.*, (2008) provide associated software as well as numerical values, but state that the products have yet to be validated. While the levels shown are comparable to those of the GGM02C and EIGEN-GL04 models, there are details that could not previously be shown due to the limited resolution available from models of order 360.

In development of geoidal models for Egypt, researchers were keen to attempt to integrate terrestrial gravity data with EGM96, a geopotential model that had already used much of the gravity data available internationally. The data sets are dependent, and effects of correlation likely ignored. The more recent models are provided at orders of 200 and 360 with the coefficients to order 200 being less dependent on gravity data. The EGM2008 geopotential model uses terrestrial gravity data and is not designed to be re-integrated with this data. However, existing terrestrial data and GPS/level data would be valuable in validation of geoidal models derived from EGM2008 locally.

DISCUSSION

The International Gravimetric Bureau (BGI) database contains values for 95 reference gravity points, most of which are of unknown date and a few date from 1970, these data are freely available. A further 335 relative data points collected during various surveys mostly undertaken between 1960 and 1980 are not publicly available. Similarly, marine gravity data from both the Arabian Gulf/Red Sea and the Mediterranean, mostly collected between 1982 and 1992 are available. However, attempts made within this research to integrate terrestrial gravity data with that from geopotential models have not been successful, and other researchers have encountered similar difficulties. Saad and Dawod (2002) produced a geoidal model based on EGM96 that also used reference gravity data, and this clearly corrupted the result, which has not been reproduced here. Problems with the data were obviously recognised as later efforts of Dawod and Ismail (2005) ignored gravity data. Merry (2003) used both reference and relative gravity data, and also included marine data in constructing AGP2003. When compared with EGM2008 in Figure 8 there is a difference of about 2 m on average in south west Egypt, and from inspection of Figures 3 and 4 a similar difference exist with EGM96. Other substantial differences of up to 5 m are also seen in Figure 8, but these are likely due to lack of data in regions where the density and topography of the terrain is changing rapidly and limitations in using a geopotential model of order 360 to model such change. Further localized work has been undertaken within Egypt, a complete bibliography is provided by Dawod (2008).

CONCLUSIONS

The ultimate objective is to produce a digital geoidal model for Egypt, with data provided on a grid of suitable density, and to make this freely available. A model of potential across the terrain would also be of value. Previous work has been based on EGM96, which was limited to order 360 and did not provide sufficient precision to model variations due to variation of the Earth's mass across Egypt. It is suggested here that attempts by previous researchers to overcome the shortfalls by incorporation of terrestrial gravity measurements have been thwarted by poor data to the extent that results have been degraded rather than improved.

The release of EGM2008 has provided a geopotential model of order 2160 that has overcome limitations of previous models. Furthermore, topographic data sets are now available to apply necessary terrain corrections. The need for gravity data may be considered redundant, but the EGM2008 geopotential model and resulting WGS84 geoidal model still need to be validated. Work undertaken on this project and results of previous research that aimed to develop geoidal models across Egypt suggest that existing gravity data and level/GPS data nationally is of insufficient quality for validation purposes.

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